Superfluid Helium-3: Universal Concepts for Condensed Matter and the Big Bang

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Helium: after hydrogen the most abundant element in the universe
Helium

Two stable Helium isotopes: $^4\text{He}$, $^3\text{He}$

$^4\text{He}$: air, oil wells, ...  
Janssen/Lockyer/Secci (1868)

\[
\frac{^4\text{He}}{\text{air}} \approx 5 \times 10^{-6} \quad \frac{^3\text{He}}{^4\text{He}} \bigg|_{\text{air}} \approx 1 \times 10^{-6}
\]

$^3\text{He}$:  
\[\beta^- + ^6\text{Li} + ^1\text{n} \rightarrow ^3\text{He} + ^1\text{H} + ^1\alpha\]

Ramsay (1895)  
Cleveit (UO$_2$)

Research on macroscopic samples of $^3\text{He}$ only since 1947

$^4\text{He}$: Coolant, Welding, Balloons
$^3\text{He}$: - Contrast agent in medicine  
- Neutron detectors  
- $^3\text{He}$-$^4\text{He}$ dilution refrigerators (quantum computers!)
Helium

Atoms: spherical, hard core diameter $\sim 2.5$ Å

Interaction: • hard sphere repulsion
  • van der Waals dipole/multipole attraction

Boiling point: 4.2 K, $^4$He  Kamerlingh Onnes (1908)

3.2 K, $^3$He  Sydoriak et al. (1949)

Dense, simple liquids $\left\{\begin{array}{l}
\text{isotropic} \\
\text{short-range interactions} \\
\text{extremely pure}
\end{array}\right\}$
Helium

Atoms: • spherical shape → weak attraction
• light mass → strong zero-point motion

\[ T \to 0, \ P \leq 3 \ \text{MPa}: \ \text{Helium remains liquid} \]

\[ \lambda \propto \frac{\hbar}{\sqrt{k_B T}} \xrightarrow{T \to 0} \text{quantum phenomena on a macroscopic scale} \]
### Helium

<table>
<thead>
<tr>
<th></th>
<th>$^4\text{He}$</th>
<th>$^3\text{He}$</th>
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</thead>
<tbody>
<tr>
<td><strong>Electron shell:</strong></td>
<td>2 e(^-), (S = 0)</td>
<td>(\lambda)</td>
</tr>
<tr>
<td><strong>Nucleus:</strong></td>
<td>$S = 0$</td>
<td>$S = \frac{1}{2}\hbar$</td>
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<tr>
<td><strong>Atom(!) is a</strong></td>
<td><strong>Boson</strong></td>
<td><strong>Fermion</strong></td>
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<tr>
<td><strong>Phase transition</strong></td>
<td>$T_\lambda = 2.2 \text{ K}$</td>
<td>$T_c = \text{??}$</td>
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Interacting Fermions (Fermi liquid): **Ground state**

Fermi surface → Fermi sphere

\[ k_x \quad k_y \quad k_z \]

Landau (1956/57)
Instability of Fermi liquid

Fermi sphere

+ 2 non-interacting fermions
Arbitrarily weak attraction $\Rightarrow$ instability

Universal fermionic property

Cooper (1956)
Arbitrarily weak attraction \( \Rightarrow \) Cooper pair \((k, \alpha; -k, \beta)\)

\[
\begin{align*}
\Psi_{L=0,2,4,...} &= \psi(r) |\uparrow\downarrow - \downarrow\uparrow\rangle \\
\Psi_{L=1,3,5,...} &= \psi_+(r) |\uparrow\uparrow\rangle \\
&\quad + \psi_0(r) |\uparrow\downarrow + \downarrow\uparrow\rangle \\
&\quad + \psi_-(r) |\downarrow\downarrow\rangle
\end{align*}
\]

S=0 (singlet)

S=1 (triplet)

L = 0 ("s-wave"): isotropic pair wave function
L > 0 ("p,d,f,... -wave"): anisotropic pair wave function

\(^3\text{He}:\) Strongly repulsive interaction \( \Rightarrow \) L > 0 expected
BCS theory

Bardeen, Cooper, Schrieffer (1957)

Generalization to macroscopically many Cooper pairs

Energy gap $\Delta(T)$
here: $L=0$ (s-wave)

Pair condensate
with transition temperature

$$T_c = 1.13 \varepsilon_c \exp\left(-1/ N(0)|V_L|\right)$$
in weak coupling theory

$\varepsilon_c, V_L$: Magnitude? Origin? $\rightarrow T_c$?

