# Surmounting Fluctuating Barriers: Basic Concepts and Results

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**Abstract.** We review some recent progress for the problem of thermally activated escape over a potential barrier that is additionally subjected to colored noise driven fluctuations. Particular emphasis is put on the general framework and qualitative physical insights. The main findings are that for colored noise of constant variance, the phenomenon of resonant activation is typical; in contrast, with colored noise of constant intensity, resonant activation is typically *not* observed. It can emerge, however, in form of a prefactor effect, if suitably fluctuating potential landscapes are chosen.

## 1 Introduction and Model

Thermally induced surmounting of a potential barrier by a Brownian particle plays an prominent role in a wide variety of physical, chemical, and biological contexts (for a review see Hänggi, Talkner, and Borkovec 1990). Correspondingly, with the word "Brownian particle" one refers to a true physical particle, a chemical reaction coordinate, or some other relevant state variable or collective coordinate of the problem under discussion. In many cases, the potential itself cannot be regarded as static but rather as subjected to random fluctuations which are correlated on a characteristic time scale that is neither negligibly fast nor slow in comparison with the other time scales governing a typical escape event. An example is the activation of a $O_2$ or $CO$ ligand molecule out of a myoglobin "pocket" after photodissociation (Beece et al. 1980). Further, a model for the ion channel kinetics in the lipid cell membrane based on fluctuations in the activation energy barriers has been proposed by Croxton (1988). Also in other strongly coupled chemical systems (Maddox 1992), the dynamics of dye lasers (Jung et al. 1987), and even for some aspects of relaxation in glasses, or during protein folding, fluctuating potentials are likely to be of relevance (Beece et al. 1980, Stein et al. 1989, Wang and Wolynes 1994). In all these examples one has in mind the picture that the potential fluctuations experienced by the Brownian particle are controlled by some collective motion of the environment with a much larger (effective) mass such that back-coupling effects can be neglected. On top of that, this collective environmental fluctuations must be *far from thermal equilibrium* since otherwise they would be negligibly small due to their large mass. In the abovementioned example of a ligand escaping from the ("heavy") myoglobin the far from equilibrium situation is created by the sudden photodissociation, while in the ion channel it is maintained by permanent chemical reactions which are themselves far from thermal equilibrium.

In this contribution we present an overview over some recent achievements in the field with the main emphasis on general qualitative concepts and insights. A few more involved quantitative methods and results will be briefly mentioned in passing. We remark that somewhat similar systems, but with *deterministic*, time-periodic rather than *random* potential fluctuations are presently much discussed under the label "stochastic resonance" (see, e.g. in Jung 1993, Moss, Pierson, and O'Gorman 1994, and the special issue J. Stat. Phys. (1993): 70, 1). It is not surprising that in all these models many, qualitatively similar features are observed.

The simplest and most often considered model consists of the following one-dimensional Langevin equation:

$$\dot{x}(t) = -U'(x(t)) - \eta(t)\, W'(x(t)) + \sqrt{2D}\, \xi(t) \ , \tag{1}$$

describing the overdamped thermal diffusion of a Brownian particle with coordinate $x$ in a time dependent potential $U(x) + \eta(t)\, W(x)$. The thermal fluctuations are modeled by the delta correlated Gaussian noise $\xi(t)$ and the coupling strength (temperature) $D$. The static part of the potential $U(x)$ is assumed to have a well (potential minimum) at $x_0$ and a barrier (potential maximum) at $x^{\#} > x_0$. Its behavior beyond $x^{\#}$ is of minor interest, i.e., $U(x)$ may be either of the single well type like $U(x) = x^2/2 - x^3/3$ or a double well potential like $U(x) = x^4/4 - x^2/2$. The dependence upon the the detailed choice of the fluctuating part of the potential $W(x)$ and its stochastic driving $\eta(t)$ in (1) is more subtle and different cases have to be distinguished. In the following we will mainly focus on stochastic processes $\eta(t)$ of the so-called dichotomous and Ornstein-Uhlenbeck types and we will further assume that they are stationary and independent of the thermal fluctuations $\xi(t)$.

