Superfluid Helium-3: From very low Temperatures to the Big Bang

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50 years of Bajas Temperaturas at Centro Atómico Bariloche
PUSHING THE LIMITS
John Wheatley (1927-1986)
Bariloche 1961-63

LOS ALAMOS SCIENCE Fall 1986
Centro Atómico Bariloche: 'Bajas Temperaturas' group in 1964
Contents:

- The quantum liquids $^3$He and $^4$He
- Superfluid phases of $^3$He
- Broken symmetries and long-range order
- Topologically stable defects
- Big Bang simulation in the low temperature lab
Two stable Helium isotopes:

$^4\text{He}$: air, oil wells, ...  
Janssen/Lockyer (1868)  
Ramsay (1895)

\[
\frac{\text{He}}{\text{air}} \approx 5 \times 10^{-6}, \quad \frac{^3\text{He}}{^4\text{He}}_{\text{air}} \approx 1 \times 10^{-6}
\]

$^3\text{He}$:

\[
^6\text{Li} + ^1\text{n} \rightarrow ^3\text{H} + ^4\text{He} \rightarrow ^2\text{He} + e^-
\]

(1939)

Research on macroscopic samples of $^3\text{He}$ since 1947
Helium

Atoms: spherical, hard core diameter $\sim 2.5$ Å

Interaction: • hard sphere repulsion
• van der Waals dipole/multipole attraction

Boiling point: 4.2 K, $^4$He Kamerlingh Onnes (1908)
3.2 K, $^3$He Sydoriak, et al. (1949)

Dense, simple liquids $\{\begin{align*}
\text{isotropic} \\
\text{short-range interactions} \\
\text{extremely pure}
\end{align*}\}$
Helium

\[ T \xrightarrow{\to} 0, \ P \lesssim 30 \ \text{bar}: \ \text{Helium remains liquid} \]

Atoms:
- spherical shape $\rightarrow$ weak attraction
- light mass $\rightarrow$ strong zero-point motion

\[ \lambda \propto \frac{\hbar}{\sqrt{k_B T}} \xrightarrow{T \to 0} \ \text{Macroscopic quantum phenomena} \]
# Helium

<table>
<thead>
<tr>
<th></th>
<th>$^4\text{He}$</th>
<th>$^3\text{He}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Electron shell:</strong></td>
<td>2 e$^-$, $S = 0$</td>
<td></td>
</tr>
<tr>
<td><strong>Nucleus:</strong></td>
<td><img src="image" alt="Nucleus" /> $S = 0$</td>
<td><img src="image" alt="Nucleus" /> $S = \frac{1}{2}\hbar$</td>
</tr>
<tr>
<td>Atom(!) is a</td>
<td>Boson</td>
<td>Fermion</td>
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<tr>
<td><strong>Phase transition</strong></td>
<td>$T_\lambda = 2.2\ \text{K}$ (&quot;BEC&quot;)</td>
<td>$T_c = ???$</td>
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**Quantum liquids**

**Fermi liquid theory**
Fermi gas: **Ground state**

- Fermi surface →
- Fermi sea

$k_x$, $k_y$, $k_z$
Fermi gas: **Excited states** \((T>0)\)

Switch on interaction adiabatically \((d=3)\)

Exact \(k\)-states ("particles"): **infinite** life time

**Switch on interaction adiabatically** \((d=3)\)
Landau Fermi liquid

“Standard model of condensed matter physics“

1-1 correspondence between one-particle states \( (k, \sigma) \)

(Quasi-) Particle
= elementary excitation

Prototype: Helium-3

- Large effective mass
- Strongly enhanced spin susceptibility
- Strongly reduced compressibility
Instability of Landau Fermi liquid

+ 2 non-interacting particles
Arbitrarily weak attraction $\Rightarrow$ Cooper instability