Thanksgiving 1971: Transition in $^3$He at $T_c = 0.0026$ K

Osheroff, Richardson, Lee (1972)
Osheroff, Gully, Richardson, Lee (1972)
The Nobel Prize in Physics 1996
"for their discovery of superfluidity in helium-3"

David M. Lee
Cornell (USA)

Douglas D. Osheroff
Stanford (USA)

Robert C. Richardson
Cornell (USA)
Phase diagram of Helium-3

P-T phase diagram

Dense, simple liquid

- isotropic
- short-range interactions
- extremely pure
- nuclear spin $S=1/2$

Solid (bcc)

Fermi liquid
Phase diagram of Helium-3

P-T phase diagram

Dense, simple liquid
- isotropic
- short-range interactions
- extremely pure
- nuclear spin $S=1/2$

Viscosity $\rightarrow$ zero

High viscosity (machine oil)

http://ltl.tkk.fi/research/theory/helium.html
Phase diagram of Helium-3

P-T-H phase diagram

“Very (ultra) low temperatures”: $T \ll T_{\text{boiling}} \sim 3\, \text{K}$
and $\ll T_{\text{backgr. rad.}} \sim 3\, \text{K}$
Superfluid phases of $^3$He

Experiment: Osheroff, Richardson, Lee, Wheatley, ...
Theory: Leggett, Wölfle, Mermin, ...

$\rightarrow$ $L=1$, $S=1$ ("p-wave, spin-triplet") in all 3 phases

Attraction due to spin fluctuations

Anderson, Brinkman (1973)

$\rightarrow$ anisotropy directions in every $^3$He Cooper pair

orbital part
spin part
NMR experiment on nuclear spins $I = \frac{1}{2} \hbar$  

$$\omega^2 - \omega_L^2 \propto \Delta^2(T)$$

$\omega_L$ Larmor frequency: $\omega_L = \gamma H$

$\omega$

3mT

$\omega_L$

superfluid

$T_{C,A}$

normal

$T$

Shift of $\omega_L$ ↔ spin-nonconserving interactions

$\rightarrow$ nuclear dipole interaction $g_D \sim 10^{-7} K \ll T_C$

... and a mystery!

Origin of frequency shift ?!

Osheroff et al. (1972)

Leggett (1973)
The superfluid phases of $^3$He
**B-phase**

All spin states $|\uparrow\uparrow\rangle$, $|\uparrow\downarrow + \downarrow\uparrow\rangle$, $|\downarrow\downarrow\rangle$ occur equally

$$\Delta(k) = \Delta_0$$

Balian, Werthamer (1963)
Vdovin (1963)

"(pseudo-) isotropic state" $\leftrightarrow$ s-wave superconductor

Weak-coupling theory: stable for all $T<T_c$
A-phase

Spin states \( |\uparrow\uparrow\rangle, |\downarrow\downarrow\rangle \) occur equally

\[ \Delta(\hat{k}) = \Delta_0 \sin(\hat{k}, \hat{l}) \]

\( \rightarrow \) strong gap anisotropy

Anderson, Morel (1961)

\[ \Delta(\hat{k}) = \Delta_0 \sin(\hat{k}, \hat{l}) \]

Cooper pair

\( \hat{l} \)

Orbital angular momentum

\[ \Delta(\hat{k}) = \Delta_0 \sin(\hat{k}, \hat{l}) \]

\[ \rightarrow \] Helped to understand unconventional pairing in

- heavy fermion superconductors (CeCu\(_2\)Si\(_2\), UPt\(_3\), ...)
- high-\( T_c \) (cuprate) superconductors

Strong-coupling effect
Volovik (1987)

$\hat{l}$

$E^2_k = v_F^2 (k - k_F)^2 + \Delta_0^2 \sin^2 (\hat{k}, \hat{l}) = g^{ij} p_i p_j$

Lorentz invariance

$g^{ij} = v_F^2 l_i l_j + \left( \frac{\Delta}{k_F} \right)^2 (\delta_{ij} - l_i l_j)$

$e = \begin{cases} +1 & \hat{k} \parallel + \hat{l} \quad \text{chirality "up"} \\ -1 & \hat{k} \parallel - \hat{l} \quad \text{chirality "down"} \end{cases}$

$A = k_F \hat{l}$

$p = k - eA$

$\hat{l}$

$\hat{l}$

$\hat{l}$

$\hat{l}$

$\hat{l}$

\[ E_{k}^{2} = v_{F}^{2}(k - k_{F})^{2} + \Delta_{0}^{2} \sin^{2}(\hat{k}, \hat{l}) = g_{ij}p_{i}p_{j} \]

