# The Superfluid Phases of Helium 3

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# 1

# Introduction

## 1.1 THE HELIUM LIQUIDS

There are two stable isotopes of the chemical element helium: helium 3 and helium 4, conventionally denoted by $^3$He and $^4$He respectively. The existence of the heavier one, $^4$He, had already been established indirectly in 1871 by its characteristic line in the solar spectrum (hence the name "helium"). Then in 1895 Ramsay succeeded in obtaining an actual sample of $^4$He gas by heating the uranium ore cleveite. It was later found that $^4$He is also part of the Earth's atmosphere, but only at a fraction of about $2 \times 10^{-5}$. Kamerlingh Onnes was the first to liquefy $^4$He in 1908. For a fascinating account of the history of early helium physics and of low-temperature physics in general, we refer the reader to the books by Keesom (1942) and Mendelssohn (1977).

The discovery and identification of the lighter isotope $^3$He was only made much later (Oliphant *et al.* 1933). In fact, at that time it was thought that the $^3$He isotope was unstable and that tritium ($^3$H) was stable. Using a cyclotron as a mass spectrograph, Alvarez and Cornog (1939a,b) then showed that $^3$He is indeed a *stable* isotopic constituent of "ordinary" helium. However, in the atmosphere $^3$He constitutes only one part in a million of the total helium content, which is so small anyway. The quantities of $^3$He needed for low-temperature experiments can only be produced by nuclear reactions such as

$$^6_3\text{Li} + ^1_0\text{n} \rightarrow ^3_1\text{H} + ^4_2\alpha \tag{1.1a}$$

and the subsequent decay of the tritium

$$^3_1\text{H} \rightarrow ^3_2\text{He} + \text{e}^- \tag{1.1b}$$

Clearly, this was not possible until after the end of the Second World War. The condensation of $^3$He gas and the first experimental work on the new liquid were achieved by Sydoriak *et al.* (1949a,b). Somewhat larger quantities of $^3$He became available only in the late 1950s. Since then, macroscopic samples of $^3$He have been investigated intensively and at lower and lower temperatures.

By the time experimental $^3He$ physics eventually started, the theoretical and experimental investigations of $^4He$ had been going on for such a long time and had yielded so many novel results that "helium" was used synonymously for $^4He$. While this tradition can be found even in today's scientific literature, one may expect the situation to change in the future. After all, the low-temperature phases (normal, superfluid, solid) of $^3He$ have led to a unique multitude of fundamentally new concepts and have thereby enlarged our knowledge of the possible states of condensed matter more than any other single-component system previously studied.

From a microscopic point of view, helium atoms are structureless spherical particles interacting via a two-body potential that is well understood (see Chapter 2). The attractive part of the potential, arising from weak van der Waals-type dipole (and higher multipole) forces, causes helium gas to condense into a liquid state at temperatures of 3.2 K and 4.2 K for $^3He$ and $^4He$ respectively, at normal pressure. The pressure versus temperature phase diagrams of $^3He$ and $^4He$ are shown in Figs. 1.1 and 1.2. When the temperature is decreased even further one finds that the helium liquids, unlike all other known liquids, do not solidify unless a pressure of around 30 bar is applied. This is the first remarkable indication of macroscopic quantum effects in these systems. The origin of this unusual behaviour lies in the quantum-mechanical uncertainty principle, which requires that a quantum particle can never be completely at rest at given position, but rather performs a zero-point motion about the average position. The smaller the mass of the particle and the weaker the binding force, the stronger these oscillations are. In most solids the zero-point motion is confined to a small volume of only a fraction of the lattice-cell volume. In the case of helium, however, two features combine to prevent the formation of a crystalline solid with a rigid lattice structure: (i) the strong zero-point motion arising from the small atomic mass (helium is the second-lightest element in the periodic table); and (ii) the weakness of the attractive interaction due to the high symmetry of these simple atoms.

It is this very property of helium—of staying liquid—that makes it such a valuable system for observing quantum behaviour on a macroscopic scale.

Quantum effects are also responsible for the strikingly different behaviours of $^4He$ and $^3He$ at even lower temperatures. Whereas $^4He$ undergoes a second-order phase transition (Kamerlingh Onnes 1911b, Kamerlingh Onnes and Boks 1924, Keesom and Wolfke 1928) into a state later shown to be superfluid (Kapitza 1938, Allen and Misener 1938), i.e. where the liquid is capable of flowing through narrow capillaries or tiny pores without friction, no such transition is observed in liquid $^3He$ in the same temperature range (see Figs. 1.1 and 1.2). The properties of liquid $^3He$ below 1 K are nevertheless found to be increasingly different from those of a classical liquid. It is only at a temperature roughly one thousandth of the transition temperature of $^4He$ that $^3He$ also becomes superfluid, and in fact forms *several* superfluid phases, each of which has a much more complex structure than that of superfluid $^4He$.

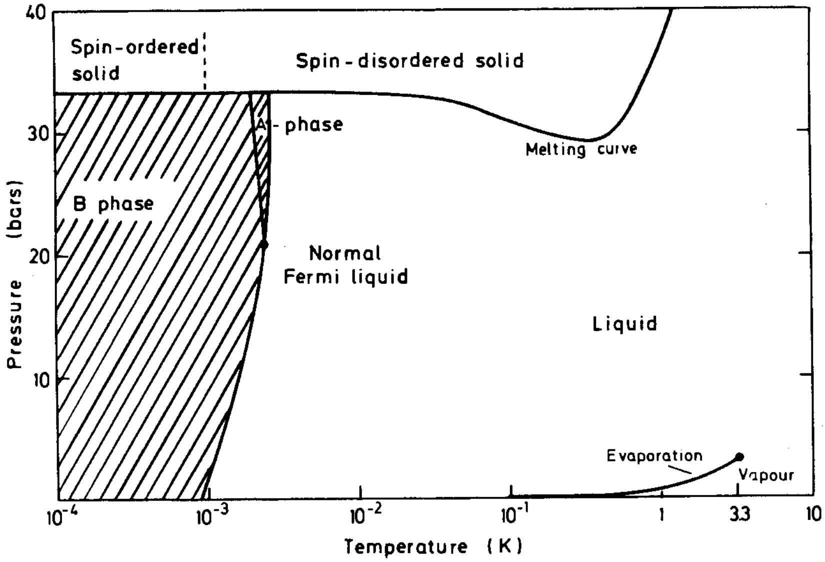


Figure 1.1 Pressure versus temperature phase diagram for $^3$He; note the logarithmic temperature scale.

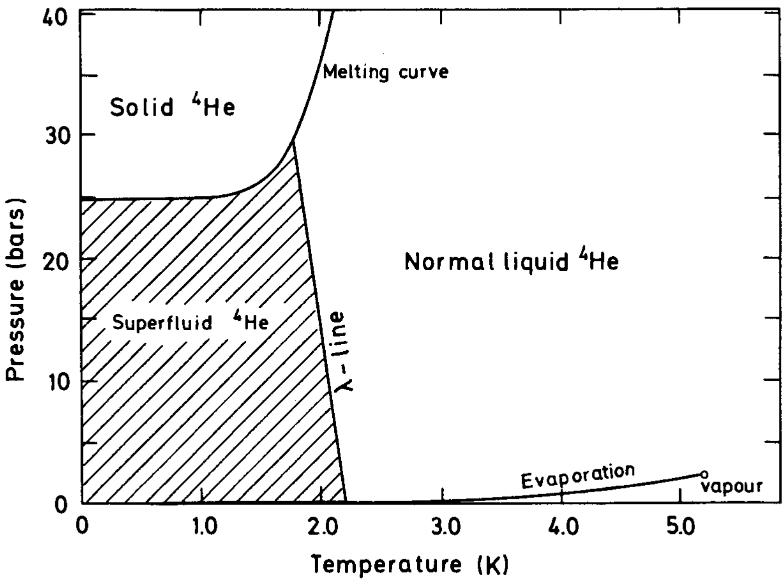


Figure 1.2 Pressure versus temperature phase diagram for $^4$He; linear temperature scale.