Once the considered model (1) is fixed, the quantity of foremost importance is the escape time distribution for an ensemble of particles starting out at, or close to the potential well $x_0$. Of particular interest is its first moment, the mean escape time $\bar{T}$. As is turns out, it is usually *not* convenient (Hänggi, Jung, and Talkner 1988) to study first passage times across the barrier $x^{\#}$ but rather those across a threshold $x_{th}$ sufficiently far *beyond* the barrier $x^{\#}$ such that particles, once they have left the region $x \leq x_{th}$, are very unlikely to return soon into the neighborhood of the well $x_0$. We finally remark that for sufficiently "weak" noises $\xi(t)$ and $\eta(t)$ the escape time distribution will often approach an exponential decay with a decay rate $k = 1/\bar{T}$ after a time that is negligibly short in comparison with $\bar{T}$. However, for general noise strengths and even in some cases with "weak" noises but extremely slow potential fluctuations ($\tau \geq \bar{T}$), the decay is non-exponential. In those cases, *a meaningful rate can no longer be defined*, whereas the concept of the mean escape time $\bar{T}$ remains still valid.

## 2 Dichotomous Potential Fluctuations

Presumably the simplest choice for the potential fluctuations $\eta(t)$ in (1) is a stationary dichotomous noise that flips at a rate $\gamma$ between two possible states $\pm\sigma$. Hence, $\langle \eta(t)\, \eta(s)\rangle = \sigma^2 e^{-|t-s|/\tau}$, where $\tau = 1/2\gamma$ is the correlation time. Note that this correlation function possesses the constant ($\tau$-independent) variance $\sigma^2$. This "archetypal" model (1) with $U(x)$ a box with a piecewise linear barrier and $W(x) = U(x)$ has been investigated in the seminal paper by Doering and Gadoua 1992. They observed a resonance-like depression in the mean escape time $\bar{T}(\tau)$ when plotted as a function of the correlation time $\tau$ of the potential fluctuations. This phenomenon, for which they coined the term "resonant activation" (RA), caused considerable attention (Maddox 1992) and stimulated a flurry of subsequent works. Most notably, their limitations to extremely large barrier fluctuations could be relaxed in the more detailed analytic studies (Zürcher and Doering 1993, Bier and Astumian 1993) of the same model. The occurrence of RA for general potentials $U(x)$ and $W(x)$ has been demonstrated in Pechukas and Hänggi 1994, Reimann 1995a by means of small- and large-$\tau$ studies. Extensive numerical results from analog simulations can be found in Marchesoni et al. 1995, Marchi et al. 1996. For weak thermal noise $D$, the asymptotically exact mean escape time $\bar{T}(\tau)$ for arbitrary $\tau$ and very general $U(x)$, $W(x)$, and $\sigma$ has been obtained very recently by Reimann and Elston (1996).

A suggestive heuristic explanation of RA, in the case that meaningful rates exist, is due to Pechukas and Hänggi, pointing out that: 'If barrier fluctuations are very fast, the rate of relaxation is determined by the average barrier. If barrier fluctuations are very slow, the ultimate rate of relaxation is determined by the highest barrier. If barrier fluctuations are slow enough that, during a correlation time, we may speak of a rate over the instantaneous barrier, but fast enough that a number of correlation times must pass before substantial relaxation occurs, the rate is the average of the "instantaneous" rates over the various barrier heights. Given the Arrhenius dependence of the rate on barrier height, this average must be greater than the rate over the average barrier and, of course, greater than the rate over the highest barrier: that is the maximum called resonant activation.'

Apparently the simplest and most general analytic demonstration that RA is a very common effect has been given by Reimann (1995a) and goes as follows: For asymptotically small correlation times $\tau$ of the potential fluctuations $\eta(t)$ the Brownian particle does no longer feel these correlations and we can approximate $\eta(t)$ by white Gaussian noise (in Stratonovich interpretation, cf.(Hänggi and Thomas 1982, Risken 1984) of the same intensity $\int dt\, \langle \eta(t)\, \eta(0)\rangle = 2\,\tau\sigma^2$. Next, the two independent Gaussian white noises $-\eta(t)\, W'(x(t))$ and $\sqrt{2\,D}\,\xi(t)$ in (1) can be replaced by an exactly equivalent single one of the form $\sqrt{2\,[D + \tau\,\sigma^2 W'(x)^2]}\,\hat{\xi}(t)$ with $\langle \hat{\xi}(t)\, \hat{\xi}(s)\rangle = \delta(t-s)$. The exact mean first passage time $T_\tau(x)$ across the threshold $x_{th}$ for a particle with seed $x < x_{th}$ now follows by standard methods (Hänggi, Talkner, and Borkovec 1990) as

$$T_\tau(x) = \int_x^{x_{th}} dy \int_{-\infty}^{y} dz \, \frac{\exp\left\{\int_z^y \frac{U'(v)}{D+\tau\,\sigma^2 W'(v)^2}\, dv\right\}}{\sqrt{[D+\tau\,\sigma^2 W'(y)^2]\,[D+\tau\,\sigma^2 W'(z)^2]}} \quad . \tag{2}$$

