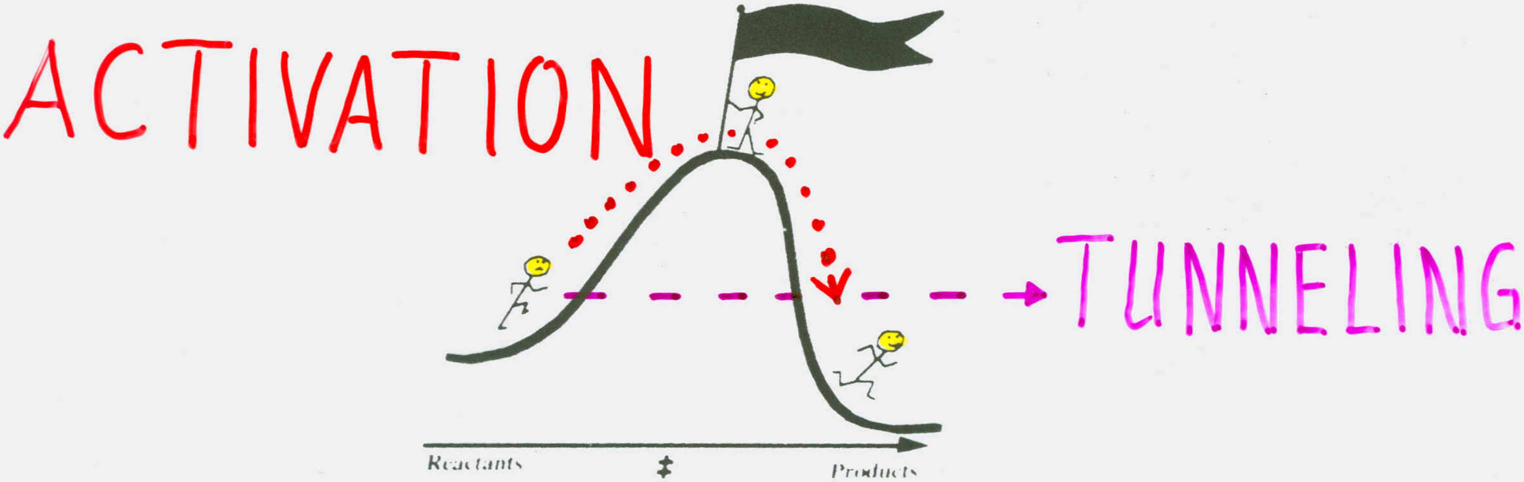
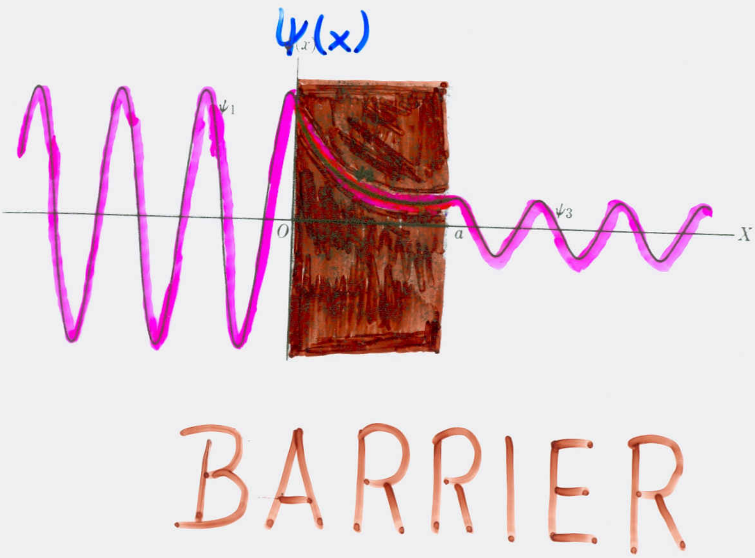
Coherent and Incoherent Quantum Stochastic Resonance

PETER HÄNGGI and MILENA GRIFONI

Institute of Physics
University of Augsburg

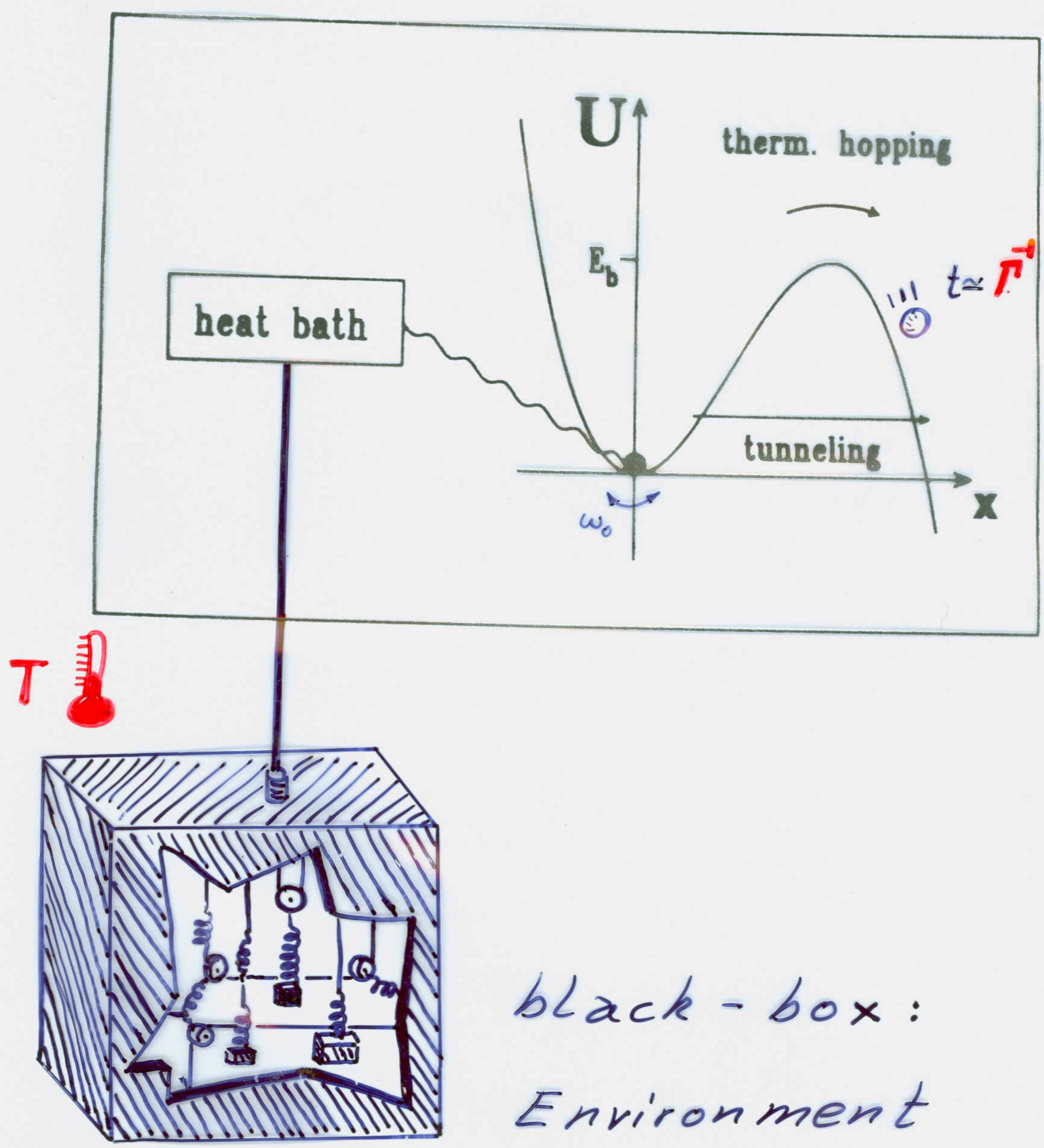
PART 1


ACTIVATION
TUNNELING
Reactants
‡
Products


ψ(x)
ψ1
ψ3
O
a
X
BARRIER

QUANTUM DISSIPATION

reaction rate
& quantum mechanics
+ dissipation

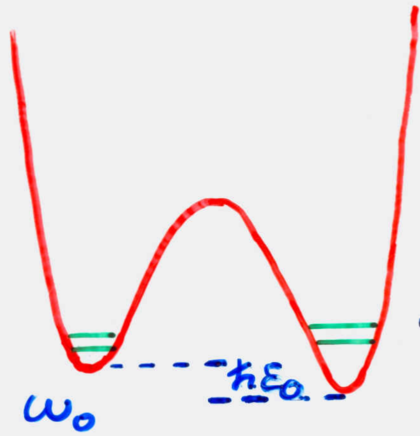


black-box:
Environment

i.e. $N \to \infty$

$$V_0 \gg \hbar\omega_0 \gg \hbar\varepsilon_0,\ kT$$

TS

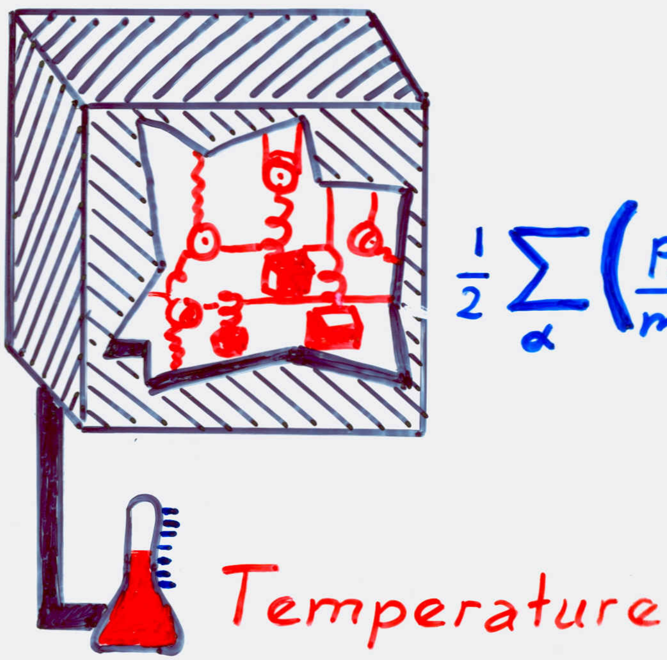


$$-\frac{\hbar}{2}\left(\varepsilon_0\,\sigma_z + \Delta\,\sigma_x\right)$$

$\oplus$

BATH

$T,\ \gamma$

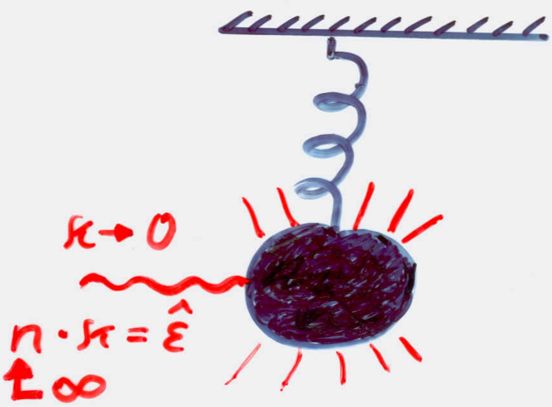


$$\frac{1}{2}\sum_\alpha\left(\frac{p_\alpha^2}{m_\alpha} + m_\alpha\omega_\alpha^2 x_\alpha^2 - c_\alpha x_\alpha \sigma_z\right)$$

$\oplus$

DRIVE

$\Omega,\ \hat{\varepsilon}$



$$\frac{\hbar\hat{\varepsilon}}{2}\cos(\Omega t)\,\sigma_z$$

SYNOPSIS QUANTUM DISSIPATION

$$H_{\substack{\text{system}\\ \oplus\text{ bath}}} = \frac{p^2}{2M} + U(q) + \sum_i \left\{ \frac{p_i^2}{2m_i} + \frac{m_i\omega_i^2}{2}\left(x_i - \frac{c_i}{m_i\omega_i^2}q\right)^2 \right\}$$

$$J(\omega) \equiv \frac{\pi}{2}\sum_i \frac{c_i^2}{m_i\omega_i}\,\delta(\omega-\omega_i)$$