The striking difference in the behaviours of $^3$He and $^4$He at low temperatures is a consequence of the laws of quantum theory as applied to systems of identical particles, i.e. the laws of quantum statistics. The $^4$He atom, being composed of an even number of electrons and nucleons, has spin zero and consequently obeys Bose–Einstein statistics. In contrast, the $^3$He nucleus consists of *three* nucleons, whose spins add up to give a total nuclear spin of $I = \frac{1}{2}$, making the total spin of the entire $^3$He atom $\frac{1}{2}$ as well. Consequently liquid $^3$He obeys Fermi–Dirac statistics. So it is the tiny nuclear spin, buried deep inside the helium atom, that is responsible for all the differences of the macroscopic properties of the two isotopes.

Since in a Bose system single-particle states may be multiply occupied, at low temperatures this system has a tendency to condense into the lowest-energy single-particle state (Bose–Einstein condensation). It is believed that the superfluid transition in $^4$He is a manifestation of Bose–Einstein condensation. The all-important qualitative feature of the Bose condensate is its phase rigidity, i.e. the fact that it is energetically favourable for the particles to condense into a single-particle state of fixed quantum-mechanical phase, such that the global gauge symmetry is spontaneously broken. As a consequence, macroscopic flow of the condensate is (meta)stable, giving rise to the phenomenon of superfluidity.

In a Fermi system, on the other hand, the Pauli exclusion principle allows only single occupation of fermion states. The ground state of the Fermi gas is therefore the one in which all single-particle states are filled up to a limiting energy, the Fermi energy $E_F$. As predicted by Landau (1956, 1957, 1958) and later verified experimentally (for a review see Wheatley 1966), the properties of $^3$He well below its Fermi temperature $T_F = E_F/k_B \approx 1$ K are similar to those of a degenerate Fermi gas. In particular, the formation of a phase-rigid condensate is not possible in this framework. Until the mid-1950s a superfluid phase of liquid $^3$He was therefore believed to be ruled out. On the other hand, it is most remarkable that the property of superfluidity (see F. London 1950, 1954) was indeed first discovered experimentally in a *Fermi* system, namely that of the "liquid" of conduction electrons in a superconducting metal (Kamerlingh Onnes 1911a). The superfluidity of $^4$He was only found more than 25 years later.

The key to the theory of superconductivity (Bardeen, Cooper and Schrieffer (BCS) 1957) turned out to be the formation of "Cooper pairs", i.e. pairs of electrons with opposite momentum $\boldsymbol{k}$ and spin projection $\sigma$: $(\boldsymbol{k}\uparrow, -\boldsymbol{k}\downarrow)$. These particular Cooper pairs are structureless objects, i.e. the two partners form a spin-singlet state in a relative s-wave orbital state. Cooper pairs may therefore be looked upon in a way as composite bosons, which all have the same pair wave function and are all in the same quantum-mechanical state. Hence in this picture the transition to the superconducting state corresponds to the formation of Cooper pairs that are automatically Bose-condensed, the condensate being characterized by macroscopic quantum coherence. Such a picture requires some qualification (see Chapter 12), but is nevertheless very helpful for an understanding of many basic properties of superconductors.

While in free space an attractive force has to be sufficiently strong to bind two electrons, inside the metal the presence of the filled Fermi sea of conduction electrons blocks the decay of a Cooper pair, so that an *arbitrarily* small attractive interaction leads to the formation of stable Cooper pairs. The attractive interaction between the electrons of a Cooper pair in a conventional superconducting metal is due to the exchange of virtual phonons (electron–phonon interaction). If the phonon-mediated interaction is strong enough to overcome the repulsive Coulomb interactions between the two electrons then a transition into a superconducting state may occur. On the other hand, any other mechanism leading to attraction between electrons at the Fermi surface is equally well suited for producing superconductivity.

## 1.2 EARLY HISTORY OF SUPERFLUID $^3$He

Given the success of the BCS theory in the case of superconductivity, it was natural to ask whether a similar mechanism might also work for liquid $^3$He. Since there is no underlying crystal lattice in the liquid that could mediate the attractive force, the attraction must clearly be an intrinsic property of the one-component $^3$He liquid itself. The main feature of the interatomic $^3$He potential is the strong repulsive component at short distances, and the weak van der Waals attraction at medium and long distances. It soon became clear that, in order to avoid the hard repulsive core and thus make optimal use of the attractive part of the potential, the $^3$He atoms would have to form Cooper pairs in a state of *nonzero* relative angular momentum $l$. In this case the Cooper-pair wave function vanishes at zero relative distance, thus cutting out the most strongly repulsive part of the potential. In a complementary classical picture one might imagine the partners of a Cooper pair revolving about their centre of gravity, thus being kept away from each other by the centrifugal force.

A first estimate of the transition temperature for Cooper pairs with large relative angular-momentum quantum number, bound by the long-range tail of the van der Waals attraction (which was argued to be essentially unrenormalized by many-body effects), yielded unattainably low values (Pitaevskii 1959). Another line of approach was based on an approximate calculation of the effective pair interaction, taking multiple scattering into account; this yielded an attraction for d-wave pairs (Brueckner *et al.* 1960) but repulsion for s-wave pairs (Cooper *et al.* 1959). Other proposals for anisotropic, i.e. non-s-wave Cooper pairing, were due to Thouless (1960), Emery and Sessler (1960) and Galasiewicz (1960, 1969). However, from a modern point of view, none of these attempts was sophisticated enough. As more experimental data on liquid $^3$He became available, it was soon realized that this is a strongly interacting system. The entities forming the Cooper pairs are not the bare $^3He$ atoms but are rather the quasiparticles of Landau's theory. These quasiparticles are single-particle excitations, which are sometimes viewed as particles surrounded by a polarization cloud of other particles. In fact, the effective mass of such quasiparticles may be as much as six times the bare atomic mass. Similarly, the interactions between quasiparticles were found to be very strong. It is then not surprising that the bare atomic potential bears little resemblance to the effective quasiparticle potential.

These difficulties notwithstanding, several authors considered the properties of non-s-wave pairing states. Anderson and Morel (1960, 1961) gave an extensive discussion of the thermodynamic properties of general anisotropic states, concentrating on d-wave states, but introducing also the socalled "axial" p-wave state (later named the Anderson–Brinkman–Morel (ABM) state). This state has the peculiar feature that the energy-gap function has nodes (i.e. zero-points) on the Fermi surface, with the orbital-angular-momentum projection pointing along the direction of these nodes. In fact this state turned out to describe one of the superfluid phases, the A phase, which was to be discovered only much later. These authors realized that the antisymmetry of the Cooper-pair wave function under exchange of the two particles requires the spin state to be the triplet state for any odd angular momentum. In contrast, any even-parity Cooper pair (e.g. a d-wave pair) is in the spin-singlet state. Clearly, a general p-wave Cooper pair must therefore have *three* spin substates—not only two, like that discussed by Anderson and Morel (1960, 1961). Indeed, Vdovin (1963) and Balian and Werthamer (1963) showed that, within weak-coupling theory, a state with an equal admixture of all three states is energetically favoured at all temperatures. The energy gap of this state (later called the Balian–Werthamer (BW) state) was found to be isotropic, just as in the case of s-wave pairing, despite the intrinsic anisotropy of the Cooper-pair wave function. Nevertheless, its magnetic properties are as complex and anisotropic as that of the ABM state. The BW state was also shown to sustain order-parameter collective oscillations of a type never encountered before (Vdovin 1963), although this work remained unknown outside the Soviet Union. The BW state turned out to describe the superfluid B phase, observed at temperatures below which the A phase is stable.