Finally, the mean escape time $\bar{T}(\tau)$ is obtained by performing an average over the initial distribution $\rho_0(x)$ of particles as $\int dx\, \rho_0(x)\, T_\tau(x)$. We remark that this simple small-$\tau$ approximation is in accordance with all known more rigorous but less general results for dichotomous barrier fluctuations $\eta(t)$ (Zürcher and Doering 1993, Bier and Astumian 1993, Pechukas and Hänggi 1994, Reimann and Elston 1996). On the other hand, in the limit $\tau \to \infty$ the potential fluctuations $\eta(t)$ get frozen to their initial values $\pm\sigma$ and the exact mean first passage time across the threshold $x_{th}$ can be easily determined as

$$T_\infty(x) = \int_x^{x_{th}} dy \int_{-\infty}^{y} dz \, \frac{\exp\left\{\frac{U(y)-U(z)}{D}\right\} \cosh\left(\frac{\sigma\,[W(y)-W(z)]}{D}\right)}{D} \quad . \tag{3}$$

From (2) and (3) we see that $T_0(x) \leq T_\infty(x)$ for all $x < x_{th}$ and therefore

$$\bar{T}(0) \leq \bar{T}(\infty) \quad . \tag{4}$$

Consequently, a sufficient condition for RA is that the asymptotics (2) decreases for increasing $\tau$ which in turn can be verified under the sufficient (but not at all necessary) conditions that either $W'(x) = 0$ whenever $U'(x) < 0$ and $x \leq x_{th}$ (with $U(x)$, $D$, and $\sigma$ arbitrary) or that $D$ is sufficiently small (with $U(x)$, $W(x)$, and $\sigma$ arbitrary), see Pechukas and Hänggi 1994. In other words, *RA is expected to occur typically.*

### 2.1 Weak Thermal Noise

We now turn to the physically most relevant case that (i) $\sigma$ is not too large such that in both realizations $U(x) \pm \sigma\, W(x)$ of the potential a particle, when starting out from the well $x_0$ of $U(x)$, cannot reach the threshold $x_{th}$ deterministically, i.e., without the presence of the thermal noise term in (1), and (ii) the thermal noise strength $D$ is small. As a consequence, the typical escape time $\bar{T}(\tau)$ becomes large and independent of the detailed initial distribution $\rho_0(x)$ of particles as well as of the exact position of the threshold $x_{th} > x^{\#}$ (Hänggi, Talkner, and Borkovec 1990). As a further consequence, a particle typically spends most of its time near the well $x_0$ before it escapes. The sojourn close to $x_0$ is interrupted by unsuccessful escape attempts and is terminated by a successful escape attempt. It can be shown that the so-called Suzuki time scale $T_s$ (Suzuki 1981) of the escape attempts increases like $\ln(1/D)$ but is still much smaller than $\bar{T}(\tau)$ for sufficiently small $D$ (Coleman 1977). We therefore have the *three relevant time scales* $\tau$, $T_s$, and $\bar{T}$ with $1 \ll T_s \ll \bar{T}(\tau)$ and $0 \leq \tau \leq \infty$ (Hänggi 1994, Reimann 1995a,b).

Let us now concentrate first on so-called conventional, or type I potentials $W(x)$ by which we mean that $W'(x)$ does not change sign on the interval between the well $x_0$ and the barrier $x^\#$ of the static $U(x)$. Without loss of generality we may assume $W'(x) \geq 0$ for $x_0 \leq x \leq x^\#$, for instance $W(x) = U(x)$. In this case a particle (1) typically escapes while the "instantaneous" potential $U(x) + \eta(t)\, W(x)$ is in a "low" state, $\eta(t) = -\sigma$, see Fig. 1. Since upon decreasing $\tau$ each realization $\eta(t)$ tends to fluctuate faster and faster, favorable escape-conditions $\eta(t) = -\sigma$ during the entire typical time $T_s$ of a successful escape attempt become less and less probable. However, this argument becomes clearly invalid as soon as $\tau$ is comparable or larger than $\bar{T}(\tau)$ since many particles would escape in such a case long before even the first "favorable" fluctuation of the potential occurs. We thus expect that $\bar{T}(\tau)$ is a decreasing function of $\tau$ provided $\tau \ll \bar{T}(\tau)$, in accordance with (2) and all exactly solvable models (Doering and Gadoua 1992, Zürcher and Doering 1993, Bier and Astumian 1993, Reimann and Elston 1996). We remark that precisely within the separation of time scales $\tau \ll \bar{T}(\tau)$ also the existence of a meaningful escape rate is always guaranteed (Hänggi, Talkner, and Borkovec 1990).

