Rudolf Peierls Centre for Theoretical Physics & Mansfield College

UNIVERSITY OF OXFORD

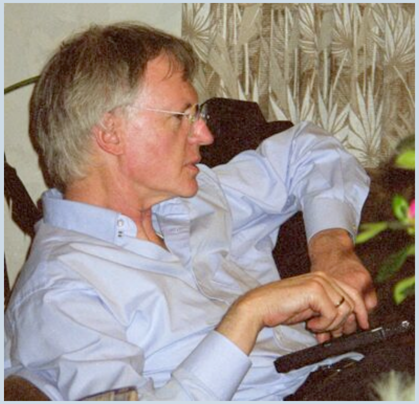
Thermodynamics & Brownian motion in special relativity

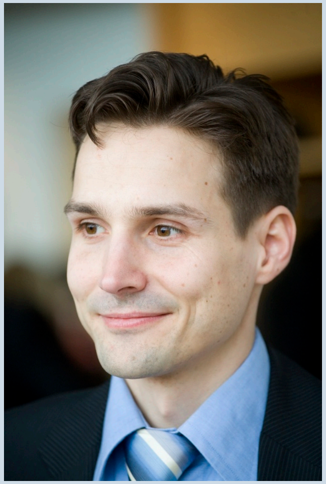
Jörn Dunkel

DPG Conference, Bonn, March 2010

Many thanks to

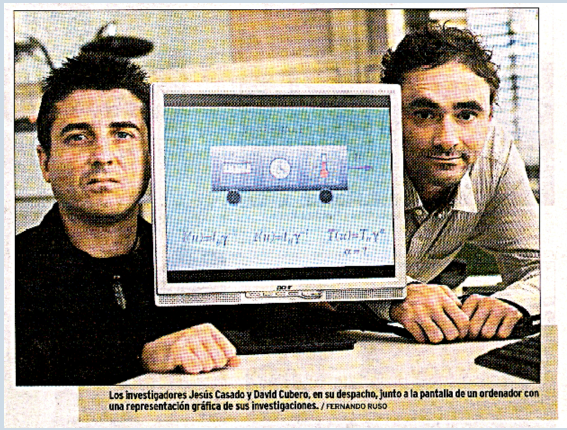
Peter Talkner (Augsburg)





Stefan Hilbert
(MPA / Bonn)

Peter Hänggi
(Augsburg)

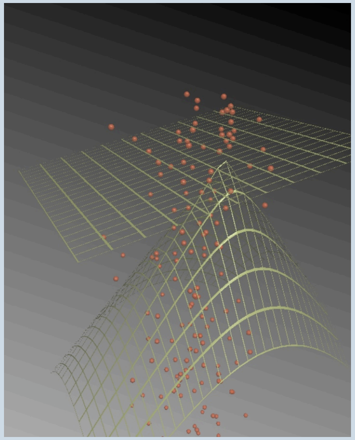


Jesus Casado & David Cubero (Sevilla)

Stefan Weber (Cornell)

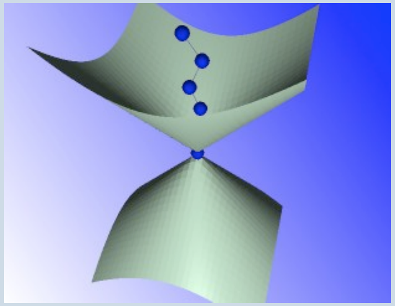
Outline

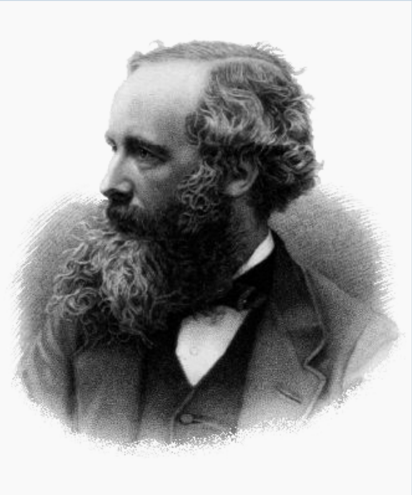
- brief history
- thermostatistical equilibrium, time parameters & entropy
- definition(s) of non-local thermodynamic quantities
- relativistic Brownian motion



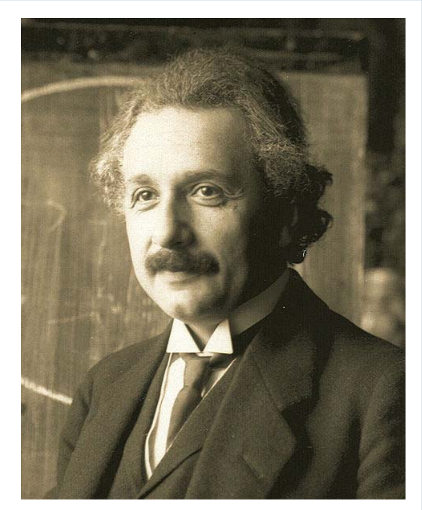
1905: (Special) Relativity

- ✓ unifies Maxwell's theory & mechanics
- ✓ inertial observers equivalent
- ✓ *c=const* for inertial observers
- ✓ *v<c* for massive particles
- ✓ Lorentz transformation
- ✓ energy-mass-momentum relation





$$E^2 = m^2 c^4 + p^2 c^2$$



- ✓ length contraction
- ✓ time dilatation

➡ What about thermodynamic quantities?

1908. № 6.

ANNALEN DER PHYSIK.

VIERTE FOLGE. BAND 26.

1. *Zur Dynamik bewegter Systeme;*

von M. Planck.

Aus den Sitzungsber. d. k. preuß. Akad. der Wissensch. vom 13. Juni 1907, mitgeteilt vom Verf.

$$p = p', \quad \frac{V}{\sqrt{1-\frac{q^2}{c^2}}} = \frac{V'}{\sqrt{1-\frac{q'^2}{c^2}}} \quad \text{und} \quad \frac{T}{\sqrt{1-\frac{q^2}{c^2}}} = \frac{T'}{\sqrt{1-\frac{q'^2}{c^2}}},$$

oder:

$$\frac{V'}{V} = \frac{T'}{T} = \sqrt{\frac{c^2-q'^2}{c^2-q^2}}, \quad p' = p, \quad S' = S \tag{17}$$

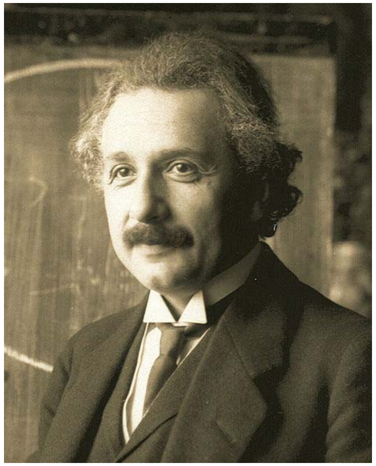
als *allgemein gültige Beziehung zwischen den gestrichenen und den ungestrichenen Variabeln.*

moving bodies appear cooler

Über das Relativitätsprinzip und die aus demselben gezogenen Folgerungen.

Von A. Einstein.

(Eingegangen 4. Dezember 1907.)



IV. Zur Mechanik und Thermodynamik der Systeme.

Einstein, Relativitätsprinzip u. die aus demselben gezog. Folgerungen. 453

$$\frac{T}{T_0} = \sqrt{1 - \frac{q^2}{c^2}}. \tag{27}$$

Die Temperatur eines bewegten Systems ist also in bezug auf ein relativ zu ihm bewegtes Bezugssystem stets kleiner als in bezug auf ein relativ zu ihm ruhendes Bezugssystem.

THE MATHEMATICAL THEORY

OF

RELATIVITY

BY

A. S. EDDINGTON, M.A., M.Sc., F.R.S.

PLUMIAN PROFESSOR OF ASTRONOMY AND EXPERIMENTAL
PHILOSOPHY IN THE UNIVERSITY OF CAMBRIDGE

CAMBRIDGE
AT THE UNIVERSITY PRESS
1923

$$\beta = (1 - u^2/c^2)^{-\frac{1}{2}},$$

$$T' = \beta T \qquad \text{..........................(14·4).}$$

In general it is unprofitable to apply the Lorentz transformation to the *constitutive equations* of a material medium and to coefficients occurring in them (permeability, specific inductive capacity, elasticity, velocity of sound). Such equations naturally take a simpler and more significant form for axes moving with the matter. The transformation to moving axes introduces great complications without any evident advantages, and is of little interest except as an analytical exercise.

moving bodies
appear hotter

Zeitschrift für Physik 175, 70—104 (1963)

Aus dem Institut für Theoretische Physik der Universität Würzburg

Lorentz-Transformation der Wärme und der Temperatur

Von
H. Ott*

Mit 1 Figur im Text

(Eingegangen am 11. Januar 1963)

In die übliche Herleitung der Lorentz-Transformation von Wärme und Temperatur haben sich zwei grundsätzliche Fehler eingeschlichen, nämlich

1. die Verwendung einer unvollkommenen Bewegungsgleichung für Körper mit veränderlicher Ruhemasse,

2. die irrtümliche Ansicht, die Leistung $(\mathfrak{L}\mathfrak{v})$ der an einem bewegten Leiter angreifenden Lorentz-Kraft $\mathfrak{L}$ trage nur zur Erhöhung der mechanischen Energie des Leiters und nichts zur Stromwärme bei.

Nach Richtigstellung ergeben sich für eine Wärmemenge dQ und für die Temperatur die Transformationen

$$dQ = \frac{dQ^0}{\sqrt{1-\beta^2}}, \qquad T = \frac{T^0}{\sqrt{1-\beta^2}}$$

(dQ^0, T^0 beziehen sich auf das Eigensystem), wenn man den 1. und 2. Hauptsatz in der üblichen Form zugrunde legt.

photo provided by
Oscar Bandtlow (QMUL)

No. 5062 NOVEMBER 5, 1966 NATURE 571

DOES A MOVING BODY APPEAR COOL?