Another important early advance in the description of the properties of these hypothetical states was made by Leggett (1965a,b), who showed that the thermodynamic properties would be strongly renormalized (in a temperature-dependent way) by Fermi-liquid interaction effects.

Meanwhile, a better understanding of the physically relevant processes leading to large renormalization effects, and in particular of the important role of spin fluctuations, began to emerge. It was realized that ferromagnetic spin fluctuations favour spin-triplet/odd-$l$ states over spin-singlet/even-$l$ states (Emery 1964). The consequences of spin fluctuations for the thermodynamic and transport properties were worked out and found to be in qualitative agreement with experiment (Berk and Schrieffer 1966, Doniach and Engelsberg 1966, Brenig and Mikeska 1967, Brenig *et al.* 1967, Rice 1967a,b, Brinkman and Engelsberg 1968, Riedel 1968). A detailed calculation of the influence of spin fluctuations on the transition temperature for anisotropic pairing gave clear preference for spin-triplet/odd-$l$ pairing (Layzer and Fay 1968, 1971, Nakajima 1973) and yielded values of $T_c$ that were rather encouraging (Layzer and Fay 1968). A quantitative prediction of $T_c$, however, was not possible on this basis. For an account of early theoretical work on superfluid $^3$He see Anderson and Brinkman (1975, 1978).

When the superfluid phases of $^3$He were finally discovered in 1971 at temperatures of about 2.6 mK and 1.8 mK respectively (Osheroff *et al.* 1972a), in an experiment actually designed to observe a magnetic phase transition in solid $^3$He, the results came as a great surprise.

## 1.3 ELEMENTARY DISCUSSION OF SUPERFLUID $^3$He

Soon after the discovery of the phase transitions by Osheroff, Richardson and Lee (1972a), it was possible to identify altogether *three* distinct stable superfluid phases of bulk $^3$He (see Chapter 4); these are referred to as the A, B and $A_1$ phases. In zero magnetic field only the A and B phases are stable. In particular, in zero field the A phase only exists within a finite range of temperatures, above a critical pressure of about 21 bar. Hence its region of stability in the pressure–temperature phase diagram has a roughly triangular shape as shown in Fig. 1.1. The B phase, on the other hand, occupies the largest part of this phase diagram and is found to be stable down to the lowest temperatures attained so far. Application of an external magnetic field has a strong influence on this phase diagram. First of all, the A phase is now stabilized down to zero pressure. Secondly, an entirely new phase, the $A_1$ phase, appears as a narrow wedge between the normal state and the A and B phases. Since the magnetic field changes the structure of the A and B phases somewhat, they are referred to as $A_2$ and $B_2$ phases respectively. Thus the $A_2$ and $B_2$ phases are just the old A and B phases in the presence of a magnetic field. This is summarized in Table 1.1, and the full $P$–$H$–$T$ phase diagram is shown in Fig. 4.2.

Owing to the theoretical work on anisotropic superfluidity that had been carried out before the actual discovery of superfluid $^3$He, progress in understanding the detailed nature of the phases was very rapid. This was clearly also due to the excellent contact between experimentalists and theorists, which greatly helped to develop the right ideas at the right time. In particular, it fairly soon became possible to identify the A phase and the B phase as realizations of the states studied previously by Anderson and Morel (1960, 1961) and Balian and Werthamer (1963) respectively. Therefore the A phase is described by the so-called "Anderson–Brinkman–

Table 1.1 Superfluid phases of $^3$He.

| External magnetic field $H$ | Stable phases |
|---|---|
| $H=0$ | A phase<br>B phase |
| $H \neq 0$ | $A_1$ phase<br>$A_2$ phase (≡A phase in a magnetic field)<br>$B_2$ phase (≡B phase in a magnetic field) |

Morel" (ABM) state, while the B phase is described by the "Balian–Werthamer" (BW) state. Consequently, "A phase" and "ABM state" are now used as synonyms; the same is true in the case of "B phase" and "BW state". (The fact that the ABM state describes the A phase and the BW state the B phase is a very fortunate concidence—if it was the other way around, it would be quite confusing!).

Although the three superfluid phases all have very different properties, they have one important thing in common: the Cooper pairs in all three phases are in a state with *parallel* spin ($S=1$) and relative orbital angular momentum $l=1$. This kind of pairing is referred to as "spin-triplet p-wave pairing". In contrast, prior to the discovery of the superfluid phases of $^3$He, Cooper pairing in superconductors was only known to occur in a state with opposite spins ($S=0$) and $l=0$, i.e. in a "spin-singlet s-wave state". It should be noted that Cooper pairs in a superconductor and in superfluid $^3$He are therefore very different entities: in the former case pairs are formed by pointlike, structureless electrons and are spherically symmetric, while in the case of $^3$He Cooper pairs are made of actual atoms (or rather of quasiparticles involving $^3$He atoms) and have an internal structure themselves.

### 1.3.1 The internal structure of Cooper pairs

Quantum-mechanically, a spin-triplet configuration ($S=1$) of two particles has three substates with different spin projection $S_z$. They may be represented as $|\uparrow\uparrow\rangle$ with $S_z=+1$, $2^{-1/2}(|\uparrow\downarrow\rangle+|\downarrow\uparrow\rangle)$ with $S_z=0$ and $|\downarrow\downarrow\rangle$ with $S_z=-1$. The pair wave function $\Psi$ is in general a linear superposition of all three spin substates, i.e.

$$\Psi = \psi_{1,+}(\boldsymbol{k})\,|\uparrow\uparrow\rangle + \psi_{1,0}(\boldsymbol{k})(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) + \psi_{1,-}(\boldsymbol{k})\,|\downarrow\downarrow\rangle, \tag{1.2}$$

where $\psi_{1,+}(\boldsymbol{k})$, $\psi_{1,0}(\boldsymbol{k})$ and $\psi_{1,-}(\boldsymbol{k})$ are the three complex-valued amplitudes of the respective substates. In the case of a superconductor, where $S=0$ and $l=0$, the pair wave function is much simpler, i.e. it is given by only a single component

$$\Psi_{sc} = \psi_0(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle), \tag{1.3}$$

with a single amplitude $\psi_0$.

So far we have only taken into account that, since $S = 1$, there are three substates for the spin. The same is of course true for the relative orbital angular momentum $l = 1$ of the Cooper pair, which also has three substates $l_z = 0, \pm 1$. This fact is important if we want to investigate the amplitudes $\psi_{1,+}(\boldsymbol{k})$ etc. further. They still contain the complete information about the space (or momentum) dependence of $\Psi$. The pair wave function $\Psi$ is therefore characterized by three spin substates and three orbital substates, i.e. by altogether $3 \times 3 = 9$ substates with respect to the spin and orbital dependence. Each of these nine substates is connected with a complex-valued parameter. Here we see the essential difference between Cooper pairs with $S = l = 0$ (conventional superconductors) and $S = l = 1$ ($^3$He): their pair wave functions are very different. In the former case a single complex-valued parameter is sufficient for its specification, in the latter case of superfluid $^3$He *nine* such parameters are required. This also expresses the fact that a Cooper pair in superfluid $^3$He has an internal structure, while that for a conventional superconductor does not: because $l = 1$, it is intrinsically *anisotropic.* This anisotropy may conveniently be described by specifying some direction with respect to a quantization axis both for the spin and the orbital component of the wave function.