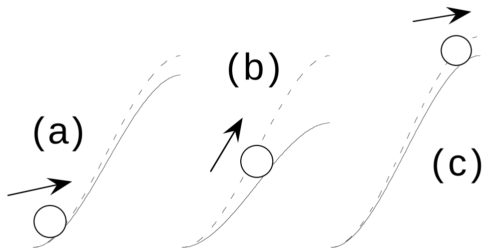


**Fig. 1.** Typical successful escape attempt for a (type I) fluctuating potential at three successive time instances. Dashed: average potential $U(x)$.

Next we address so-called "breathing", or type II potentials $W(x)$ defined by the property that $W(x)$ vanishes at the well $x_0$ and the barrier $x^\#$ of the static $U(x)$ (Reimann 1995a). In this case the basic escape mechanism is sketched in Fig. 2.: Typically, a successful escape attempt starts while the potential $U(x) + \eta(t)\, W(x)$ is in a "low" state (Fig. 2a), then the particle is lifted by an appropriate flip of $\eta(t)$ (Fig. 2b), and finally it moves in a in a "high" state of the "instantaneous" potential across the saddle $x^\#$ (Fig. 2c). Since this mechanism works best if one single flip of $\eta(t)$ occurs during a successful escape attempt we expect that $\bar{T}(\tau)$ exhibits a minimum (RA) at a $\tau$-value comparable to the time that the particle needs to pass through the domain where $W(x)$ notably differs from zero during its successful escape attempt. Closer inspection shows (Coleman 1977) that this time is comparable to the one which the particle would need to pass deterministically through the same domain but in the opposite direction. This yields as a rough estimate that RA will occur at $\tau = O([x^{\#} - x_0]^2/[U(x^{\#}) - U(x_0)])$.

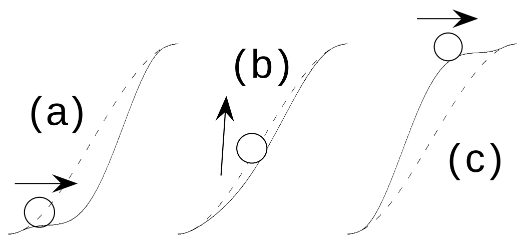


**Fig. 2.** Same as Fig. 1 but for a breathing fluctuating potential (type II).

We finally address the regime $\tau \gg T_s$ which overlaps with the previous one $\tau \ll \bar{T}(\tau)$. In this regime, so-called kinetic models (Zürcher and Doering 1993, Bier and Astumian 1993, Van den Broeck 1993, Pechukas and Hänggi 1994, Reimann 1995a, Gaveau and Moreau 1996) provide a very accurate description of the problem since a particle (1) approximately sees a static potential $U(x) + \eta(t)\, W(x)$ during any escape attempt and is thus successful at the well-known Smoluchowski-rate (Hänggi, Talkner, and Borkovec 1990) $k_\pm$ corresponding to the "instantaneous" static potential $U(x) \pm \sigma\, W(x)$ it experiences. Recalling that $\gamma = 1/2\tau$ is the flip rate of the dichotomous noise $\eta(t)$, the populations $\pi_\pm$ of not yet escaped particles that feel an "instantaneous" potential $U(x) \pm \sigma\, W(x)$ thus obey the "kinetic equations" $\dot{\pi}_\pm(t) = -k_\pm \pi_\pm(t) - \gamma\pi_\pm(t) + \gamma\pi_\mp(t)$ with initial conditions $\pi_\pm(0) = 1/2$. Disregarding the trivial case that $k_+ = k_-$ it follows that the decay of the total population $\pi(t) = \pi_+(t) + \pi_-(t)$ is given by the sum of two exponentials and cannot be rewritten as a simple exponential law. Furthermore, for the mean escape time $\bar{T}(\tau) = \int_0^\infty t[-\dot{\pi}(t)]dt$ one obtains (Van den Broeck 1993)

$$\bar{T}(\tau) = \frac{2 + \tau\,[k_+ + k_-]}{2\,\tau\, k_+ k_- + k_+ + k_-} \; . \tag{5}$$

Hence, $\bar{T}(\tau)$ is a strictly monotonically increasing function of $\tau$ with $\bar{T}(\tau) =$

$2/[k_+ + k_-]$ for very small $\tau$ (but still respecting $\tau \gg T_s$) and $\bar{T}(\tau) = [k_+^{-1} + k_-^{-1}]/2$ for asymptotically large $\tau$. Verifying the agreement with (3) is possible but somewhat tedious. In the asymptotic regime $\tau \gg \bar{T}(\tau)$ the rate determining process is given by the escape over the higher of the two barriers (Pechukas and Hänggi 1994).