Lorentz invariance

\[ e = \begin{cases} +1 & \hat{k} \parallel + \hat{l} \quad \text{chirality "up"} \\ -1 & \hat{k} \parallel - \hat{l} \quad \text{chirality "down"} \end{cases} \]

\[ g_{ij} = v_{F}^{2}l_{i}l_{j} + \left( \frac{\Delta}{k_{F}} \right)^{2}(\delta_{ij} - l_{i}l_{j}) \]

Massless, chiral leptons, e.g., neutrino \( E(p) = cp \)

Chiral (Adler) anomaly measured

Bevan et al. (1997)
A$_1$-phase

In finite magnetic field

Only spin state $|\uparrow\uparrow\rangle$

Long-range ordered magnetic liquid
Cooper pairing of Fermions vs. Bose-Einstein condensation

Conventional superconductors

Superfluid $^3$He: 
Cooper pair: “Quasi-boson“

High-$T_C$ superconductors

Superfluid $^4$He:
Tightly packed fermions (boson)

BCS

$\xi_0 \approx 10000 \, \text{Å}$

$\xi_0 \approx 150 \, \text{Å}$

$\xi_0 \approx 10 \, \text{Å}$

$\xi_0 \approx 1 \, \text{Å}$

BEC

Leggett (1980)

New insights from BEC of cold atoms
Broken Symmetries & Long-Range Order
Broken Symmetries & Long-Range Order

Normal $^3$He ↔ $^3$He-A, $^3$He-B: 2nd order phase transition

$T<T_c$: higher order, lower symmetry of ground state

I. Ferromagnet

$T>T_c$: SO(3) rotation symmetry in spin space spontaneously broken

$T<T_c$: average magnetization: $\langle M \rangle = 0$

Symmetry group: $SO(3)$

$T<T_c$: $\langle M \rangle \neq 0$ Order parameter

Symmetry group: $U(1) \subset SO(3)$
**Broken Symmetries & Long-Range Order**

Normal $^3$He ↔ $^3$He-A, $^3$He-B: 2nd order phase transition

$T<T_c$: higher order, lower symmetry of ground state

**II. Liquid crystal**

$T>T_c$: SO(3) rotation symmetry in real space spontaneously broken
Broken Symmetries & Long-Range Order

Normal $^3$He $\leftrightarrow$ $^3$He-A, $^3$He-B: 2\textsuperscript{nd} order phase transition

$T<T_c$: higher order, lower symmetry of ground state

III. Conventional superconductor

Pair amplitude $\langle c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger \rangle = 0$

Gauge transf. $c_{k\sigma}^\dagger \rightarrow c_{k\sigma}^\dagger e^{i\phi}$: gauge invariant

Symmetry group $U(1)$

$\Delta e^{i\phi}$ complex order parameter

not gauge invariant
Broken Symmetries & Long-Range Order

Normal $^3\text{He} \leftrightarrow ^3\text{He-}\text{A, }^3\text{He-}\text{B}$: 2\textsuperscript{nd} order phase transition

$T<T_c$: higher order, lower symmetry of ground state

III. Conventional superconductor

$T>T_c$: normal state

$T<T_c$: superfluid state

$T<T_c$: $U(1)$ "gauge symmetry" spontaneously broken
Broken symmetries in superfluid $^3$He

$L=1$, $S=1$ in all 3 phases

Cooper pair: \( \hat{l} \) orbital part, \( \hat{d} \) spin part

Quantum coherence in \( \{ \) phase (complex order parameter) 
\hspace{1cm} \text{anisotropy direction in real space} 
\hspace{1cm} \text{anisotropy direction in spin space} \} \hspace{1cm} \text{Superfluid, liquid crystal magnet}

Characterized by $2 \times (2L + 1) \times (2S + 1) = 18$ real numbers

3x3 order parameter matrix $A_{ij\mu}$

\( \text{SO}(3)_S \times \text{SO}(3)_L \times U(1)_\varphi \) symmetry spontaneously broken

\( \cong \text{SU}(2)_L \times \text{SU}(2)_R \times U(1)_Y \) for electroweak interactions

Leggett (1975)

Pati, Salam (1974)
Broken symmetries in superfluid $^3\text{He}$

Mineev (1980)
Bruder, Vollhardt (1986)

$3\text{He-A} \quad \text{SO}(3)_S \times \text{SO}(3)_L \times U(1)_\varphi$ symmetry broken

$U(1)_{S_z} \times U(1)_{L_z-\varphi}$ Unconventional pairing

Cooper pairs

Fixed absolute orientation
... solution of the NMR mystery
Superfluid $^3\text{He}$ - a quantum amplifier

Cooper pairs in $^3\text{He}$-A

Fixed absolute orientation

What fixes the relative orientation of $\hat{d}, \hat{l}$?