In order to understand the novel properties of superfluid $^3$He, it is therefore important to keep in mind that there are two characteristic directions that specify a Cooper pair. Here lies the substantial difference from a superconductor and the origin of the multitude of unusual phenomena occurring in superfluid $^3$He: the structure of the Cooper pair is characterized by *internal degrees of freedom.* Nevertheless, in both cases the superfluid/superconducting state can be viewed as the condensation of a macroscopic number of these Cooper pairs into the same quantum-mechanical state, similar to a Bose–Einstein condensation.

### 1.3.2 Broken symmetry and the order parameter

In the normal liquid state Cooper pairs do not exist. Obviously, in the superfluid a new state of order appears, which spontaneously sets in at the critical temperature $T_c$. This particular transition from the normal to the superfluid, i.e. into the ordered state, is called "continuous", since the condensate—and hence the state of order—builds up continuously. This fact may be expressed quantitatively by introducing an "order parameter" that is finite for $T < T_c$ and zero for $T \geqslant T_c$. A well-known example of such a transition is that from a paramagnetic to a ferromagnetic state of a metal when the system is cooled below the Curie temperature. In the paramagnetic regime the spins of the particles are disordered such that the average magnetization $\langle \boldsymbol{M} \rangle$ of the system is zero. By contrast, in the ferromagnetic phase the spins are more or less aligned and $\langle \boldsymbol{M} \rangle$ is thus finite. In this case the system exhibits long-range order of the spins. The degree of ordering is quantified by $|\langle \boldsymbol{M} \rangle|$, the magnitude of the magnetization. Hence $\boldsymbol{M}$ is called the "order parameter" of the ferromagnetic state. Clearly, the existence of a preferred direction $\boldsymbol{M}$ of the spins implies that the symmetry of the ferromagnet under spin rotations is reduced ("broken") when compared with the paramagnet: the directions of the spins are no longer isotropically distributed, and the system will therefore no longer be invariant under a spin rotation. This phenomenon is called "spontaneously broken symmetry"; it is of fundamental importance in the theory of phase transitions. It describes the property of a macroscopic system (i.e. a system in the thermodynamic limit) that is in a state that does not have the full symmetry of the microscopic dynamics.

The concept of spontaneously broken symmetry also applies to superconductivity and superfluid $^3$He. In this case the order parameter measures the existence of Cooper pairs and is given by the probability amplitude for a pair to exist at a given temperature. It follows from the discussion of the possible structure of a Cooper pair in superfluid $^3$He that the associated order parameter will reflect this structure and the allowed internal degrees of freedom. What then are the spontaneously broken symmetries in superfluid $^3$He?

As already mentioned, the interparticle forces between the $^3$He atoms are rotationally invariant in spin and orbital space and, of course, conserve particle number. The latter symmetry gives rise to a somewhat abstract symmetry called "gauge symmetry". Nevertheless, gauge symmetry is spontaneously broken in any superfluid or superconductor (see Chapter 3). In addition, in an odd-parity pairing superfluid as in the case of $^3$He, where $l = 1$, the pairs are necessarily in a spin-triplet state, implying that rotational symmetry in spin space is broken, just as in a magnet. At the same time, the anisotropy of the Cooper-pair wave function in orbital space calls for a spontaneous breakdown of orbital rotation symmetry, as in liquid crystals. All three symmetries are therefore simultaneously broken in superfluid $^3$He. This implies that the A phase, for example, may be considered as a "superfluid nematic liquid crystal with (anti)ferromagnetic character". One might think that a study of the abovementioned broken symmetries could be performed much more easily by investigating them separately, i.e. within the isotropic superfluid, the magnet, the liquid crystal etc. itself. However, the combination of several *simultaneously* broken continuous symmetries is more than just the simple sum of the properties of all these known systems. Some of the symmetries broken in superfluid $^3$He are "relative" symmetries, such as spin–orbit rotation symmetry or gauge–orbit symmetry (Leggett 1972, 1973b, Liu and Cross 1978). Because of this, a rigid connection is established between the corresponding degrees of freedom of the condensate, leading to long-range order only in the combined (and not in the individual) degrees of freedom. This particular kind of broken symmetry, for example the so-called "spontaneously broken spin–orbit symmetry", gives rise to very unusual behaviour, as will be discussed later. The whole concept of broken symmetries in superfluid $^3He$ and its consequences will be discussed in detail in Chapter 6.

It is clear that in principle the internal degrees of freedom of a spin-triplet p-wave state allow for many different Cooper-pair states and hence superfluid phases. (This is again different from ordinary superconductivity with $S = 0$, $l = 0$ pairing, where only a *single* phase is possible.) Of these different states, the one with the lowest energy for given external parameters will be realized. In fact, Balian and Werthamer (1963) showed, that, within a conventional "weak-coupling" approach, of all possible states there is precisely one state (the BW state) that has the lowest energy at *all* temperatures. This state is the one that describes the B phase of superfluid $^3He$. The state originally discussed by these authors is one in which the orbital angular momentum $\boldsymbol{l}$ and spin $\boldsymbol{S}$ of a Cooper pair couple to a total angular momentum $\boldsymbol{J} = \boldsymbol{l} + \boldsymbol{S} = 0$. This $^3P_0$ state is, however, only a special case of a more general one with the same energy (in the absence of spin–orbit interaction), obtained by an arbitrary rotation of the spin axes relative to the orbital axes of the Cooper-pair wave function. Such a rotation may be described mathematically by specifying a rotation axis $\hat{\boldsymbol{n}}$ and a rotation angle $\theta$. In the BW state all three spin substates in (1.2) occur with equal measure. This state has a rather surprising property: in spite of the intrinsic anisotropy, the state has an *isotropic* energy gap (see Fig. 3.4a). (The energy gap is the amount by which the system lowers its energy in the condensation process, i.e. it is the minimum energy required for the excitation of a single particle out of the condensate.) Therefore the BW phase resembles ordinary superconductors in several ways. On the other hand, even though the energy gap is isotropic, the BW state *is* intrinsically anisotropic. This is clearly seen in dynamic experiments in which the Cooper-pair structure is distorted. For this reason the BW state is sometimes referred to as "pseudo-isotropic". Owing to the quantum coherence of the superfluid state, the rotation axis $\hat{\boldsymbol{n}}$ and angle $\theta$ characterizing a Cooper pair in the BW state are macroscopically defined degrees of freedom, whose variation is physically measurable.

Since in weak-coupling theory the BW state always has the lowest energy, an explanation of the existence of the A phase of superfluid $^3He$ obviously requires one to go beyond such an approach and to include "strong-coupling effects", as will be discussed in Chapter 5. In view of the fact that at present microscopic theories are not capable of computing transition temperatures for $^3He$, it is helpful to single out a particular effect that can explain the stabilization of the A phase over the B phase. As shown by Anderson and Brinkman (1973), there is such a conceptually simple effect, which is based on a feedback mechanism: the pair correlations in the condensed state change the pairing interaction between the $^3$He quasiparticles, the modification depending on the actual state itself. As a specific mechanism, these authors considered the role of spin fluctuations and showed that a stabilization of the state first considered by Anderson and Morel (1960, 1961) is indeed possible. This only happens at somewhat elevated pressures, since spin fluctuations become more pronounced only at higher pressures. This state, which is now referred to as the "ABM state" (from the initials of these three authors), does indeed describe the A phase. It has the property that, in contrast with $^3$He-B, its magnetic susceptibility is essentially the same as that of the normal liquid. This is a clear indication that in this phase the spin substate with $S_z = 0$, which is the only one that can be reduced appreciably by an external magnetic field, is absent. Therefore $^3$He-A is composed only of $|\uparrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle$ Cooper pairs. This implies that the anisotropy axis of the spin part of the Cooper-pair wave function, called $\hat{\boldsymbol{d}}$, has the same, fixed, direction in every pair. (More precisely, $\hat{\boldsymbol{d}}$ is the direction along which the total spin of the Cooper pair vanishes: $\hat{\boldsymbol{d}} \cdot \boldsymbol{S} = 0$.) Likewise, the direction of the relative orbital angular momentum $\hat{\boldsymbol{l}}$ is the same for all Cooper pairs. Therefore in the A phase the anisotropy axes $\hat{\boldsymbol{d}}$ and $\hat{\boldsymbol{l}}$ of the Cooper-pair wave function are long-range-ordered, i.e. are preferred directions in the whole macroscopic sample. This implies a pronounced anisotropy of this phase in all its properties. In particular, the value of the energy gap now explicitly depends on the direction in $\boldsymbol{k}$ space on the Fermi sphere and takes the form

$$\Delta_{\hat{k}}(T) = \Delta_0(T)[1 - (\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{l}})^2]^{1/2}. \tag{1.4}$$

