

Center for
Electronic Correlations and Magnetism
University of Augsburg

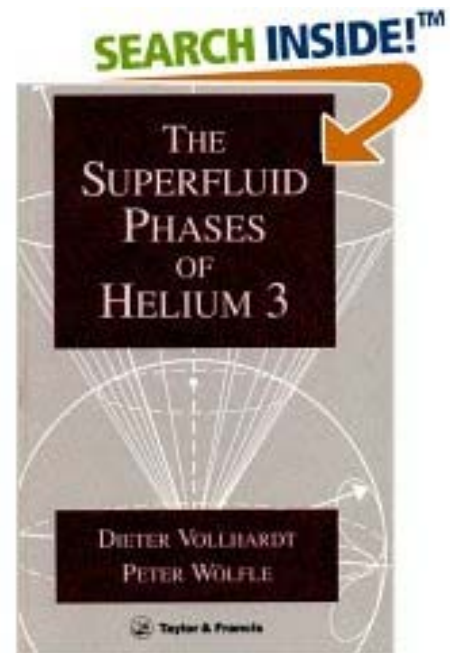
Superfluid Helium-3: From very low Temperatures to the Big Bang

Dieter Vollhardt

University of Warwick, Coventry; February 13, 2008

Contents:

- The quantum liquids ^3He and ^4He
- Superfluid phases of ^3He
- Broken symmetries and long-range order
- Topological defects
- Big Bang simulation in the low temperature lab



The Superfluid Phases of Helium 3
Dieter Vollhardt and Peter Wölfle
(Taylor and Francis, 1990)

Helium

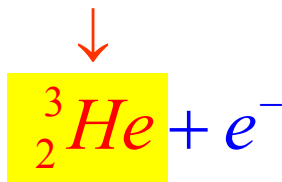
Two stable Helium isotopes:

^4He : air, oil wells, ...

Janssen/Lockyer (1868)

Ramsay (1895)

^3He : $^6_3\text{Li} + ^1_0n \rightarrow ^3_1\text{H} + \alpha$ (1939)



$\frac{\text{He}}{\text{Luft}} \approx 5 \times 10^{-6}, \quad \frac{^3\text{He}}{^4\text{He}} \Big _{\text{Luft}} \approx 1 \times 10^{-6}$
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Research on macroscopic samples of ^3He since 1947

Helium

Atoms: spherical, hard core diameter $\sim 2.5 \text{ \AA}$

Interaction:

- hard sphere **repulsion**
- van der Waals dipole (+ ...) **attraction**

Boiling point: 4.2 K, ^4He Kamerlingh Onnes (1908)

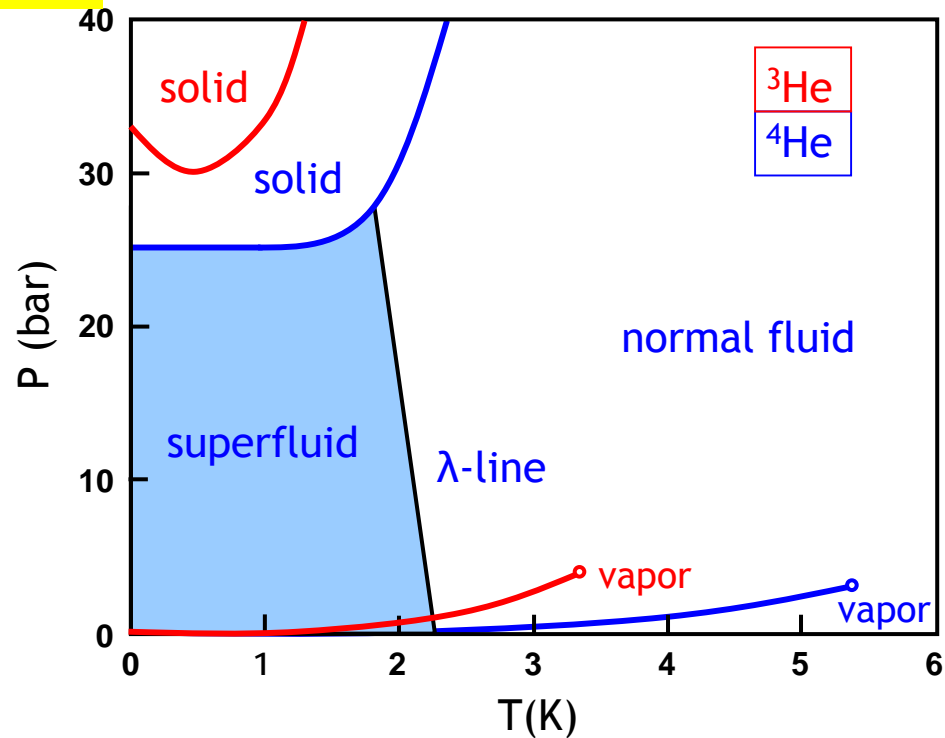


Nobel Prize 1913

3.2 K, ^3He Sydoriak, *et al.* (1949)

Dense, simple liquid { isotropic
short-range interactions
extremely pure

Helium

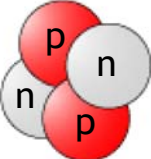
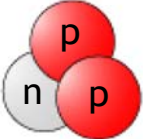


- Atoms:
- spherical shape \rightarrow weak attraction
 - light mass \rightarrow strong zero-point motion

$T \rightarrow 0$, $P \lesssim 30$ bar: Helium remains liquid

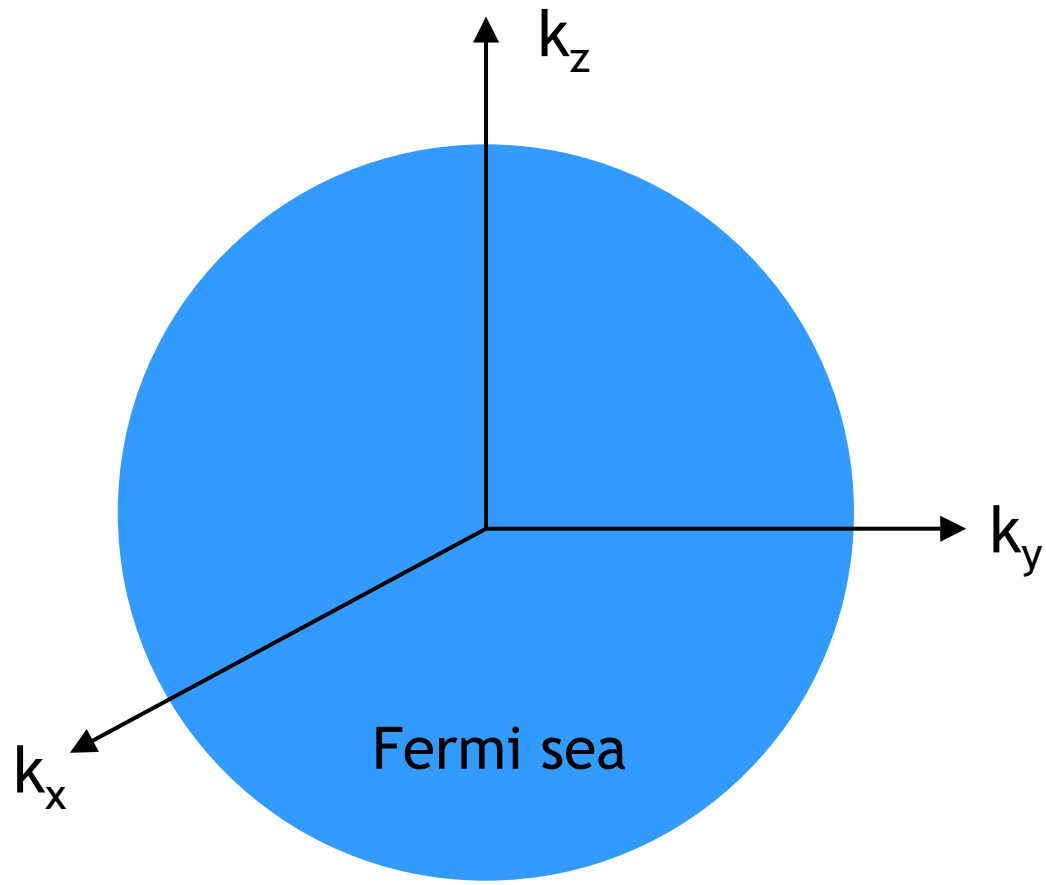
$$\lambda \propto \frac{\hbar}{\sqrt{k_B T}} \xrightarrow{T \rightarrow 0} \text{Macroscopic quantum phenomena}$$

Helium

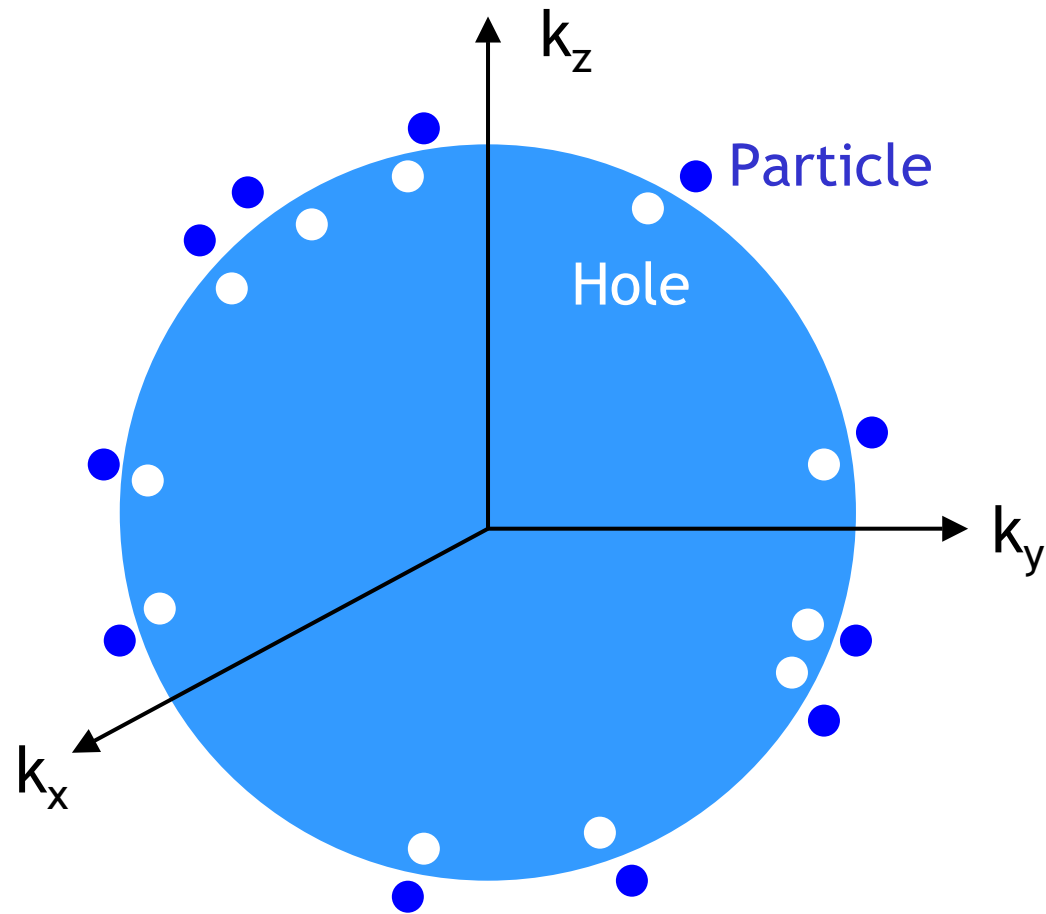
	^4He	^3He
Electron shell:	$2 e^-, S = 0$	
Nucleus:	 $S = 0$	 $S = \frac{1}{2}\hbar$
Atom(!) is a	Boson	Fermion
Phase transition	$T_\lambda = 2.2 \text{ K}$ ("BEC")	$T_c = ???$ Fermi liquid theory