By PROF. P. T. LANDSBERG
Department of Applied Mathematics and Mathematical Physics, University College, Cathays Park, Cardiff

We wish to ensure that the temperature is invariant, so that the equilibrium condition (8) becomes $T_1 = T_2$ even in a relativistic theory.

Relativistic Thermodynamics of Moving Systems

N. G. van Kampen
Physics Department, Howard University, Washington, D.C.
(Received 10 May 1968)

The second law (5)

$$TdS = \text{đ}Q$$

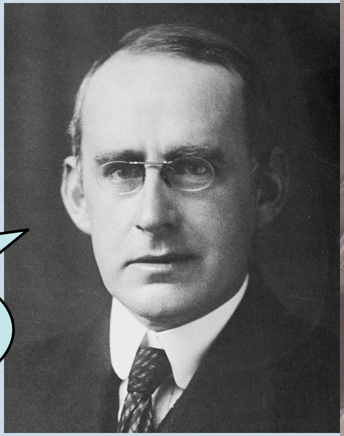
remains valid when T is defined as equal to the temperature T^0 in the rest frame.

Thus we have obtained a different formulation of relativistic thermodynamics, in which both $\text{đ}Q$ and T are scalars.[13] However, the first law is now replaced with the covariant equation

$$\text{đ}Q_\mu = dU_\mu + \text{đ}A_\mu$$

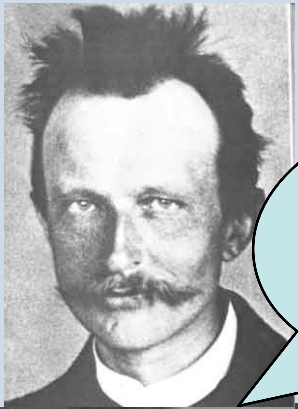
$$u_\mu \text{đ}Q_\mu = \text{đ}Q^0$$

"Temperature" problem in RTD?




1923/1963

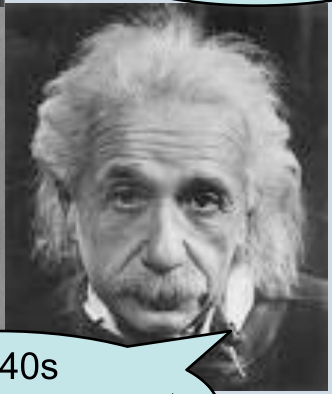
.. hotter!



1907/08

moving bodies appear cooler

$$T'(w) = T\,(1-w^2)^{\alpha/2} \qquad \alpha = \begin{cases} +1 & \text{Planck, Einstein} \\ 0 & \text{Landsberg, van Kampen} \\ -1 & \text{Ott} \end{cases}$$

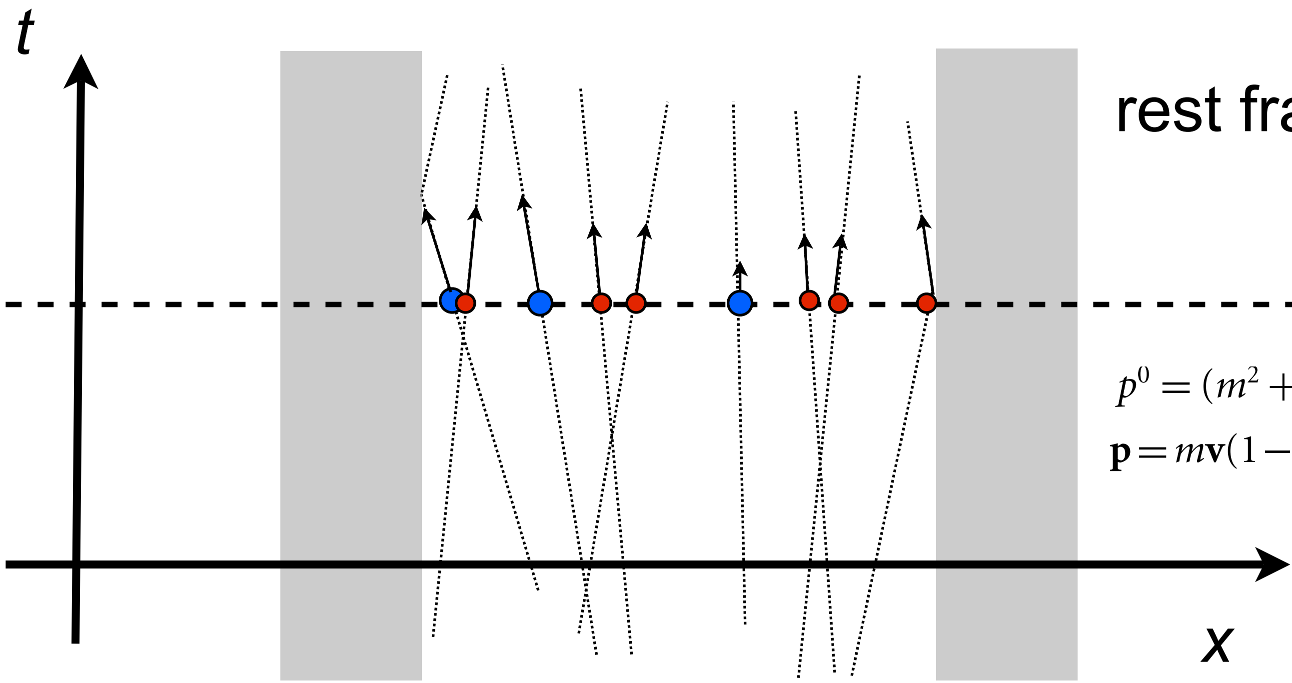
1940s

maybe ... not

T=T' 1966-69


CK Yuen, Amer. J. Phys. 38:246 (1970)

Warm-up: 1D two-component gas model



$$p^0 = (m^2 + \mathbf{p}^2)^{1/2}$$
$$\mathbf{p} = m\mathbf{v}(1-\mathbf{v}^2)^{-1/2}$$

$$\epsilon(m_A, p_A) + \epsilon(m_B, p_B) = \epsilon(m_A, \tilde{p}_A) + \epsilon(m_B, \tilde{p}_B),$$
$$p_A + p_B = \tilde{p}_A + \tilde{p}_B,$$

+ BC!

PRL 99: 170601(2007)

Measure one-particle phase space PDFs ... but how?

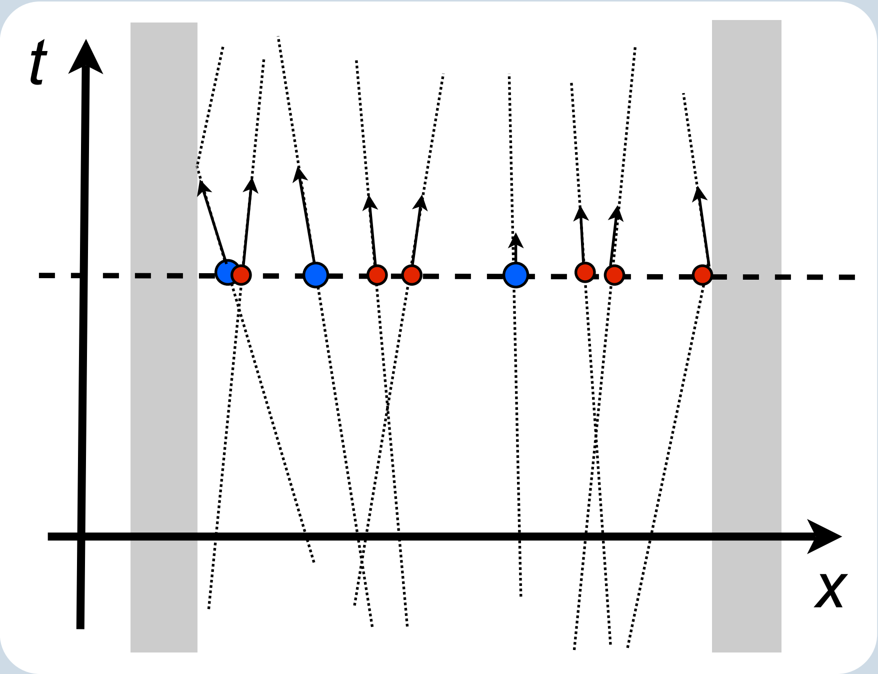
a. histogram over all particles at

- constant t
- constant τ

$$d\tau(t) = (M/P^0)dt$$

b. time average over single particle

- constant Δt
- constant $\Delta\tau$



PRL 99: 170601(2007), EPL 87: 30005 (2009)

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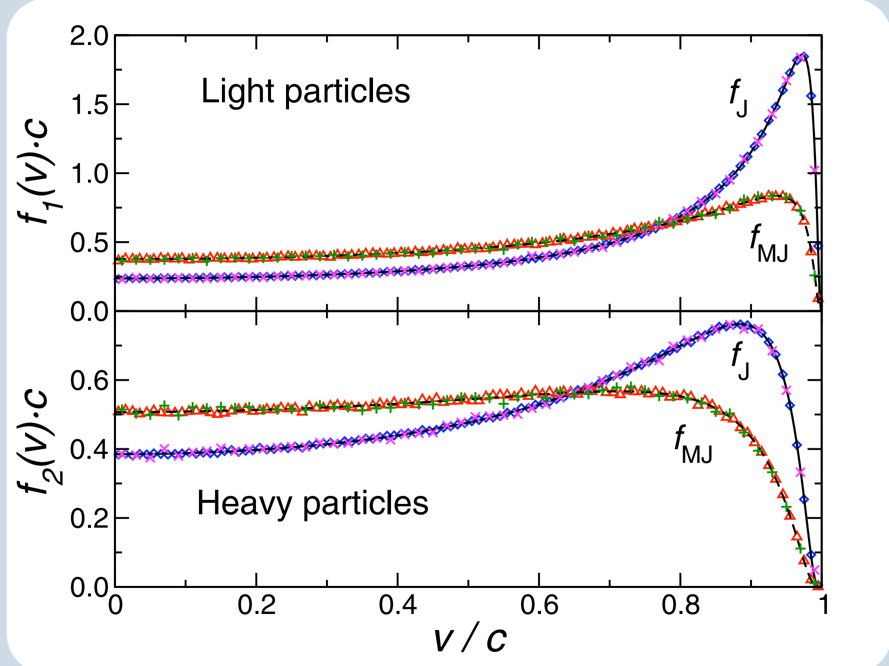


$$f(t,\mathbf{x},\mathbf{p}) = \varphi(\mathbf{x},\mathbf{p}) = \varrho(\mathbf{x})\,\phi_J(\mathbf{p})$$

$$\varrho(\mathbf{x}) = \begin{cases} V^{-1}, & \text{if } \mathbf{x}\in\mathbb{V} \\ 0, & \text{if } \mathbf{x}\notin\mathbb{V} \end{cases}$$

$$\phi_J \propto \exp(-\beta p^0)$$

$$\phi_{MJ} \propto \frac{\exp(-\beta p^0)}{p^0}$$

same β !