Hence the gap vanishes at two points on the Fermi sphere, namely along $\pm\hat{\boldsymbol{l}}$ (see Fig. 3.4b). Because of the existence of an axis $\hat{\boldsymbol{l}}$, the ABM state is also called the "axial state". The existence of nodes implies that in general quasiparticle excitations may take place at arbitrarily low temperatures. Therefore, in contrast with $^3$He-B or ordinary superconductors, there is a finite density of states for excitations with energies below the average gap energy, leading for example to a specific heat proportional to $T^3$ at low temperatures (see Chapters 3 and 7).

The third experimentally observable superfluid phase of $^3$He, the $A_1$ phase, is only stable in the presence of an external magnetic field. In this phase Cooper pairs are all in a single spin substate, the $|\uparrow\uparrow\rangle$ state, corresponding to $S_z = +1$; the components with $|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle$ states are missing. It is therefore a *magnetic* superfluid, the first ever observed in nature.

### 1.3.3 Orientational effects

For a pair-correlated superfluid, the pairing interaction is the most important interaction, since it is responsible for the formation of the condensate itself. Nevertheless, there also exist other, much weaker, interactions, which may not be important for the actual transition to the pair-condensed state, but which do become important if their symmetry differs from the aforementioned. In particular, they may be able to break remaining degeneracies.

### *The dipole–dipole interaction and other weak effects*

The dipole–dipole interaction between the nuclear spins of the $^3$He atoms leads to a very weak, spatially strongly anisotropic, coupling. The relevant coupling constant $g_D(T)$ is given by

$$g_D(T) \approx \frac{\mu_0^2}{a^3}\left(\frac{\Delta(T)}{E_F}\right)^2 n. \tag{1.5}$$

Here $\mu_0$ is the nuclear magnetic moment, such that $\mu_0^2/a^3$ is the average dipole energy of two particles at relative distance $a$ (the average atomic distance), while the second factor measures the probability for these two particles to form a Cooper pair, and $n$ is the overall particle density. Since $\mu_0^2/a^3$ corresponds to about $10^{-7}$ K, this energy is extremely small and the resulting interaction of quasiparticles at temperatures of the order of $10^{-3}$ K might be expected to be completely swamped by thermal fluctuations. This is indeed true in a normal system. However, the dipole–dipole interaction implies a spin–orbit coupling and thereby has a symmetry different from that of the pairing interaction. In the condensate the symmetries with respect to a rotation in spin and orbital space are spontaneously broken, leading to long-range order (for example of $\hat{\mathbf{d}}$ and $\hat{\mathbf{l}}$ in the case of $^3$He-A). Nevertheless, the pairing interaction does not fix the *relative* orientation of these preferred directions, leaving a continuous degeneracy. As pointed out by Leggett (1973a,b, 1974a), in this situation the tiny dipole interaction is able to lift the degeneracy, namely by choosing that particular relative orientation for which the dipolar energy is minimal. Thereby this interaction becomes of *macroscopic* importance. One may also view this effect as a macroscopic amplification of a microscopic interaction via the quantum coherence of the pair condensate. This correlation is equivalent to a *permanent* local magnetic field of about 30 G at any point in the superfluid (in a liquid!). In $^3$He the dipolar interaction is minimized by a parallel orientation of $\hat{\mathbf{d}}$ and $\hat{\mathbf{l}}$; for details see Section 6.3.

Besides the dipole interaction, there are also other, even weaker, interactions, which—although completely unimportant in a normal system—may be amplified to macroscopic size by the long-range order in the superfluid (Leggett 1977a,b, 1978a). For example, the electromagnetic interaction of the two $^3$He atoms in a Cooper pair leads to a minute distortion of the electronic shell of the $^3$He atom, thereby shifting the centres of the positive and negative charges relative to each other. The orbital motion within the Cooper pair then leads to a magnetic moment along $\hat{l}$ in every Cooper pair. Because of the long-range order of $\hat{l}$, these magnetic moments add up to an equivalent magnetic field of about 0.02 G. Although this is very small, it makes the system behave differently upon reversal of the direction of an external magnetic field. Such a change has indeed been detected (Paulson and Wheatley 1978a). Therefore $^3$He-A is a *liquid* orbital ferromagnet!

Leggett (1977a) also predicted that the macroscopic quantum coherence in superfluid $^3$He will raise weak-interaction effects in elementary particle physics to a macroscopic level. For example, the interaction between electrons and protons within a $^3$He atom has a parity-violating part given by the exchange of a neutral $Z^0$ boson. This effect is expected to induce a finite electric dipole moment along the preferred direction $\hat{n}$ in $^3$He-B, with an overall magnitude of about $10^{-12} e\,\mathrm{cm}^{-2}$, where $e$ is the electronic charge. This is still a very small effect, but it does not seem to be hopelessly outside experimental reach. If an experiment succeeded in measuring this effect, it would be the first detection of parity violation on a macroscopic scale.

The amplification of these and other effects due to the specific long-range order in superfluid $^3$He is discussed in Chapter 12. Clearly, of all those interactions the nuclear dipole interaction is by far the most important. Its effect on the relative orientation of the spontaneously preferred directions in spin and orbital space has to be included in any description of low-energy phenomena in superfluid $^3$He (i.e. where the order-parameter structure itself remains unchanged).

### *Effect of a magnetic field*

An external magnetic field acts on the nuclear spins and thereby leads to an orientation of the preferred direction in spin space. In the case of $^3$He-A the orientation energy is minimal if $\hat{d}$ is perpendicular to the field $\boldsymbol{H}$, since (taking into account $\hat{d} \cdot \boldsymbol{S} = 0$) this orientation guarantees $\boldsymbol{S} \parallel \boldsymbol{H}$.

### *Walls*

Every experiment is performed in a volume of finite size. Clearly, the walls will have some effect on the liquid inside. In superfluid $^3$He this effect may readily be understood by using a simple picture. Let us view the Cooper pair as a kind of giant "molecule" of two $^3$He quasiparticles orbiting around each other. For a pair not to bump into a wall, this rotation will have to take place in a plane parallel to the wall. In the case of $^3$He-A, where the orbital angular momentum $\hat{l}$ has the same direction in all Cooper pairs (standing perpendicular on the plane of rotation), this means that $\hat{l}$ has to be oriented perpendicular to the wall. So there exists a strict orientation of $\hat{l}$ caused by the walls (Ambegaokar *et al.* 1974). In the B phase, with its (pseudo)isotropic order parameter, the orientational effect is not as pronounced, but there are qualitatively similar boundary conditions.