Quantum liquids

Fermi gas: Ground state



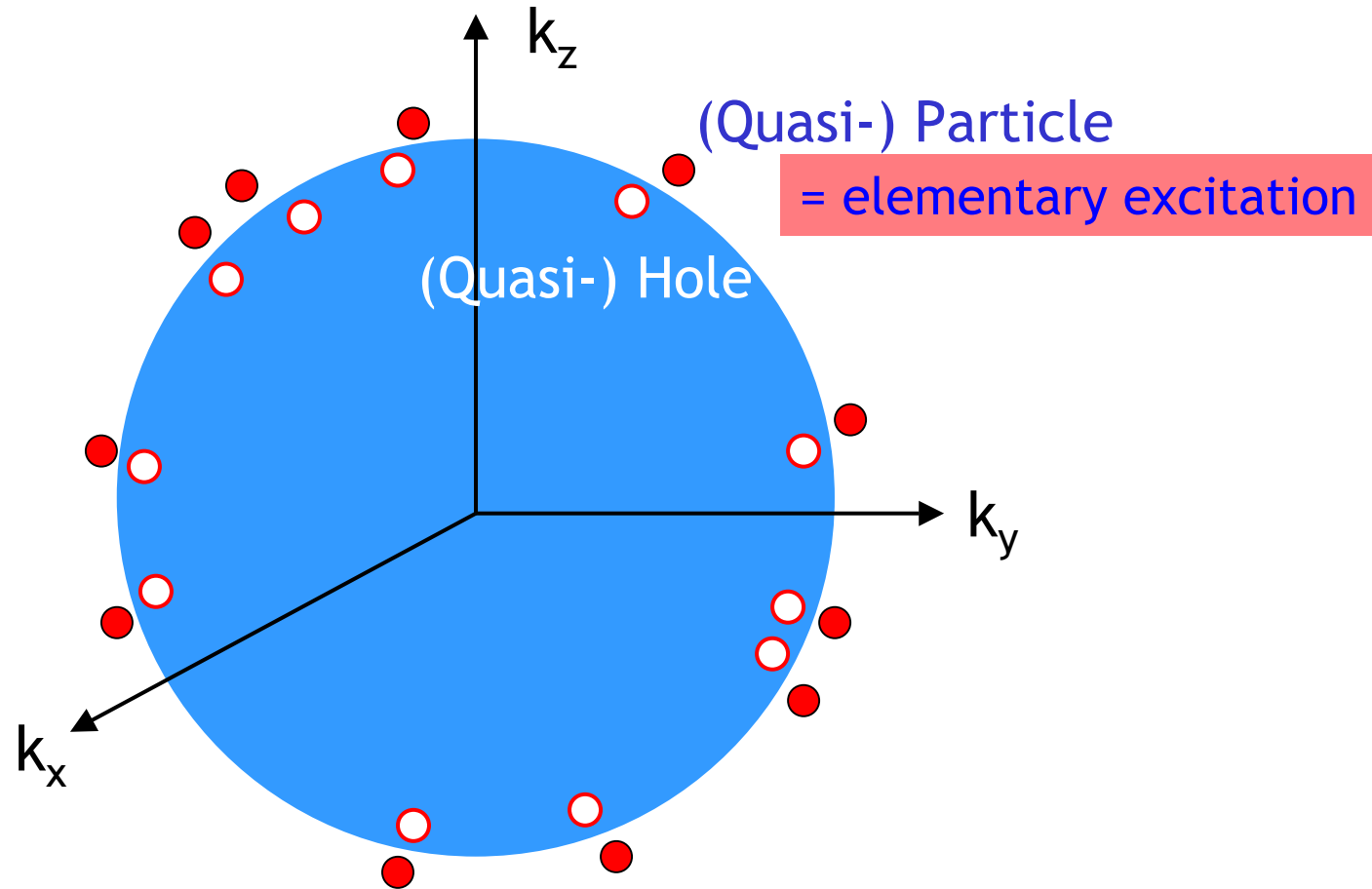
Fermi gas: Excited states ($T > 0$)



Switch on interaction adiabatically

Landau Fermi liquid

1-1 correspondence
between states

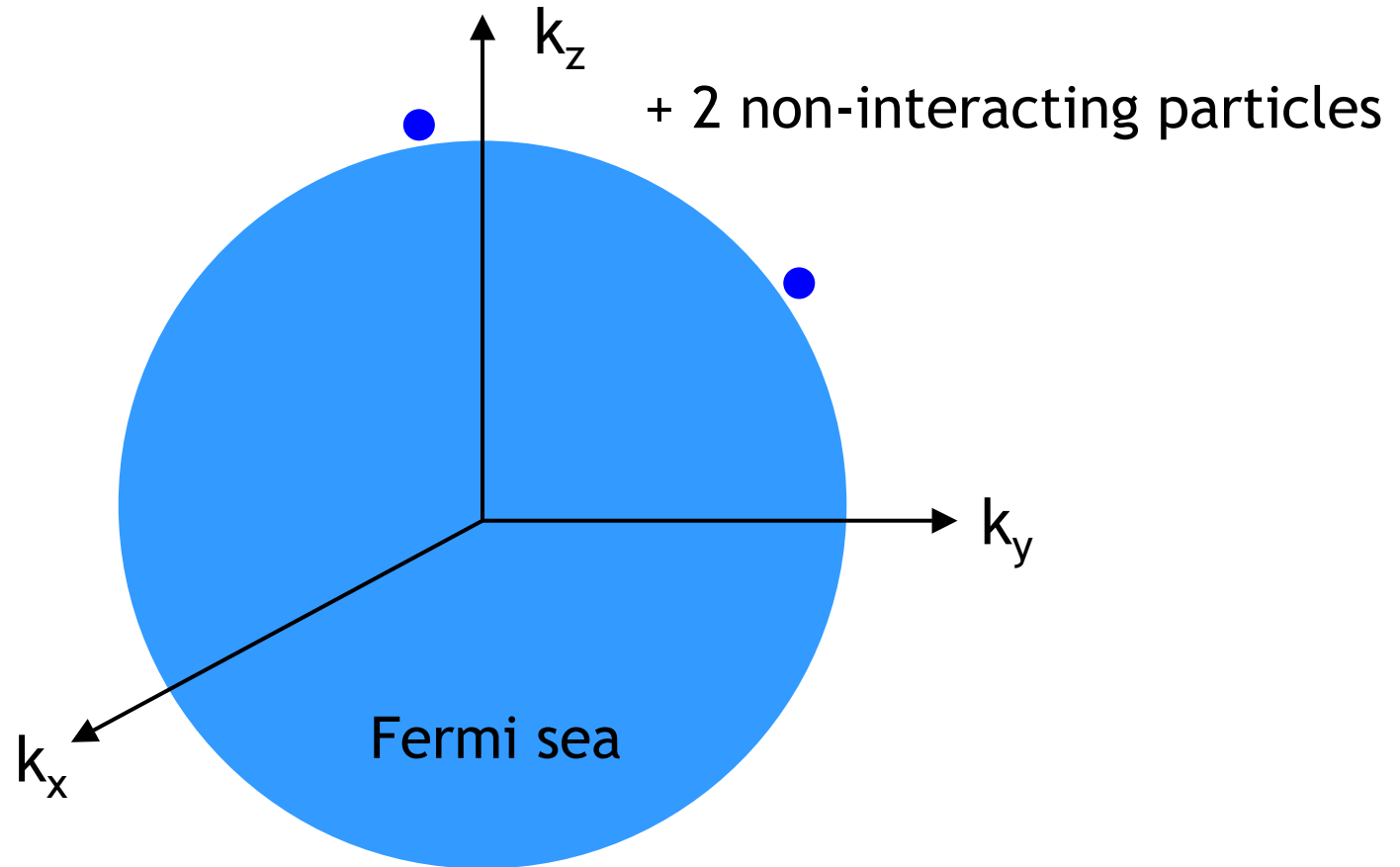


Prototype: Helium-3

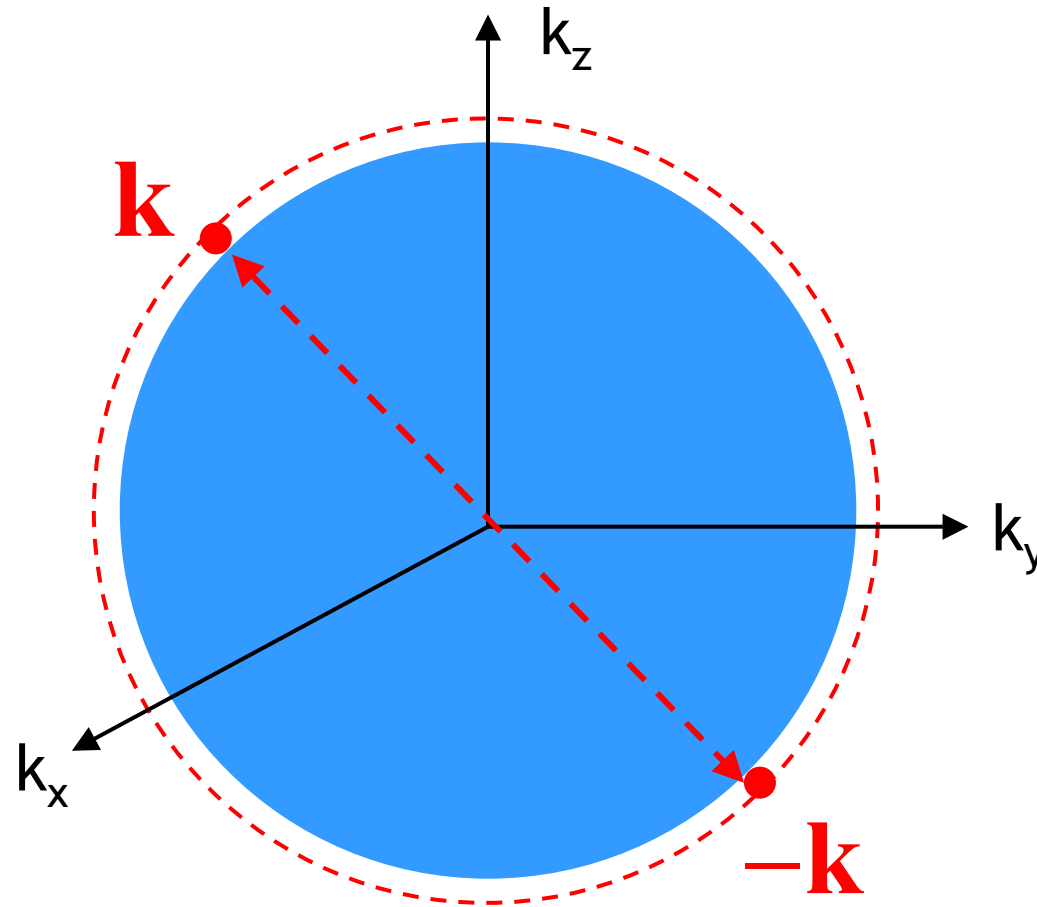
- Large effective mass
- Strongly enhanced spin susceptibility
- Strongly reduced compressibility

Instability ?