Universal fermionic property
Arbitrarily weak attraction \( \Rightarrow \) Cooper pair \((k, \alpha; -k, \beta)\)

\[ \Psi_{L=0,2,4,...} = \psi(r) | \uparrow \downarrow - \downarrow \uparrow \rangle \]

\[ \Psi_{L=1,3,5,...} = \psi_+(r) | \uparrow \uparrow \rangle \]

\[ + \psi_0(r) | \uparrow \downarrow + \downarrow \uparrow \rangle \]

\[ + \psi_-(r) | \downarrow \downarrow \rangle \]

\( S=0 \) (singlet)

\( S=1 \) (triplet)
Arbitrarily weak attraction \( \Rightarrow \) Cooper pair \((k, \alpha; -k, \beta)\)

\[ \Psi_{L=0,2,4,...} = \psi(r)|\uparrow\downarrow - \downarrow\uparrow\rangle \]

S=0 (singlet)

\[ \Psi_{L=1,3,5,...} = \psi_+(r)|\uparrow\uparrow\rangle + \psi_0(r)|\uparrow\downarrow + \downarrow\uparrow\rangle + \psi_-(r)|\downarrow\downarrow\rangle \]

S=1 (triplet)

L = 0: isotropic wave function
L > 0: anisotropic wave function

Helium-3: Strongly repulsive interaction \( \Rightarrow \) L > 0 expected
Generalization to macroscopically many Cooper pairs
Generalization to macroscopically many Cooper pairs

\[ \varepsilon_c \ll E_F \]\n
\[ \Rightarrow \text{"Pair condensate"} \]

with macroscopically coherent wave function

Transition temperature

\[ T_c = 1.13 \varepsilon_c \exp\left(-\frac{1}{N(0)|V_L|}\right) \]

"weak coupling theory"

\[ \varepsilon_c, V_L: \text{Magnitude? Origin?} \Rightarrow T_c? \]

Thanksgiving 1971: Transition in $^3$He at $T_c = 0.0026$ K

Osheroff, Richardson, Lee (1972)
The Nobel Prize in Physics 1996
"for their discovery of superfluidity in helium-3"

David M. Lee
Cornell (USA)

Douglas D. Osheroff
Stanford (USA)

Robert C. Richardson
Cornell (USA)
Phase diagram of Helium-3

P-T phase diagram

- Normal Fermi liquid
- Superfluid A phase
- Superfluid B phase
- Solid (bcc)
- Ordered spins
- Disordered spins

Temperature (K) vs. Pressure (MPa)
Phase diagram of Helium-3

P-T phase diagram

Dense, simple liquid

- isotropic short-range interactions
- extremely pure
- nuclear spin S=1/2

Normal Fermi liquid

Superfluid B phase

Solid (bcc)

ordered spins
disordered spins

Superfluid A phase

Gas

Pressure (MPa)

Temperature (K)
Phase diagram of Helium-3

P-T-H phase diagram

“Very low temperatures”: $T \ll T_{\text{boiling}} \sim 3-4 \text{ K}$
$\ll T_{\text{backgr. rad.}} \sim 3 \text{ K}$
Superfluid phases of $^3$He

Experiment: Osheroff, Richardson, Lee, Wheatley, ...
Theory: Leggett, Wölfle, Mermin, ...

→ $L=1$, $S=1$ in all phases

Attraction due to spin fluctuations

Anderson, Brinkman (1973)

→ anisotropy directions in a $^3$He Cooper pair

orbital part

spin part
... and a mystery!

NMR experiment on nuclear spins $I = \frac{1}{2} \hbar$  

$$\omega^2 - \omega_L^2 \propto \Delta^2(T)$$

$\omega_L$  

$3mT$  

Larmor frequency: $\omega_L = \gamma H$

Shift of $\omega_L$  

$\leftrightarrow$ spin-nonconserving interactions  

$\to$ nuclear dipole interaction $g_D \sim 10^{-7} K \ll T_C$

Origin of frequency shift ?!  