### 1.3.4 Textures

From the above discussion, it is clear that the preferred directions $\hat{\boldsymbol{l}}$ and $\hat{\boldsymbol{d}}$ in $^3$He-A are in general subject to different, often competing, orientational effects (for simplicity, we shall limit our description to $^3$He-A). At the same time, the condensate will oppose any spatial variation of its long-range order. Any "bending" of the order-parameter field will therefore increase the energy, thus giving an internal stiffness or rigidity to the system. While the orientational effects might want $\hat{\boldsymbol{d}}$ and $\hat{\boldsymbol{l}}$ to adjust on the smallest possible lengthscale, the bending energy wants to keep the configuration as uniform as possible. Altogether, the competition between these two opposing effects will lead to a smooth spatial variation of $\hat{\boldsymbol{d}}$ and $\hat{\boldsymbol{l}}$ throughout the sample, called a "texture". This nomenclature is borrowed from the physics of liquid crystals, where similar orientational effects of the preferred directions occur (de Gennes 1974). The largest part of Chapter 7, which itself is by far the longest chapter in this book, is devoted to the discussion of textures in superfluid $^3$He.

The bending energy and all quantitatively important orientational energies are invariant under the replacement $\hat{\boldsymbol{d}} \rightarrow -\hat{\boldsymbol{d}}$, $\hat{\boldsymbol{l}} \rightarrow -\hat{\boldsymbol{l}}$. A state where $\hat{\boldsymbol{d}}$ and $\hat{\boldsymbol{l}}$ are parallel therefore has the same energy as one where $\hat{\boldsymbol{d}}$ and $\hat{\boldsymbol{l}}$ are antiparallel. This leads to two different, degenerate, ground states. There is then the possibility that in one part of the sample the system is in one ground state and in the other in a different ground state. Where the two configurations meet they form a planar "defect" in the texture, called a "domain wall" (Maki 1977a) (see Fig. 7.22). This is in close analogy to the situation in a ferromagnet composed of domains with a different orientations of the magnetization. Domain walls are spatially localized and are quite stable against external perturbations. In fact, their stability is guaranteed by the specific nature of the order-parameter structure of $^3$He-A. Mathematically, this structure may be analysed according to its topological properties. The stability of a domain wall can then be traced back to the existence of a conserved "topological charge". Using the same mathematical approach, one can show that the order-parameter fields of the superfluid phases of $^3$He not only allow for planar defects but also for point and line defects, called "monopoles" and "vortices" respectively. Defects can be "nonsingular" or "singular", depending on whether the core of the defect remains superfluid or whether it is forced to become normal liquid. The concept of vortices is of course well known from superfluid $^4$He. However, since the order-parameter structure of superfluid $^3$He is so much richer than that of superfluid $^4$He, there exist a wide variety of different vortices in these phases. Their detailed structure has been the subject of intensive investigation, in particular in the context of experiments on rotating superfluid $^3$He, where they play a central role (Hakonen and Lounasmaa 1987); see Chapter 7.

### 1.3.5 Superfluid mass currents in $^3$He-A

Superfluids owe their name to their ability to flow through tiny pores and narrow slabs, apparently without friction. This extraordinary property, which is in sharp contrast with our usual experience with liquids, is certainly the most impressive manifestation of the macroscopic quantum coherence of the pair condensate in a Fermi system. Mathematically, superfluidity finds its expression in a complex-valued order parameter, i.e. the existence of *macroscopic* quantum-mechanical phase variables.

To understand the flow properties of a superfluid, it is helpful to consider it as a system of two independent, interpenetrating components: a "superfluid" and a "normal" component, with densities $\rho_s$ and $\rho_n$ respectively, such that $\rho_s + \rho_n = \rho$, with $\rho$ the total density. At $T \geqslant T_c$ one has $\rho_s = 0$ and $\rho_n = \rho$, while at $T = 0$ one has $\rho_s = \rho$ and $\rho_n = 0$. This "two-fluid model" is useful in many ways (but, of course, it is only a *model,* since in reality the liquid is not composed of distinct superfluid and normal particles). In this model a mass current $\boldsymbol{g}$ is the sum of a superfluid and a normal part $\boldsymbol{g}_s = \rho_s \boldsymbol{v}_s$ and $\boldsymbol{g}_n = \rho_n \boldsymbol{v}_n$ respectively. Here $\boldsymbol{v}_s$ and $\boldsymbol{v}_n$ are the velocities of the two components. In a superfluid with isotropic order parameter, $\Psi = \psi_0 e^{i\phi}$, as in $^4$He, the superfluid velocity $\boldsymbol{v}_s$ is simply given by the gradient of the macroscopic, i.e. long-range-ordered, phase of the order parameter:

$$\boldsymbol{v}_s = \frac{\hbar}{m_4} \nabla \phi, \tag{1.6}$$

where $m_4$ is the mass of a $^4$He atom. This simple dependence implies

$$\nabla \times \boldsymbol{v}_s = 0, \tag{1.7}$$

i.e. the flow is "curl-free"—there is no rotational motion (we do not consider the rotation of the system as a whole). For any closed contour $\mathscr{C}$ in the liquid, we may define the "circulation" $\kappa$ as the line integral over the velocity:

$$\kappa = \oint_{\mathscr{C}} d\boldsymbol{s} \cdot \boldsymbol{v}_s = \int_{\text{area}} d\boldsymbol{f} \cdot (\nabla \times \boldsymbol{v}_s). \tag{1.8}$$

Using (1.7), it follows that $\kappa = 0$. On the other hand, in a container with a hole (e.g. a vessel with the shape of a doughnut—a "torus") the situation is quite different. The circulation $\kappa$ along a contour $\mathscr{C}$ enclosing the hole will clearly be finite as long as $\boldsymbol{v}_s \neq 0$. Since the phase of the order parameter is a macroscopic quantity, it is well defined along $\mathscr{C}$ and can only change by multiplies of $2\pi$ when $\mathscr{C}$ is traversed. In this case $\kappa$ is *quantized* and (1.8) takes the well-known form of the Bohr–Sommerfeld quantization condition for electronic orbits in an atom:

$$\kappa = \frac{h}{m_4} N. \tag{1.9}$$

Here $N = 0, \pm 1, \pm 2, \ldots$, and $h/m_4$ is the so-called "flux quantum". This means that supercurrents in a toroidal geometry are quantized and hence cannot decay continuously: they are (meta)stable, their lifetime being of the order of cosmological times.

In an anisotropic superfluid such as $^3$He-A the situation is very different. Here the order parameter not only has a phase but also has an *orientation*, namely the preferred direction $\hat{\boldsymbol{l}}$ (the spin structure is unimportant in this discussion). The anisotropy implies that the system can now distinguish directions relative to $\hat{\boldsymbol{l}}$. In this case the superfluid velocity will depend not only on the spatial change of the phase $\phi$ but also on that of $\hat{\boldsymbol{l}}$. Parametrizing $\hat{\boldsymbol{l}}$ by azimuthal ($\beta$) and polar ($\alpha$) angles, $\boldsymbol{v}_s$ now takes the form

$$\boldsymbol{v}_s = \frac{\hbar}{2m_3}(\boldsymbol{\nabla}\phi - \cos\beta\, \boldsymbol{\nabla}\alpha) \tag{1.10}$$

(note that the mass of a Cooper pair, $2m_3$, appears in (1.10)). The fact that $\boldsymbol{v}_s$ is no longer given by a phase gradient has an immediate, drastic, consequence: $\boldsymbol{\nabla} \times \boldsymbol{v}_s \neq 0$. This means that $\boldsymbol{\nabla} \times \boldsymbol{v}_s$, and thereby $\kappa$ in (1.7), can in general take *any* value, depending on how $\hat{\boldsymbol{l}}$ changes. There is no longer any flux quantization! Consequently, a superfluid mass current in $^3$He-A is unstable (Mermin and Ho 1976). Therefore $^3$He-A does not seem to be a "superfluid" at all. However, we have so far neglected the surface of the container. Since $\hat{\boldsymbol{l}}$ has to be perpendicular to the wall, i.e. it has a fixed orientation there, the circulation at the wall is quantized and the stability of a supercurrent is guaranteed at least very near to the surface. These and related questions are discussed in detail in Chapter 7.