Time parameters and (relative) entropy

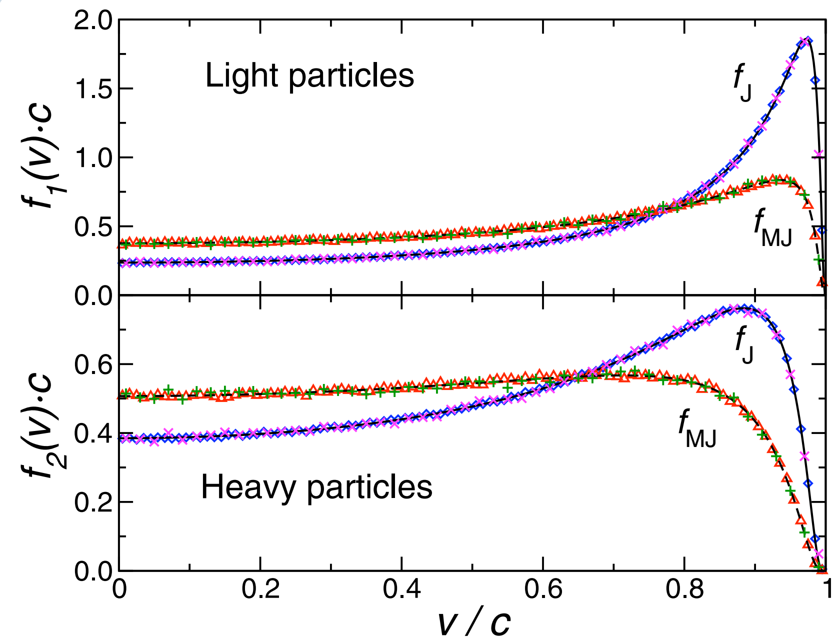
observer time *vs.* proper time

$$f_t(\boldsymbol{v}) = \frac{1}{t}\int_0^t dt'\,\delta[\boldsymbol{v}-\boldsymbol{V}(t')],$$

$$\hat{f}_\tau(\boldsymbol{v}) = \frac{1}{\tau}\int_0^\tau d\tau'\,\delta[\boldsymbol{v}-\hat{\boldsymbol{V}}(\tau')]$$

maximum relative entropy principle

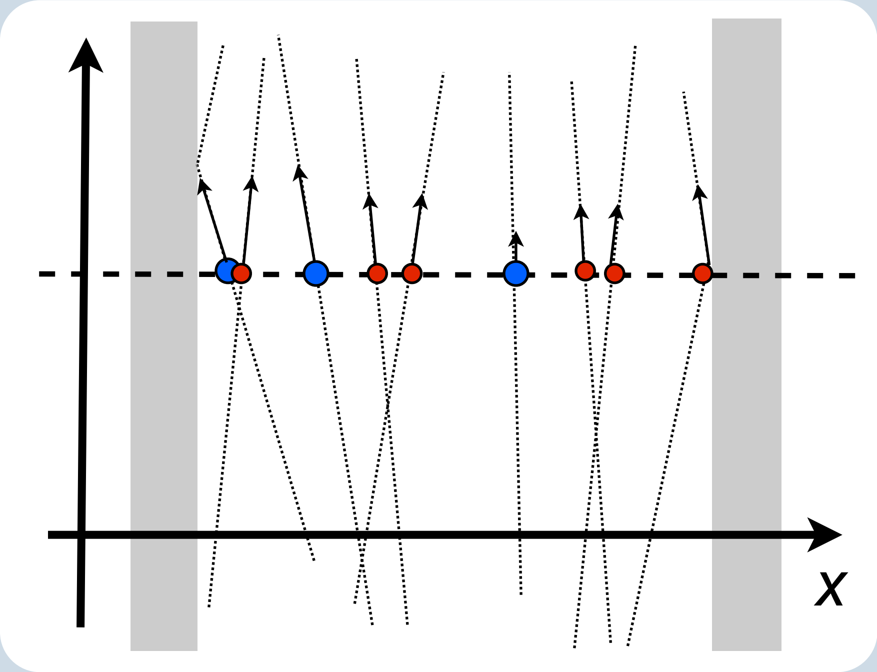
$$\begin{aligned} S[\phi|\rho] &= \int \mathrm{d}^d p\,\phi(\boldsymbol{p})\ln\left[\frac{\phi(\boldsymbol{p})}{\rho(\boldsymbol{p})}\right] \\ \epsilon_\rho &= \int \mathrm{d}^d p\,\phi(\boldsymbol{p})\,p^0 \\ 1 &= \int \mathrm{d}^d p\,\phi(\boldsymbol{p}) \end{aligned}$$



$$\begin{aligned} \rho = \rho_0 : &\quad \phi_\mathrm{J} \propto \exp(-\beta p^0) &\Leftrightarrow&\quad f(t\to\infty) \\ \rho \propto \frac{1}{p^0} : &\quad \phi_\mathrm{MJ} \propto \frac{\exp(-\beta p^0)}{p^0} &\Leftrightarrow&\quad \hat{f}(\tau\to\infty) \end{aligned}$$

EPL 87: 30005 (2009)

Summary (part 1)



Simple gas model

- ✓ thermostatistical equilibrium (in rest frame)
- ✓ "ergodicity" for different time parameters
- ✓ Hamiltonian system ⇒ microcanonical equipartition theorem
- ✓ relative entropy principle: (modified) Juettner distribution
- ✓ time parameter ⇔ symmetry of reference measure

Part 2: Relativistic thermodynamics

What “is” thermodynamics?

“Good” *vs* ”bad” starting points in relativistic thermodynamics?

Origin of different temperature transformation laws?

How can (or should) one define thermodynamic observables in relativity?

What “is” thermodynamics?

... attempt to efficiently describe an extended physical system by means of a few macroscopic control parameters

What "is" thermodynamics?

... attempt to efficiently describe an extended physical system by means of a few macroscopic control parameters

... more precisely

- mathematical theory of differential forms $\mathrm{d}\mathcal{S} = a_i \mathrm{d}\mathcal{Z}_i$
- theory of symmetry (breaking)
  - Energy E ⇔ time translation invariance
  - Volume V ⇔ breaking of spatial translation invariance
  - Magnetic field B ⇔ breaking of spatial isotropy
- non-local description

Herbert B Callen
(1920-1993)

Starting points in RTD?

"BAD" macroscopic variables $\mathcal{N},\ \mathcal{U}^0,\ \boldsymbol{\mathcal{U}},\ \ldots$

- ambiguous definition
- non-local character not explicit

"GOOD" mesoscopic tensor densities $j^\mu(t,\boldsymbol{x}),\ \theta^{\mu\nu}(t,\boldsymbol{x}),\ \ldots$

- uniquely defined, e.g., can be derived from Lagrangians
- encode symmetries $\Leftrightarrow$ conserved Noether currents

Example: Relativistic gas (lab frame)

$$p^0 = (m^2 + \mathbf{p}^2)^{1/2}$$
$$\mathbf{p} = m\mathbf{v}(1 - \mathbf{v}^2)^{-1/2}$$

$$f(t, \mathbf{x}, \mathbf{p}) = \varphi(\mathbf{x}, \mathbf{p}) = \varrho(\mathbf{x})\, \phi_J(\mathbf{p})$$

$$\phi_J(\mathbf{p}) = Z^{-1} \exp(-\beta p^0), \qquad \beta > 0$$

$$\varrho(\mathbf{x}) = \begin{cases} V^{-1}, & \text{if } \mathbf{x} \in \mathbb{V} \\ 0, & \text{if } \mathbf{x} \notin \mathbb{V} \end{cases}$$

Tensor densities

$$j^\mu(t, \mathbf{x}) = N \int \frac{\mathrm{d}^3 p}{p^0} f\, p^\mu$$

$$\theta^{\mu\nu}(t, \mathbf{x}) = N \int \frac{\mathrm{d}^3 p}{p^0} f\, p^\mu p^\nu$$

$$s^\mu(t, \mathbf{x}) = -N \int \frac{\mathrm{d}^3 p}{p^0} p^\mu f \ln(h^3 f)$$

Nature Physics 5: 741 (2009)

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$$j^\mu(t, \mathbf{x}) = N \int \frac{d^3p}{p^0} f\, p^\mu \quad = N\varrho \begin{cases} 1, & \mu = 0 \\ 0, & \mu > 0 \end{cases} \quad \Rightarrow \quad \partial_\nu j^\nu \equiv 0$$