Leggett (1973)
The superfluid phases of $^3$He
B-phase

\[ \Psi = |\uparrow\uparrow\rangle + |\uparrow\downarrow + \downarrow\uparrow\rangle + |\downarrow\downarrow\rangle \]

\[ \Delta(k) = \Delta_0 \]

Balian, Werthamer (1963)
Vdovin (1963)

(pseudo-) isotropic state \(\leftrightarrow\) s-wave superconductor

Weak-coupling theory: stable for all \(T<T_c\)
A-phase

$$\Psi = |\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle \rightarrow \text{strong anisotropy}$$

$$\Delta(\hat{k}) = \Delta_0 \sin(\hat{k}, \hat{l})$$  Anderson, Morel (1961)

Cooper pair
orbital angular momentum

“Axial state” has point nodes

→ “unconventional” pairing in
  • heavy fermion/high-$T_c$ superconductors
  • $\text{Sr}_2\text{RuO}_4$

Strong-coupling effect
$A_1$-phase finite magnetic field

$\Psi = |\uparrow\uparrow\uparrow\rangle$

Long-range ordered magnetic liquid
Broken Symmetries, Long Range Order
Broken Symmetries, Long Range Order

Normal $^3$He ↔ $^3$He-A, $^3$He-B: 2. order phase transition

$T<T_c$: higher order, lower symmetry of ground state

I. Ferromagnet

$T>T_c$ : Average magnetization: $\langle M \rangle = 0$
Symmetry group: $\text{SO}(3)$

$T<T_c$ : $\langle M \rangle \neq 0$ Order parameter
Symmetry group: $\text{U}(1) \subset \text{SO}(3)$

$T<T_c$: $\text{SO}(3)$ rotation symmetry in spin space spontaneously broken
Broken Symmetries, Long Range Order

2. order phase transition

\( T < T_c \): higher order, lower symmetry of ground state

II. Liquid crystal

\( T > T_c \)

\( T < T_c \)

Symmetry group: \( SO(3) \)

\( U(1) \subseteq SO(3) \)

\( T < T_c \): \( SO(3) \) rotation symmetry in real space spontaneously broken
Broken Symmetries, Long Range Order

2. order phase transition

$T<T_c$: higher order, lower symmetry of ground state

III. Conventional superconductor

$T>T_c$

Pair amplitude $\langle c_{k \uparrow}^\dagger c_{-k \downarrow}^\dagger \rangle = 0$

Gauge transf. $c_{k \sigma}^\dagger \rightarrow c_{k \sigma}^\dagger e^{i\phi}$: gauge invariant

Symmetry group $U(1)$

$T<T_c$

$\Delta e^{i\phi}$ “Order parameter“

not gauge invariant
Broken Symmetries, Long Range Order

2. order phase transition

\( T < T_c \): higher order, lower symmetry of ground state

III. Conventional superconductor

\( T > T_c \)

\( T < T_c \)

\( T < T_c \): \( U(1) \) "gauge symmetry" spontaneously broken

\( \rightarrow \) \( U(1) \) gauge symmetry also broken in BEC
Broken symmetries in superfluid $^3$He

$L=1$, $S=1$ in all phases

Cooper pair: $\hat{d}$ orbital part, $\hat{l}$ spin part

Quantum coherence in

- phase
- anisotropy direction for spin
- anisotropy direction in real space

Superfluid, magnetic liquid crystal

Characterized by $2 \times (2L + 1) \times (2S + 1) = 18$ real numbers

$3 \times 3$ order parameter matrix $A_{ij\mu}$

$SO(3)_S \times SO(3)_L \times U(1)_\phi$ symmetry spontaneously broken

Leggett (1975)
**Broken symmetries in superfluid $^3$He**

Mineev (1980)  
Bruder, DV (1986)

$3\text{He-A}$  
$SO(3)_S \times SO(3)_L \times U(1)_\varphi$ symmetry broken

\[
U(1)_{S_z} \times U(1)_{L_z - \varphi}
\]

„Unconventional“ pairing

Cooper pairs

Fixed absolute orientation
Resolution of the NMR puzzle

\[ \omega^2 - \omega_L^2 \propto \Delta^2(T) \]

- \( \omega_L \)
- 3mT
- superfluid
- \( T_{C,A} \)
- normal
Superfluid $^3$He - a quantum amplifier

Cooper pairs in $^3$He-A

Fixed absolute orientation

What determines the actual relative orientation of $\hat{d}$, $\hat{l}$?