↳ Heisenberg eq. for $\hat{q}(t)$

$$M\ddot{\hat{q}} + M\int_{t_0}^{t}\gamma(t-s)\,\dot{\hat{q}}(s) + \frac{\partial U}{\partial \hat{q}} = \hat{\xi}(t) - M\gamma(t-t_0)\,\hat{q}(t_0)$$

$$\gamma(t) = M^{-1}\sum_i \frac{c_i^2}{m_i\omega_i^2}\cos(\omega_i t) = \frac{2}{\pi}\int_0^\infty \frac{J(\omega)}{\omega}\cos(\omega t)\,d\omega$$

$$\hat{\xi}(t) = \sum_i c_i\left[\hat{x}_i(t_0)\cos[\omega_i(t-t_0)] + \frac{\hat{p}_i(t_0)}{m_i\omega_i}\sin[\omega_i(t-t_0)]\right]$$

$$\frac{1}{2}\left\langle \hat{\xi}(t)\hat{\xi}(0) + \hat{\xi}(0)\hat{\xi}(t)\right\rangle_0 = \frac{M}{\pi}\int_0^\infty (\hbar\omega)\,\mathrm{Re}\,\hat{\gamma}(\omega+i0^+)\cos\omega t \cdot \coth(\hbar\omega/(2kT))\,d\omega$$

$$\left\langle \hat{\xi}(t)\hat{\xi}(0)\right\rangle_0 = \frac{\hbar M}{\pi}\int_0^\infty \tilde{\gamma}(\omega)\,\omega\,\frac{\cosh[\omega(\hbar\beta/2 - it)]}{\sinh(\hbar\beta\omega/2)}\,d\omega$$

$$\equiv K(t) \equiv \frac{d^2W}{dt^2}$$

$$W(t) = S(\tau) + i\,R(\tau)$$

QUANTUM CORRECTIONS TO SR

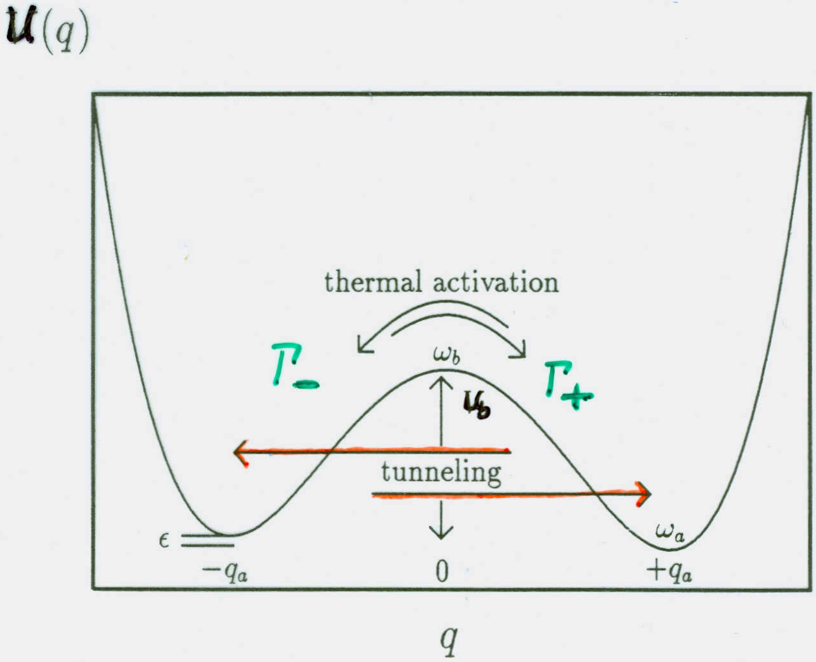
DESCRIBE POPULATION IN WELLS IN TERMS OF A 2-STATE DYNAMICS

$$q(t) = q_a\,\sigma(t) \; ; \quad \sigma(t) = \pm 1$$

$$n_R(t) = \int_{-\infty}^{0} p(q,t)\,dq \; ; \quad n_L(t) = 1 - n_R(t)$$

$$n_R(t) + n_L(t) = 1$$

QUANTUM CORRECTIONS TO SR



DESCRIBE POPULATION IN WELLS IN TERMS OF A 2-STATE DYNAMICS

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$$n_R(t) + n_L(t) = 1$$

$$\langle q(t)\rangle \sim q_a \underbrace{[n_R(t) - n_L(t)]}_{P(t)} = q_a P(t)$$

$$\dot{n}_R = -\Gamma_- n_R + \Gamma_+ n_L$$

$$-\dot{n}_L = +\Gamma_+ n_L \mp \Gamma_- n_R$$

$$\therefore \dot{P} = \dot{n}_R - \dot{n}_L$$

$$= -2\Gamma_- n_R + 2\Gamma_+ n_L$$

$$= -[2\Gamma_- n_R - 2\Gamma_+ (1-n_R)] = -2\Gamma_- (1-n_L) + 2\Gamma_+ n_L$$

$$\dot{P} = -[2\Gamma_- n_R + 2\Gamma_+ n_R - 2\Gamma_+] = (2\Gamma_+ + 2\Gamma_-) n_L - 2\Gamma_-$$

$$\cancel{2}\dot{P} = -\cancel{2}[\underbrace{\Gamma_+ + \Gamma_-}_{\bar{\Gamma}}] n_R + \cancel{2}\Gamma_+ + \cancel{2}(\Gamma_+ + \Gamma_-) n_L - \cancel{2}\Gamma_-$$

$$\dot{P} = -\bar{\Gamma}[n_R - n_L] + (\Gamma_+ - \Gamma_-)\frac{\bar{\Gamma}}{\bar{\Gamma}}$$

$$\underline{\underline{\underline{\dot{P} = -\bar{\Gamma}[P(t) - \Gamma_0]}}} \quad ; \quad \Gamma_0 = \frac{\Gamma_+ - \Gamma_-}{\bar{\Gamma}}$$

2-state process; see P. H. & H. Thomas, Phys. Rep. 88:207–319(82); p. 221

$$S(\tau) = \langle(\delta q)^2\rangle \exp(-\bar{\Gamma}\tau)$$

$$= q_a^2 (1 - \langle\sigma(t)\rangle^2) \exp(-\bar{\Gamma}|\tau|)$$

$$= q_a^2 (1 - P_0^2) \exp(-\bar{\Gamma}|\tau|)$$

$$= q_a^2 \frac{4\Gamma_+ \Gamma_-}{\bar{\Gamma}^2} \exp(-\bar{\Gamma}|\tau|)$$

$$\therefore S(\omega = \Omega) = q_a^2 \frac{4\Gamma_+ \Gamma_-}{\bar{\Gamma}^2} \cdot \frac{2\bar{\Gamma}}{\bar{\Gamma}^2 + \Omega^2}$$

$$\Rightarrow \operatorname{Im} \chi(\Omega) = S(\omega)\, \hbar^{-1} \tanh(\hbar\beta\Omega/2)$$

with $\hbar\beta\Omega/2 \ll 1$; $\tanh x \sim x - x^3/3$, $x \to 0$

$$\to \beta\Omega \frac{\bar{\Gamma}}{\bar{\Gamma}^2 + \Omega^2} \cdot \frac{4\Gamma_+ \Gamma_-}{\bar{\Gamma}^2} q_a^2 + O(\hbar\Omega\beta)^2$$

$$\therefore \chi(\Omega) = \left(\frac{4q_a^2}{kT}\right)\left(\frac{\Gamma_+ \Gamma_-}{\bar{\Gamma}^2}\right) \frac{\bar{\Gamma}}{\bar{\Gamma} - i\Omega} + O(\hbar\Omega\beta)^2$$

with detailed balance: $\Gamma_- = \Gamma_+ \exp(-\varepsilon/kT)$

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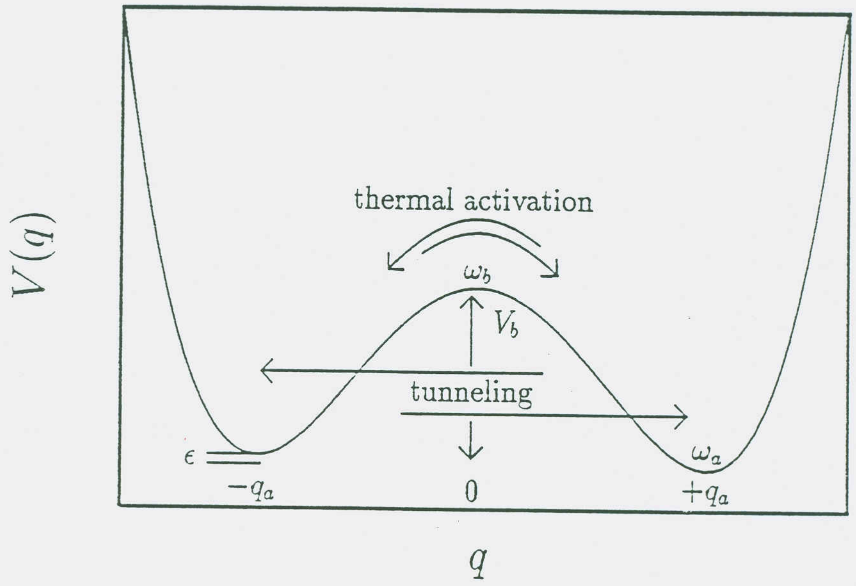
with detailed balance: $\Gamma_- = \Gamma_+ \exp(-\varepsilon/kT)$