Instability of Landau Fermi liquid

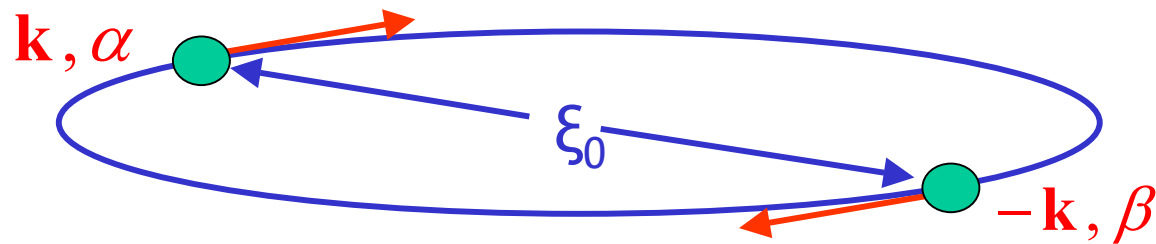


Arbitrarily weak attraction \Rightarrow Cooper instability



Universal fermionic property

Arbitrarily weak attraction \Rightarrow Cooper pair $(\mathbf{k}, \alpha; -\mathbf{k}, \beta)$



$$\Psi_{L=0,2,4,\dots} = \psi(\mathbf{r}) |\uparrow\downarrow - \downarrow\uparrow\rangle$$

$S=0$ (singlet)

$$\begin{aligned} \Psi_{L=1,3,5,\dots} = & \psi_+(\mathbf{r}) |\uparrow\uparrow\rangle \\ & + \psi_0(\mathbf{r}) |\uparrow\downarrow + \downarrow\uparrow\rangle \\ & + \psi_-(\mathbf{r}) |\downarrow\downarrow\rangle \end{aligned}$$

$S=1$ (triplet)

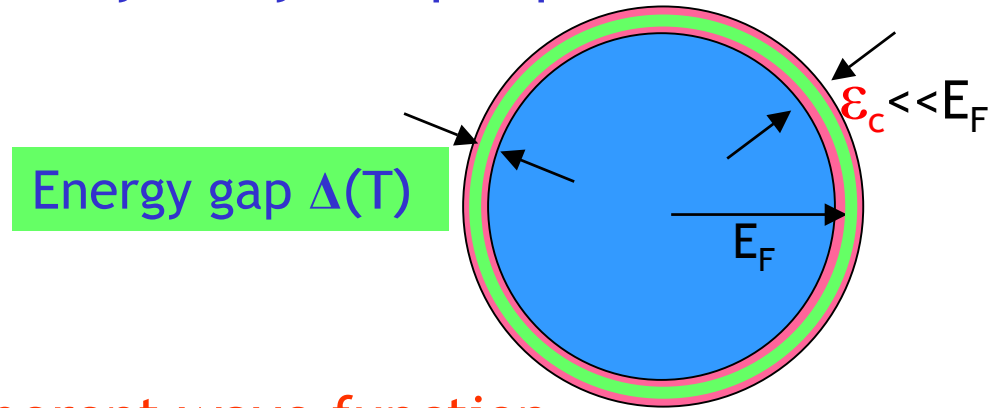
$L = 0$: isotropic wave function
 $L > 0$: anisotropic wave function

Helium-3: Strongly repulsive interaction $\rightarrow L > 0$ expected

BCS theory

Bardeen, Cooper, Schrieffer (1957)

Generalization to macroscopically many Cooper pairs



→ "Pair condensate"
with macroscopically coherent wave function

Transition temperature

$$T_c = 1.13 \epsilon_c \exp(-1 / N(0) |V_L|)$$

"weak coupling theory"

ϵ_c, V_L : Magnitude ? Origin ? → T_c ? Many predictions...

Thanksgiving 1971: Transition in ^3He at $T_c = 0.0026 \text{ K}$

Osheroff, Richardson, Lee (1972)

The Nobel Prize in Physics 1996
"for their discovery of superfluidity in helium-3"



David M. Lee
USA, Cornell
b. 1931



Douglas D.
Osheroff USA,
Stanford
b. 1945



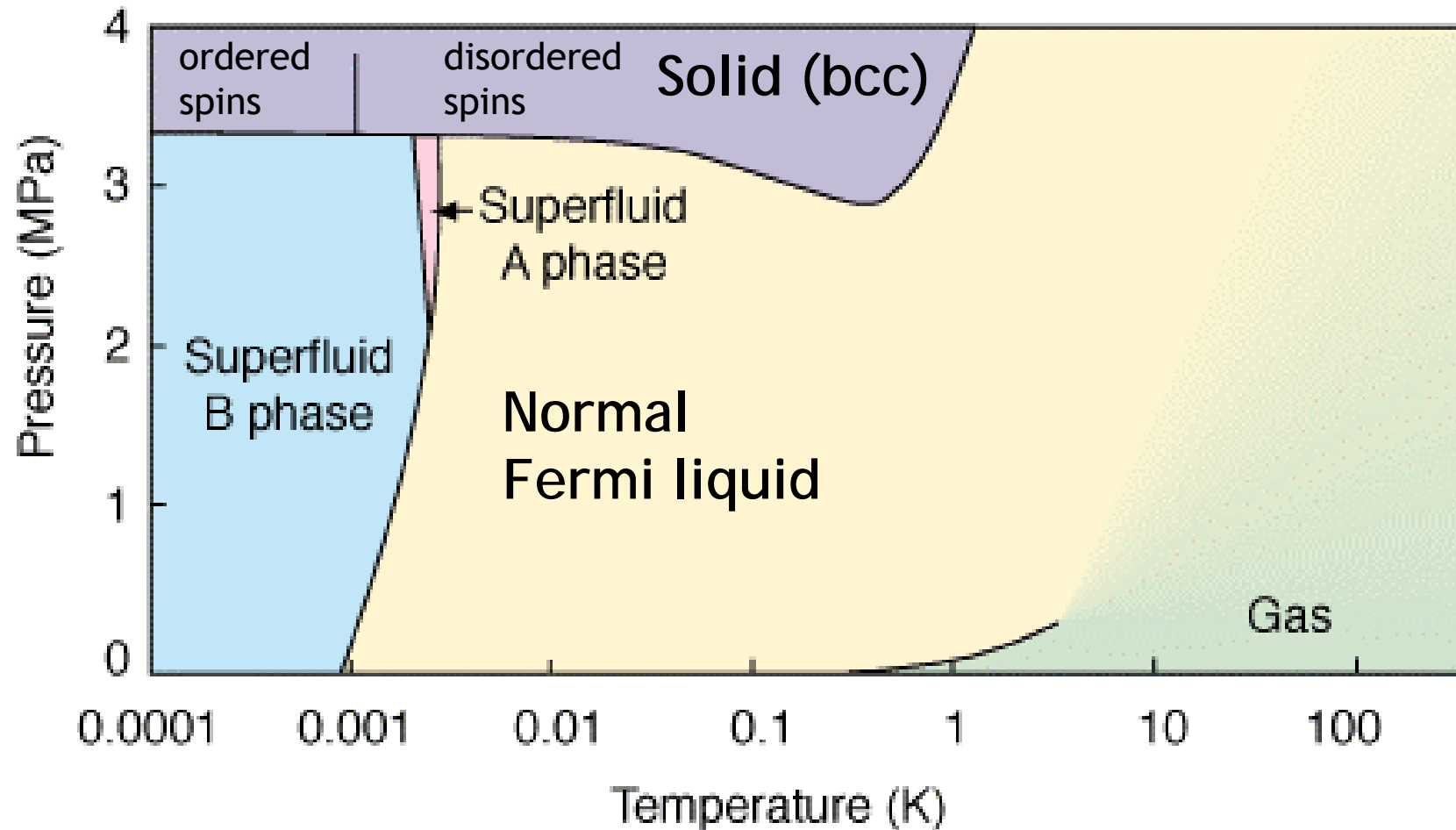
Robert C.
Richardson
USA, Cornell
b. 1937

Phase diagram of Helium-3

P-T phase diagram

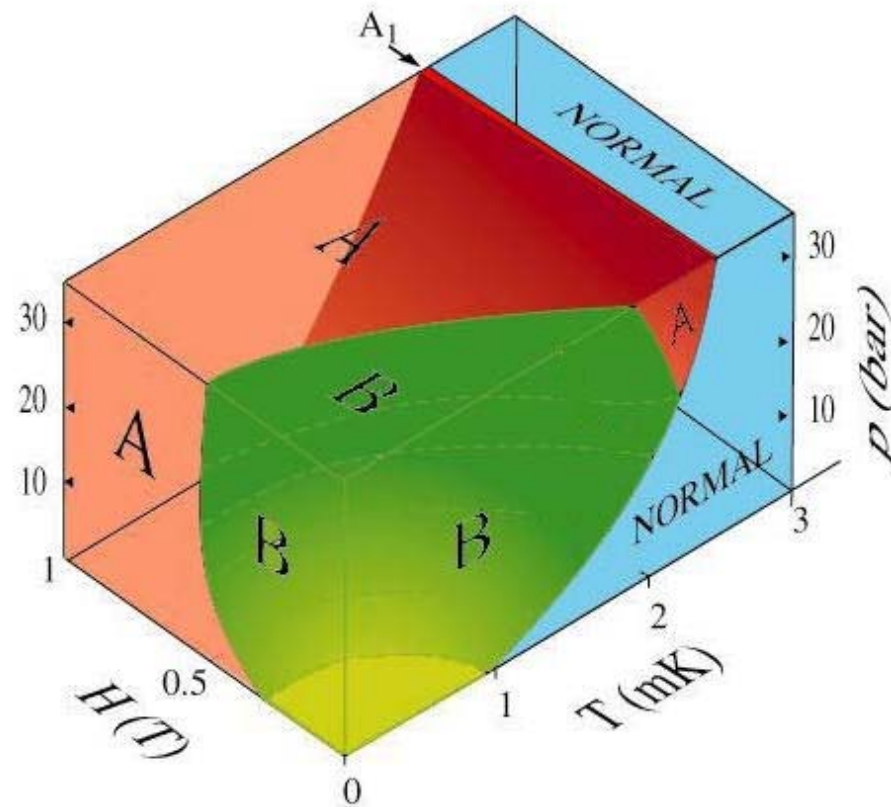
Dense, simple liquid

isotropic
short-range interactions
extremely pure
nuclear spin $S=1/2$



Phase diagram of Helium-3

P-T-H phase diagram



“Very low temperatures”: $T \ll T_{\text{boiling}} \sim 3\text{-}4\text{ K}$
 $\ll T_{\text{backgr. rad.}} \sim 3\text{ K}$