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$$\theta^{\mu\nu}(t, \mathbf{x}) = N \int \frac{\mathrm{d}^3 p}{p^0}\, f\, p^\mu p^\nu \quad = N \varrho \begin{cases} \langle p^0 \rangle, & \mu = \nu = 0 \\ \beta^{-1}, & \mu = \nu = 1, 2, 3 \\ 0, & \mu \neq \nu \end{cases} \quad \boxed{\Rightarrow \quad \partial_\mu \theta^{\mu i} \not\equiv 0}$$

$$s^\mu(t, \mathbf{x}) = -N \int \frac{\mathrm{d}^3 p}{p^0}\, p^\mu f \ln(h^3 f)$$

$$\langle p^0 \rangle = 3\beta^{-1} + m K_1(\beta m)/K_2(\beta m)$$

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Thermodynamic variables

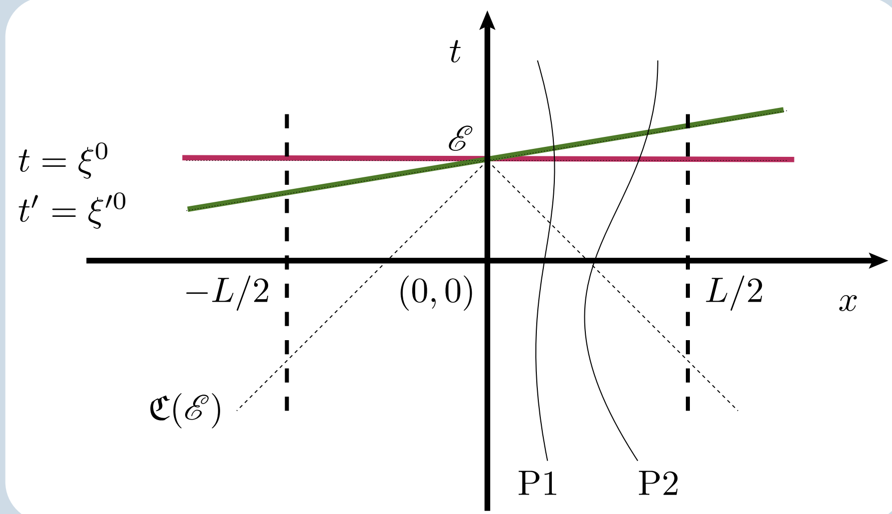
$$\partial_\mu j^\mu \equiv 0 \qquad \partial_\nu s^\nu \equiv 0 \qquad \boxed{\partial_\mu \theta^{\mu i} \not\equiv 0}$$

non-local TD variables $\Leftrightarrow$ surface integrals in space-time

$$\mathcal{N}[\mathfrak{H}] := \int_{\mathfrak{H}} \mathrm{d}\sigma_\nu \, j^\nu = \mathcal{N}[\mathfrak{H}'] \qquad \text{scalar}$$

$$\mathcal{S}[\mathfrak{H}] := \int_{\mathfrak{H}} \mathrm{d}\sigma_\nu \, s^\nu = \mathcal{S}[\mathfrak{H}'] \qquad \text{scalar}$$

$$\mathcal{U}^\nu[\mathfrak{H}] := \int_{\mathfrak{H}} \mathrm{d}\sigma_\mu \, \theta^{\mu\nu} \boxed{\neq \mathcal{U}^\nu[\mathfrak{H}']} \qquad \text{4-vector}$$



explicitly:

$$\mathcal{N}[\{t=\xi^0\}] = \int_{t=\xi^0} \mathrm{d}\sigma_0 \, j^0 = \int_{t=\xi^0} \mathrm{d}^3x \, j^0 = N$$

$$\mathcal{S}[\{t=\xi^0\}] = \int_{t=\xi^0} \mathrm{d}\sigma_0 \, s^0 = \int_{t=\xi^0} \mathrm{d}^3x \, s^0 = N \ln(VZ/h^3) + \beta N \langle p^0 \rangle$$

Thermodynamic variables

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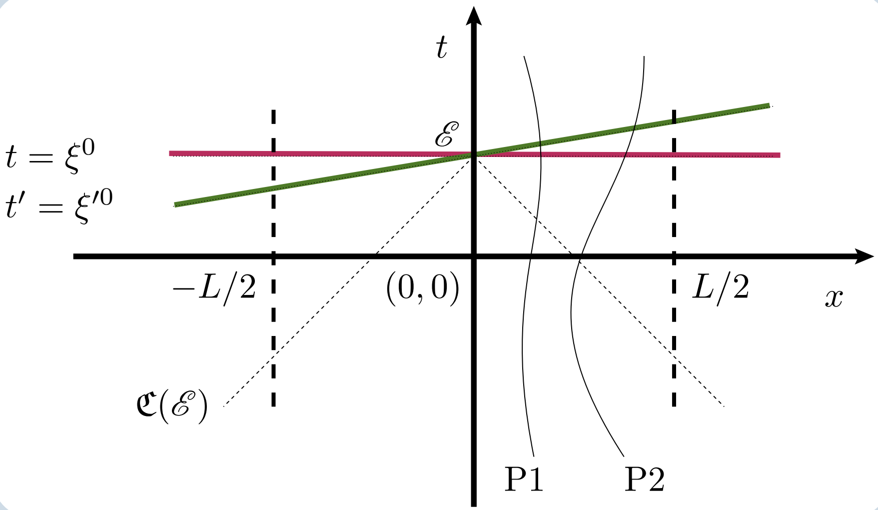
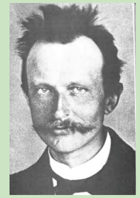


$$\mathcal{N}[\mathfrak{H}] = \mathcal{N}'[\mathfrak{H}] = \mathcal{N}'[\mathfrak{H}'] = \mathcal{N}[\mathfrak{H}'] \qquad \mathcal{S}[\mathfrak{H}] = \mathcal{S}'[\mathfrak{H}] = \mathcal{S}'[\mathfrak{H}'] = \mathcal{S}[\mathfrak{H}'] \qquad \Lambda^\nu{}_\mu \mathcal{U}^\mu[\mathfrak{H}] = \boxed{\mathcal{U}'^\nu[\mathfrak{H}] \neq \mathcal{U}'^\nu[\mathfrak{H}']} = \Lambda^\nu{}_\mu \mathcal{U}^\mu[\mathfrak{H}']$$

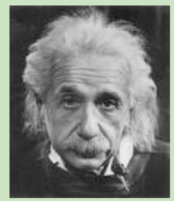
Heat, second law & temperature

Planck-Einstein



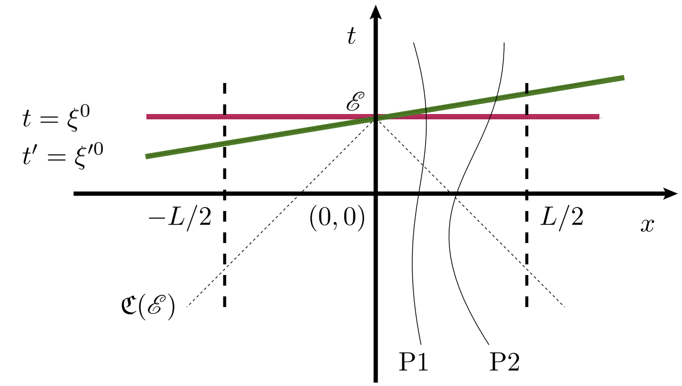
$$\text{đ}\mathcal{Q}'[\mathfrak{I}'] := \mathcal{T}'\mathrm{d}\mathcal{S}' := \mathrm{d}\mathcal{U}'^0[\mathfrak{I}'] - w'\mathrm{d}\mathcal{U}'^1[\mathfrak{I}'] + \mathcal{P}'\mathrm{d}V'$$

$$\gamma = (1 - w'^2)^{-1/2}$$



$$\mathcal{T}'\,\mathrm{d}\mathcal{S}' := \text{đ}\mathcal{Q}'^0[\mathfrak{I}'] = \gamma^{-1}\text{đ}\mathcal{Q}^0[\mathfrak{I}] = \gamma^{-1}\,\mathcal{T}\,\mathrm{d}\mathcal{S}$$

cooler



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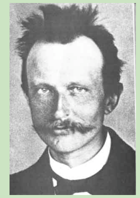
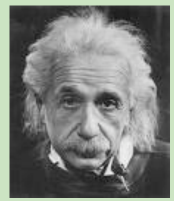