To summarize our results for asymptotically *small* $D$, we find that for potentials $W(x)$ of the conventional type, $\bar{T}(\tau)$ is monotonically decreasing for sufficiently small $\tau$ and increasing for sufficiently large $\tau$, respectively. In the rather extended intermediate regime $T_s \ll \tau \ll \bar{T}(\tau)$, where both the rate concepts and the kinetic models are valid approximations but predict actually opposite signs for $d\bar{T}(\tau)/d\tau$, we must conclude that $\bar{T}(\tau)$ will be almost constant. In particular, the absolute minimum of $\bar{T}(\tau)$ (RA) will occur within this "plateau" regime and thus diverges at least like $T_s = \ln(1/D)$ when $D \to 0$. On the other hand, for potentials $W(x)$ of the breathing type, the RA minimum of $\bar{T}(\tau)$ will converge to a finite $\tau$-value when $D \to 0$, while an extended "plateau" about this minimum is not to be expected. For an illustration of this qualitatively different behavior of $\bar{T}(\tau)$ for conventional and breathing potentials $W(x)$ the reader may glance ahead to Figs. 3 and 4. Though in those numerical simulations Ornstein-Uhlenbeck noise $\eta(t)$, as discussed in the following section has been used, the qualitative features are representative for the dichotomous case as well.

## 3 Ornstein-Uhlenbeck Potential Fluctuations

Because Gaussian distributed noise is abundant in natural systems as well as technical applications, this is clearly a type of potential fluctuations that warrants to be investigated in detail. The case that they are uncorrelated (white noise) has been considered in the very early study by Hänggi 1980. Here, we admit the more general situation of arbitrary correlations (colored noise). In the simplest case, the Gaussian fluctuations are furthermore stationary and Markovian, thus a so-called Ornstein-Uhlenbeck process (Hänggi and Thomas 1982, Risken 1984), characterized by the probability distribution

$$\rho(\eta) = (\sqrt{2\,\pi\,\sigma^2(\tau)})^{-1/2}\ \exp\{-\eta^2/2\,\sigma^2(\tau)\} \tag{6}$$

and the time correlation

$$\langle \eta(t)\,\eta(s)\rangle = \sigma^2(\tau)\ \exp\{-|t-s|/\tau\}\ , \tag{7}$$

with the variance $\sigma^2(\tau)$ and the correlation time $\tau$ as free parameters.

### 3.1 Constant Variance Scaling

In analogy to Sect. 2 we assume that *the variance remains constant upon variation of the correlation time* $\tau$, $\sigma^2(\tau) = \sigma^2$ (constant variance scaling).

This model has been extensively studied by Hänggi (1994), Pechukas and Hänggi (1994), Reimann (1994), Reimann (1995a,b), Marchesoni et al. (1995), Marchi et al. (1996). Both the typical features of $\bar{T}(\tau)$ (see Figs. 3,4) and the heuristic explanation from Sect. 2 remain robust in this case. Moreover, the small-$\tau$ approximation (2) remains in perfect agreement with rigorous results by Pechukas and Hänggi (1994), Reimann (1994), Reimann (1995b), Madureira et al. (1995), Iwaniszewski (1996). The necessary modification of the large-$\tau$ asymptotics (3) is straightforward, yielding

$$T_\infty(x) = \int_x^{x_{th}} dy \int_{-\infty}^{y} dz \, \frac{1}{D} \exp\left\{ \frac{U(y) - U(z) + \frac{\sigma^2}{2D}[W(y) - W(z)]^2}{D} \right\} . \quad (8)$$

Eqs. (2) and (8) again imply (4), whence the ubiquity of RA follows by the same line of reasoning as in Sect. 2.