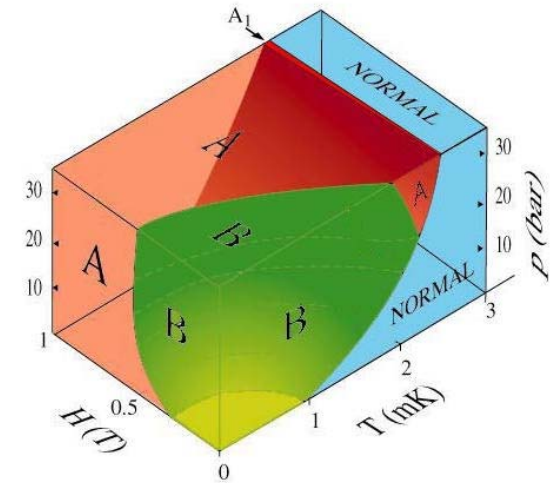
Superfluid phases of ^3He

Theory + experiment: $L=1$, $S=1$ in all phases

Leggett

Wölfle

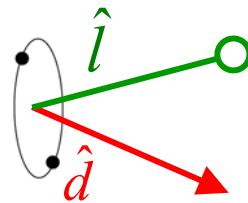
Mermin, ...



Pairing mechanism: Spin fluctuations

Anderson, Brinkman (1973)

→ anisotropy directions
in a ^3He Cooper pair



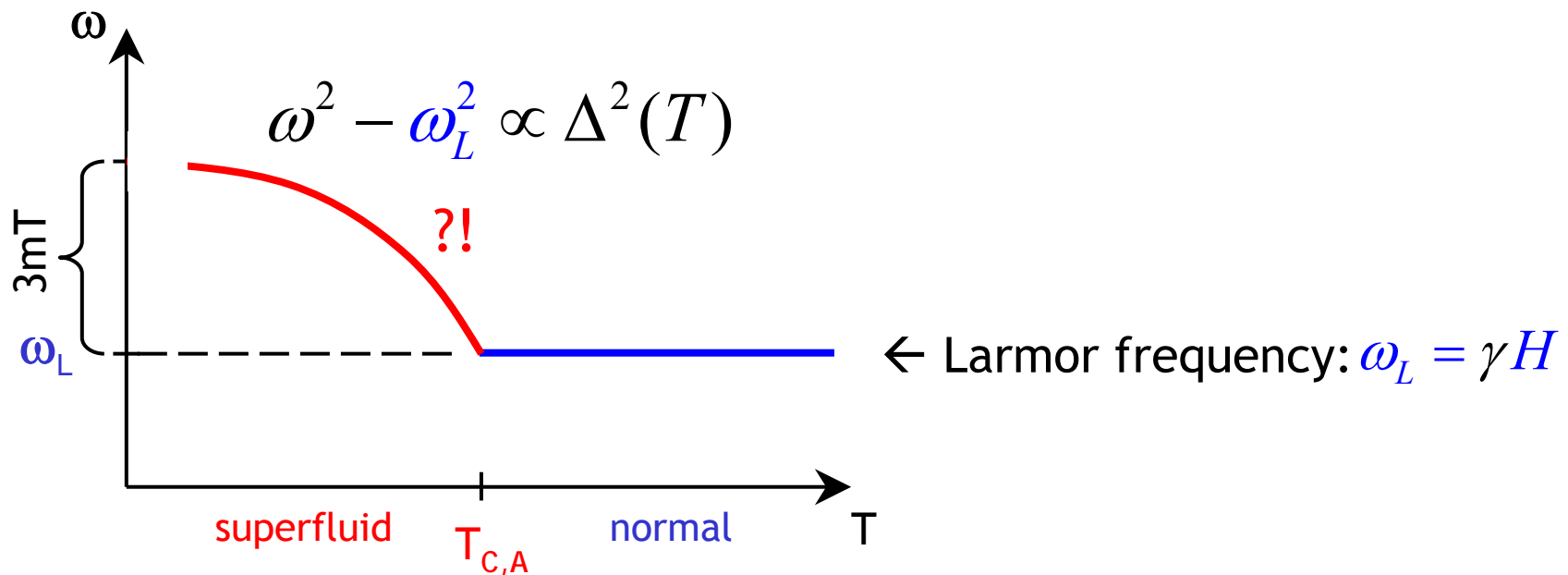
orbital part

spin part

... and a mystery!

NMR experiment on nuclear spins $I = \frac{1}{2}\hbar$

Osheroff *et al.* (1972)



Shift of ω_L \Leftrightarrow spin-nonconserving interactions
 \rightarrow nuclear dipole interaction $g_D \sim 10^{-7} K \ll T_C$

Origin of frequency shift ?!

Leggett (1973)

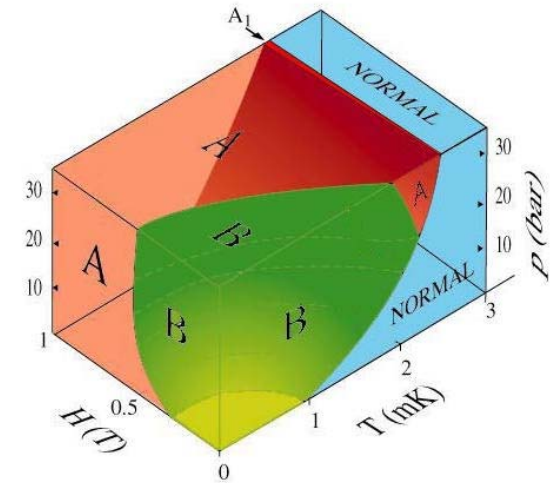
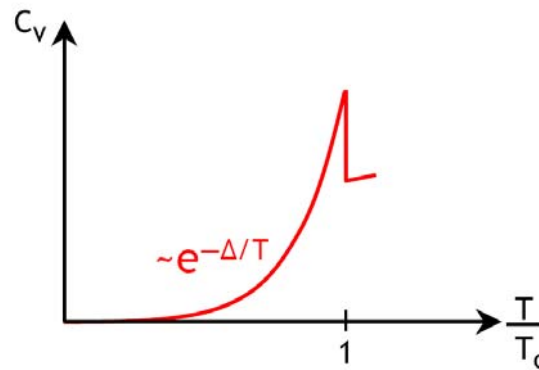
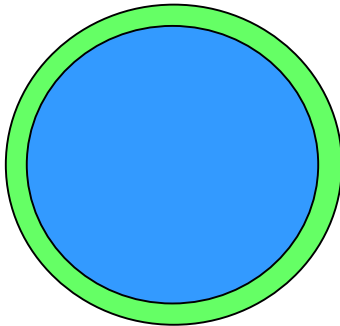
The superfluid phases of ^3He

B-phase

$$\Psi = |\uparrow\uparrow\rangle + |\uparrow\downarrow + \downarrow\uparrow\rangle + |\downarrow\downarrow\rangle$$

$$\Delta(\mathbf{k}) = \Delta_0$$

Balian, Werthamer (1963)
Vdovin (1963)



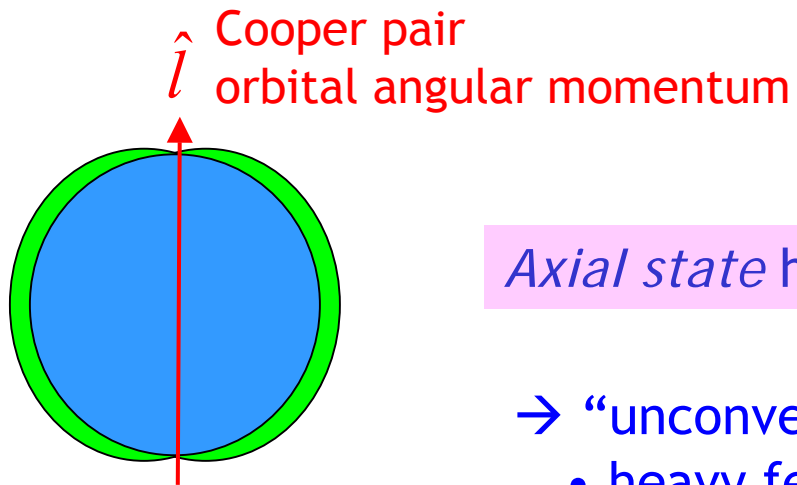
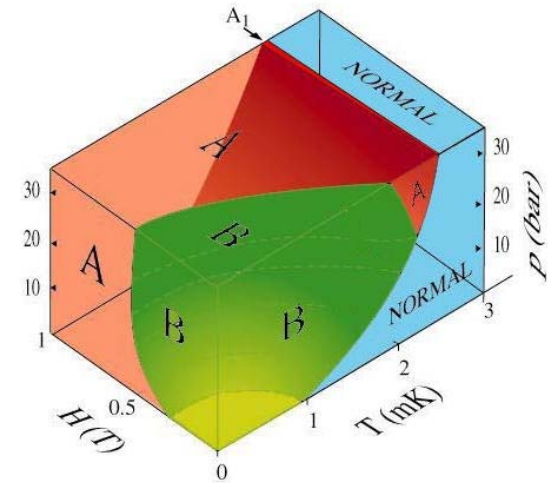
(pseudo-) isotropic state \leftrightarrow s-wave superconductor

Weak-coupling theory: stable for all $T < T_c$

A-phase

$$\Psi = |\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle \rightarrow \text{strong anisotropy}$$

$$\Delta(\hat{k}) = \Delta_0 \sin(\hat{k}, \hat{l}) \quad \text{Anderson, Morel (1961)}$$