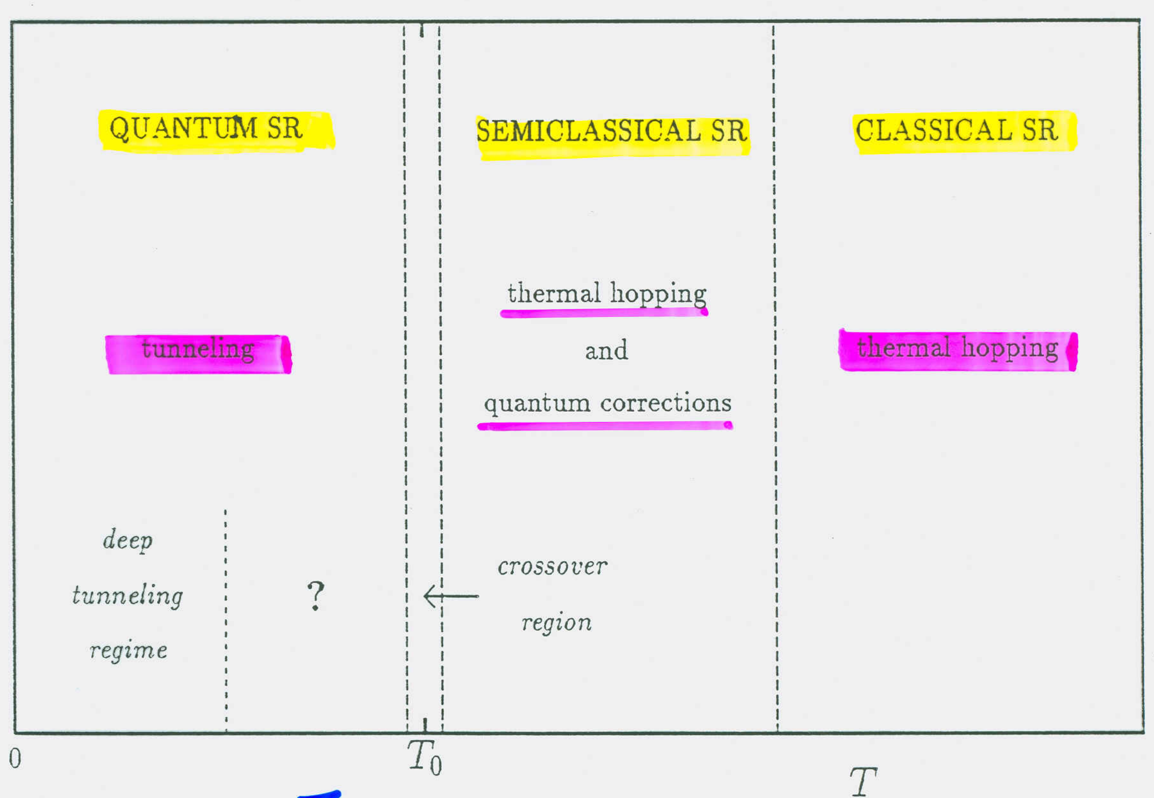
2 LIMITS

above ~ near crossover to thermal hopping

AT LOW T

• SR AND TUNNELING

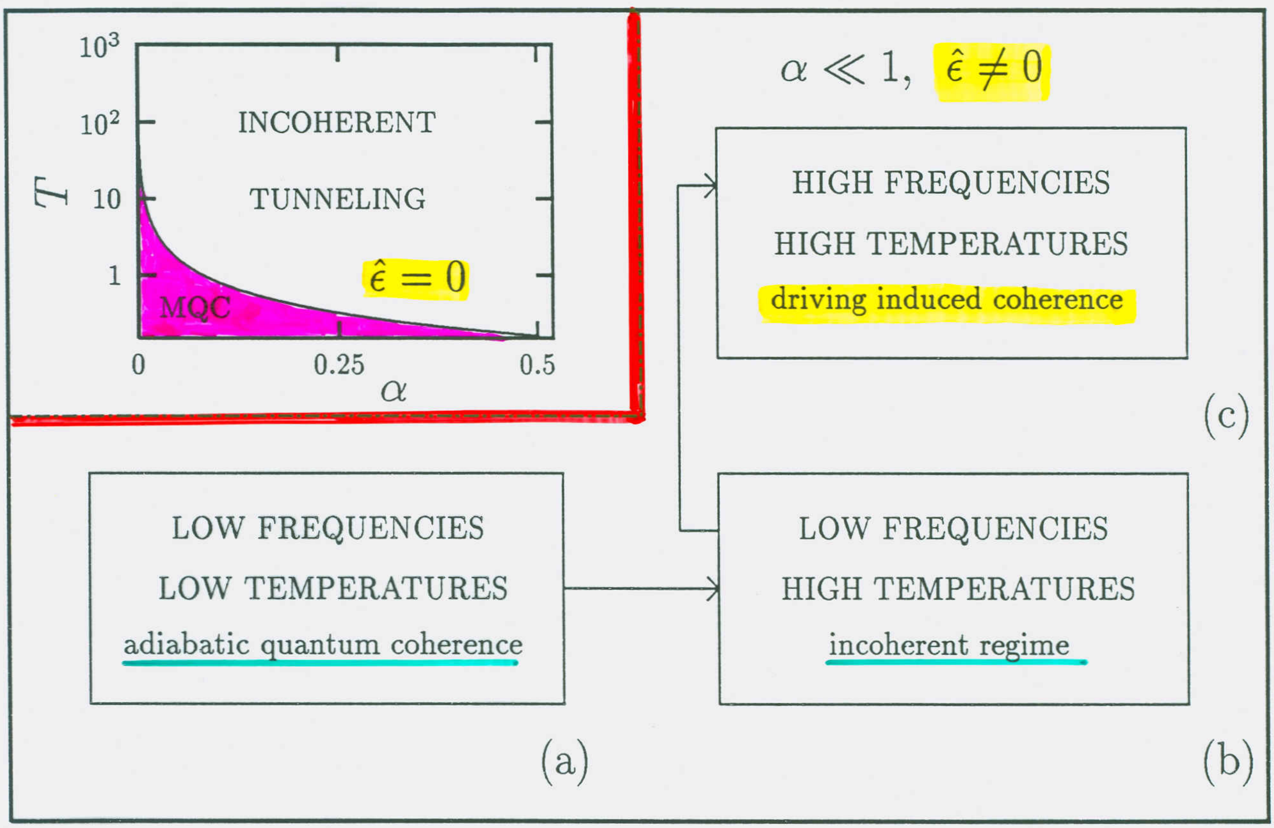




$$T_0 = \hbar\omega_R/(2\pi k)$$

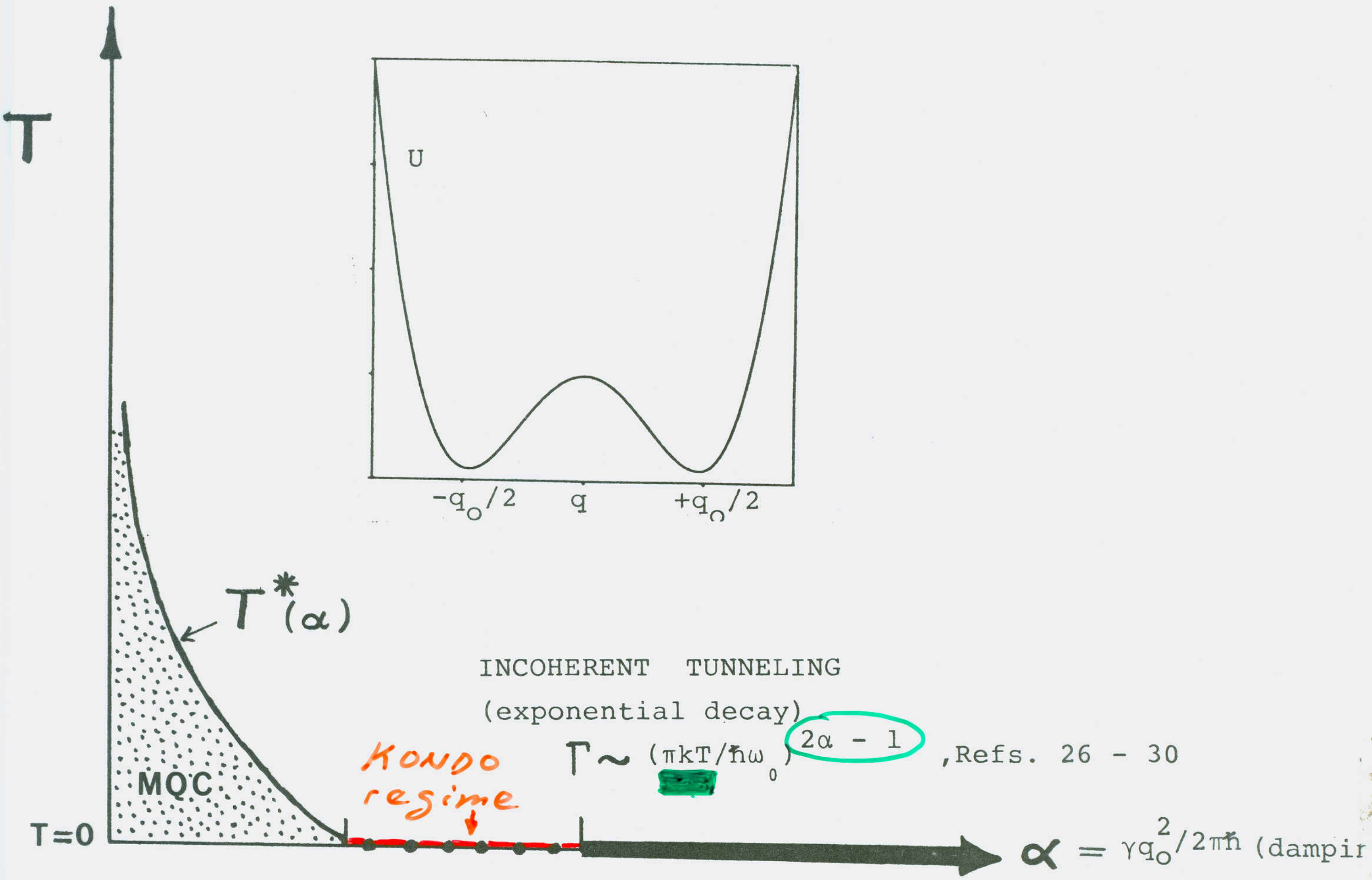
$$\omega_R = \left(\omega_b^2 + \frac{\gamma^2}{4}\right)^{1/2} - \frac{\gamma}{2} \xrightarrow{\gamma \to \infty} \frac{\omega_b^2}{\gamma}$$

QUANTUM SR



10^3
10^2
10
1
T
INCOHERENT
TUNNELING
$\hat{\epsilon} = 0$
MQC
0
0.25
0.5
α
$\alpha \ll 1,\ \hat{\epsilon} \neq 0$
HIGH FREQUENCIES
HIGH TEMPERATURES
driving induced coherence
(c)
LOW FREQUENCIES
LOW TEMPERATURES
adiabatic quantum coherence
LOW FREQUENCIES
HIGH TEMPERATURES
incoherent regime
(a)
(b)

QUANTUM COHERENCE



: damped oscillations (MQC for $T < T^*$) + small incoherent power law background at T = 0 (∞ sum of exponentials for finite T)

1/2 monotonous incoherent decay

■: *spontaneous symmetry breaking at T = 0*

– "2" exponentials: $\Gamma_1 = \Gamma > \Gamma_2 \sim \Gamma/2$.

Egger + Weiss, Z. Phys. B89: 97 (92)

non-Ohmic couplings

$J(\omega) \sim \omega^s$ ($s = 1$: Ohmic)

zero bias $\sigma = 0$

$s < 1$ $P(t) = 1$, localized at $T = 0$

fractal

$T > 0$: $\Gamma(T) \sim \exp(-a/T^{1-s})$

incoherent tunneling

$1 < s < 2$ crossover underdamped ($T = 0$)

$\rightarrow$ incoherent tunn. as T increases

$T > 0$

$s > 2$ underdamped coherent

oscillations ($T \ll \hbar\omega_0/k$)

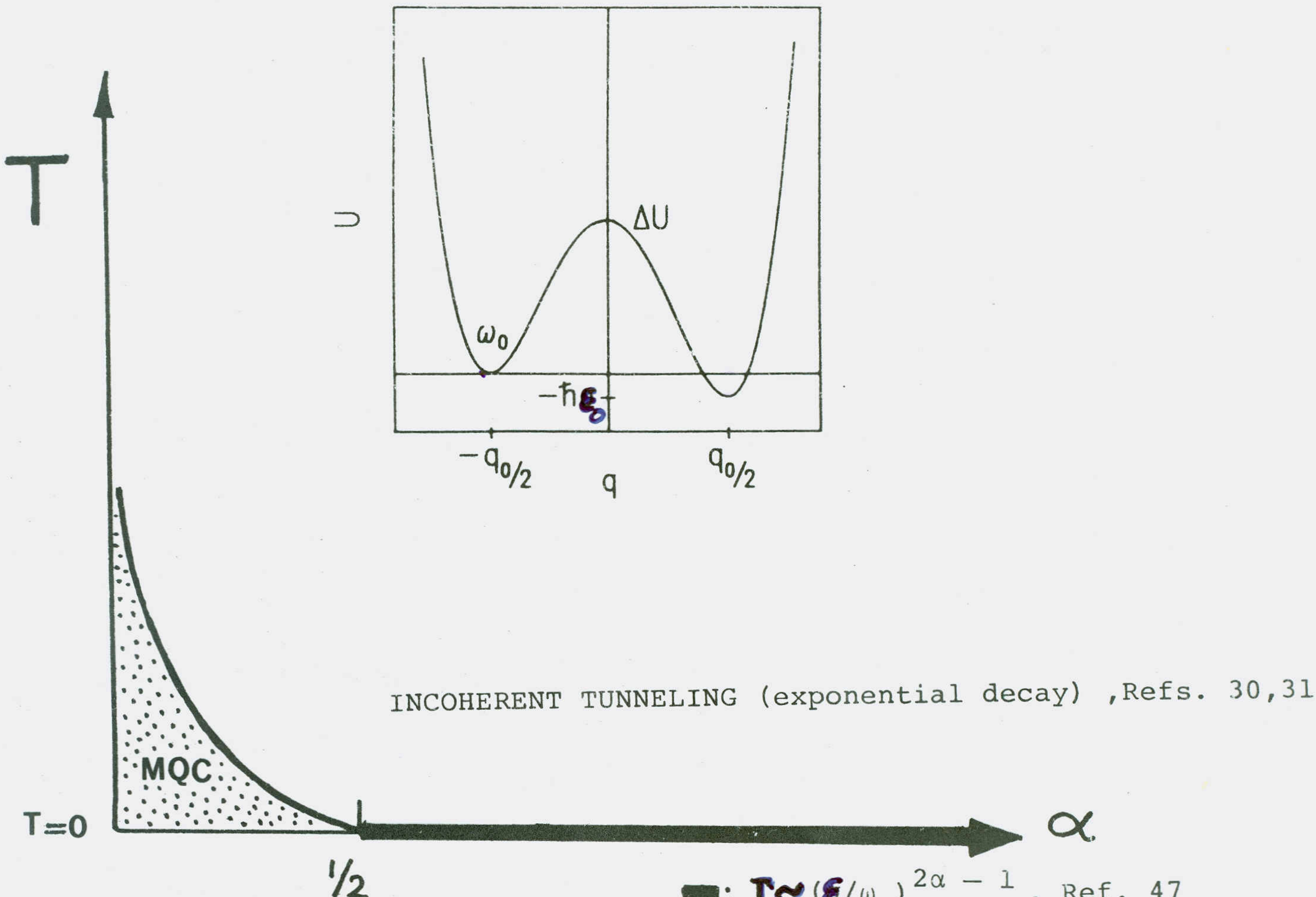
---

finite bias

$s \geq 1$, σ: large enough

coherence oscillations

are suppressed $\rightarrow$ incoherent decay



: *damped oscillations*

\+ *exponential decay into lower well*

\+ *small power law background* (T=0)

(∞ *sum of exponentials at finite* T)

$$\frac{\Gamma(T)}{\Gamma(T=0)} = \exp\left\{\frac{4}{3}\pi^2\omega_0^2\,\frac{\alpha^3}{|\sigma|^2}\,\frac{(kT)^2}{(\hbar\omega_0)^2}\right\}$$

QSR

LINEAR RESPONSE

$q(t) = a\,\sigma_z(t)/2 \qquad H_{INT}(t) = -\dfrac{\hbar\hat{\varepsilon}}{2}\cos(\Omega t)\,\sigma_z$

$$P_{as}(t) = \langle \sigma_z(t \to \infty) \rangle$$

$$= P_0 + \hbar\hat{\varepsilon}\,\chi(\Omega)\,e^{-i\Omega t} + c.c.$$

$$\chi(t) = \frac{i}{4\hbar}\,\theta(t)\,\langle [\sigma_z(t), \sigma_z(0)] \rangle_\beta$$ KUBO

LINEAR RESPONSE IS DETERMINED BY THERMAL EQUILIBRIUM PROPERTIES

QUANTUM LINEAR RESPONSE THEORY

$$\langle \delta \hat{x}(t) \rangle_\beta = \int_{-\infty}^{t} \chi(t-t')\, f(t')\, dt'$$

$$S(\omega) = |\chi(\omega)|^2 S_{ff}(\omega) + S^{0}_{xx}(\omega)$$

$$= \hbar \coth\left(\frac{\hbar\omega}{2k_BT}\right) \mathrm{IM}\, \chi(\omega)$$

$$\chi^{TLS}(\omega) = \beta \frac{x_0^2}{\cosh^2(\varepsilon_0 \beta/2)} \frac{\Gamma}{\Gamma - i\omega}$$

QUANTUM RATE

$$\Gamma = \Delta^2 \int_0^\infty dt\, \exp(-Q'(t)) \cos(Q''(t)) \cos(\varepsilon_0 t/\hbar)$$

INFLUENCE OF BATH

$$\pi k_B T \ll \hbar\omega_c \quad \& \quad \varepsilon_0 \ll \hbar\omega_c$$

$$\Gamma \sim \frac{\Delta^2}{2\omega_c \hat{\Gamma}(2\alpha)} \left(\frac{2\pi k_B T}{\hbar\omega_c}\right)^{2\alpha-1} \left|\hat{\Gamma}\left(\alpha + i\frac{\varepsilon_0}{2\pi k_B T}\right)\right|^2$$

$$\cosh(\varepsilon_0/2k_BT)$$

NOT VALID APPROX. FOR PROTON TRANSFER IN NONPOLAR MEDIA $\omega_0 \sim 400\,\mathrm{cm}^{-1}$ & $\omega_c \sim 80\,\mathrm{cm}^{-1}$

• LINEAR RESPONSE

$$S(\omega) = |\chi(\omega)|^2 S_{ff}(\omega) + S^{(0)}_{xx}(\omega)$$

QFDT: $= \hbar \coth\left(\frac{\hbar\omega}{2k_B T}\right) \mathrm{IM}\, \chi(\omega)$

QUANTUM RATE

$$\Gamma = \Delta^2 \int_0^\infty dt \exp\left(-\underline{Q'(t)}\right) \cos\left(\underline{Q''(t)}\right) \cos(\varepsilon_0 t/\hbar)$$

INFLUENCE BATH

$\pi k_B T \ll \hbar\omega_c$ & $\varepsilon_0 \ll \hbar\omega_c$

$\Downarrow$

$$\Gamma \sim \frac{\Delta^2}{2\omega_c \hat{\Gamma}(2\alpha)} \left(\frac{2\pi k_B T}{\hbar\omega_c}\right)^{2\alpha-1} \left|\hat{\Gamma}\left(\alpha + i\frac{\varepsilon_0}{2\pi k_B T}\right)\right|^2$$

$$\cdot \cosh(\varepsilon_0 / 2k_B T)$$

ARRHENIUS-LIKE

LINEAR RESPONSE & QSR

with $P_1 = \frac{A}{2} \chi_{qq}(\Omega) \equiv \frac{A}{2} \chi(\Omega)$

$$\eta_1 = 4\pi |P_1|^2 = \pi A^2 |\chi(\Omega)|^2$$

$$\mathrm{SNR} = \frac{\pi A^2 |\chi(\Omega)|^2}{S_{qq}(\Omega; A=0)} = \frac{\pi A^2 |\chi(\Omega)|^2}{\mathrm{Im}\,\chi(\Omega)\,\hbar \coth(\hbar\Omega\beta/2)}$$

¡valid at all temperatures!