### 1.3.6 Dynamic properties

From the discussion presented so far, we have already seen that the static properties of an anisotropic superfluid are very unusual. Clearly, the dynamic properties can be expected to be at least as new and diverse. Indeed, the fact that in superfluid $^3$He Cooper pairs have an internal structure can only be investigated in detail by studying the dynamics, i.e. the frequency and momentum dependence, of the condensate. One may roughly distinguish between magnetic and nonmagnetic dynamic properties, depending on whether the magnetization of the system is probed or whether properties such as mass transport (see Chapter 10) or the propagation of sound are studied.

For the investigation of dynamical effects, it is instructive to have an idea of the typical frequencies inherent to the superfluid condensate. For this, we again employ the two-fluid model. Both the normal and the superfluid components are essentially characterized by a single timescale each: for the normal component this is the quasiparticle lifetime $\tau$, and for the superfluid component it is $\hbar/\Delta(T)$, where $\Delta(T)$ is the average of the temperature-dependent energy gap. The orders of magnitude of the equivalent frequencies are given by $\tau^{-1} \approx 10\,\text{MHz}$ and $\Delta(T)/\hbar \approx 10^3(1 - T/T_c)^{1/2}\,\text{MHz}$, i.e. usually one has $\tau^{-1} \ll \Delta(T)/\hbar$. For frequencies $\omega$ much smaller than either of these characteristic values, the liquid is always in local thermodynamic equilibrium, since the system always has sufficient time to adjust to any change induced on the timescale $\omega^{-1}$. This is called the "hydrodynamic regime", which is important for a couple of reasons: (i) in this regime knowledge of the conserved quantities and of those describing the broken symmetries is sufficient to describe the properties of the system (see Chapter 9); and (ii) this regime is experimentally well accessible. The multitude of broken symmetries in superfluid $^3$He consequently leads to very rich hydrodynamics, which describes the various low-frequency collective excitations of the system. Here the word "collective" (as opposed to "single-particle") means that a macroscopic number of particles is involved in a coherent fashion.

### *Spin dynamics*

Investigations of the collective magnetic (i.e. spin-dependent) properties of the superfluid phases of $^3$He by nuclear magnetic resonance (NMR) were particularly useful in identifying the explicit order-parameter structure of these phases. In usual NMR experiments the system under investigation is brought into a strong constant external magnetic field $\boldsymbol{H}_0 = H_0\hat{\boldsymbol{z}}$, which forces the (nuclear) spin $\boldsymbol{S}$ to precess about $\boldsymbol{H}_0$. By applying a weak high-frequency magnetic field $\boldsymbol{H}_{\text{rf}}$ perpendicular to $\boldsymbol{H}_0$, one is able to induce transitions in $S_z$, the component along $\boldsymbol{H}_0$, of magnitude $\pm\hbar$. This effect is observed as an energy absorption from the magnetic field. In the case of noninteracting spins these transitions occur *exactly* at the energy $\gamma\hbar H_0$, i.e. at the Larmor frequency $\omega_L = \gamma H_0$, where $\gamma$ is the gyromagnetic ratio of the nucleus. How does this change in the presence of interactions? For a spin of magnitude $\frac{1}{2}\hbar$, as in the case of the $^3$He nucleus, a very general statement is possible (Leggett 1972, 1973b): as long as the interactions are spin-*conserving*, there is no change at all—the resonance remains at $\omega_L$. On the other hand, for spin-nonconserving interactions, such as the spin–orbit interaction caused by the dipole coupling of the nuclear spins, a frequency shift may indeed occur. However, such a "nonsecular" shift will usually be very small, namely at most of the order of the linewidth. The experimental data obtained by Osheroff *et al.* (1972b) in connection with their discovery of the superfluid phases therefore came as a great surprise—they found that the resonance, although still very sharp, occurred at frequencies substantially higher than $\omega_L$. The origin of this large shift was especially mysterious, since it obviously corresponded to a *constant* local magnetic field of order 30 G surrounding the nuclear spins in the liquid.

The solution to this puzzle was found by Leggett (1972, 1973b, 1974a), who showed that the NMR shifts are a consequence of the broken symmetries of the spin-triplet p-wave condensate, which he named "spontaneously broken spin–orbit symmetry". As explained earlier, the meaning of this concept is that the preferred directions in spin and orbital space are long-range-ordered (individually so, or in a combined way) and the tiny dipole interaction may take advantage of this situation by lifting the remaining degeneracy. The macroscopic quantum coherence of the condensate therefore raises the dipole coupling to macroscopic importance. In this way, Leggett (1974a) was able to calculate the general NMR response of a spin-triplet p-wave condensate. In particular, in the A phase the transverse NMR frequency $\omega_t$ is given by

$$\omega_t^2 = \omega_L^2 + \Omega_A^2(T), \tag{1.11}$$

where $\Omega_A^2(T)$ is proportional to the dipole coupling constant (see (1.5)). It should be noted that the field and temperature dependences of $\omega_t$ are neatly separated in a "Pythagorean" form: $\omega_L$ only depends on $H_0$ and $\Omega_A$ only on $T$! In fact, Leggett (1974a) worked out a complete theory of spin dynamics, whose predictions were experimentally confirmed in every detail; this will be discussed in Chapter 8. For example, the equation of motion of the total spin $\boldsymbol{S}$ is given by

$$\dot{\boldsymbol{S}} = \gamma \boldsymbol{S} \times \boldsymbol{H} + \boldsymbol{R}_D, \tag{1.12}$$

where $\boldsymbol{H} = \boldsymbol{H}_0 + \boldsymbol{H}_{rf}$ is the total external magnetic field and a dot over a symbol indicates the time derivative. Here $\boldsymbol{R}_D$ is the anisotropic "dipole torque", which itself depends on the change of the dipole energy under a reorientation of the order parameter. In the normal phase $\boldsymbol{R}_D$ is always zero. In the superfluid one has $\boldsymbol{R}_D \neq 0$, except for static situations. If the system is displaced from static equilibrium (for example by applying $\boldsymbol{H}_{rf}$), $\boldsymbol{R}_D$ acts as a restoring force. For example, in the A phase a periodic oscillation of $\boldsymbol{S}$ will lead to an oscillation of $\hat{\boldsymbol{d}}$, the preferred direction in spin space, around the orbital degree of freedom $\hat{\boldsymbol{l}}$ (which may be assumed to remain fixed because it cannot move very quickly). Equation (1.12) led Leggett to a spectacular prediction: even if the high-frequency field $\boldsymbol{H}_{rf}$ is oriented *parallel* to $\boldsymbol{H}_0$, there is a resonance, i.e. there exists a longitudinal spin resonance! Since in this case $(\boldsymbol{S} \times \boldsymbol{H})_z = 0$, (1.12) yields $dS_z/dt = R_{Dz}$. In a normal system there can be no resonance since there is no restoring force: the $z$ component of the magnetization will simply relax exponentially but will not oscillate. How then can we understand the nature of the longitudinal oscillation in the case of superfluid $^3$He-A? The A phase only consists of the two spin substates $|\uparrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle$. They may be viewed as essentially independent interpenetrating superfluids, which are only very weakly coupled by the spin-nonconserving dipole coupling. This coupling allows for a transition of $|\uparrow\uparrow\rangle$ pairs into $|\downarrow\downarrow\rangle$ pairs, and vice versa. (The situation is quite similar to a pair of weakly coupled superconductors, where Cooper pairs can tunnel from one superconductor to the other (the

"Josephson effect"); the difference here is that the two subsystems fill the same volume, i.e. they are not spatially separated). Applying a high-frequency magnetic field parallel to the static field $\boldsymbol{H}_0$ leads to oscillatory nonequilibrium between the two spin subsystems, with the dipole interaction acting as a restoring force. The resonant frequency of this longitudinal oscillation occurs at

$$\omega_\ell = \Omega_A(T), \tag{1.13}$$

where $\Omega_A$ is the frequency that has already appeared in the expression for the transverse frequency (1.11).