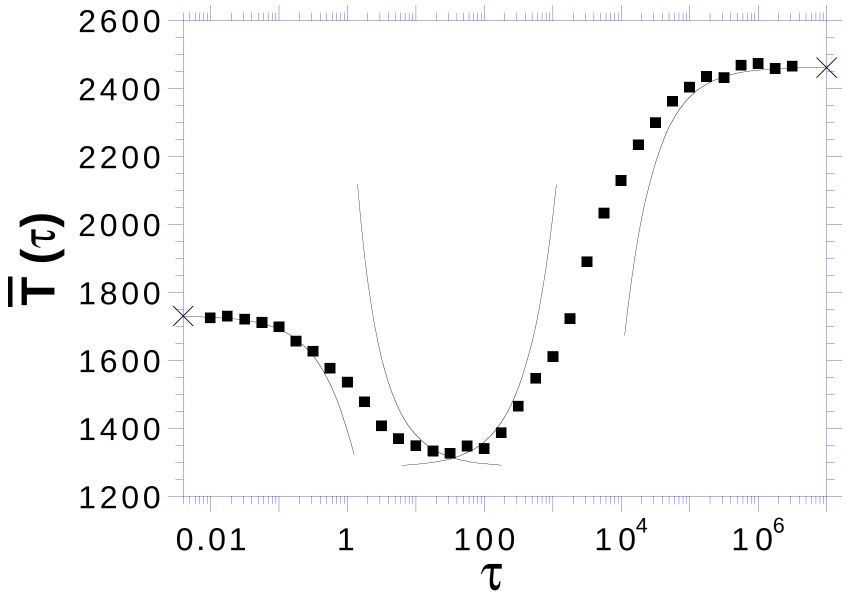


**Fig. 3.** Mean escape time from the region $x \le x_{th} = 1.5$ (filled squares) by simulating Eq. (1) with Ornstein-Uhlenbeck noise $\eta(t)$ of constant variance, $U(x) = x^2/2 - x^3/3$, $W(x) = U(x)$ (type I), and $D = \sigma^2 = 0.03$. Crosses: exact $\bar{T}(0)$ and $\bar{T}(\infty)$ from (2) and (8). The leftmost line is the small-$\tau$ approximation (2). For the approximations used for the remaining three lines see main text.

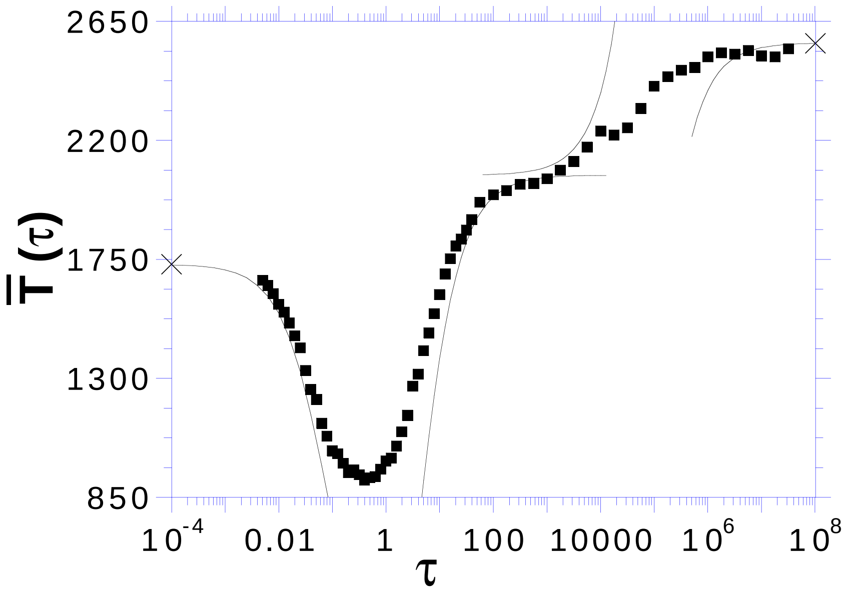


**Fig. 4.** Same as Fig. 3 but for the "breathing" (type II) potential $W(x) = 0.4 \cos^2(2\pi x)$ for $0.25 \leq x \leq 0.75$ and $W(x) = 0$ otherwise.

In the absence of the thermal noise term, Eq. (1) reduces to the much studied escape problem with colored noise alone, see Hänggi and Jung 1995 for a review (though constant variance scaling is somewhat uncommon in this context). Since $\eta(t)$ becomes a white noise of vanishing intensity $\int \langle \eta(t)\, \eta(0) \rangle \, dt$ for $\tau \to 0$ we can infer that $\bar{T}(0) = \infty$ for $D = 0$. Similarly, one finds that $\bar{T}(\infty) = \infty$ for $D = 0$. However, as remarked by Reimann 1995b, for intermediate $\tau$ one will have a finite $\bar{T}(\tau)$ provided $|W'(x)|$ is strictly positive for $x_0 \leq x \leq x^{\#}$ (thus a subclass of the "type I" potentials $W(x)$ introduced in Sect 2.1). We therefore recover a very pronounced form of RA even in the absence of the thermal noise $\xi(t)$ that is only modified and actually diminished by including $\xi(t)$. So, RA for such potentials $W(x)$ is not the effect of an interplay between thermal and potential fluctuations but rather a property of the escape problem with colored noise alone that survives in the presence of additional white noise (Reimann 1994, Reimann 1995b). For dichotomous noise, an analogous conclusion is possible under the additional proviso that $\sigma$ is rather large. For all other potential types $W(x)$, *the presence of two noise sources in (1) is crucial.*