PROBLEM: QUANTUM $\chi_{qq}(\Omega) \overset{FDT}{\leftrightarrow} S_{qq}(\Omega)$

$$S_{qq}(t) = \frac{1}{2} \langle \delta\hat{q}(t)\delta\hat{q}(0) + \delta\hat{q}(0)\delta\hat{q}(t) \rangle_\beta$$

¡DIFFICULT!

DIMENSIONLESS QSR-MEASURES

$$\tilde{\eta}_1 = \frac{\eta_1}{(A q_a^2 / U_b)^2} = \pi \left(\frac{U_b}{kT}\right)^2 \frac{1}{\cosh^4(\varepsilon/2kT)} \frac{\bar{\Gamma}^2}{\Omega^2 + \bar{\Gamma}^2}$$

$$\widetilde{SNR} = \frac{SNR/\omega_b}{(A q_a^2 / U_b)^2} = \frac{\pi}{2} \left(\frac{U_b}{kT}\right)^2 \frac{\bar{\Gamma}/\omega_b}{\cosh^2(\varepsilon/2kT)}$$

M. Grifoni, P.H. PRE **53**: 5890 (96)

EVALUATE QUANTUM – $\bar{\Gamma}$

see: P. H., P. TALKNER, M. BORKOVEC

REV. MOD. PHYS **62**: 251 – 341 (90)

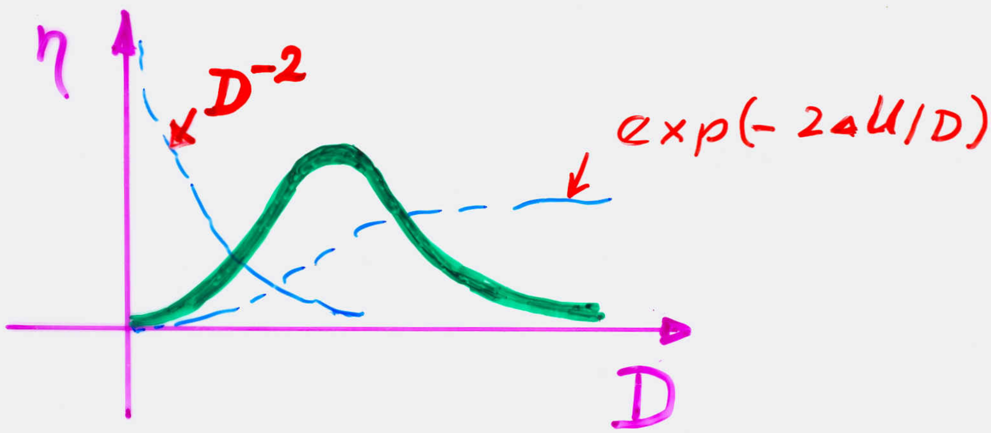
SECT. IX QUANTUM RATE THEORY

RESULTS

linear response $\frac{A}{D} \to 0$

$$\eta \overset{A\to 0}{\simeq} \frac{1}{D^2} \frac{g_1^2}{1 + \Omega^2/\lambda^2_{min}}$$

$$\boxed{\eta \simeq \frac{1}{D^2} \frac{1}{1 + \frac{\Omega^2}{\pi \omega_0^2} \exp\left(\frac{2\Delta U}{D}\right)}}$$



LRT (IN & FAR FROM EQ.)

"STOCHASTIC PROCESSES: RESPONSE THEORY AND FLUCTUATION THEOREMS
Helv. Acta (78)

$\frac{A}{D} \ll 1$

$$\chi_{FAE}(t) = -\frac{1}{D}\frac{d}{dt}\langle \delta x(t)\delta x(0)\rangle$$

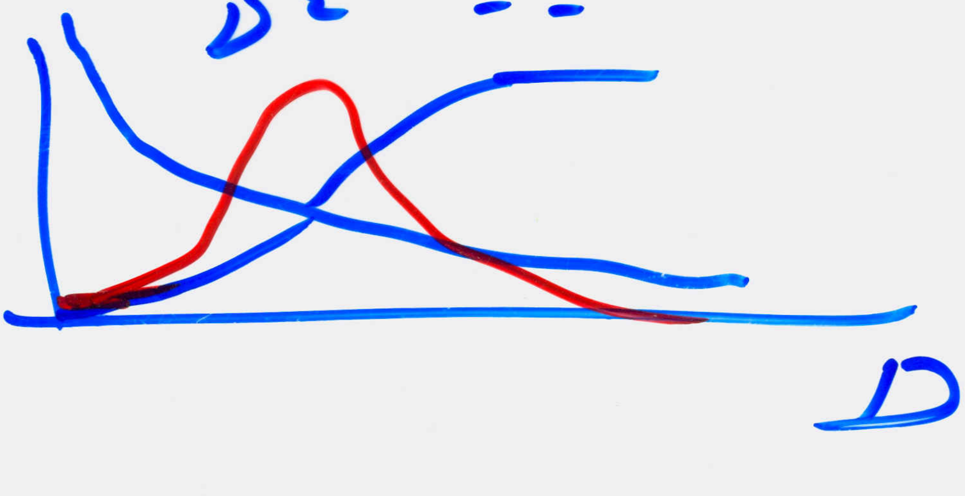
$$\chi(t) = \alpha \frac{1}{D}\frac{d}{dt}\langle \delta x(t)\delta \psi(x(0))\rangle$$

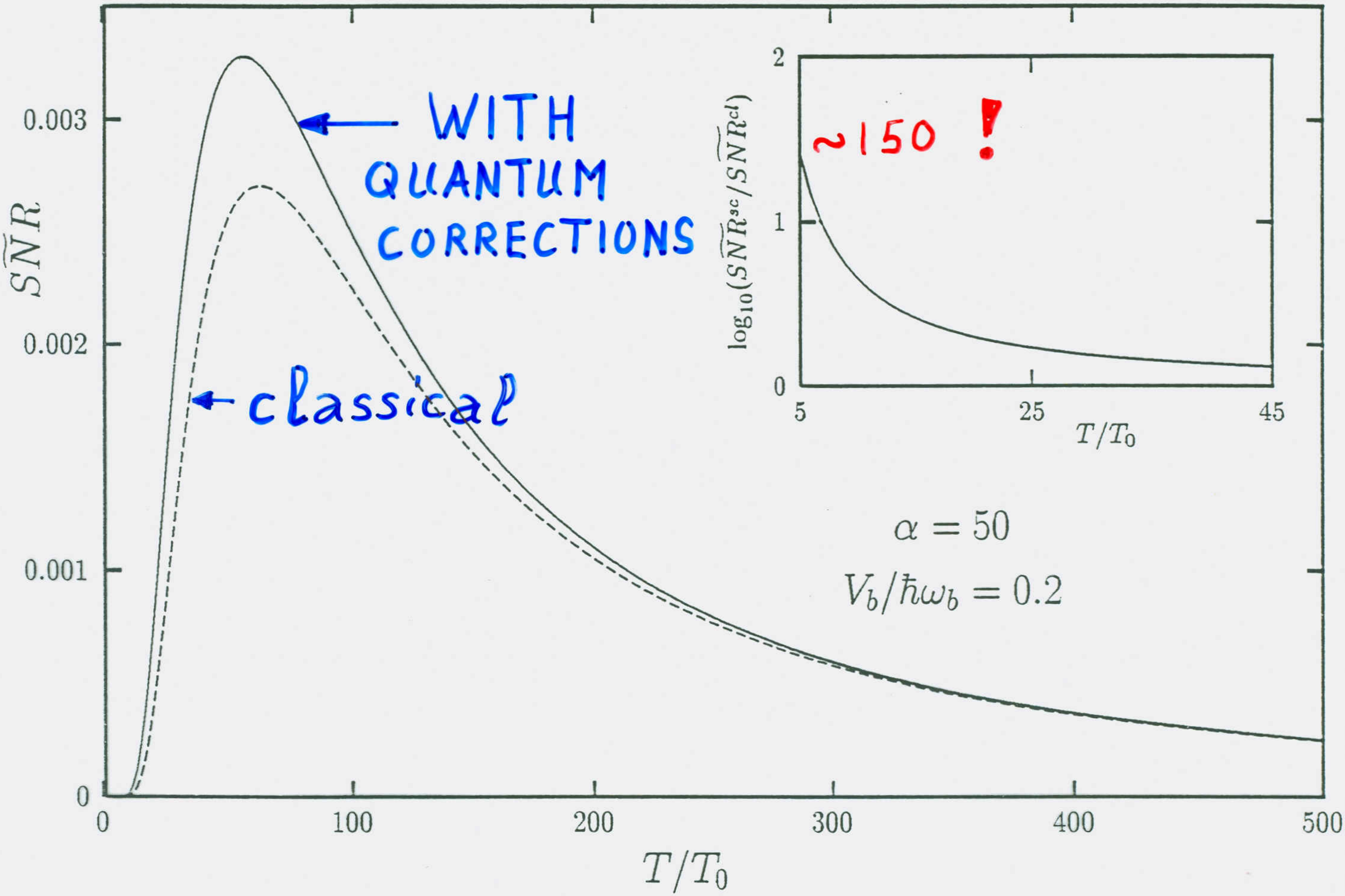
$$e^{-\lambda_1 t}$$

$$|M_1|^2 \propto |\chi(\omega=\Omega)|^2 \quad \frac{1}{D^2}\frac{\cancel{\lambda_1^2}}{\cancel{\lambda_1^2}+\Omega^2} \qquad \lambda_1 \sim 2\Gamma_K$$

$$= \frac{1}{D^2}\frac{1}{1+\Omega^2/\lambda_1^2}\, e^{-\Delta U/D}$$

$$\frac{1}{D^2}\frac{\quad}{--}\, e^{-2\Delta U/D}$$





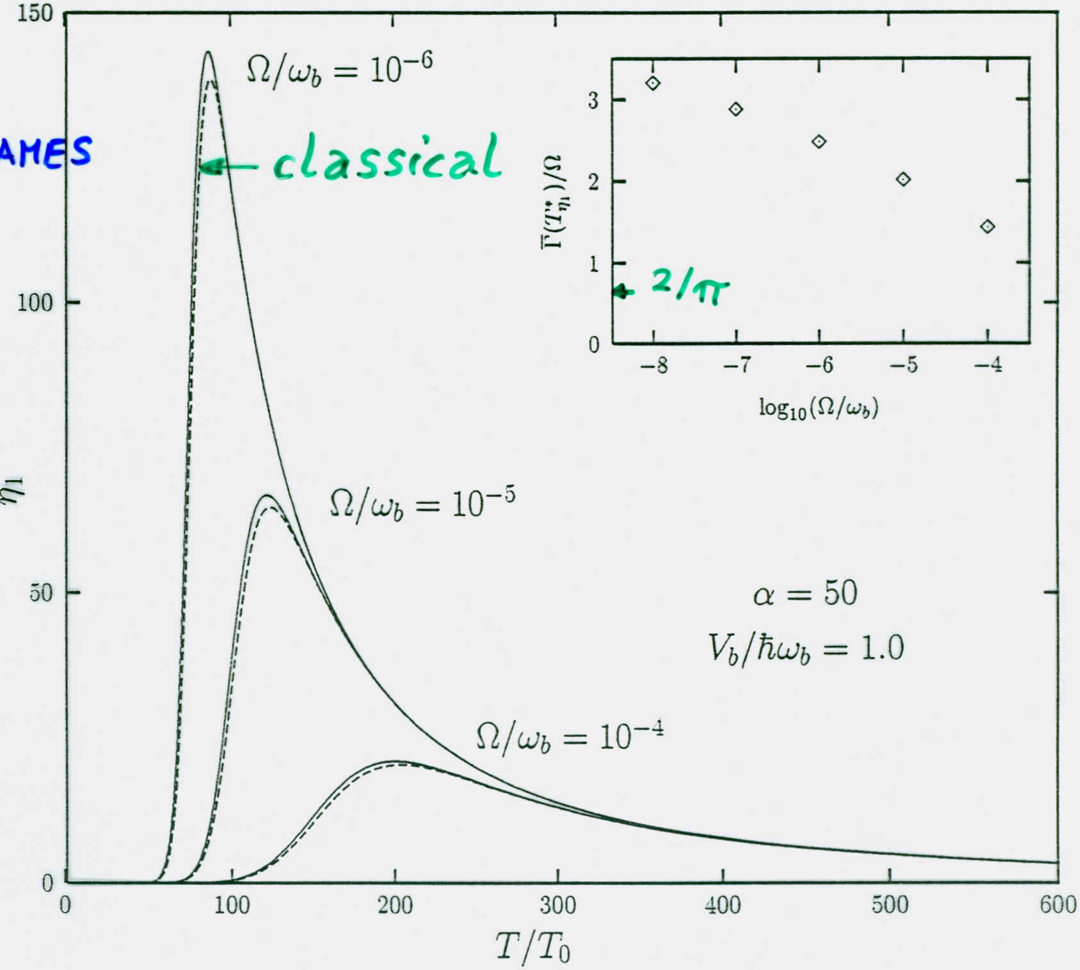
0.003
0.002
0.001
0
0
100
200
300
400
500
$\widetilde{SNR}$
T/T_0
WITH QUANTUM CORRECTIONS
classical
$\alpha = 50$
$V_b/\hbar\omega_b = 0.2$
$\log_{10}(\widetilde{SNR}^{sc}/\widetilde{SNR}^{cl})$
2
1
0
5
25
45
~150 !