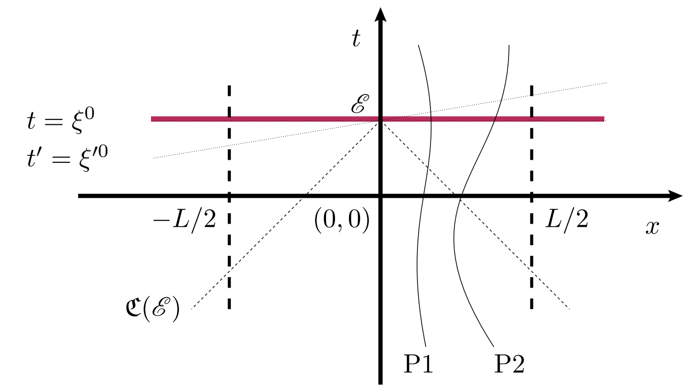
$$\text{đ}\mathcal{Q}'[\mathfrak{I}'] := \mathcal{T}'\mathrm{d}\mathcal{S}' := \mathrm{d}\mathcal{U}'^{0}[\mathfrak{I}'] - w'\mathrm{d}\mathcal{U}'^{1}[\mathfrak{I}'] + \mathcal{P}'\mathrm{d}V'$$

$$\gamma = (1 - w'^{2})^{-1/2}$$

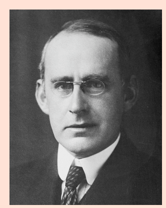
$$\mathcal{T}'\,\mathrm{d}\mathcal{S}' := \text{đ}\mathcal{Q}'^{0}[\mathfrak{I}'] = \gamma^{-1}\text{đ}\mathcal{Q}^{0}[\mathfrak{I}] = \gamma^{-1}\,\mathcal{T}\,\mathrm{d}\mathcal{S}$$



cooler



Eddington / Ott



$$\text{đ}\mathcal{Q}^{\mu}[\mathfrak{I}] := \mathrm{d}\mathcal{U}^{\mu}[\mathfrak{I}] - \text{đ}\mathcal{A}^{\mu}[\mathfrak{I}],$$

$$(\text{đ}\mathcal{A}^{\mu}[\mathfrak{I}]) := (-\mathcal{P}\mathrm{d}V, \mathbf{0}).$$

$$\mathcal{T}'\,\mathrm{d}\mathcal{S}' := \text{đ}\mathcal{Q}'^{0} = \gamma\text{đ}\mathcal{Q}^{0} = \gamma\,\mathcal{T}\,\mathrm{d}\mathcal{S},$$

hotter

Landsberg / van Kampen

$$\text{đ}\mathcal{Q}^{\mu}[\mathfrak{I}] := \mathrm{d}\mathcal{U}^{\mu}[\mathfrak{I}] - \text{đ}\mathcal{A}^{\mu}[\mathfrak{I}],$$

$$\text{đ}\mathcal{Q}' := -w'_{\mu}\text{đ}\mathcal{Q}'^{\mu} = -w_{\mu}\text{đ}\mathcal{Q}^{\mu} = \text{đ}\mathcal{Q} = \text{đ}\mathcal{Q}^{0}.$$

$$\mathcal{T}'\mathrm{d}\mathcal{S}' := \text{đ}\mathcal{Q}' = \text{đ}\mathcal{Q} = \mathcal{T}\mathrm{d}\mathcal{S},$$

T'=T

“Isochronous” RTD useful at all?

Nature Physics 5: 741 (2009)

shortcomings

(i) not feasible experimentally

(ii) extension to General Relativity problematic

"Isochronous" RTD useful at all?

Nature Physics 5: 741 (2009)

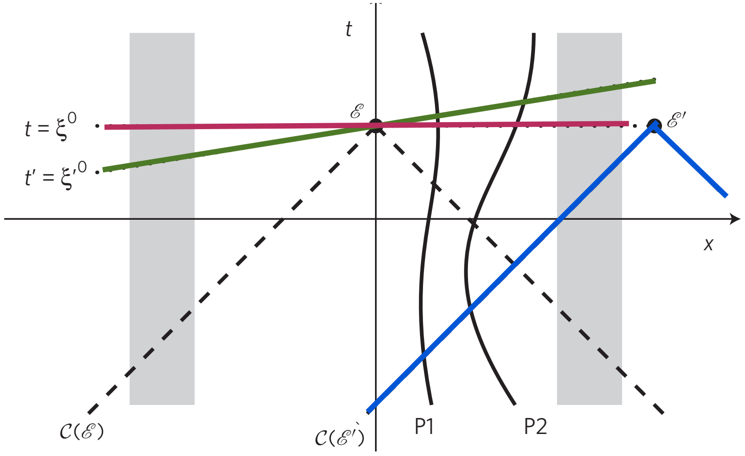
shortcomings

(i) not feasible experimentally
(ii) extension to General Relativity problematic

SOLUTION: replace isochronous hyperplanes by lightcones

Why?

✓ photographic measurements
✓ equivalent for moving observers
✓ GR extension
✓ nonrelativistic limit



“Photographic” thermodynamics

Nature Physics 5: 741 (2009)

light-cone averaging

$$\mathfrak{C}(\mathscr{E}) \;:=\; \{\, (t, \boldsymbol{x}) \mid t = \xi^0 - |\boldsymbol{x} - \boldsymbol{\xi}| \,\}$$

photographic state variables

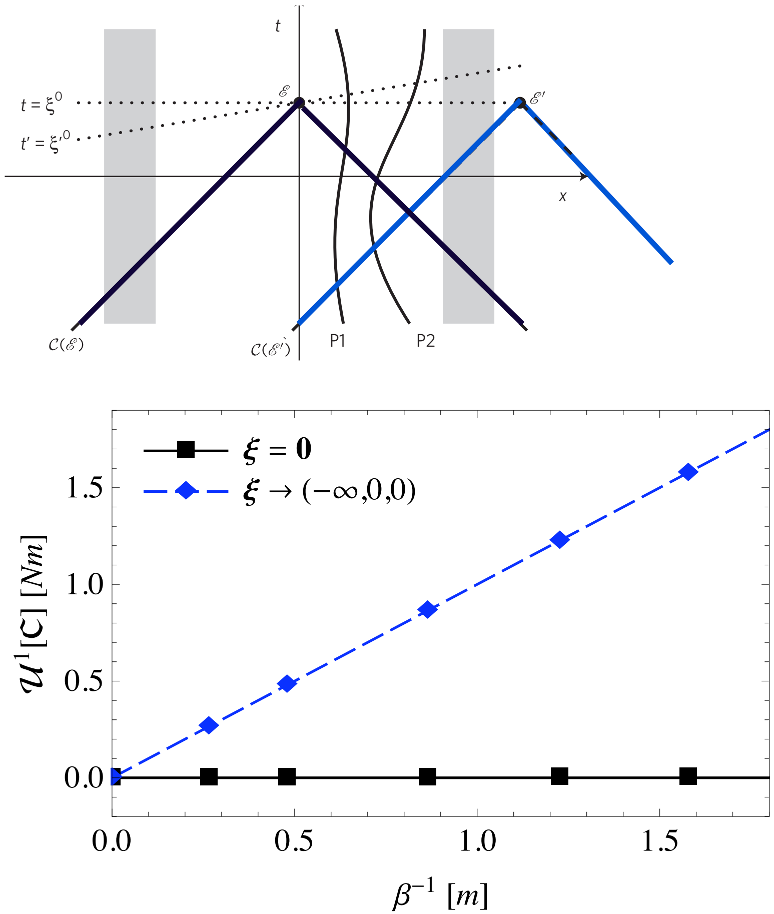
$$\mathfrak{U}^0[\mathfrak{C}] \;=\; N \, \langle p^0 \rangle$$

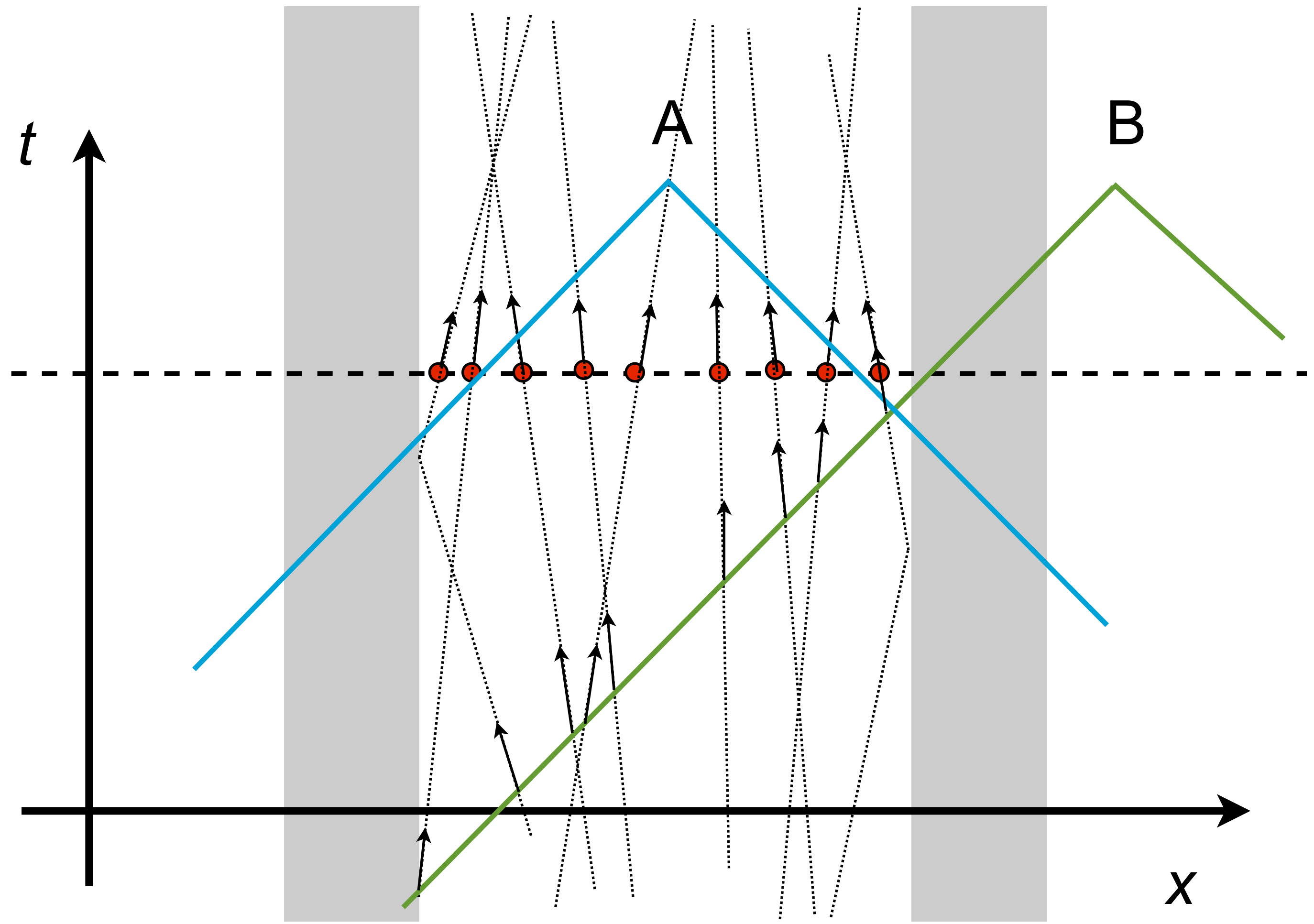
$$\mathfrak{U}^i[\mathfrak{C}] \;=\; \frac{N}{\beta} \int \mathrm{d}^3 x \, \frac{x^i - \xi^i}{|\boldsymbol{x} - \boldsymbol{\xi}|} \, \varrho(\boldsymbol{x})$$

distant observer

$$\mathfrak{U}^i[\mathfrak{C}] = -\frac{\xi^i}{|\boldsymbol{\xi}|} N k_{\mathrm{B}} \mathfrak{T}$$

