# What is temperature?

Peter Hänggi

Universität Augsburg


In collaboration with
Dr. Gerhard Schmid

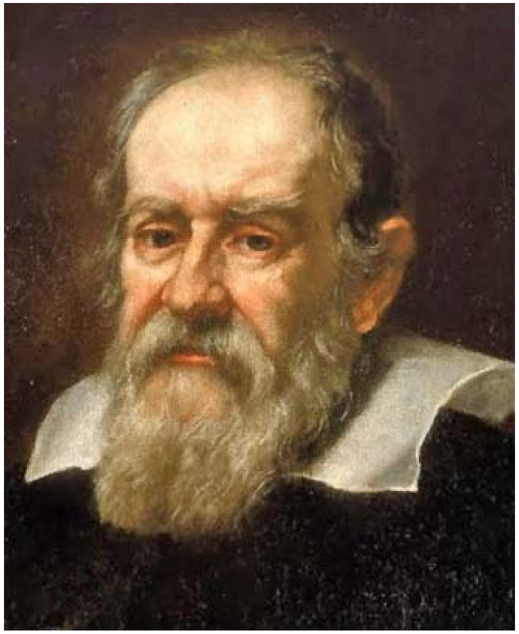
# Temperature sensation

- Subjective temperature sensation
- Memory effects
- Measuring the heat flow
- Heat – substance $\leftrightarrow$ Cold – substance

→ **What is temperature?**

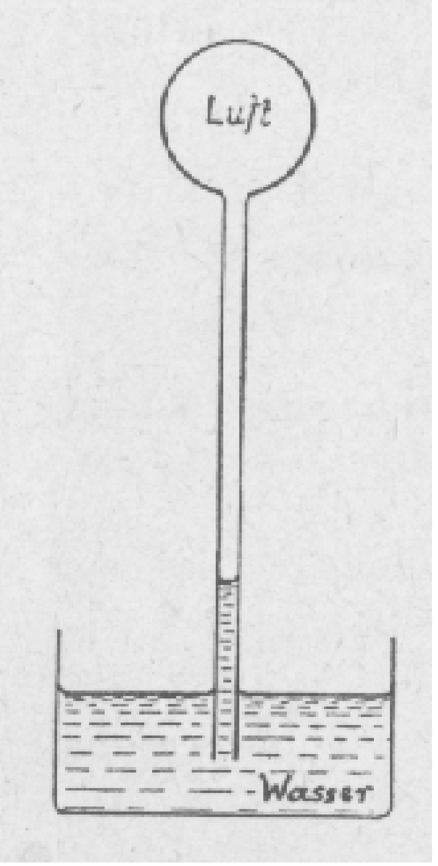
# Early temperature measurements

No scale for measuring
temperature between some reference points



**Galileo Galilei**
(1564 – 1642)

a thermoscope




1592: first documented (gas-) thermometer invented by Galileo Galilei

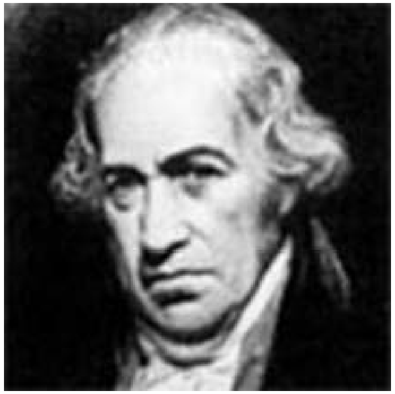
# First thermometer & temperature scales

1638: Robert Fludd – air thermometer &scale
~1700: linseed oil thermometer by Newton
1701: red wine as temperature indicator by Rømer
1702: Guillaume Amontons:  Absolute zero temperature?
1714: mercury and alcohol thermometer by Fahrenheit