IF $T_\Omega \sim 2\,T_{KRAMERS}$

$\Rightarrow \dfrac{\bar{\Gamma}}{\Omega} \sim \dfrac{2}{\pi}$

NOT OBEYED

AT LOW Ω



M. GRIFONI + P. H., PRE 53: 5890 – 5898 (96)

DISCUSSION

- QSR $\leftrightarrow$ QUANTUM RATES Γ_+ & Γ_-
- VALID FOR ADIABATIC & NONADIABATIC Ω

— but $\hbar\beta\Omega \ll 1$; i.e β NOT TOO LARGE —

- $\tilde{\eta}_1$: FUNCT. OF Ω

  EXTERNAL CONTROL OF ¡MAX! VIA Ω

- ASYMMETRY $\varepsilon \neq 0$ YIELDS EXPON. SUPPRESSION
- FAR BELOW CROSSOVER

  NO ARRHENIUS LAW FOR $\bar{\Gamma}$

  QSR NEEDS $\varepsilon \neq 0$

  ¡ T^{-2} vs. DET. BALANCE FACTOR!

REGIME OF VALIDITY

(i) WEAK NOISE $\quad kT << U_b$

$\therefore$ rate behavior

(ii) SEMICLASSICAL APPROXIMATION

$$\hbar\omega_R(\alpha) << U_b$$

$\alpha > 1: \quad U_b > 5\hbar\omega_0/\alpha$

$\alpha \leq 1: \quad U_b \gtrsim 5\hbar\omega_0$

(iii) NO ENERGY-DIFFUSION CONTROLLED ESCAPE; i.e. moderate-to-strong friction

(i) – (iii):

1) $U_b \gg (T/T_0)(\hbar\omega_b/4\pi\alpha)$

2) $\omega_0^2 + \omega_b^2 \gg 2\,\Omega^2$

Coherent and Incoherent Quantum Stochastic Resonance

PETER HÄNGGI and MILENA GRIFONI

Institute of Physics
University of Augsburg

PART 2

S R

IN THE DEEP QUANTUM REGIME

MILENA GRIFONI AND P. H.

PRL 76 : 1611 (1996)

PRE 54 : 1390 (1996)

B. QSR in the Deep Quantum Regime

B. QSR in the Deep Quantum Regime

$$\boxed{\text{MEASURE FOR QSR}}$$

$$\eta_m(\Omega, A) = 4\pi \, | P_m(\Omega, A)|^2$$

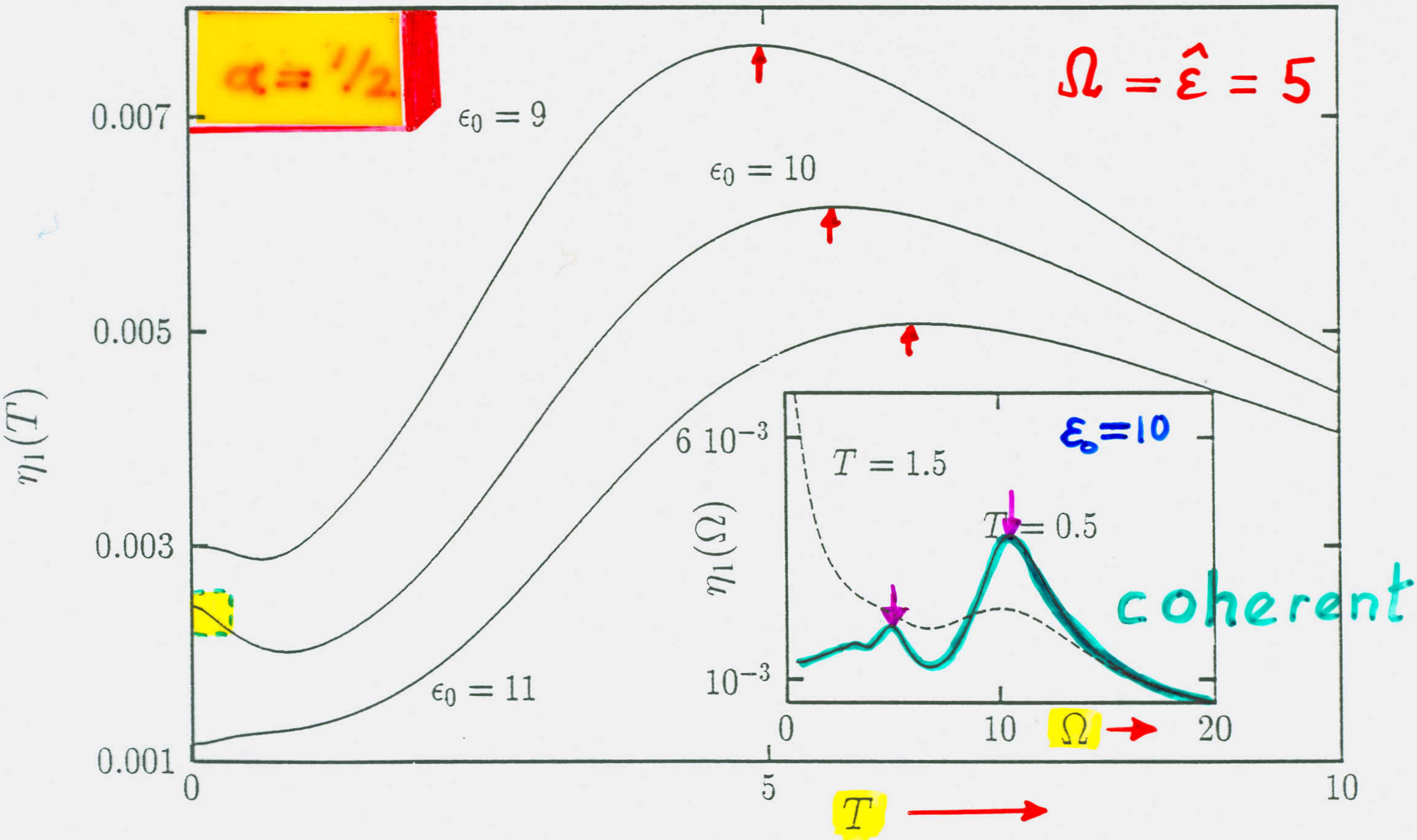
$$\varphi_m(\Omega, A) = \tan^{-1}\{\mathrm{Im}\, P_m(\Omega, A)/\mathrm{Re}\, P_m(\Omega, A)\}$$

$$\bar{S}(\tau) = \frac{\Omega}{2\pi} \int_0^{2\pi/\Omega} dt \, \tfrac{1}{2} \langle \hat{q}(t+\tau)\hat{q}(t) + \hat{q}(t)\hat{q}(t+\tau)\rangle$$

$$\xrightarrow{\tau \to \infty} S_{as}(\tau) = \sum_{m=-\infty}^{\infty} |P_m|^2 e^{-im\Omega\tau}$$

$$S_{as}(\omega) = 2\pi \sum_{m=-\infty}^{\infty} |P_m|^2 \delta(\omega - m\Omega)$$

EXACT



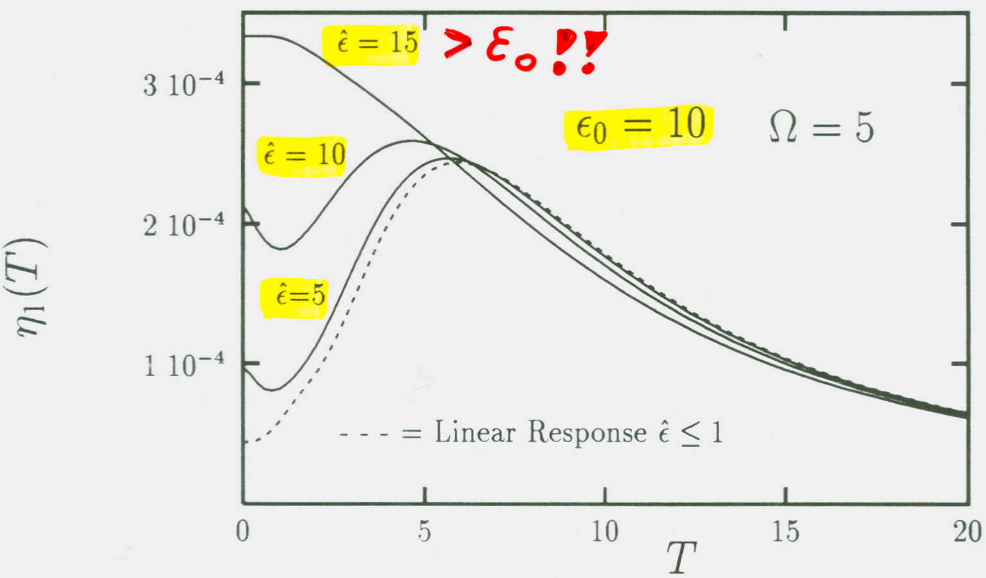
$$J(\omega) = \underbrace{\frac{2\pi\hbar}{a^2}\,\alpha}_{\Rightarrow \text{ Ohmic friction}}\,\omega$$

in the tunneling transitions. The series can be summed up *exactly in analytic form* for the special value $\alpha = 1/2$ of the Ohmic strength, to give

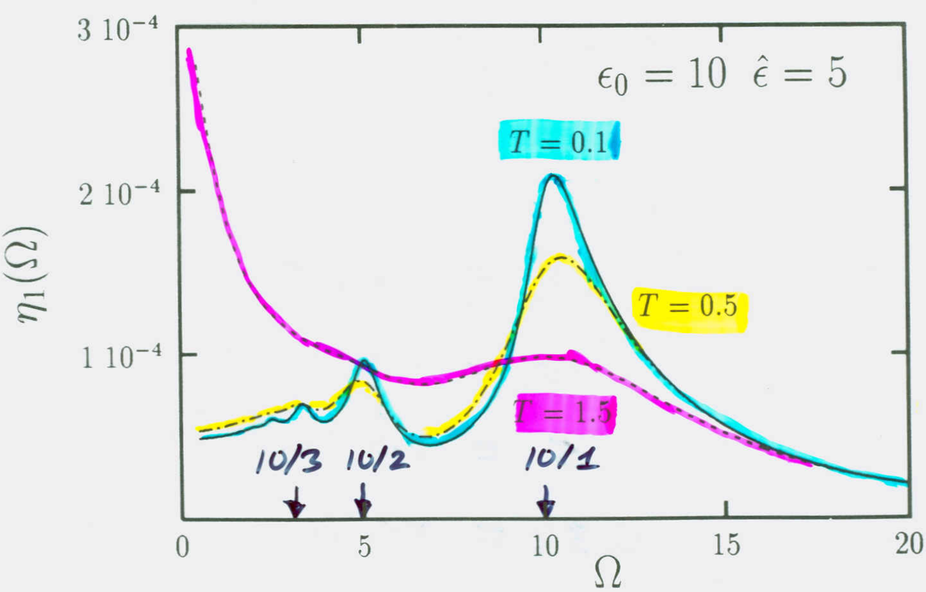
$$P_m(\Omega, \hat{\epsilon}) = \frac{\gamma_e}{\gamma_e - im\Omega}\frac{2\omega_c}{\pi}h_m(-im\Omega, \gamma) \tag{6}$$

with

$$\begin{aligned} h_{2k}(\lambda, \gamma) &= (-1)^k \sin(\pi\alpha)\int_0^\infty d\tau\, e^{-\lambda\tau - \gamma_e\tau/2 - S(\tau)} \\ &\times e^{-ik\Omega\tau}\sin(\epsilon_0\tau)J_{2k}\left(\frac{2\hat{\epsilon}}{\Omega}\sin\frac{\omega\tau}{2}\right), \\ h_{2k+1}(\lambda, \gamma) &= (-1)^k \sin\pi\alpha\int_0^\infty d\tau\, e^{-\lambda\tau - \gamma\tau/2 - S(\tau)} \\ &\times e^{-i(k+1/2)\Omega\tau}\cos(\epsilon_0\tau)J_{2k+1}\left(\frac{2\hat{\epsilon}}{\Omega}\sin\frac{\Omega\tau}{2}\right). \end{aligned} \tag{7}$$