More quantitative insight can be gained, e.g., by means of the so-called generalized unified colored noise approximation (GUCNA) put forward by Madureira et al. (1995), Bartussek, Madureira and Hänggi (1995). Based on appropriate (but non-systematic) approximations for small and large $\tau$ and an ad-hoc crossover formula, it's agreement with numerical simulations is surprisingly good, see Fig. 5 and Bartussek, Madureira, and Hänggi 1995, but it's asymptotical predictions can not always be trusted (Madureira et al. 1995). All other known approximations schemes are only valid for small thermal noise strengths $D$. Measuring also the variance of the potential fluctuations in units of $D$,

$$\sigma^2 = R_{CV}\, D \ , \tag{9}$$

the ($\tau$-independent) coefficient $R_{CV}$ should remain finite when $D \to 0$. Otherwise, either the barrier fluctuations or the thermal noise would become negligible, which is of little interest in our present context.

For small $R_{CV}$, asymptotically exact predictions when $D \to 0$ can be derived by path integral methods for rather general $U(x)$ and $W(x)$ and arbitrary $\tau$-values compatible with the separation of time scales $\tau \ll \bar{T}(\tau)$ (Reimann 1995b). A different path integral approximation is possible for more general $R_{CV}$ and large $\tau$ (but still respecting $\tau \ll \bar{T}(\tau)$), shown as the second curve from the left in Figs. 3 and 4. Theories for general $R_{CV}$ *and* $\tau$ are plagued with the same difficulties as the single colored noise problem and no truly satisfactory approximation scheme is known to us.

Finally, for $\tau \gg T_s = \ln(1/D)$, "kinetic models" analogous to those for dichotomous noise can be invoked when $D$ becomes small and $R_{CV}$ stays finite. In contrast to the dichotomous case, explicit solutions can be given only for small $\tau$ (but sill $\tau \gg T_s$) and for asymptotically large $\tau$ (Reimann 1995b). These predictions are displayed in Figs. 3 and 4 as the two rightmost curves. What can further be proven rigorously (Van den Broeck and Bouten 1986) is that $\bar{T}(\tau)$ is strictly monotonically increasing within the validity of the "kinetic models". Combining these findings for $\tau \gg T_s$ with those from the previously mentioned path integral approach for $\tau \ll \bar{T}(\tau)$, the two qualitatively different types of RA for "conventional" and "breathing" potentials $W(x)$ are recovered, see Figs. 3 and 4, respectively. The two different corresponding escape mechanisms can again be understood by means of Figs. 1 and 2.

### 3.2 Constant Intensity Scaling

As pointed out by Hänggi 1994, Reimann 1994, Marchesoni et al. 1995 the occurrence (or not) of RA crucially depends on how the distribution of the potential fluctuations $\eta(t)$ at any given time instance changes upon variation of the correlation time. In particular, the phenomenon of RA seems to require that $\tau\, \sigma^2(\tau)$ increases with $\tau$ (Hänggi 1994). In this section, we focus on the case that the *"intensity"* $\int \langle\eta(t)\, \eta(0)\rangle\, dt$ *remains constant upon variation of* *the correlation time* $\tau$ (constant intensity scaling). With (7) this implies the $\tau$-dependent variance of the form

$$\sigma^2(\tau) = Q/\tau \ , \qquad (10)$$

where $Q$ *is now* $\tau$*-independent.* The motivation for this scaling is a sensible white noise limit of $\eta(t)$ for $\tau \to 0$, and that it is commonly employed in the single colored noise context (Hänggi and Jung 1995). This type of potential fluctuations has been studied already early by Stein et al. 1989, Stein et al. 1990 in the weak color limit and subsequently extended to full color by Rattray and McKane 1991, Hänggi 1994, Reimann 1994, Madureira et al. 1995, Bartussek, Madureira, and Hänggi 1995, Marchesoni et al. 1995; but RA has never been observed.