Interaction of nuclear dipoles ("spin-orbit coupling"):

Dipole-dipole coupling of $^3\text{He}$ nuclei: $g_D \sim 10^{-7} K \ll T_C$

Unimportant?!
Superfluid $^3$He - a quantum amplifier

Cooper pairs in $^3$He-A

- Long-range order in $\hat{d}, \hat{l}$
- $g_D \sim 10^{-7} K$: tiny, but lifts degeneracy of relative orientation

$\hat{d},\hat{l}$ locked in all Cooper pairs at a fixed angle

Quantum coherence

NMR frequency increases: $\omega^2 = (\gamma H)^2 + g_D \Delta^2(T)$  
Leggett (1973)

$\rightarrow$ Nuclear dipole interaction is macroscopically measurable
The Nobel Prize in Physics 2003
"for pioneering contributions to the theory of superconductors and superfluids"

Alexei A. Abrikosov
USA and Russia

Vitaly L. Ginzburg
Russia

Anthony J. Leggett
UK and USA
Order-parameter textures and topological defects
1) Walls $\rightarrow \hat{i}$

Orientation of the anisotropy directions $\hat{d}, \hat{l}$:

2) Magnetic field $\rightarrow \hat{d}$

$\Rightarrow$ Textures in $\hat{d}, \hat{l}$ $\leftrightarrow$ liquid crystals

Order-parameter textures in $^3$He-A

Order-parameter textures and topological defects in $^3$He-A

$D=2$: domain walls in $\hat{d}$ or $\hat{i}$

Cannot be removed by local surgery $\rightarrow$ topological defect
**Order-parameter textures and topological defects in $^3$He-A**

**D=1: Vortices**

Vortex formation by rotation

http://ltl.tkk.fi/research/theory/vortex.html

e.g., Mermin-Ho vortex (non-singular)

Thin film of $^3$He-A (chiral)

Skyrmion vortex  Volovik (2003), Sauls (2013)
Order-parameter textures and topological defects in $^3$He-A

$D=0$: Monopoles

"Boojum" in $\hat{l}$-texture of $^3$He-A

Defect formation by
- rotation
- geometric constraints
- rapid crossing through continuous phase transition
Big bang simulation in the low-temperature lab
Universality in continuous phase transitions

High symmetry, short-range order

\[ T > T_c \]

Phase transition

Broken symmetry, long-range order

\[ T = T_c \]

\[ T < T_c \]

Spins:
- paramagnetic
- ferromagnetic

Helium:
- normal liquid
- superfluid

Universe:
- Unified forces and fields
- elementary particles, fundamental interactions
- cosmic strings, etc. Kibble (1976)

Defects:
- domain walls
- vortices, etc.

nucleation of galaxies?
Rapid thermal quench through 2nd order phase transition

1. Local temperature $T \gg T_c$
   $\rightarrow$ Expansion + rapid cooling

2. Nucleation of independently ordered regions
   Clustering of ordered regions
   $\rightarrow$ Defects

3. Defects overlap

4. $T < T_c$ : Vortex tangle

Estimate of density of defects: Zurek (1985)

"Kibble-Zurek mechanism" of defect formation: How to test?
Big-bang simulation in the low-temperature laboratory

Grenoble: Bäuerle et al. (1996), Helsinki: Ruutu et al. (1996)

Measured vortex tangle density:
Quantitative support for Kibble-Zurek mechanism
Current research on superfluid $^3$He

1. **Influence of disorder on superfluidity**

2. **Quantum Turbulence** (=Turbulence in the absence of viscous dissipation)
   Origin of dissipation in the absence of friction?
   Test systems: $^4$He-II, $^3$He-B

3. **Majorana fermions** (e.g., zero-energy Andreev bound states at surfaces in $^3$He-B)
The Superfluid Phases of Helium 3
D. Vollhardt and P. Wölfle
(Taylor & Francis, 1990), 656 pages
Reprinted by Dover Publications (2013)
Superfluid Helium-3:

- **Anisotropic superfluid (p-wave, spin-triplet pairing)**
  - Cooper pairs with internal structure
  - 3 different bulk phases with many novel properties

- **Large symmetry group broken**
  - Close connections with particle physics
  - Zoo of topological defects

Conclusion