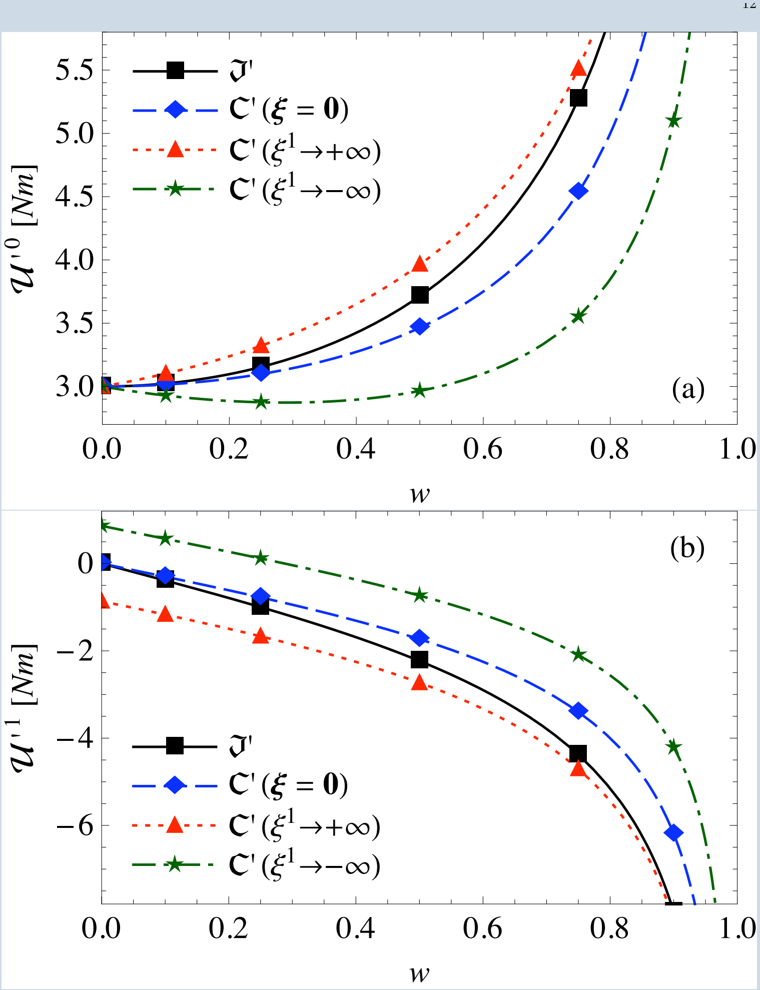
apparent drift !





t
A
B
x

Apparent drift & Lorentz transformations



Summary (part 2)

What “is” thermodynamics?

- ✓ non-local description in terms of symmetry (breaking) parameters

“Good” starting point in relativistic thermodynamics?

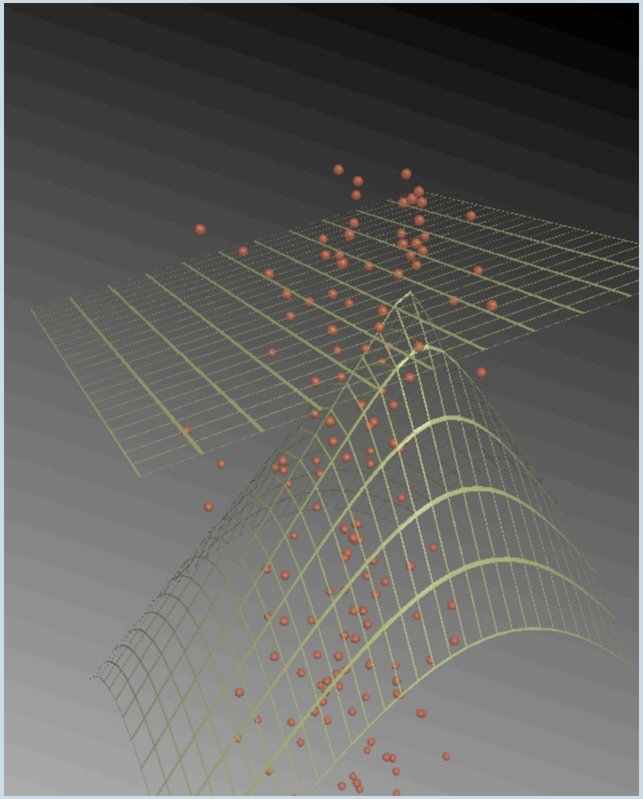
- ✓ (non-)conserved tensor densities, Noether currents

Origin of different temperature transformation laws?

- ✓ choice of space-time hyperplanes
- ✓ definition of heat, formulation of 1st/2nd law

How should one define thermodynamic observables in special and general relativity?

- ✓ invariant manifolds, lightcone integrals



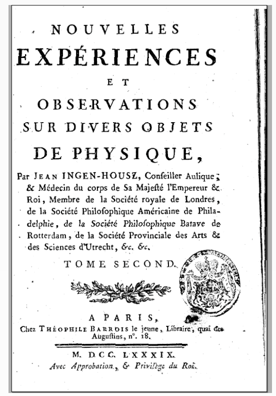
Observable consequence: temperature-induced apparent drift

PRL 99: 170601(2007), Physics Reports 471:1 (2009), EPL 87: 30005 (2009), Nature Physics 5: 741 (2009)

Part 3: Relativistic Brownian motion

“Brownian” motion

Jan Ingen-Housz (1730-1799)



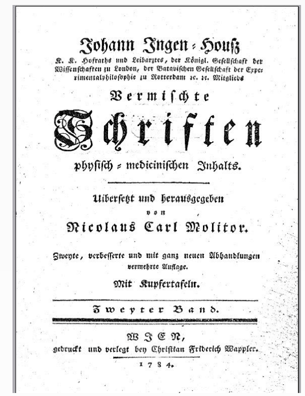
NOUVELLES
EXPÉRIENCES
ET
OBSERVATIONS
SUR DIVERS OBJETS
DE PHYSIQUE,

Par JEAN INGEN-HOUSZ, Conſeiller Aulique; & Médecin du corps de Sa Majeſté l'Empereur & Roi, Membre de la Société royale de Londres, de la Société Philoſophique Américaine de Philadelphie, de la *Société Philoſophique* Batave de Rotterdam, de la Société Provinciale des Arts & des Sciences d'Utrecht, *&c. &c.*

TOME SECOND.

A PARIS,
Chez THÉOPHILE BARROIS le jeune, Libraire, quai des Auguſtins, n°. 18.

M. DCC. LXXXIX.
Avec Approbation, & Privilège du Roi.



Johann Ingen=Houſz
K. K. Hofraths und Leibarztes, der Königl. Geſellſchaft der Wiſſenſchaften zu London, der Bataviſchen Geſellſchaft der Experimentalphiloſophie zu Rotterdam ꝛc. ꝛc. Mitglieds

Vermiſchte
Schriften
phyſiſch = mediciniſchen Inhalts.

Uiberſetzt und herausgegeben
von
Nicolaus Carl Molitor.

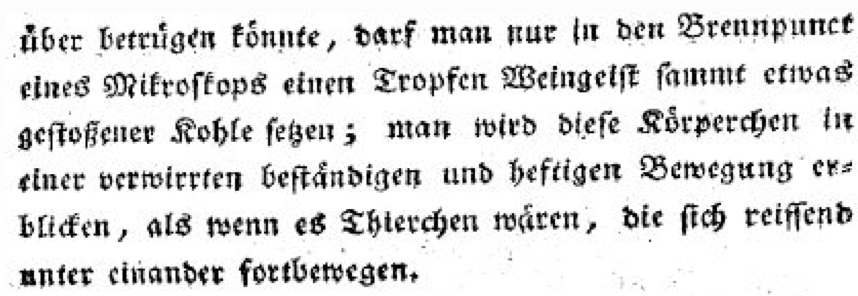
Zweyte, verbeſſerte und mit ganz neuen Abhandlungen vermehrte Auflage.

Mit Kupfertafeln.

Zweyter Band.

WIEN,
gedruckt und verlegt bey Chriſtian Friderich Wappler.
1784.

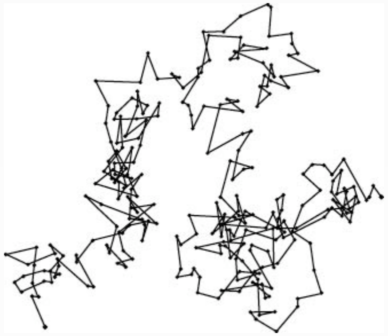
1784/1785:



über betrügen könnte, darf man nur in den Brennpunct eines Mikroſkops einen Tropfen Weingeiſt ſammt etwas geſtoßener Kohle ſetzen; man wird dieſe Körperchen in einer verwirrten beſtändigen und heftigen Bewegung er= blicken, als wenn es Thierchen wären, die ſich reiſſend unter einander fortbewegen.