Definition of temperature scales

**Sir Isaac Newton**
(1643  – 1727)

**Olaf Christensen Römer**
(1644 – 1710)



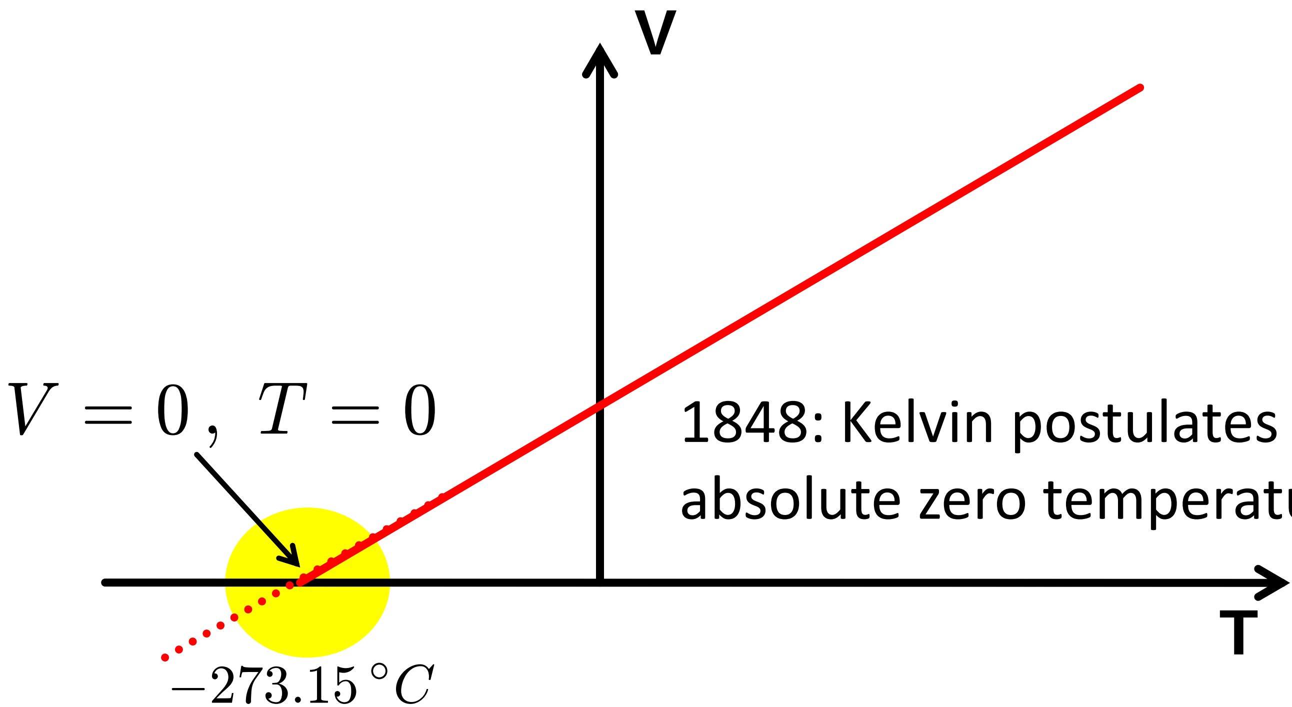
**Daniel Gabriel Fahrenheit**
(1686 – 1736)

**René Antoine Ferchault de Réaumur**
(1683 – 1757)

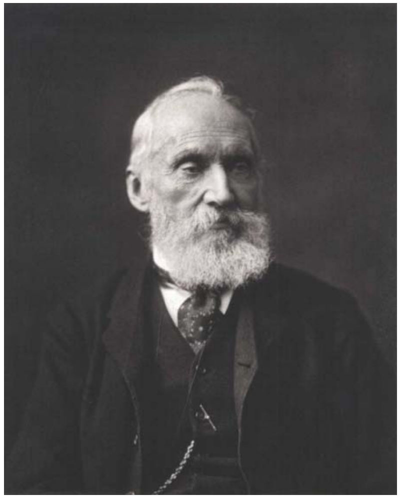
**Anders Celsius**
(1701 – 1744)

# Absolute Zero

Ideal gas law: $pV = N\,k_{\mathrm{B}}\,T$




1848: Kelvin postulates an absolute zero temperature



**William Thomson — Lord Kelvin**
(1824 – 1907)

*It is impossible by any procedure to reduce the temperature of a system to zero in a finite number of operations.*

# Absolute temperature scale

## Kelvin – scale (SI)

Reference: absolute zero temperature
& triple point of water (273.16K; 611.73 Pa)

Scale division: 1K is equal to the fraction $(273.16)^{-1}$ of the thermodynamic temperature of the triple point of water

$$\Delta T = 1K \equiv 1^\circ C \ (\text{degree Celsius})$$

# How to determine the absolute thermodynamic temperature


VORLESUNGEN

ÜBER

THERMODYNAMIK

VON

DR. MAX PLANCK,
PROFESSOR DER THEORETISCHEN PHYSIK
AN DER UNIVERSITÄT BERLIN.

VIERTE AUFLAGE.

MIT FÜNF FIGUREN IM TEXT.

LEIPZIG,
VERLAG VON VEIT & COMP.
1913

$$\log \frac{T}{T_0} = \int_{t_0}^{t} \frac{\left(\frac{\mathrm{d}v}{\mathrm{d}t}\right)_p \mathrm{d}t}{v + c'_p \left(\frac{\mathrm{d}t}{\mathrm{d}p}\right)}$$