$$\boxed{\alpha = 1/2}$$



$\Omega = \varepsilon_0 / n$

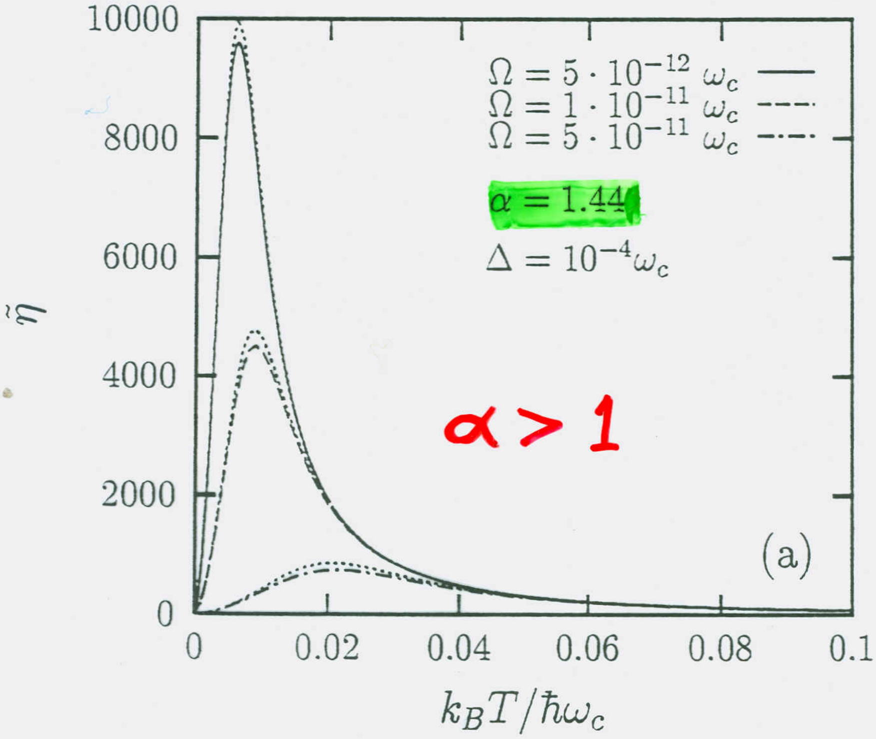
$10 = 10/1$

$5 = 10/2$

$3.33 = 10/3$

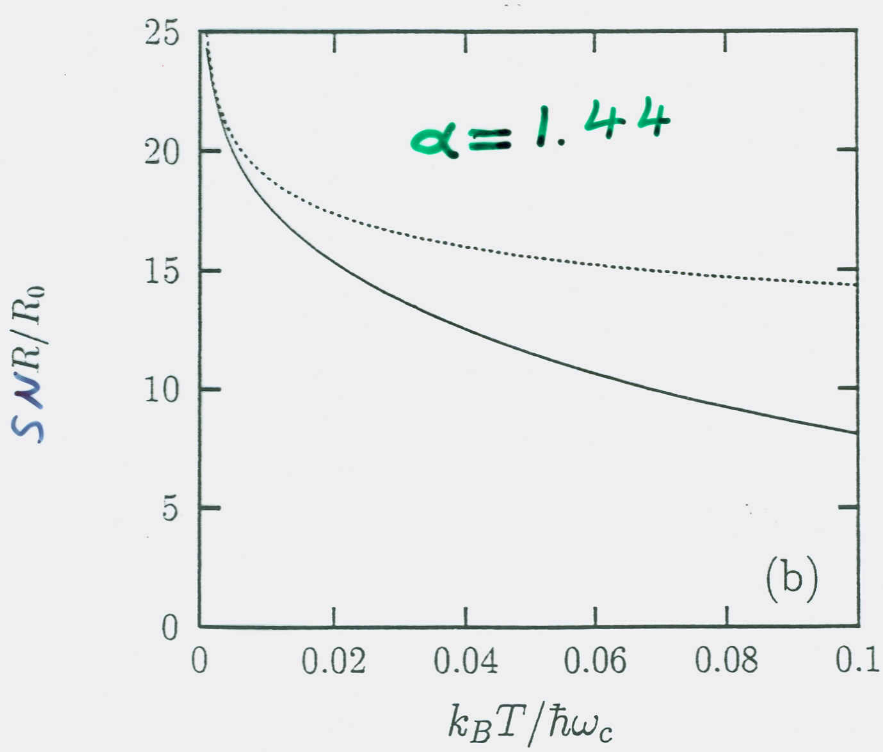
PEAKS: $\Omega = \varepsilon_0 / n$

• QSR FOR SYMMETRIC SYSTEMS $\varepsilon_0 = 0$!



$$\eta = \pi A_0^2 |\chi(\Omega)|^2 \sim \frac{\Gamma^2}{(k_B T)^2(\Gamma^2 + \Omega^2)}$$

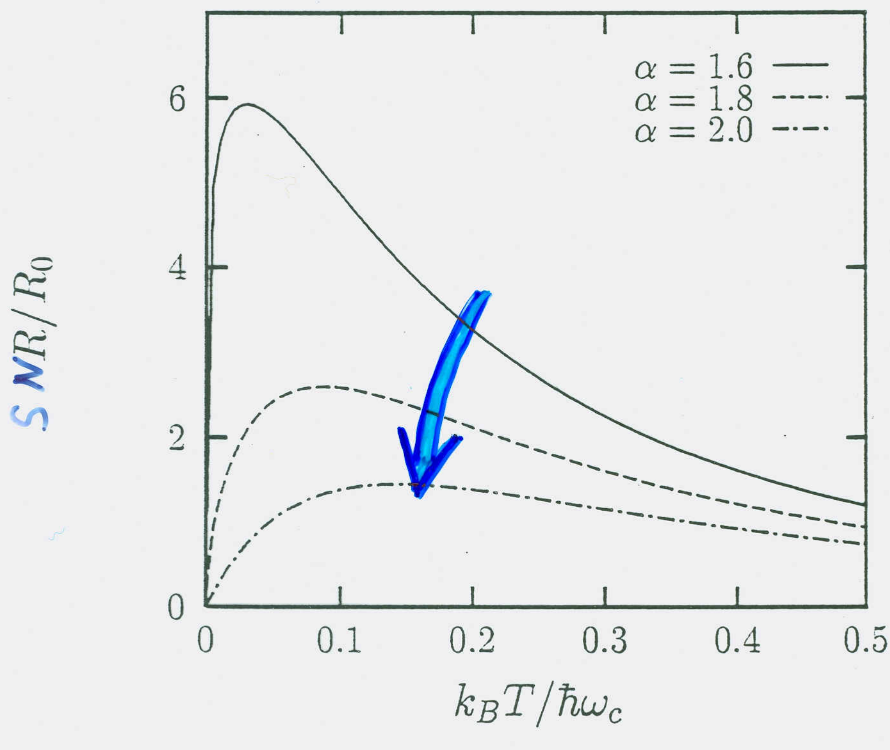
Q.-RATE $\Gamma \propto T^{2\alpha - 1}$

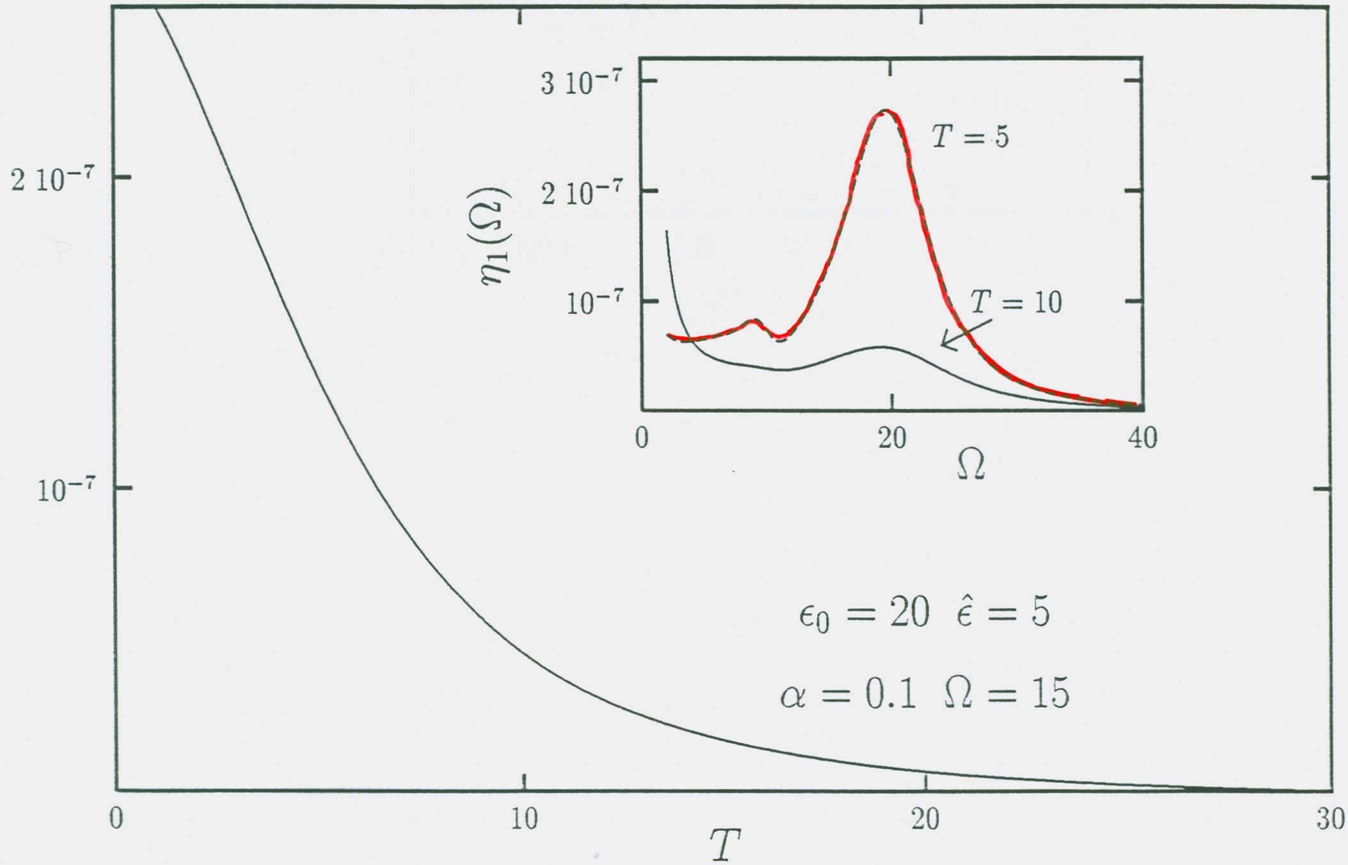


$$R = \frac{\pi A_0^2 |\chi(\Omega)|^2}{S^0_{xx}(\Omega)} \sim \frac{\Gamma}{(k_B T)^2}$$

BUT QSR FOR SNR

$\alpha > 3/2$



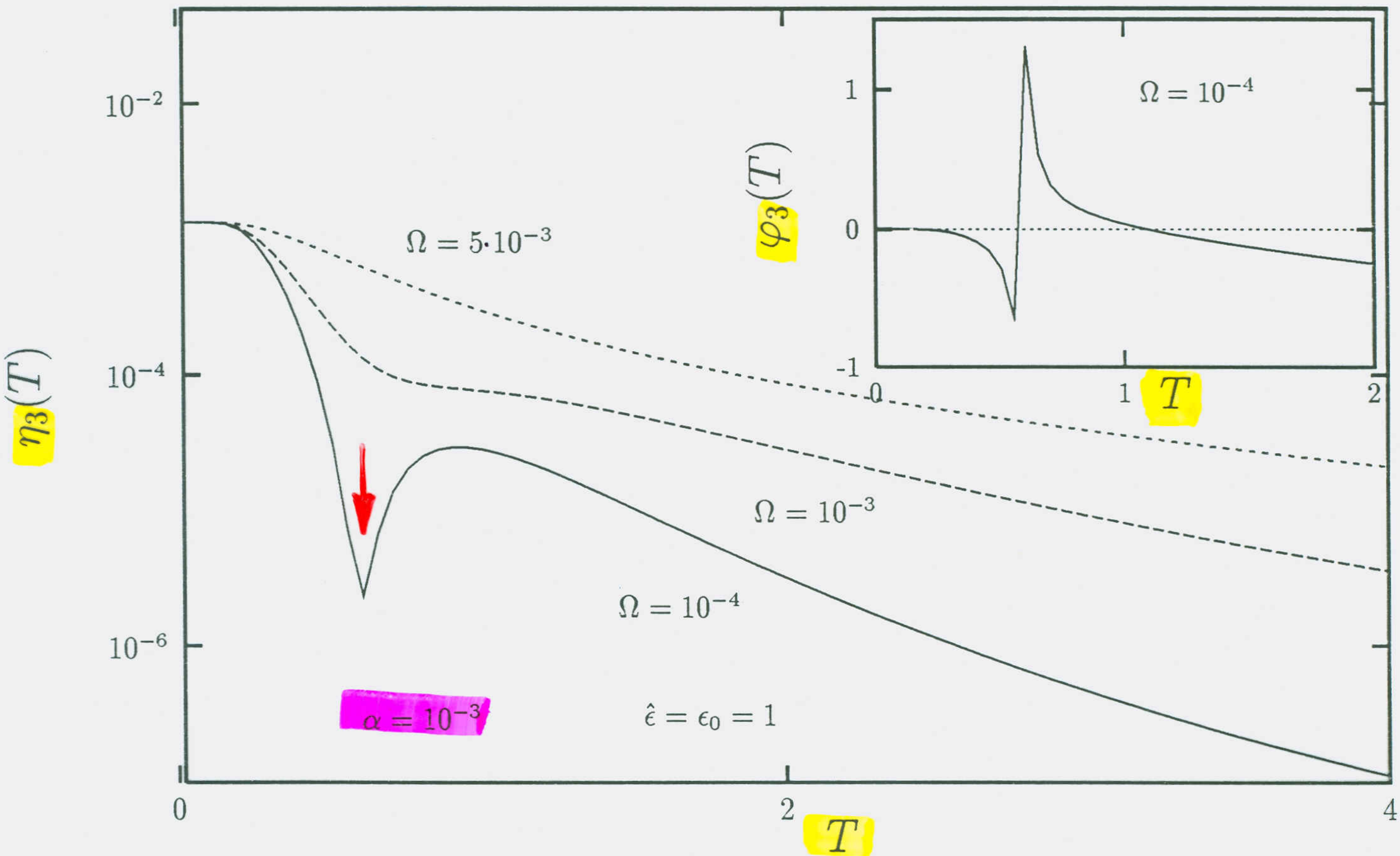


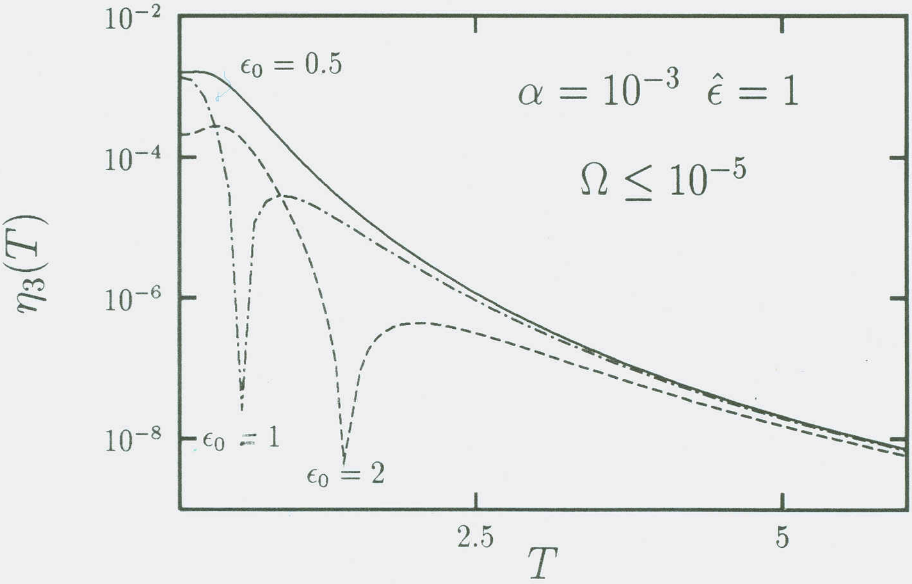
HIGH FREQUENCY

WITHIN NIBA

QUANTUM NOISE INDUCED SUPPRESSION

¡ $\mathcal{E}_0 \neq 0$!