`http://www.physik.uni-augsburg.de/theo1/hanggi/History/BM-History.html`

Robert Brown (1773-1858)

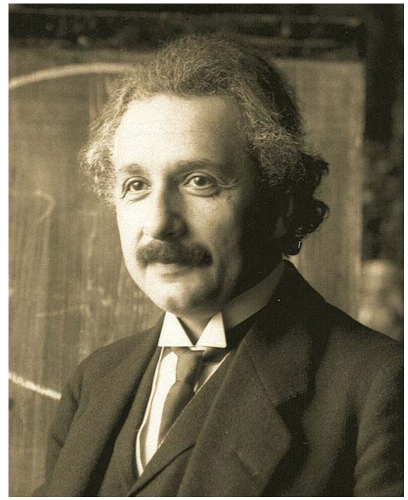


1827: irregular motion of pollen in fluid

Non-relativistic Brownian motion theory

W. Sutherland (1858-1911)	A. Einstein (1879-1955)	M. Smoluchowski (1872-1917)

Source: www.theage.com.au



Source: wikipedia.org

Source: wikipedia.org

$$\langle x^2(t) \rangle = 2Dt$$

$$D = \frac{RT}{6\pi\eta a C}$$

Phil. Mag. **9**, 781 (1905)

$$D = \frac{RT}{N}\frac{1}{6\pi k P}$$

Ann. Phys. **17**, 549 (1905)

$$D = \frac{32}{243}\frac{mc^2}{\pi\mu R}$$

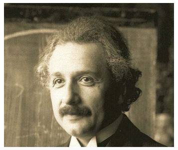
Ann. Phys. **21**, 756 (1906)

Theorie der Brownschen Bewegung

W. Sutherland (1858-1911)

Source: www.theage.com.au

A. Einstein (1879-1955)

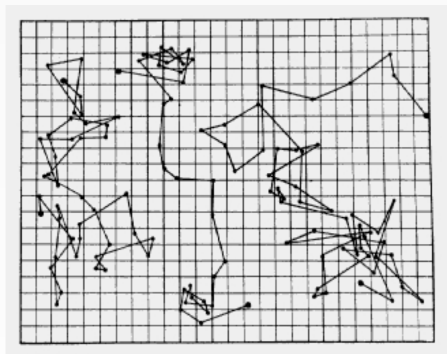


M. Smoluchowski (1872-1917)

$$D = \frac{RT}{6\pi\eta a C}$$

Phil. Mag. **9**, 781 (190

Jean Baptiste Perrin (1870-1942, Nobel 1926)



Mouvement brownien et réalité moléculaire, Annales de chimie et de physique VIII 18, 5-114 (1909)

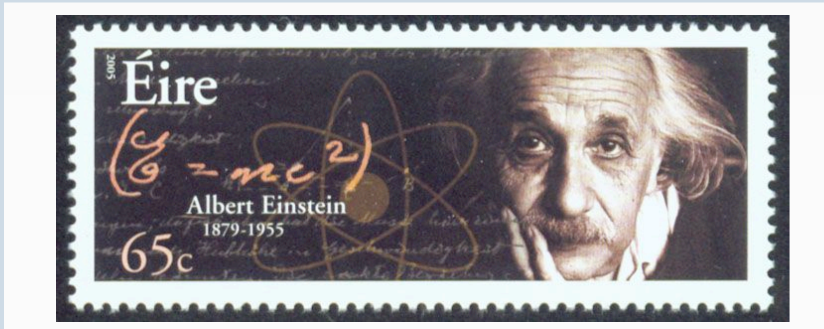
Les Atomes, Paris, Alcan (1913)

- colloidal particles of radius 0.53μm
- successive positions every 30 seconds joined by straight line segments
- mesh size is 3.2μm

atomistic structure of matter

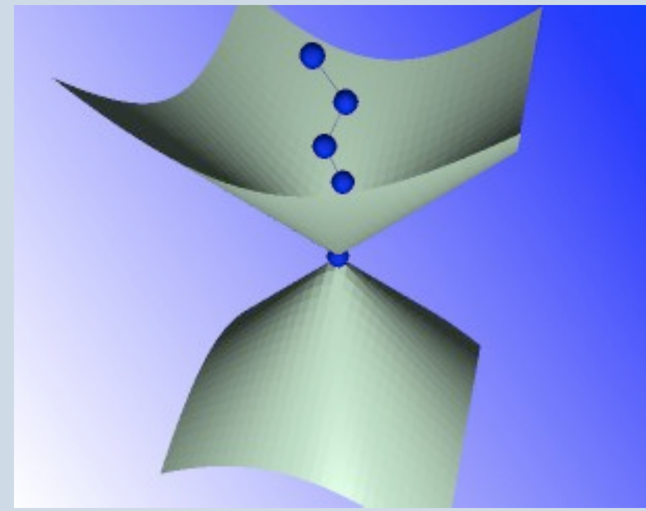
Relativistic diffusion processes

1905:



$v < c$!

- conceptually interesting
- random motion of particles in hot plasmas, heavy ion collision experiments, astrophysics, ...



Lopuszanski, Acta Phys. Polon. **12**: 87 (1953)
Schay, PhD thesis (Princeton, 1961)
Hakim, J. Math. Phys. **6**(10): 1482 (1965)
Dudley, Ark. Mat. Astron. Fis. **6**: 241 (1965)

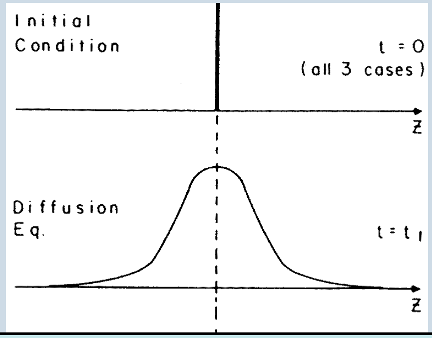
Debbasch et al., J. Stat. Phys. **88**: 945 (1997)

Physics Reports 471 (1): 1 (2009)

Diffusion: space-time only

classical diffusion (Markovian)

$$p(t, x|t_0, x_0) = \left[\frac{1}{4\pi\, \mathcal{D}(t - t_0)}\right]^{1/2} \exp\left[-\frac{(x - x_0)^2}{4\mathcal{D}(t - t_0)}\right].$$



$$\boxed{\frac{\partial}{\partial t}\varrho = \mathcal{D}\, \nabla^2 \varrho,}$$

Conflict with Einstein's postulate *v* < *c* !

PRD ***75***:*043001 (2007)*

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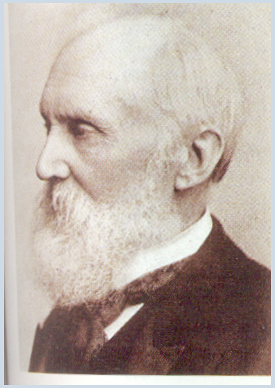
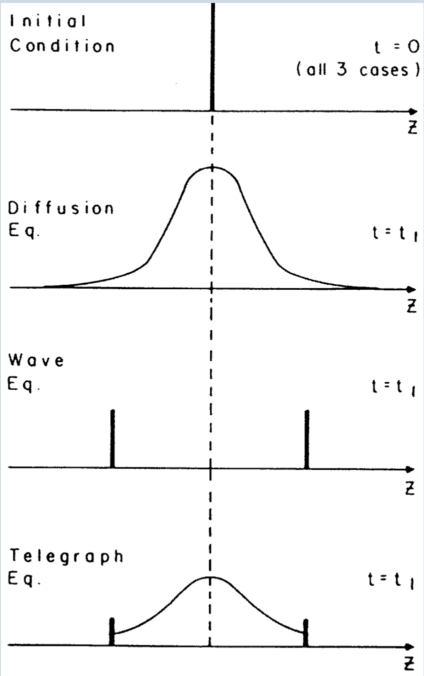
telegraph equation (non-Markovian)

$$\boxed{\tau_v \frac{\partial^2}{\partial t^2}\varrho + \frac{\partial}{\partial t}\varrho = \mathcal{D}\,\nabla^2\varrho,}$$





J Masoliver & G H Weiss
*Eur J Phys **17**:190 (1996)*

*PRD **75**:043001 (2007)*

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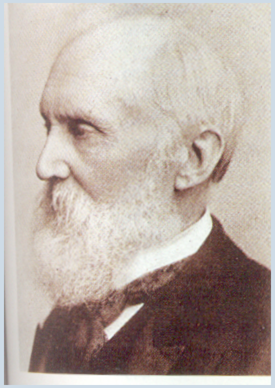
telegraph equation (non-Markovian)

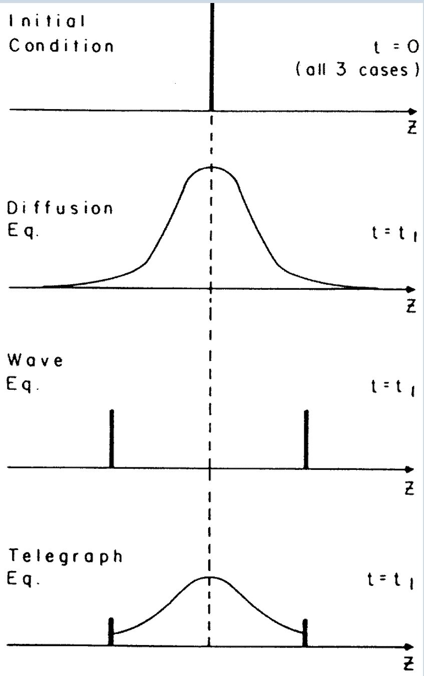
$$\tau_v \frac{\partial^2}{\partial t^2}\varrho + \frac{\partial}{\partial t}\varrho = \mathcal{D}\,\nabla^2\varrho,$$

alternative approach

$$\boxed{p(\bar{x}|\bar{x}_0) \propto \int_{a_-(\bar{x}|\bar{x}_0)}^{a_+(\bar{x}|\bar{x}_0)} \mathrm{d}a\ \exp\left(-\frac{a}{2\mathcal{D}}\right)}$$