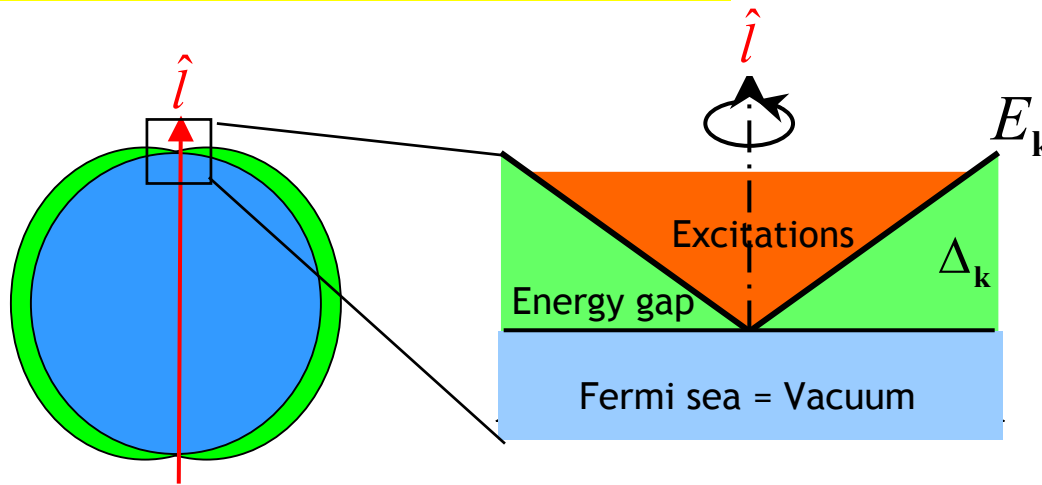
Axial state has point nodes

- “unconventional” pairing in
- heavy fermion/high- T_c superconductors
 - Sr_2RuO_4

Strong-coupling effect

³He-A: Spectrum near poles

Volovik (1987)



$$E_{\mathbf{k}}^2 = v_F^2 (k - k_F)^2 + \Delta_0^2 \sin^2(\hat{\mathbf{k}}, \hat{\mathbf{l}}) = g^{ij} p_i p_j$$

$$e = \begin{cases} +1 & \hat{\mathbf{k}} \parallel +\hat{\mathbf{l}} \\ -1 & \hat{\mathbf{k}} \parallel -\hat{\mathbf{l}} \end{cases} \quad \text{2 chiralities}$$

$$g^{ij} = v_F^2 l_i l_j + \left(\frac{\Delta}{k_F} \right)^2 (\delta_{ij} - l_i l_j)$$

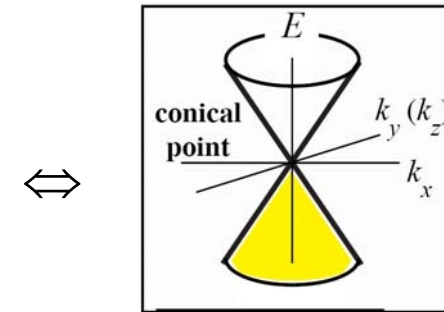
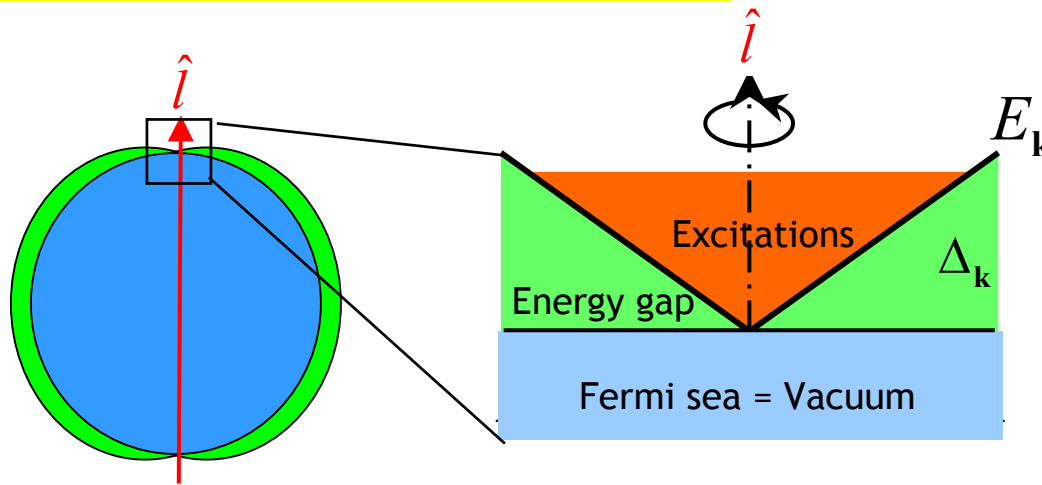
$$\mathbf{A} = k_F \hat{\mathbf{l}}$$

$$\mathbf{p} = \mathbf{k} - e\mathbf{A}$$

Lorentz invariance:
Symmetry enhancement
at low energies

³He-A: Spectrum near poles

Volovik (1987)



Fermi point:
spectral flow

$$E_{\mathbf{k}}^2 = v_F^2 (\mathbf{k} - \mathbf{k}_F)^2 + \Delta_0^2 \sin^2(\hat{\mathbf{k}}, \hat{\mathbf{l}}) = g^{ij} p_i p_j$$

$$e = \begin{cases} +1 & \hat{\mathbf{k}} \parallel +\hat{\mathbf{l}} \\ -1 & \hat{\mathbf{k}} \parallel -\hat{\mathbf{l}} \end{cases} \quad \text{2 chiralities}$$

$$g^{ij} = v_F^2 l_i l_j + \left(\frac{\Delta}{k_F} \right)^2 (\delta_{ij} - l_i l_j)$$

⇔ Massless, chiral leptons, e.g., neutrino $E(\mathbf{p}) = cp$

→ Chiral anomaly of standard model

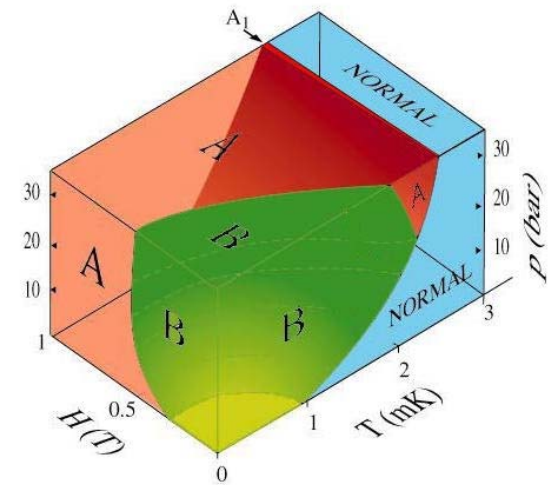
The Universe in a Helium Droplet,
Volovik (2003)

A₁-phase

finite magnetic field

$$\Psi = |\uparrow\uparrow\rangle$$

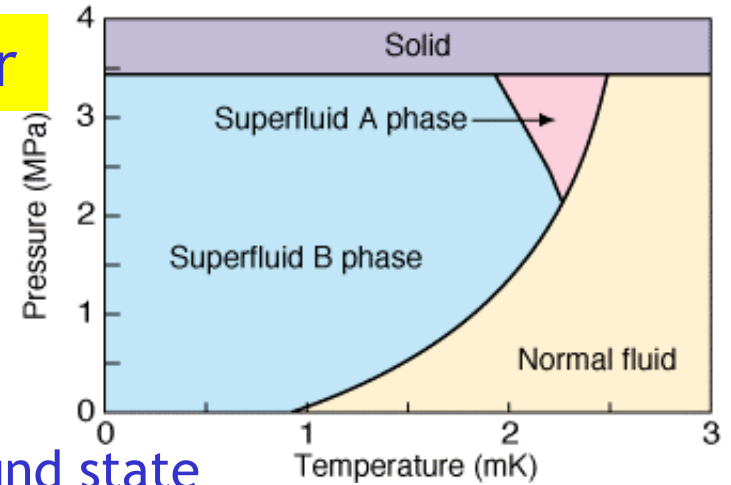
Long-range ordered magnetic liquid



Broken Symmetries, Long Range Order

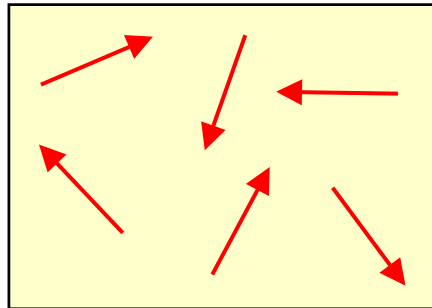
Broken Symmetries, Long Range Order

Normal $^3\text{He} \leftrightarrow ^3\text{He-A}, ^3\text{He-B}$:
 2. order phase transition



$T < T_c$: higher order, lower symmetry of ground state

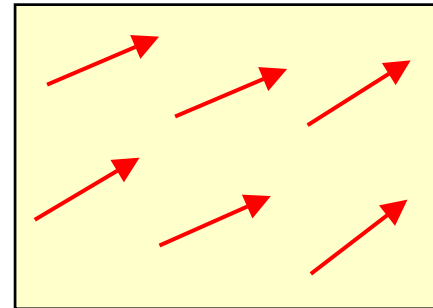
I. Ferromagnet



$T > T_c$

Average magnetization:
 Symmetry group:

$\langle \mathbf{M} \rangle = 0$
 $SO(3)$



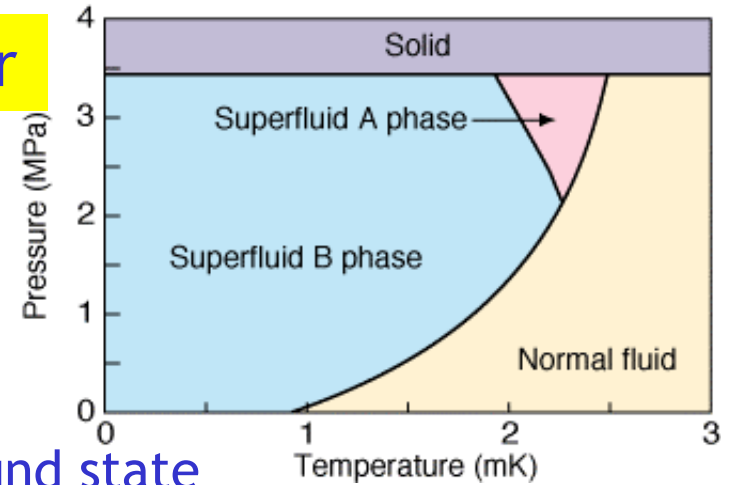
$T < T_c$

$\langle \mathbf{M} \rangle \neq 0$ Order parameter
 $U(1) \subset SO(3)$

$T < T_c$: $SO(3)$ rotation symmetry in spin space spontaneously broken

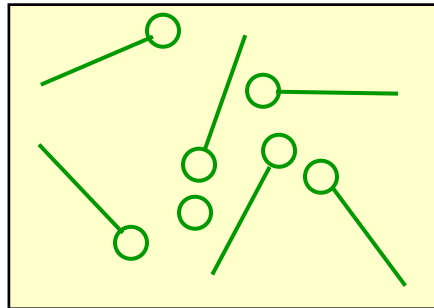
Broken Symmetries, Long Range Order

2. order phase transition

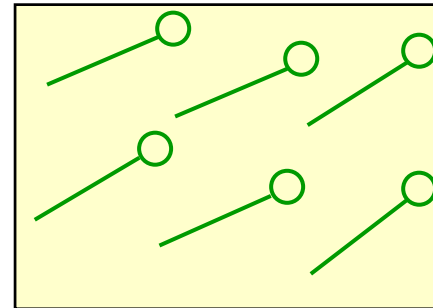


$T < T_c$: higher order, lower symmetry of ground state

II. Liquid crystal



$T > T_c$
 $SO(3)$



$T < T_c$
 $U(1) \subset SO(3)$

Symmetry group:

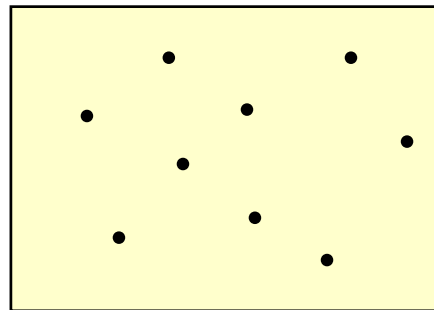
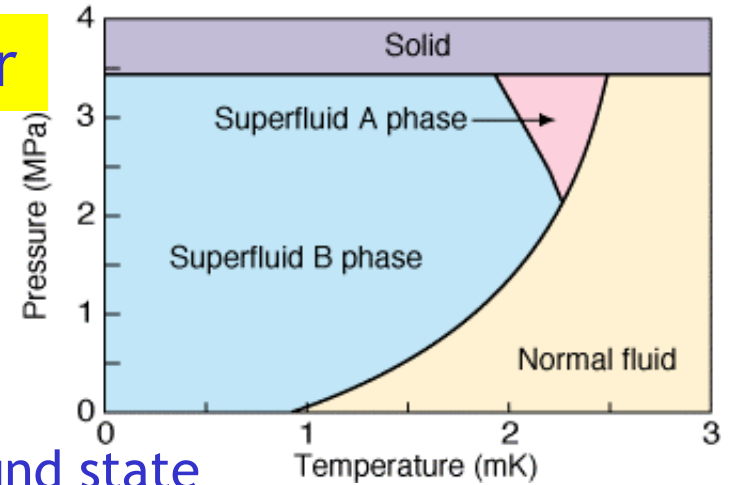
$T < T_c$: $SO(3)$ rotation symmetry in real space spontaneously broken

Broken Symmetries, Long Range Order

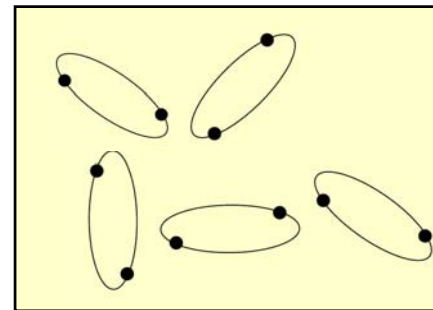
2. order phase transition

$T < T_c$: higher order, lower symmetry of ground state

III. Conventional superconductor



$T > T_c$



$T < T_c$

Pair amplitude $\langle c_{\mathbf{k}\uparrow}^\dagger c_{-\mathbf{k}\downarrow}^\dagger \rangle = 0$

$\Delta e^{i\phi}$ Order parameter

Gauge transf.

$$c_{\mathbf{k}\sigma}^\dagger \rightarrow c_{\mathbf{k}\sigma}^\dagger e^{i\phi}$$

gauge invariant

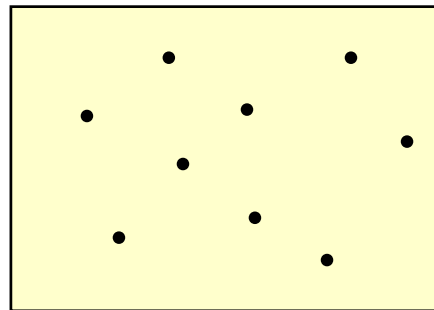
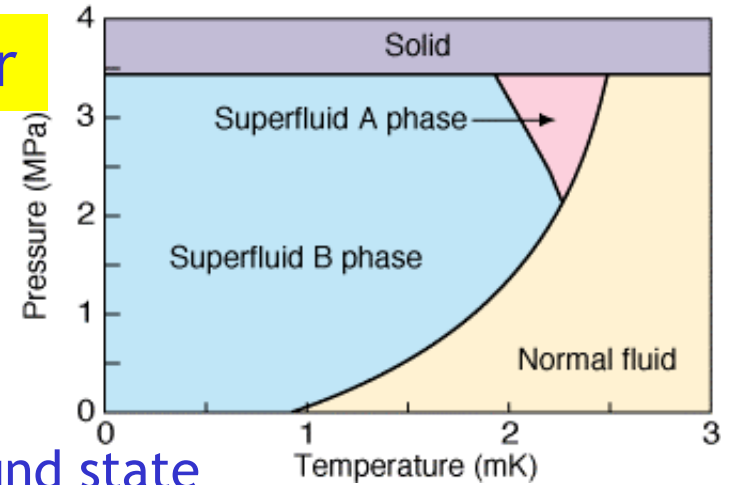
not gauge invariant

Broken Symmetries, Long Range Order

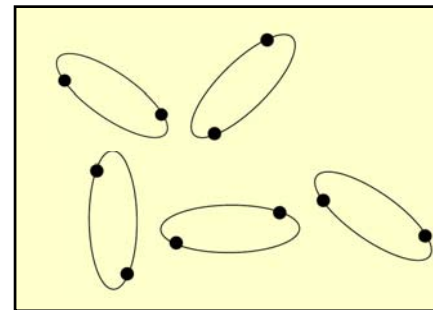
2. order phase transition

$T < T_c$: higher order, lower symmetry of ground state

III. Conventional superconductor



$T > T_c$



$T < T_c$

Pair amplitude $\langle c_{\mathbf{k}\uparrow}^\dagger c_{-\mathbf{k}\downarrow}^\dagger \rangle = 0$

$\Delta e^{i\phi}$ Order parameter

Gauge transf.

$$c_{\mathbf{k}\sigma}^\dagger \rightarrow c_{\mathbf{k}\sigma}^\dagger e^{i\phi}$$

Symmetry group:

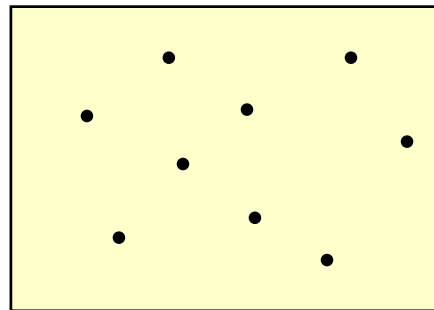
U(1)

Broken Symmetries, Long Range Order

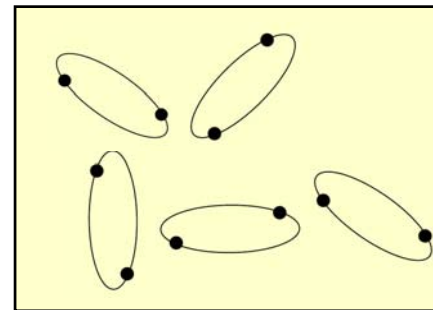
2. order phase transition

$T < T_c$: higher order, lower symmetry of ground state

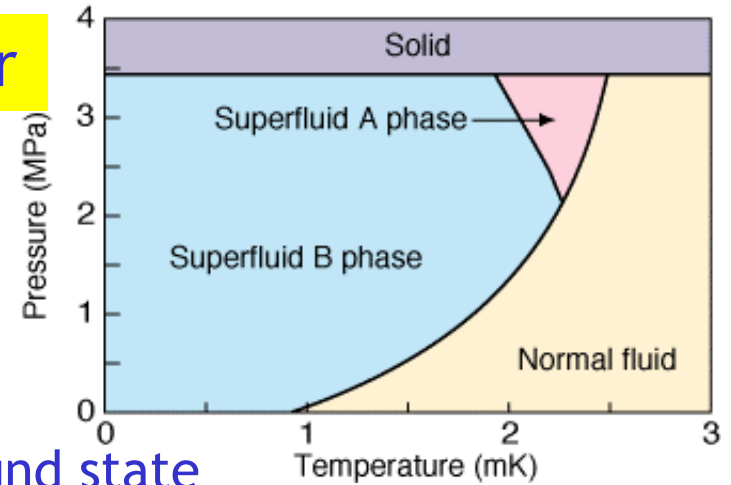
III. Conventional superconductor



$T > T_c$



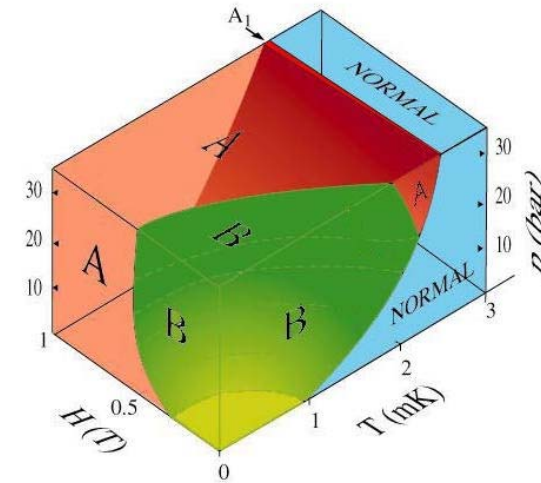
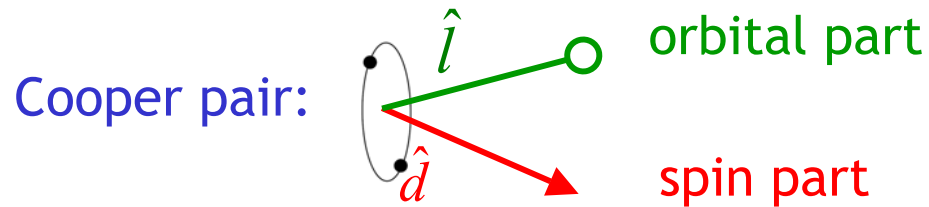
$T < T_c$



$T < T_c$: U(1) “gauge symmetry” spontaneously broken

Broken symmetries in superfluid ^3He

$L=1$, $S=1$ in all phases



Quantum coherence in	$\left\{ \begin{array}{l} \text{phase} \\ \text{anisotropy direction for spin} \\ \text{anisotropy direction in real space} \end{array} \right.$	<div style="background-color: #e0ffe0; padding: 5px;"> Superfluid, magnetic liquid crystal </div>
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Characterized by $2 \times (2L + 1) \times (2S + 1) = 18$ real numbers

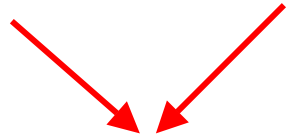
3x3 order parameter matrix $A_{j\mu}$

$SO(3)_S \times SO(3)_L \times U(1)_\varphi$ symmetry spontaneously broken Leggett (1975)

Broken symmetries in superfluid ^3He

Mineev (1980)
Bruder, DV (1986)

3He-B $\text{SO}(3)_S \times \text{SO}(3)_L \times \text{U}(1)_\varphi$ symmetry broken