Any texture formed by the order-parameter field changes the dipole torque $\boldsymbol{R}_D$ in a very specific way. Therefore the measurement of NMR shifts, in combination with the corresponding theory, provides the most versatile, and at the same time sensitive, tool for the investigation of order-parameter textures.

NMR frequencies are generally considerably smaller than the characteristic frequencies $\tau^{-1}$ and $\Delta(T)/\hbar$ of the normal and superfluid components. Hence such experiments take place in the hydrodynamic regime. At such low frequencies, i.e. energies, the magnitude of the order parameter $\Delta(T)$ does not change at all—only the orientation of its spin part varies. Hence the *structure* of the order parameter is left intact—the dynamics is due to a "rigid" excitation of the order parameter. At higher frequencies, $\omega \approx \Delta(T)$, this changes dramatically. To understand the consequences of this, it is again helpful to view a Cooper pair as some kind of diatomic molecule. As in the case of a molecule, an energy of the order of the binding energy will lead to internal excitations such as rotational and vibrational states.

### *Ultrasound excitations*

Such a situation occurs in experiments measuring the attenuation of ultrasound at sound frequencies close to $\Delta(T)/\hbar$. Quite unexpectedly, one finds that the sound attenuation of the superfluid has a sharp maximum directly below the transition temperature $T_c$; this maximum depends strongly on the frequency $\omega$. These and other phenomena are explained by collective excitations of the order-parameter structure of the condensate. They owe their existence to pair correlations in a state with nonzero relative orbital angular momentum, which imply an internal structure of the Cooper pair (Wölfle 1973a, 1978a). This structure allows for the excitation of high-frequency ($\omega \approx \Delta(T)/\hbar$) collective oscillations (pair-vibration modes). Besides this, there is also the possibility of a break-up of the Cooper pair. Pair breaking is only possible if the energy $\hbar\omega$ of the sound wave is larger than the minimum energy for breaking a pair, $2\Delta_{\hat{k}}(T)$. Here $\Delta_{\hat{k}}(T)$ is the energy gap, which in general depends on $\hat{\boldsymbol{k}}$, the position on the Fermi sphere. For smaller energies, only vibrations can be excited. A detailed theory of sound absorption, including damping effects etc., has been developed (see Chapter 11) and is in good agreement with experiments.

In particular, the existence of isotropic and anisotropic energy gaps in the B and A phase respectively led to early identification of these two phases. Indeed, in the B phase sound attenuation is independent of the direction of the sound entering the probe. By contrast, in the A phase it strongly depends on the relative orientation of the sound wave to the anisotropy axis $\hat{l}$. This orientation dependence is very remarkable: by coupling to the nuclear spins, a weak external magnetic field of the order of 30 G is able to change the direction of $\hat{l}$ and thereby to modify the sound absorption. It is the coherent ordering of *nuclear* spins that is ultimately responsible for the anisotropy of sound absorption!

This concludes our elementary discussion of the properties of superfluid $^3$He. There are many other astonishing aspects of these anisotropic phases, which will also be discussed in this book.

## 1.4 RELATION TO OTHER FIELDS

Why spend so much effort on sorting out and explaining the strange behaviour of states of matter that are not even found in nature, at temperatures well outside the reach of even a well-equipped low-temperature laboratory? Partly, of course, "because it's there", and because—like any other system—superfluid $^3$He deserves to be studied in its own right. However, what is even more important is that superfluid $^3$He is a model system that exemplifies many of the concepts of modern theoretical physics and, as such, has given us, and will further provide us, with new insights into the functioning of quantum-mechanical many-body systems close to their ground state.

As discussed in Section 1.3, the key to understanding superfluid $^3$He is "spontaneously broken symmetry". In this respect there are also very fundamental connections with particle physics, deriving from the interpretation of the order-parameter field as a quantum field with a rich group structure. The collective modes of the order parameter as well as the localized topological defects in a given ground-state configuration are the particles of this quantum field theory. Various anomalies known from particle physics can be identified in the $^3$He model system, and one may hope that insights gained from the study of superfluid $^3$He will turn out to be useful in elementary particle theory (Volovik 1987).

There are several other physical systems for which the ideas developed in the context of superfluid $^3$He are relevant or may be relevant in the future. Dilute solutions of $^3$He in $^4$He constitute a system of fermions moving in a background of superfluid $^4$He. It is expected that a transition into a pair-correlated state should take place, which would make this the first system of two interpenetrating superfluids (Bashkin and Meyerovich 1981). Unfortunately, the transition temperature is estimated to be rather low, around 5 μK or even much lower. The symmetry of the pairing is predicted to be s-wave at low concentrations (<3%) of $^3$He, with the possibility of p-wave formation at higher concentrations (3–6%) (Pfitzner 1984, Hsu 1984, Hsu and Pines 1985). The transition has not yet been observed, despite considerable effort (Owers-Bradley *et al.* 1983).

Another anisotropic superfluid system that does already exist in nature is not accessible for laboratory experiments: this is the nuclear matter forming the cores of neutron stars. There the pairing of neutrons has been calculated to be of p-wave symmetry. Because of the strong spin–orbit nuclear force, the total angular momentum of the Cooper pairs is $J = 2$ (Hoffberg *et al.* 1970, Muzikar *et al.* 1980, Sauls *et al.* 1982, Pines and Alpar 1985).

Above all, an anisotropic superconducting state would be most exciting. There are now strong indications that superconductivity in the so-called "heavy-fermion" systems, first discovered by Steglich *et al.* (1979), is, at least in some cases, due to the formation of anisotropic pairs with d-wave or possibly p-wave symmetry (for reviews see Stewart 1984, Lee *et al.* 1986, Gorkov 1987, Ott 1987, Fulde *et al.* 1988). Many of the concepts and ideas developed for superfluid $^3$He have been adapted to these systems. One must keep in mind, however, that the charge of the electrons and the presence of the underlying crystal lattice may lead to qualitatively new behaviour (Leggett 1987). Other systems where relations to superfluid $^3$He may eventually appear are the recently discovered high-$T_c$ oxide superconductors (Bednorz and Müller 1986, Wu *et al.* 1987), which are now being investigated with an intensity unparalleled in the history of condensed-matter physics.

The above discussion shows that superfluid $^3$He is a field of continuing interest. We hope that this book will contribute to the future development and expansion of our understanding of anisotropic superfluid systems by generating the interest of the beginner, educating the serious student of the subject and providing reference material for the researcher in this field.

## 1.5 REVIEWS AND INTRODUCTORY ARTICLES ON SUPERFLUID $^3$He

### 1.5.1 General reviews

*Theory*

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## 1.5.2 Reviews of specific topics

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Salomaa M M and Volovik G E 1987 Quantized vortices in superfluid $^3He$. *Rev. Mod. Phys.* **59** 533

### 1.5.3 Discussions in books on related subjects

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### 1.5.4 Introductory articles

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