PRD **75**:043001 (2007)





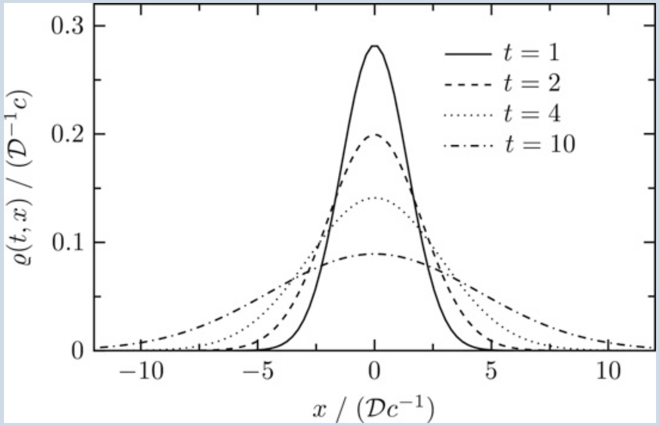
J Masoliver & G H Weiss
Eur J Phys ***17****:190 (1996)*

Diffusion propagator

*PRD **75**:043001 (2007)*

$$a(\bar{x}|\bar{x}_0) = \frac{1}{2}\int_{t_0}^{t} \mathrm{d}t'\ v(t')^2,$$

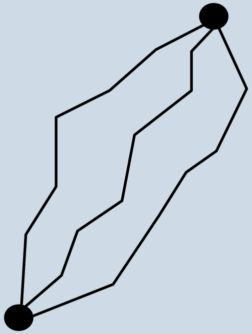
$$p(\bar{x}|\bar{x}_0) \propto \int_{a_-(\bar{x}|\bar{x}_0)}^{a_+(\bar{x}|\bar{x}_0)} \mathrm{d}a\ \exp\left(-\frac{a}{2\mathcal{D}}\right)$$



$$a_+ = +\infty$$

$$a_-(\bar{x}|\bar{x}_0) = \frac{(x - x_0)^2}{2(t - t_0)}$$

$\bar{x} = (t, x)$



$\bar{x}_0 = (t_0, x_0)$

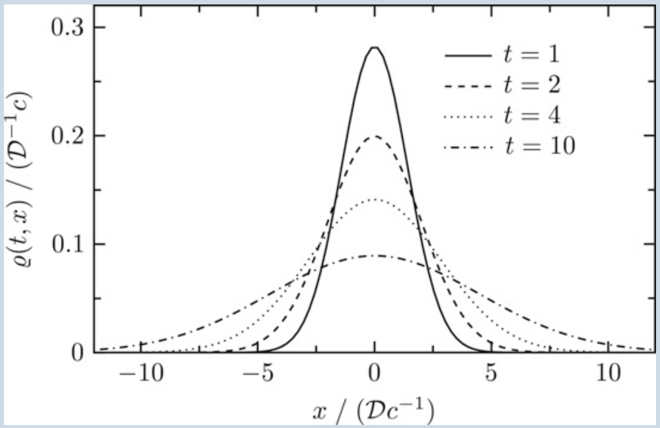
Diffusion "propagator"

*PRD **75**:043001 (2007)*

$$a(\bar{x}|\bar{x}_0) = \frac{1}{2}\int_{t_0}^{t} \mathrm{d}t'\ v(t')^2,$$

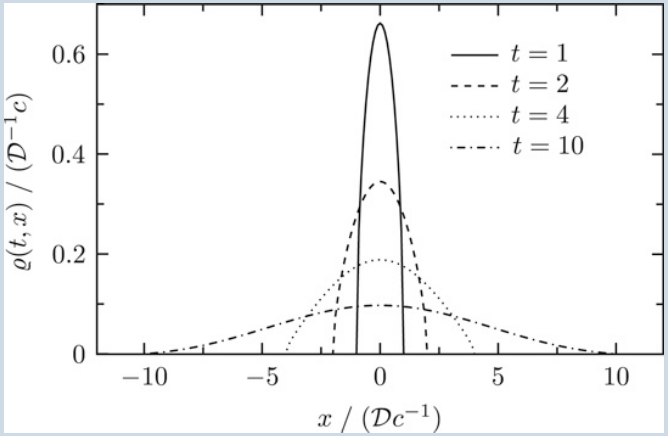
$$a(\bar{x}|\bar{x}_0) = -\int_{t_0}^{t} \mathrm{d}t'\ \left[1 - v(t')^2\right]^{1/2}$$

$$p(\bar{x}|\bar{x}_0) \propto \int_{a_-(\bar{x}|\bar{x}_0)}^{a_+(\bar{x}|\bar{x}_0)} \mathrm{d}a\ \exp\left(-\frac{a}{2\mathcal{D}}\right)$$



$$a_+ = +\infty$$

$$a_-(\bar{x}|\bar{x}_0) = \frac{(x - x_0)^2}{2(t - t_0)}$$



$$a_+ = 0$$

$$a_-(\bar{x}|\bar{x}_0) = -\left[(t - t_0)^2 - (x - x_0)^2\right]^{1/2}$$

R-Markov processes in phase space

Lab-time R-LEs

$$\mathrm{d}X(t) = (P/P^0)\ \mathrm{d}t,$$

$$\mathrm{d}P(t) = -\alpha_\bullet(P)\, P\ \mathrm{d}t + [2D(P)]^{1/2} \bullet \mathrm{d}B(t),$$

Debbasch et al, J Stat Phys 88:645 (1997)

FDR

$$0 \equiv \hat{\alpha}(p^0)\ p^0 - \beta\ \hat{D}(p^0),$$

$$\phi_{\mathrm{J}}(p) \propto \exp[\beta(p^2 + M^2)^{1/2}]$$

Diffusion constant

$$\mathcal{D}_\infty = \lim_{t\to\infty} \left\langle [X(t) - X(0)]^2 \right\rangle / (2t)$$

*Lindner, New J Phys **9**:136 (2007)*

Physics Reports 471 (1): 1 (2009)

R-Markov processes in phase space

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Diffusion constant

$$\mathcal{D}_\infty = \lim_{t\to\infty} \left\langle [X(t) - X(0)]^2 \right\rangle / (2t)$$

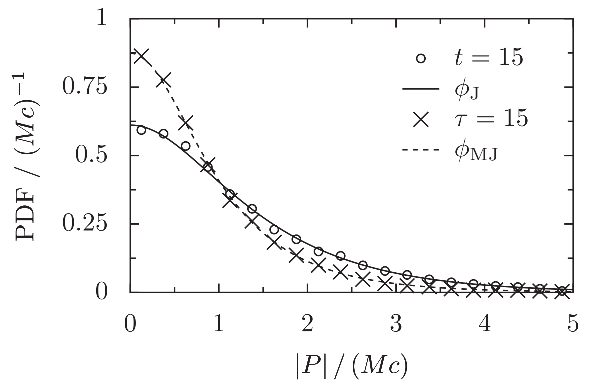
*Lindner, New J Phys **9**:136 (2007)*

Proper-time R-LEs

$$d\tau(t) = (M/P^0)dt.$$

$$dB^j(t) \simeq \sqrt{dt} = \left(\frac{\hat{P}^0}{M}\right)^{1/2} \sqrt{d\tau} \simeq \left(\frac{\hat{P}^0}{M}\right)^{1/2} d\hat{B}^j(\tau)$$

PRE 79: 010101(R) (2009)



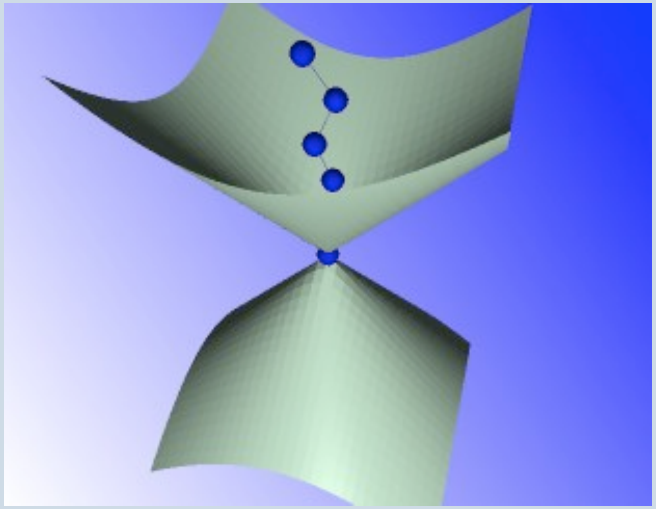
Physics Reports 471 (1): 1 (2009)

Summary (part 3)

Relativistic Brownian motion models

✓ non-Markovian in position space(time)

✓ Markovian in phase space



Many thanks for your attention !

Debbasch et al, J Stat Phys 88:645 (1997)

Physics Reports 471 (1): 1 (2009)

(Very) brief history

1905	special relativity	
1907/08	”moving bodies ... cooler”	[von Laue, Planck, Einstein]
1911/27	R-equilibrium distributions	[Jüttner]
1951-63	R-diffusion processes	[Lopuszanski, Schay, Dudley, Hakim]
1963	”moving bodies ... hotter”	[Ott]
1966/69	”... neither hotter nor cooler”	[Landsberg, Van Kampen]
1997	R-OUP	[Debbasch et al]
2005...	R-BM & applications	

*Phys Rep **471**:1 (2009)*