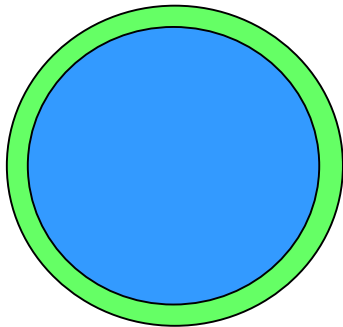


$\text{SO}(3)_{S+L}$



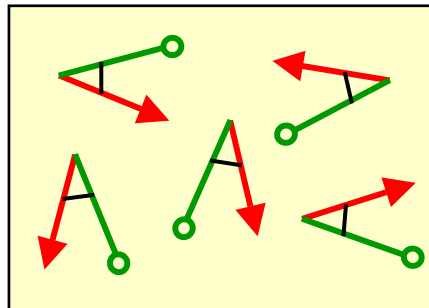
—

„Unconventional“ pairing



“Spontaneously broken spin-orbit
Symmetry“
Leggett (1972)

Cooper pairs

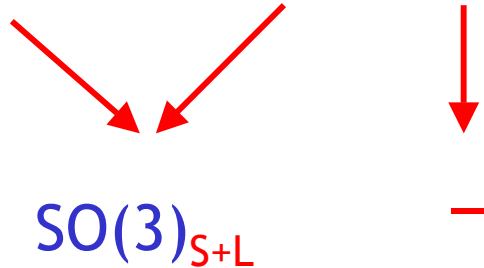


Fixed relative orientation

Broken symmetries in superfluid ^3He

Mineev (1980)
Bruder, DV (1986)

3He-B $\text{SO}(3)_S \times \text{SO}(3)_L \times \text{U}(1)_\varphi$ symmetry broken



„Unconventional“ pairing

Relation to high energy physics

Isodoublet

$\begin{pmatrix} u \\ d \end{pmatrix}_L$, $\begin{pmatrix} u \\ d \end{pmatrix}_R$ chiral invariance

Global symmetry

$\text{SU}(2)_L \times \text{SU}(2)_R$

$q\bar{q}$ condensation (“Cooper pairing”)

$\text{SU}(2)_{L+R}$

Goldstone bosons

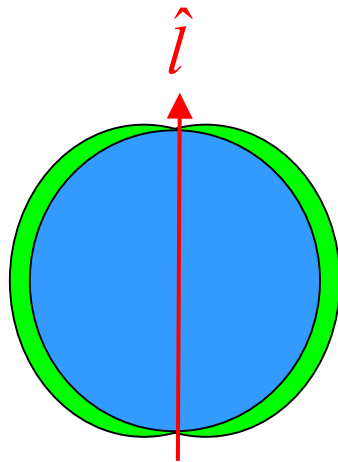
3 pions

Broken symmetries in superfluid ^3He

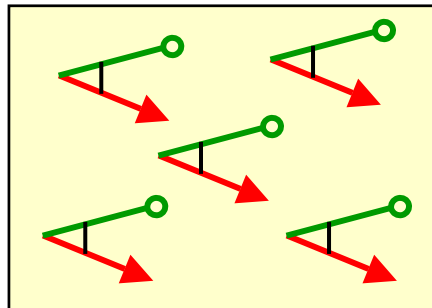
Mineev (1980)
Bruder, DV (1986)

$^3\text{He-A}$ $\text{SO}(3)_S \times \text{SO}(3)_L \times \text{U}(1)_\varphi$ symmetry broken

$\text{U}(1)_{S_z} \times \text{U}(1)_{L_z - \varphi}$ „Unconventional” pairing



Cooper pairs

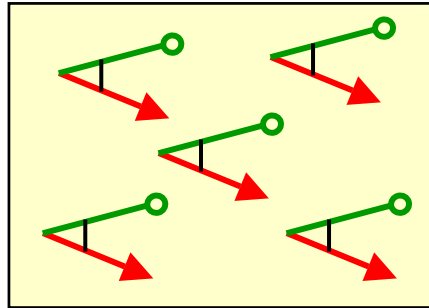


Fixed absolute orientation

Resolution of the NMR puzzle:
Macroscopic quantum amplification

Superfluid ^3He as quantum amplifier

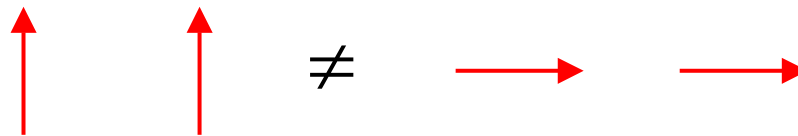
Cooper pairs in $^3\text{He-A}$



Fixed absolute orientation

What determines the actual relative orientation of \hat{d} , \hat{l} ?

→ Anisotropic spin-orbit interaction of nuclear dipoles:

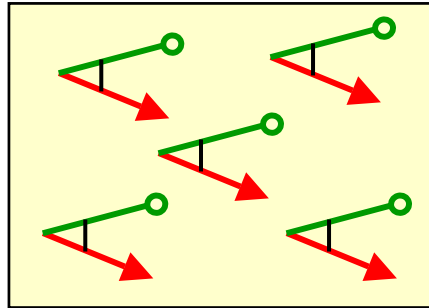


Dipole-dipole coupling of ^3He nuclei: $g_D \sim 10^{-7} K \ll T_C$

Unimportant ?!

Superfluid ^3He as quantum amplifier

Cooper pairs in $^3\text{He-A}$



Fixed absolute orientation

- Long-range order in \hat{d}, \hat{l}
- $g_D \sim 10^{-7} K$: tiny, but lifts degeneracy of relative orientation

Quantum \Downarrow coherence

\hat{d}, \hat{l} locked in all Cooper pairs

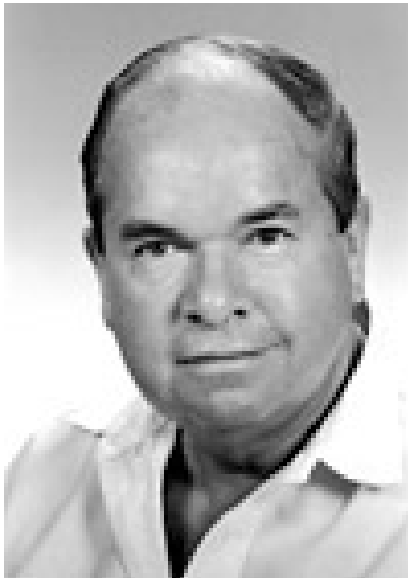


NMR frequency increases: $\omega^2 = (\gamma H)^2 + g_D \Delta^2(T)$ Leggett (1973)

→ Nuclear dipole interaction macroscopically measurable

The Nobel Prize in Physics 2003

"for pioneering contributions to the theory of superconductors and superfluids"



Alexei A.
Abrikosov, USA
and Russia

b. 1928



Vitaly L.
Ginzburg,
Russia

b. 1916



Anthony J.
Leggett, UK
and USA

b. 1938

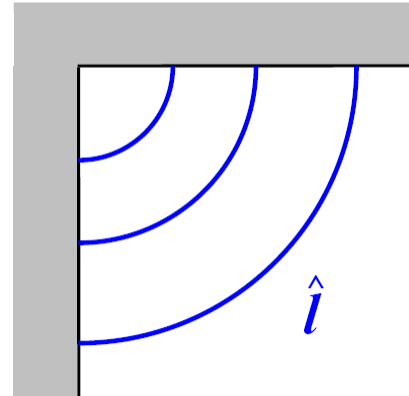
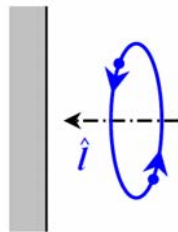
Order parameter textures and topological defects

Order parameter textures

Orientation of anisotropy directions \hat{d}, \hat{l} in ${}^3\text{He-A}$?

Magnetic field $\rightarrow \hat{d}$

Walls $\rightarrow \hat{l}$

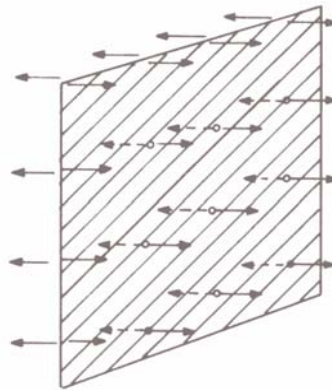


\rightarrow “Textures“ in \hat{d}, \hat{l} \leftrightarrow liquid crystals

\rightarrow Topologically stable defects from homotopy theory

Order parameter textures and topological defects

D=2: domain walls in \hat{d} or \hat{l}



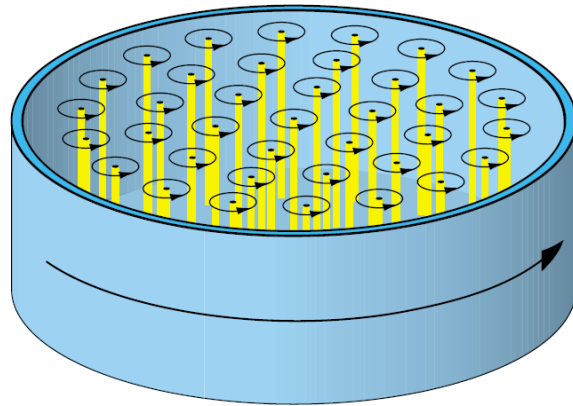
Single domain wall



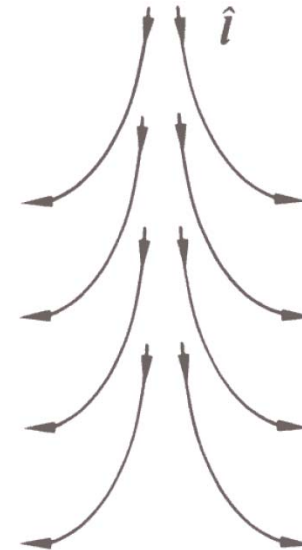
Domain wall lattice

Order parameter textures and topological defects

D=1: Vortices



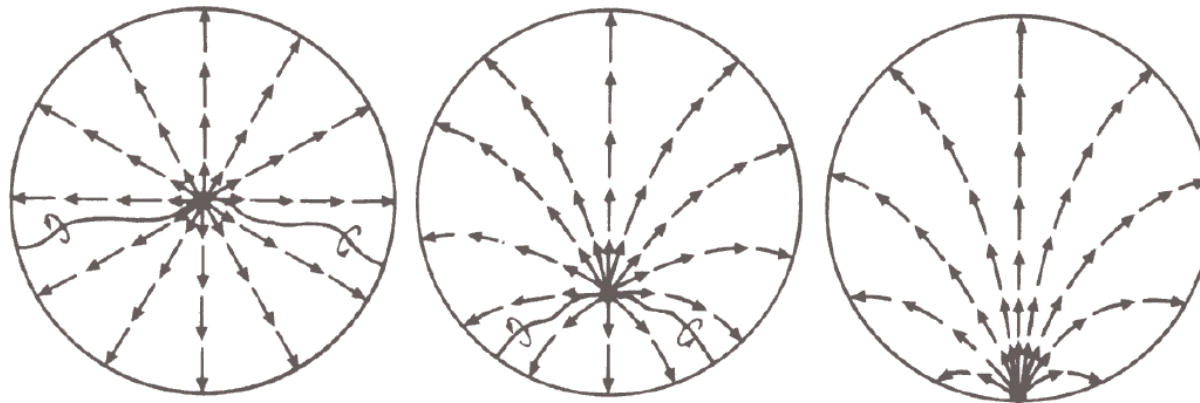
Vortex formation in rotation experiments



e.g., Mermin-Ho vortex
(non-singular)

Order parameter textures and topological defects

D=0: Monopoles



e.g., “boojum” in \hat{l} -texture of $^3\text{He-A}$

Defect formation by, e.g.,

- geometric constraints
- rotation
- rapid crossing through phase transition

Universality in continuous phase transitions



High symmetry,
short-range order

$T > T_c$

Spins:
para-
magnetic

Helium:
normal
liquid

Universe:
Unified forces
and fields

$T = T_c$

Phase transition

Broken symmetry,
long-range order

ferromagnetic

superfluid

elementary
particles,
fundamental
interactions

Defects: domain
walls

vortices,
etc.

cosmic strings,
etc. Kibble (1976)

$T < T_c$

nucleation of galaxies?