→ Anisotropic spin-orbit interaction of nuclear dipoles:

\[ \uparrow \uparrow \neq \rightarrow \rightarrow \]

Dipole-dipole coupling of $^3$He nuclei: $g_D \sim 10^{-7} K \ll T_C$

Unimportant ?!
Superfluid $^3$He - a quantum amplifier

Cooper pairs in $^3$He-A

- Long-range order in $\hat{d}, \hat{l}$
- $g_D \sim 10^{-7} K$: tiny, but lifts degeneracy of relative orientation

Quantum coherence $\downarrow$

$\hat{d}, \hat{l}$ locked in all Cooper pairs

NMR frequency increases:

$$\omega^2 = (\gamma H)^2 + g_D \Delta^2(T)$$

Leggett (1973)

$\rightarrow$ Nuclear dipole interaction macroscopically measurable
The Nobel Prize in Physics 2003
"for pioneering contributions to the theory of superconductors and superfluids"

Alexei A. Abrikosov
USA and Russia

Vitaly L. Ginzburg
Russia

Anthony J. Leggett
UK and USA
Order parameter textures and topological defects
Order parameter textures

Orientation of anisotropy directions $\hat{d}, \hat{l}$ in $^3$He-A?

Magnetic field $\rightarrow \hat{d}$

Walls $\rightarrow \hat{l}$

$\rightarrow$ “Textures“ in $\hat{d}, \hat{l}$ $\leftrightarrow$ liquid crystals

$\rightarrow$ Topologically stable defects: Classification by homotopy theory
Order parameter textures and topological defects

D=2: domain walls in $\hat{d}$ or $\hat{i}$

Single domain wall

Domain wall lattice
Order parameter textures and topological defects

D=1: Vortices

- Vortex formation (rotation experiments)

- e.g., Mermin-Ho vortex (non-singular)
Order parameter textures and topological defects

D=0: Monopoles

"Boojum" in $\hat{l}$-texture of $^3$He-A
(geometric constraint)

Defect formation by, e.g.,
- rotation
- geometric constraints
- rapid crossing through phase transition
Big bang simulation in the low temperature lab

BANG!
Universality in continuous phase transitions

\[ T > T_c \]

High symmetry, short-range order

\[ T = T_c \]

Phase transition

\[ T < T_c \]

Broken symmetry, long-range order

Spins: paramagnetic

Helium: normal liquid

Universe: Unified forces and fields

Defects: domain walls

ferromagnetic superfluid

cosmic strings, etc. Kibble (1976)

nucleation of galaxies?

Heleum: normal, superfluid

vortices, etc.
Rapid thermal quench through 2. order phase transition  

1. Local temperature $T \gg T_C$
   - Expansion + rapid cooling

2. Nucleation of independently ordered regions
   - Clustering of ordered regions
   - $\rightarrow$ Defects

3. Defects overlap

4. $T < T_C$: Vortex tangle

Estimate of density of defects  

"Kibble-Zurek mechanism": How to test?
Big bang simulation in the low temperature laboratory

Grenoble: Bäuerle et al. (1996), Helsinki: Ruutu et al. (1996)

Measured vortex tangle density: Quantitative support for Kibble-Zurek mechanism
Present research on superfluid $^3$He: Quantum Turbulence

Classical Turbulence

Leonardo da Vinci (1452-1519)
Flow through grid

Quantum Turbulence = Turbulence in the absence of viscous dissipation (superfluid at $T\to0$)

- Why are quantum and classical turbulence so similar?
- What provides dissipation in the absence of friction?

Test system: $^3$He-B

Vinen, Donnelly: Physics Today (April, 2007)
Superfluid Helium-3:

- **Anisotropic superfluid**
  - 3 different bulk phases
  - Cooper pairs with internal structure

- **Large symmetry group broken**
  - Close connections to particle theory
  - Zoo of topological defects
  - Kibble-Zurek mechanism quantitatively verified

Conclusion