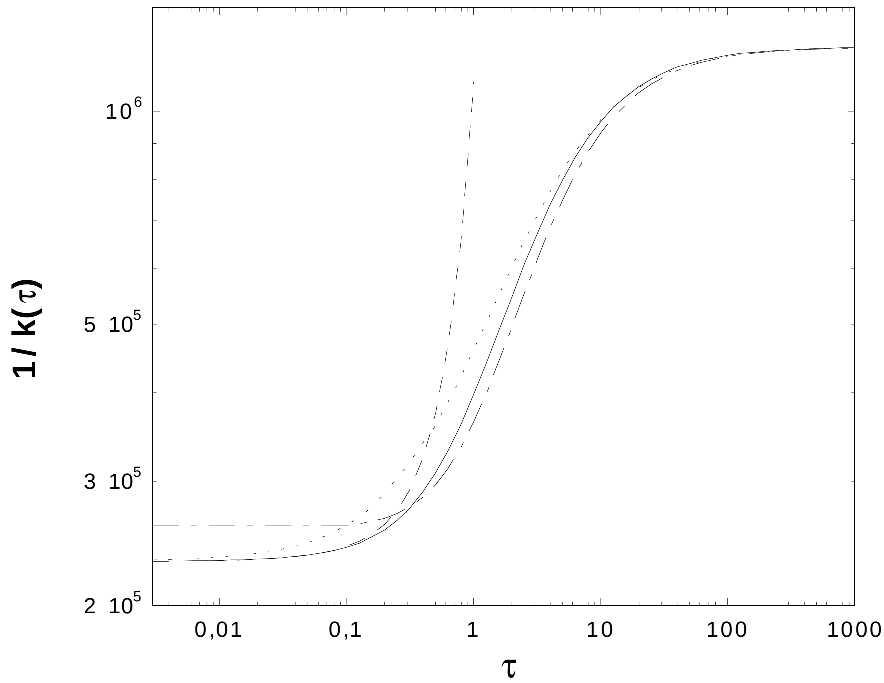


**Fig. 5.** Inverse numerical escape rate (solid line) $k(\tau) \simeq 1/\bar{T}(\tau)$ for Ornstein-Uhlenbeck noise $\eta(t)$ of constant intensity (cf. (1), (6), (7), (10), (13)) with $U(x) = x^4/4 - x^2/2$, $W(x) = x^2/2$, $D = 0.02$, and $Q = 0.01$ ($R_{CI} = 0.5$). The other three lines are $\bar{T}(\tau)$ from small-$\tau$ path integral approximation (dashed), small-$R_{CI}$ path integral approximation (dashed-dotted), and GUCNA (dotted).

Simple intuitive arguments as for constant variance scaling in order to explain qualitative features are apparently not available. However, in the white noise limit $\tau = 0$ the mean escape escape time for a particle starting out from $x < x_{th}$ follows like in (2) as

$$T_0(x) = \int_x^{x_{th}} dy \int_{-\infty}^{y} dz \, \frac{\exp\left\{\int_z^y \frac{U'(v)}{D+Q\,W'(v)^2}\,dv\right\}}{\sqrt{[D+Q\,W'(y)^2]\,[D+Q\,W'(z)^2]}} \; . \qquad (11)$$

On the other hand, for large $\tau$ one finds similarly as in (8) that

$$T_\tau(x) = \int_x^{x_{th}} dy \int_{-\infty}^{y} dz \, \frac{1}{D} \exp\left\{\frac{U(y) - U(z) + \frac{Q}{2D\tau}[W(y) - W(z)]^2}{D}\right\} \; . \quad (12)$$

This result is exact for $\tau = \infty$ and remains a valid approximation as long as particles have typically escaped already before notable potential fluctuations can occur, i.e., for $\tau \gg \bar{T}(\tau)$. We note that the $\tau$-dependent contribution in (12) yields at $\tau$ finite, but large, and $Q$ not very small, a non-negligible correction that mimics a maximum for $\bar{T}(\tau)$. This effect has been predicted by Iwaniszewski 1996 and observed numerically by Marchi et al. 1996. With decreasing $D$ and $Q$ the effect is, however, expected to practically disappear.

Similarly as in Sect. 3.1, additional quantitative insight can be gained by means of GUCNA and path integral methods provided $D$ becomes small and with $R_{CI}$ from

$$Q = R_{CI}\, D \qquad (13)$$

kept finite such that a rate exists and $\bar{T}(\tau) = 1/k(\tau)$. Those approximations are compared with numerical results in Fig. 5. Both of them predict that $\bar{T}(\tau)$ is typically an increasing function of $\tau$. However, for suitably tailored $U(x)$ and $W(x)$ one can also achieve a decreasing $\bar{T}(\tau)$ for small $\tau$ (a so-called prefactor effect, see in Iwaniszewski 1996, Reimann, Bartussek, and Hänggi 1996). In such a case, the complete $\bar{T}(\tau)$-curve thus exhibits, for suitably chosen $Q$, both RA and the abovementioned maximum (called "inhibition of activation" in Iwaniszewski 1996).

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