Rapid thermal quench through 2. order phase transition

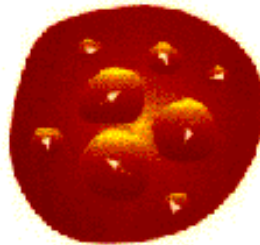
Kibble (1976)

1. Local temperature $T \gg T_c$

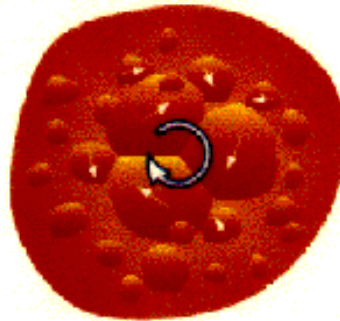


Expansion + rapid cooling

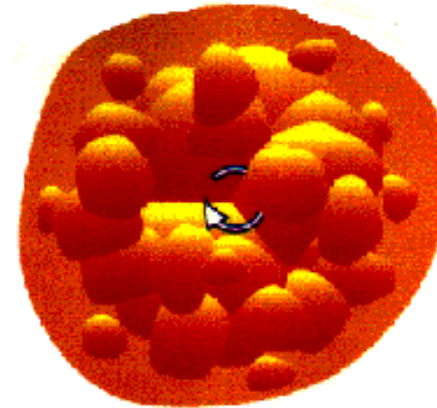
2. Nucleation of independently ordered regions



Clustering of ordered regions
→ Defects

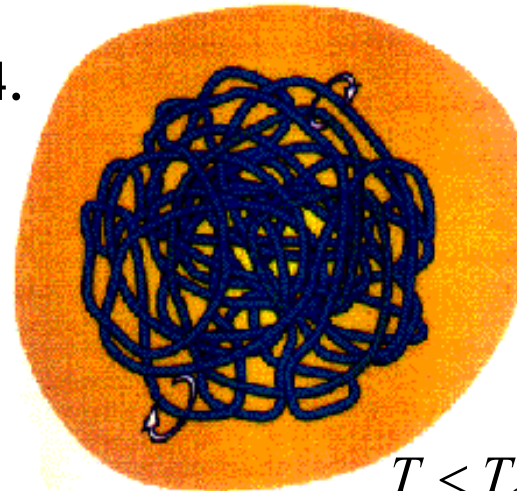


3.



Defects overlap

4.



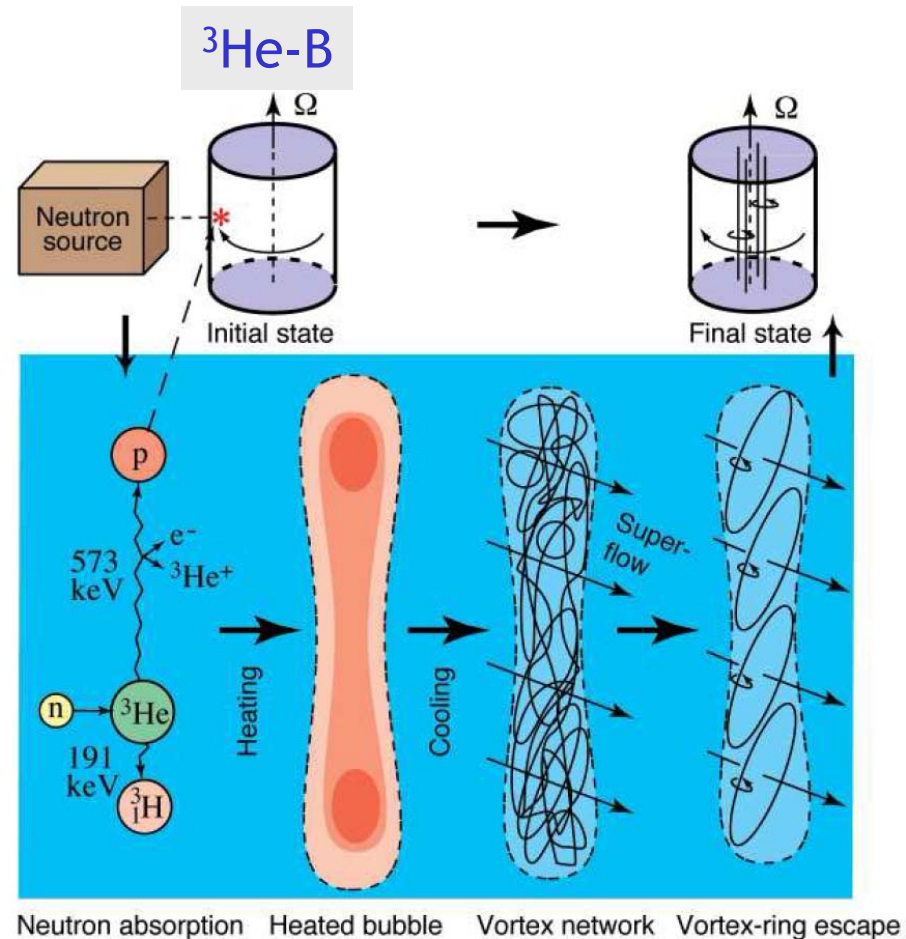
$T < T_c$: Vortex tangle

Estimate of density of defects Zurek (1985)

”Kibble-Zurek mechanism”: How to test?

Big Bang simulation in the low temperature laboratory

Grenoble: Bäuerle *et al.* (1996), Helsinki: Ruutu *et al.* (1996)

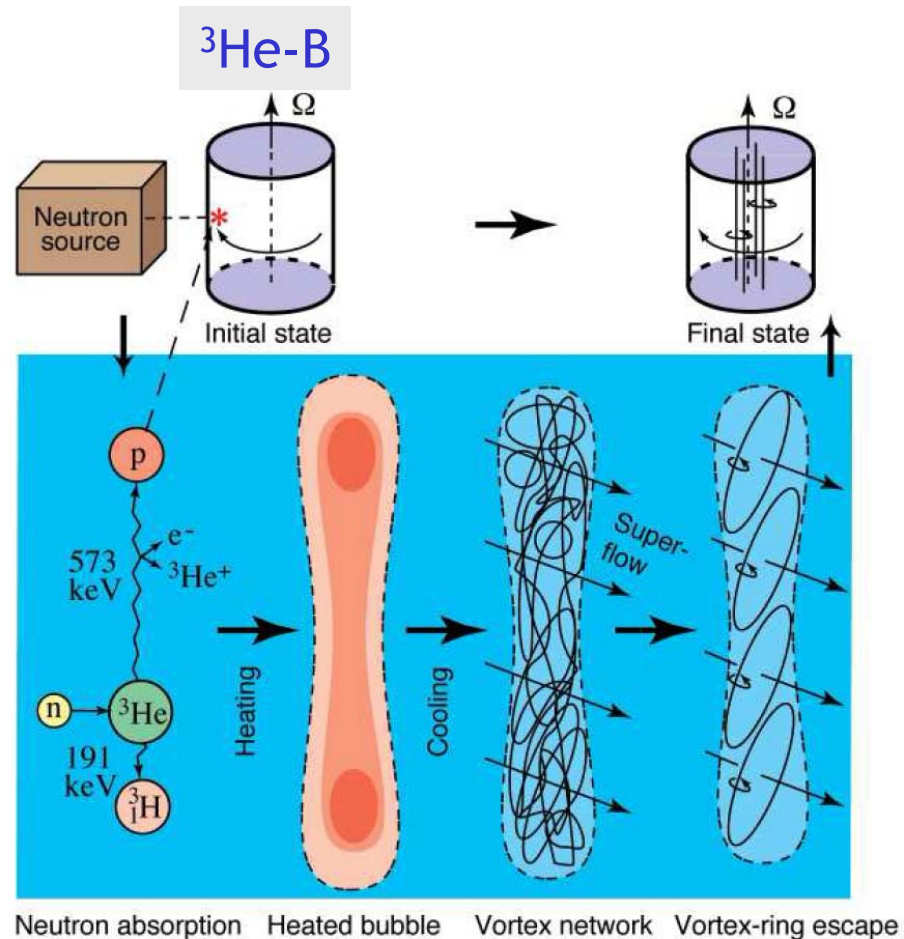


Measured vortex tangle density:

Quantitative support for Kibble-Zurek mechanism

Big Bang simulation in the low temperature laboratory

Grenoble: Bäuerle *et al.* (1996), Helsinki: Ruutu *et al.* (1996)



Cosmology in the Laboratory (COSLAB)



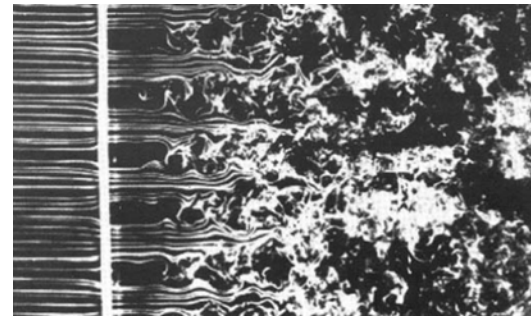
2001-2006

Present research on superfluid ^3He : Quantum Turbulence

Classical Turbulence



Leonardo da Vinci (1452-1519)



Flow through grid

Quantum Turbulence

= Turbulence in the **absence** of viscous dissipation
(superfluid at $T \rightarrow 0$)

- Microscopic mechanism of decay of quantum turbulence?
- What provides dissipation in the absence of friction?

Test system: $^3\text{He-B}$

Vinen, Donnelly: *Physics Today* (April, 2007)

Conclusion

Superfluid Helium-3:

- Anisotropic superfluid
 - 3 different bulk phases
 - Cooper pairs with internal structure
- Large symmetry group broken
 - Close connections to particle theory
 - Zoo of topological defects
 - Kibble-Zurek mechanism quantitatively verified