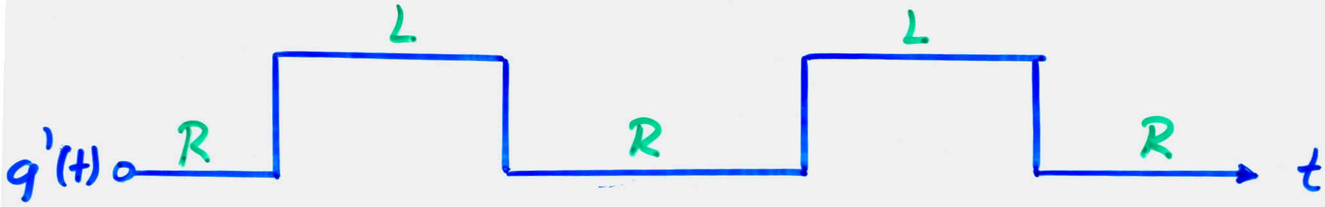
10^{-2}
10^{-4}
10^{-6}
10^{-8}
$\epsilon_0 = 0.5$
$\epsilon_0 = 1$
$\epsilon_0 = 2$
$\alpha = 10^{-3}$ $\hat{\epsilon} = 1$
$\Omega \leq 10^{-5}$
$\eta_3(T)$
2.5
5
T

$$q(t) = \sigma_t \frac{a}{2} \quad , \quad q'(t) = \sigma'_t \frac{a}{2}$$

$$\langle q(t) | \rho_r(t, t_0) | q'(t) \rangle$$

$$= \int \mathcal{D} q(t) \int \mathcal{D} q'(t) \, A[q] \, B_t[q] \, A^*[q'] \, B_t^*[q'] \, \mathcal{F}[q, q']$$

TLS DRIVING DISSIPATION

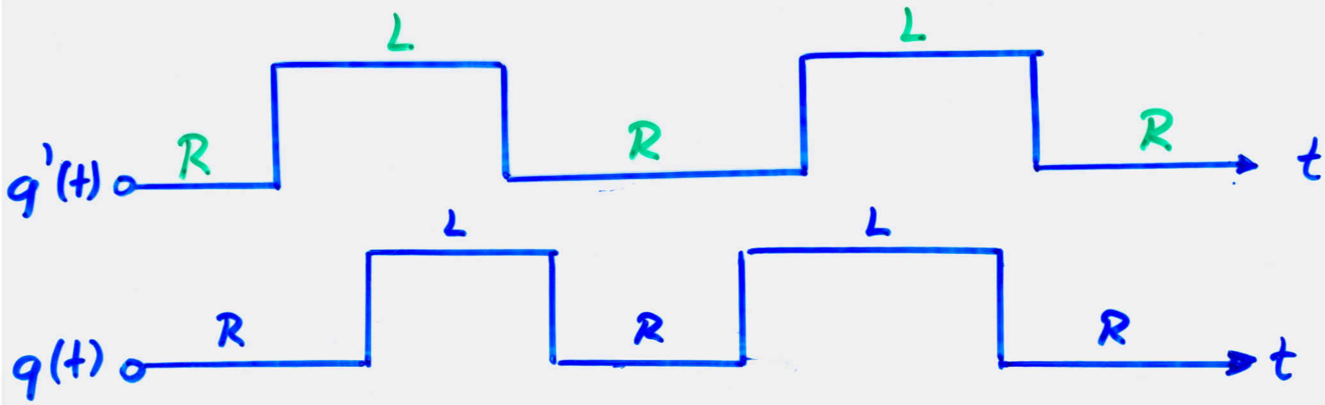


$$q(t) = \sigma_t \frac{a}{2} \quad , \quad q'(t) = \sigma'_t \frac{a}{2}$$

$$\langle q(t) | \rho_r(t, t_0) | q'(t) \rangle$$

$$= \int \mathcal{D} q(t) \int \mathcal{D} q'(t') A[q] B_t[q] A^*[q'] B_t^*[q'] \mathcal{F}[q, q']$$

TLS DRIVING DISSIPATION

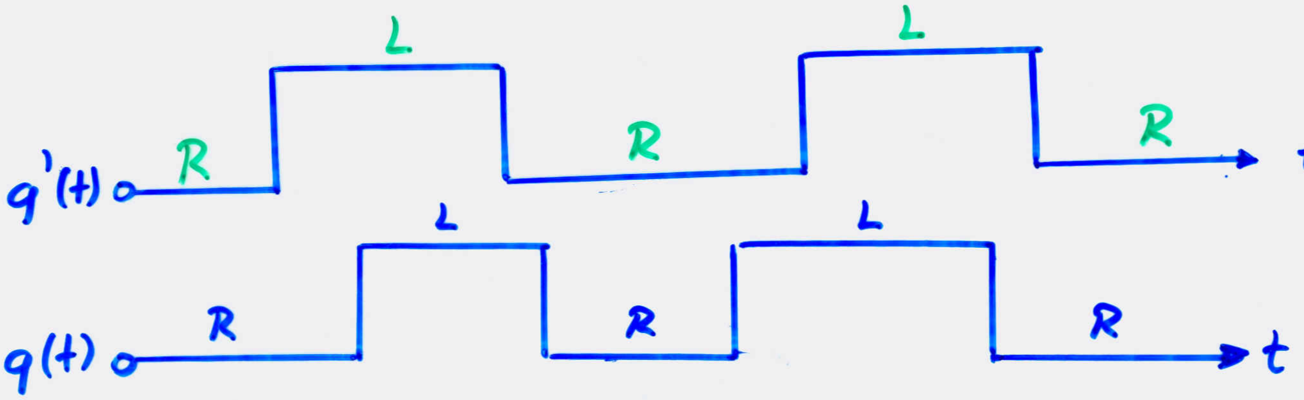


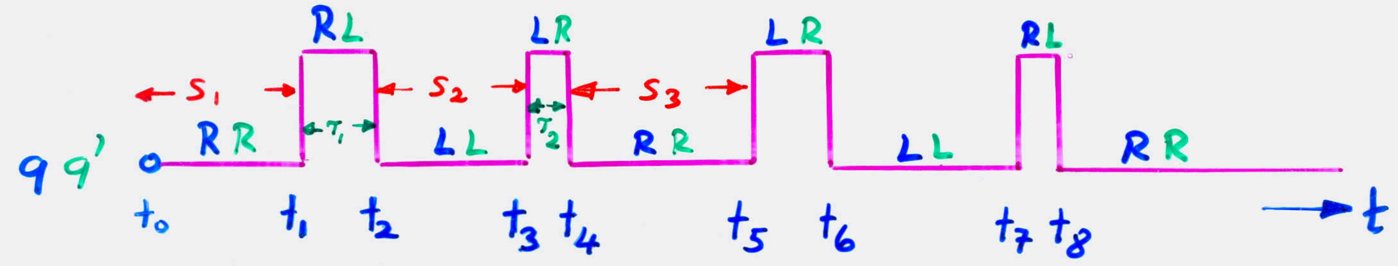
$q(t)=\sigma_t \frac{a}{2}$, $q'(t)=\sigma'_t \frac{a}{2}$

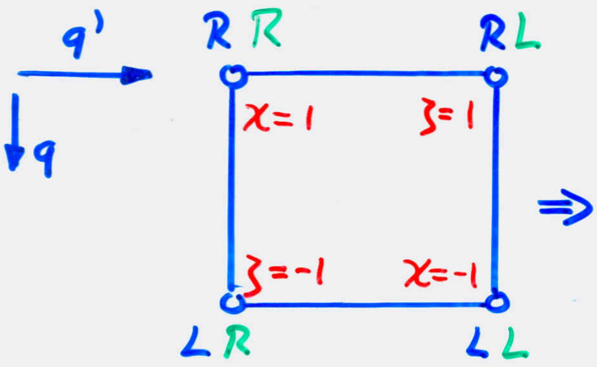
$\langle q(t)|\rho_r(t,t_0)|q'(t)\rangle$

$= \int \mathcal{D}q(t) \int \mathcal{D}q'(t')\, A[q]\, B_t[q]\, A^*[q']\, B_t^*[q']\, \mathcal{F}[q,q']$

TLS DRIVING DISSIPATION







$\int \mathcal{D}q \int \mathcal{D}q' \dots$

$\Rightarrow \sum_{n=0}^{\infty} \left(\int_0^t dt_{2n} \int_0^{t_{2n}} dt_{2n-1} \dots \int_0^{t_1} \right) \sum_{\{\xi_i\}} \sum_{\{\chi_i\}} \dots$

ρ_{LL} & ρ_{RR}

$\sum_{n=0}^{\infty} \left(\int_0^t dt_{2n+1} \int_0^{t_{2n+1}} dt_{2n} \dots \int_0^{t_2} dt_1 \right) \sum_{\{\xi_i\}} \sum_{\{\chi_i\}} \dots$

ρ_{LR} & ρ_{RL}

$S_j = t_{2j+1} - t_{2j}$

SOJOURNS

TIME IN DIAGONAL STATES

$\tau_j = t_{2j} - t_{2j-1}$

BLIPS

TIME IN OFF-DIAGONAL STATES

NONINTERACTING BLIP APPR.

$$\boxed{\text{NIBA}}$$

$\langle \tau \rangle \ll \langle s \rangle$ ∴ ONE CAN NEGLECT BATH-INDUCED BLIP-BLIP & BLIP-SOJOURN INT.

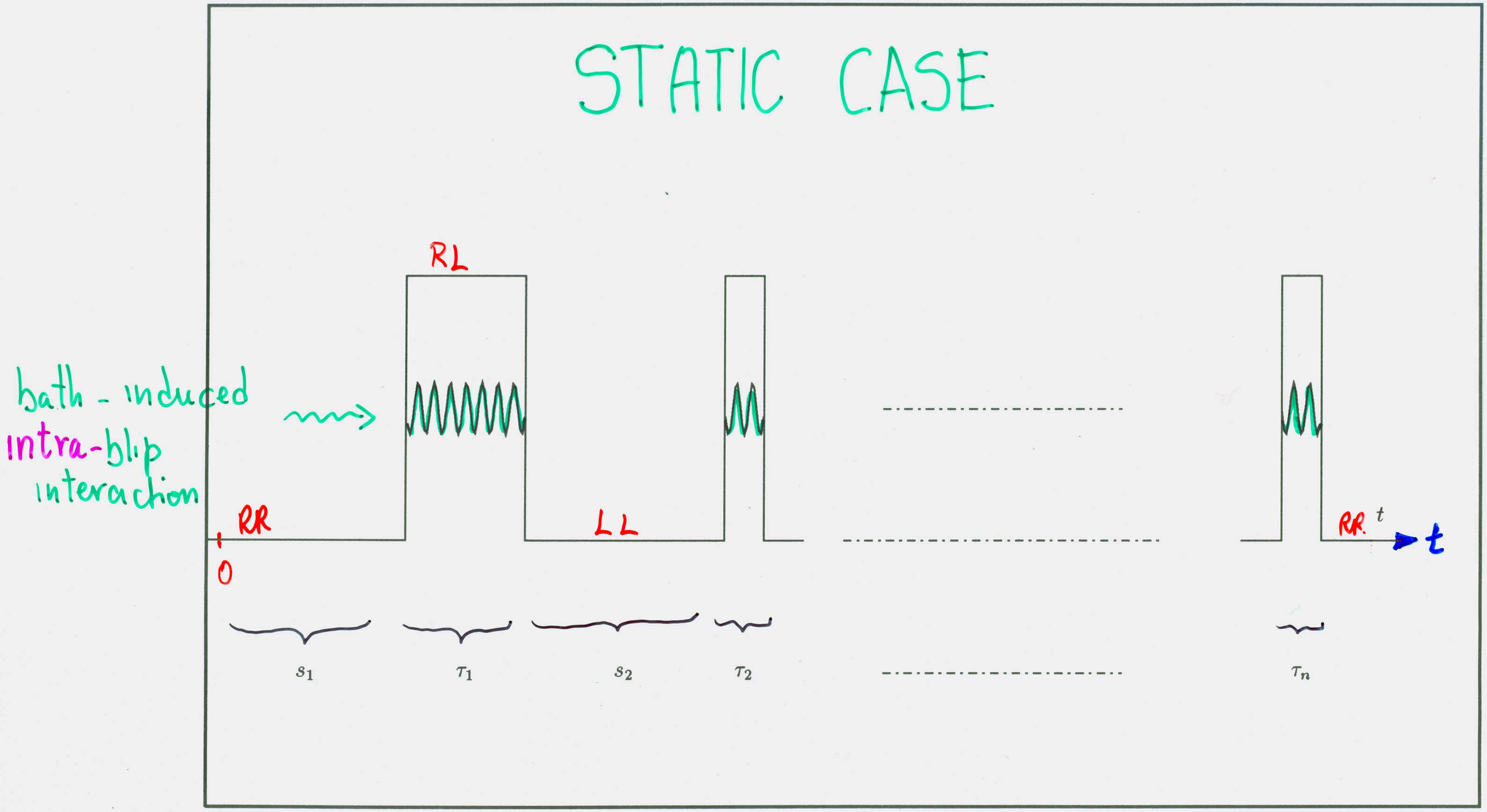
$\langle \tau \rangle \sim \frac{\hbar\beta}{\pi} \operatorname{arccot}\left\{\frac{\hbar\beta\varepsilon_0}{2\pi\alpha}\right\}$

$\langle s \rangle \sim \Gamma^{-1}$

ONE KEEPS INTRA-BLIP INT.

& DRIVING INDUCED $(\hat{\varepsilon} \neq 0)$ BLIB-SOJOURN & INTER-BLIB INT.

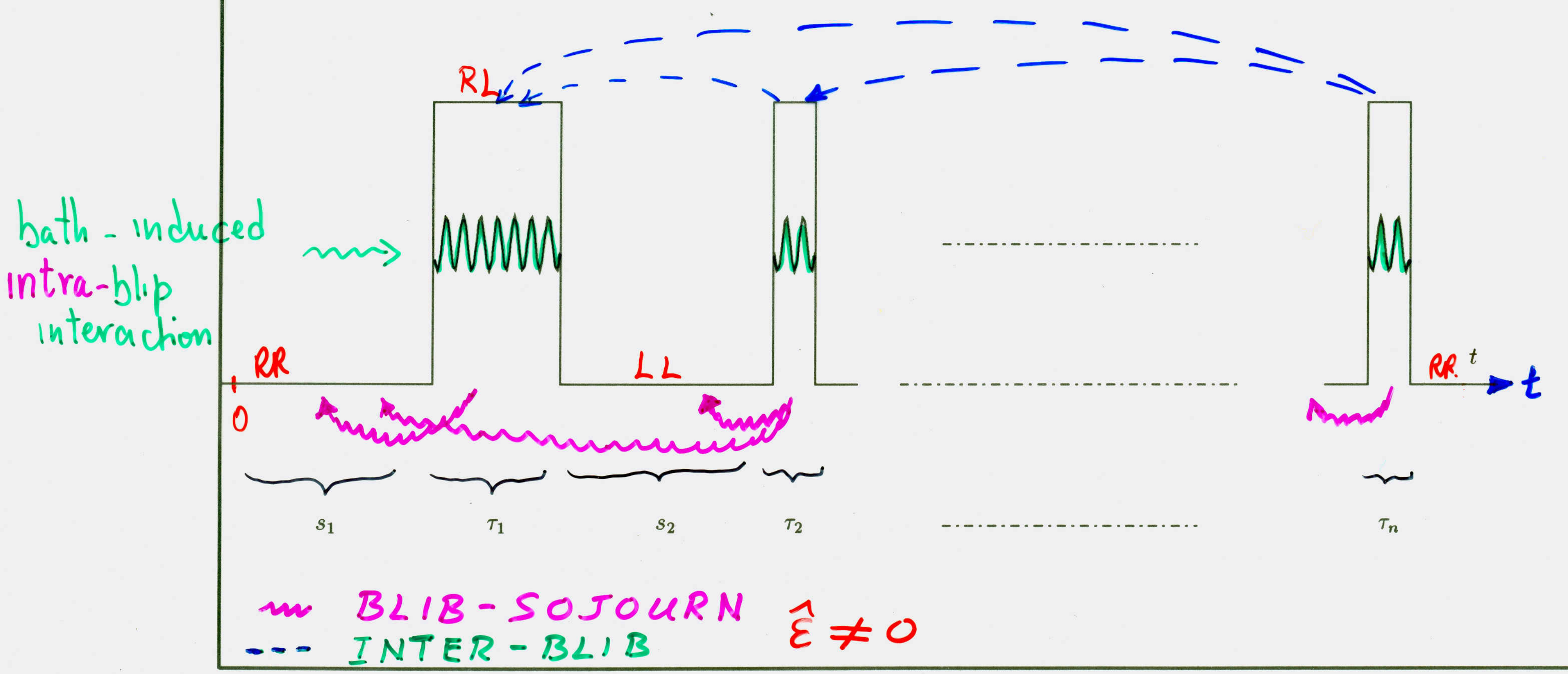
NONINTERACTING BLIP APPROXIMATION (NIBA)



$\langle \tau \rangle \ll \langle s \rangle \Rightarrow$ neglect bath-induced blip-blip and blip-sojourn interactions $\Rightarrow$ convolutive equations for $P(t)$

NONINTERACTING BLIP APPROXIMATION (NIBA)

NON-STATIC CASE



$\langle \tau \rangle \ll \langle s \rangle$ $\Rightarrow$ neglect bath-induced blip-blip and blip-sojourn interactions $\Rightarrow$ convolutive equatione for $P(t)$

NIBA

$$\langle \dot{\sigma}_z(t)\rangle = -\Delta^2\int_0^t ds\langle\sigma_z(s)\rangle\,\mathrm{Re}\,\{\Phi(t-s)\}\cos(g(t,s)) + + \Delta^2\int_0^t ds\,\mathrm{Im}\,\{\Phi(t-s)\}\sin(g(t,s)).$$

WHERE $\Phi(t) = \exp[-4Q_2(t) - 4iQ_1(t)]$,

$$Q_1(t) = \int_0^\infty d\omega\,\frac{J(\omega)}{\omega^2}\sin(\omega t),$$

$$Q_2(t) = \int_0^\infty d\omega\,\frac{J(\omega)}{\omega^2}(1-\cos(\omega t))\,\mathrm{ctgh}\left(\frac{\omega}{2k_B T}\right).$$

$$g(t,s) = \int_s^t d\tau[\varepsilon_0 + A\cos(\Omega\tau)] = \varepsilon_0(t-s) + \frac{A}{\Omega}\sin(\Omega t) - \frac{A}{\Omega}\sin(\Omega s)$$

M. GRIFONI, P. H., ... $\boxed{\text{PRE } \underline{52}\text{: 3596 (1995)}}$

↳ ANALYTIC SOLUTION

NO MARKOV   NO ROTAING WAVE

$$\mathcal{D}q\,\mathcal{D}q' \longrightarrow \sum_{n=0}^{\infty} \int_{t_0}^{t} dt_{2n} \int_{t_0}^{t_{2n-1}} dt_{2n-1} \cdots \int_{t_0}^{t_2} dt_1 \sum_{\{\xi_j=\pm 1\}} \sum_{\{\eta_j=\pm 1\}}$$

$$\Downarrow$$

$C_m^{\pm}$ driving correlat.

ξ_j

$\langle \sigma_z(t;t_0)\rangle$

$$\begin{cases} P(t,t_0) = 1 + \displaystyle\sum_{m=1}^{\infty} \left(-\frac{\Delta^2}{2}\right)^m \int_{t_0}^{t} dt_{2m} \cdots \int_{t_0}^{t_2} dt_1 \sum_{\{\xi_j=\pm 1\}} \left(F_m^{(+)} C_m^{(+)} + F_m^{(-)} C_m^{(-)} \right) \\[2ex] C_m^{(+)} = \cos\Phi_m \,, \quad C_m^{(-)} = \sin\Phi_m \,, \quad \Phi_m = \displaystyle\sum_{j=1}^{m} \xi_j \int_{t_{2j-1}}^{t_{2j}} dt'\,\eta(t') \end{cases}$$

$F_m^{(\pm)}$ influence functional

$$\Downarrow$$

$$\tilde{F}_m^{(\pm)} = F_m^{(\pm)} - \sum_{j=2}^{m} (-1)^j \sum_{m_1,\dots,m_j} F_{m_1}^{(+)} \cdots F_{m_j}^{(\pm)} \, \delta_{m_1+\dots+m_j,\, m}$$

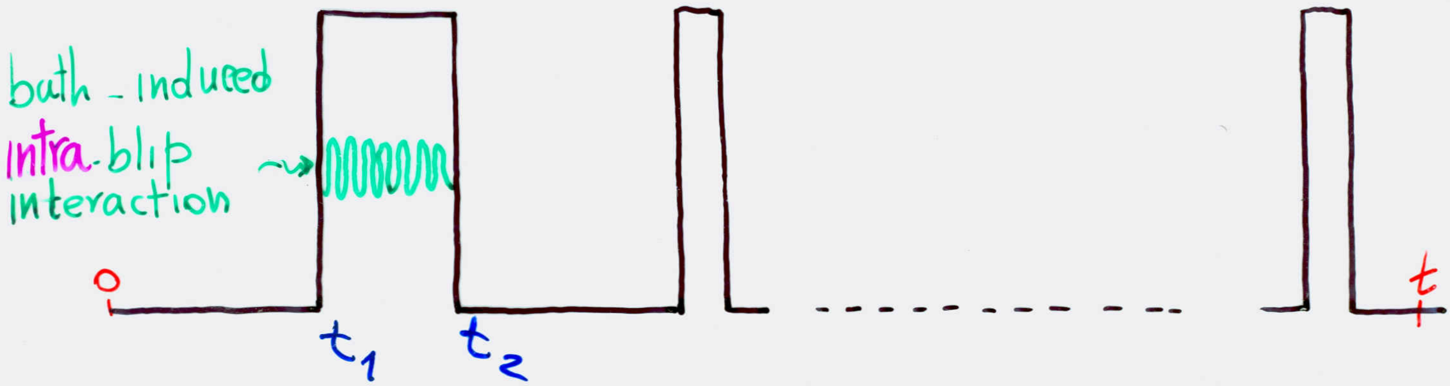
$$\boxed{\begin{aligned} \dot{P}(t,t_0) &= \int_{t_0}^{t} dt'\, K^{(+)}(t,t')\, P(t',t_0) - \int_{t_0}^{t} dt'\, K^{(-)}(t,t') \\ K^{(\pm)}(t,t') &= \sum_{m=1}^{\infty} \left(-\frac{\Delta^2}{2}\right)^m \int_{t'}^{t} dt_{2m-1} \cdots \int_{t'}^{t_3} dt_2 \sum_{\{\xi_j=\pm 1\}} \tilde{F}_m^{(\pm)} C_m^{(\pm)} \end{aligned}}$$

EXACT NONCONVOLUTIVE MASTER EQUATION !!!

NON INTERACTING BLIB APPROXIMATION (NIBA)

$\langle \tau \rangle \ll \langle s \rangle \Rightarrow$ neglect bath-induced blip-blip and blip-sojourn interactions

STATIC CASE



• $P(t)$ in convolution form $\longleftrightarrow$ $\dot{P}(t) = \int_0^t f^{(0)}(t')dt' - \int_0^t dt' g^{(0)}(t-t')P(t')$

$$\hat{P}(\lambda) = \frac{1 + F_\lambda^{(0)}(\varepsilon_0)/\lambda}{\lambda + G_\lambda^{(0)}(\varepsilon_0)}$$

$$\lim_{t\to\infty} P(t) = \frac{F_0^{(0)}}{G_0^{(0)}}$$

$$\begin{cases} F_\lambda^{(0)}(\varepsilon_0) = \Delta^2 \int_0^\infty d\tau\, e^{-\lambda\tau - S(\tau)} \operatorname{sen} \varepsilon_0 \tau \operatorname{sen} R(\tau) \\ G_\lambda^{(0)}(\varepsilon_0) = \Delta^2 \int_0^\infty d\tau\, e^{-\lambda\tau - S(\tau)} \cos \varepsilon_0 \tau \cos R(\tau) \end{cases}$$

$$S(\tau) + iR(\tau) = W(\tau)$$

$$\frac{d^2 W(\tau)}{d\tau^2} = K(\tau) = \langle \xi(\tau)\xi(0) \rangle_\beta$$

$$\xi(\tau) = \sum_\alpha c_\alpha x_\alpha(\tau)$$

RECURSIVE SOLUTION FOR $\hat{P}(\lambda)$ FOR GENERAL $\hat{\varepsilon} f(t)$

$$\begin{cases} F_\lambda(t) = \sum_m e^{-im\omega t} f_m(\lambda) \\ G_\lambda(t) = \sum_m e^{-im\omega t} g_m(\lambda) \end{cases}$$

$$\Downarrow$$

$$\begin{cases} \hat{P}(\lambda) = \dfrac{1}{\lambda + g_0(\lambda)} \sum_{j=0}^{\infty} H_j(\lambda) \\ H_{j+1}(\lambda) = \sum_{m\neq 0} \dfrac{-g_m(\lambda)}{\lambda + im\omega + g_0(\lambda+im\omega)} H_j(\lambda+im\omega) \\ H_0 = 1 + \sum_m \dfrac{f_m(\lambda)}{\lambda + im\omega} \end{cases}$$

- $\lambda + im\omega + g_0(\lambda+im\omega) = 0$ ⇝ transient dynamics
- $\lambda + im\omega = 0$ ⇝ long time dynamics

$$\lim_{t\to\infty} P(t) = P^{(as)}(t) = \sum_m P_m(\omega, \hat{\varepsilon})\, e^{-im\omega t}$$

$$P_m = \frac{i}{m\omega}\left[f_m(-im\omega) - \sum_{m'} g_{m-m'}(-im\omega) P_{m'} \right]$$

CONCLUSIONS

DYNAMICS OF THE <u>DRIVEN</u>

DISSIPATIVE TWO-STATE SYSTEM

$\Downarrow$

CONTROL OF TUNNELING

CONCLUSIONS

DYNAMICS OF THE <u>DRIVEN</u> DISSIPATIVE TWO-STATE SYSTEM

$\Downarrow$

CONTROL OF TUNNELING

Ω

HIGH

* DYNAMICAL DESTRUCTION OF TUNNELING
* UNIVERSAL DECAY RATES

MODERATE

* RESONANCE PHENOMENA
* LARGE OSCILLATIONS $\rightsquigarrow$ DELOCALIZATION
* FINITE ASYMMETRY $\varepsilon_0 \neq 0$ $\rightarrow$ QSR

LOW

* NOISE-INDUCED SUPPRESSION
* QSR CONTROLED BY INCOHERENT DYNAMICS