$t$ : arbitrary temperature scale

$v$ : volume

$c'_p$ : specific heat at constant pressure

**" ... wo nun wieder unter dem Integralzeichen lauter direkt und verhältnismäßig bequem meßbare Größen stehen. ..."**

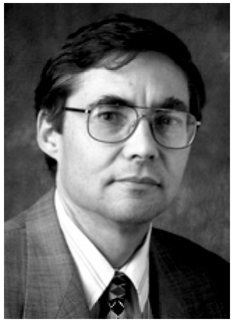
M. Planck

# Low temperature milestones

1908: liquid helium; 5 K

**Heike Kamerlingh Onnes**
(1853 – 1926)

1995: Bose-Einstein-Condensate; 20 nK

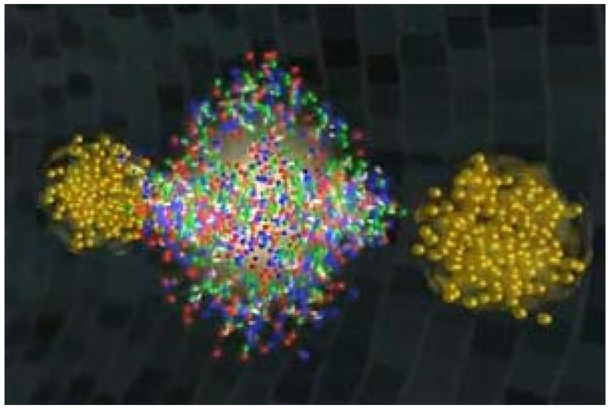

Eric A. Cornell
Carl E. Wieman
Wolfgang Ketterle

2003: BEC; 450 pK – W. Ketterle

# Temperature Extrema

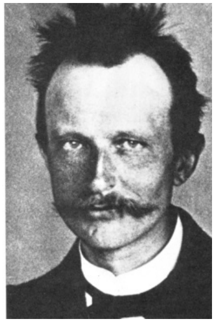
| | | | | |
|---|---|---|---|---|
| **-89,2°C** | 21. Juli 1983 | Wostok-Station | Antarktis | Gilt als der Kältepol der Erde; nicht offizielle bestätigter Wert: -91,5°C von 1997 |

| | | | | |
|---|---|---|---|---|
| **70,7°C** | 2007 | Iranische Wüste | Lut, Iran | Hitzeweltrekord übertroffen; Messung allerdings per Satellitenbeobachtung |



p + n -> quark gluon plasma
with gold ion collisions in
Relativistic Heavy Ion Collider (RHIC)

$4 \times 10^{12}$ °C



# Planck units

| Constant | Symbol | Value in SI units |
|---|---|---|
| Speed of light | c | 299 792 458 m $s^{-1}$ |
| Gravitational constant | G | 6.67429 · $10^{-11}$ $m^3$ $kg^{-1}$ $s^{-2}$ |
| reduced Planck's constant | $\hbar$ | 1.054571628 ·$10^{-34}$ J s |
| Coulomb force constant | $(4\pi\epsilon_0)^{-1}$ | 8 987 551 787.368 kg $m^3$ $s^{-2}$ $C^{-2}$ |
| Boltzmann constant | $k_{\mathrm{B}}$ | 1.3806504·$10^{-23}$ J $K^{-1}$ |

| Name | Expression | SI equivalents |
|---|---|---|
| **Planck temperature** | $T_{\mathrm{P}} = \sqrt{\hbar c^5 G^{-1} k^{-2}}$ | 1.41168· $10^{32}$ K |
| Planck length | $l_{\mathrm{P}} = \sqrt{\hbar G c^{-3}}$ | 1.61625· $10^{-35}$ m |
| Planck mass | $m_{\mathrm{P}} = \sqrt{\hbar c G^{-1}}$ | 2.17644· $10^{-8}$ kg |
| Planck time | $t_{\mathrm{P}} = \sqrt{\hbar G c^{-5}}$ | 5.39124· $10^{-44}$ s |

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**… ihre Bedeutung für alle Zeiten und für alle, auch außerirdische und außermenschliche Kulturen notwendig behalten unnd welche daher als natürliche Maßeinheiten bezeichnet werden können …**

**… These necessarily retain their meaning for all times and for all civilizations, even extraterrestrial and non-human ones, and can therefore be designated as natural units …**

(Planck, 1899)

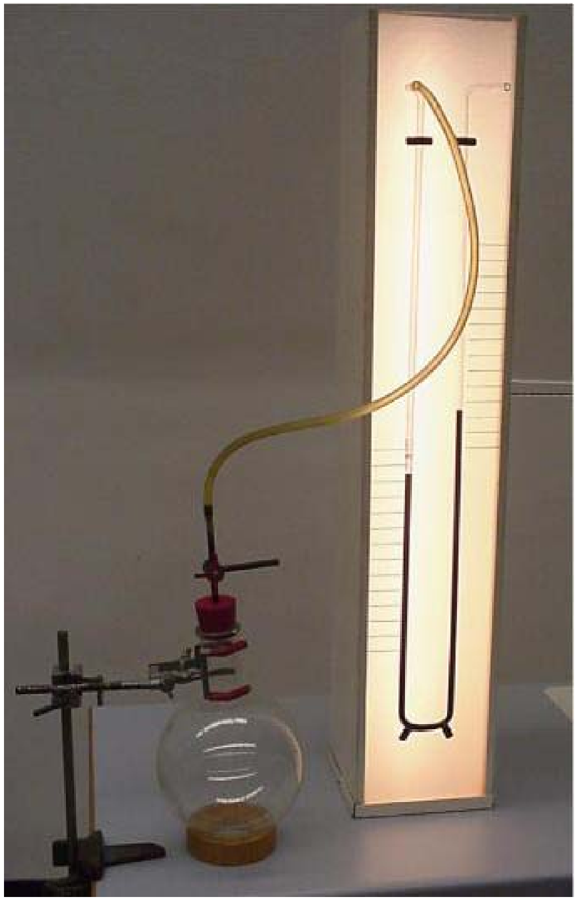
# *The highest temperature you can see*

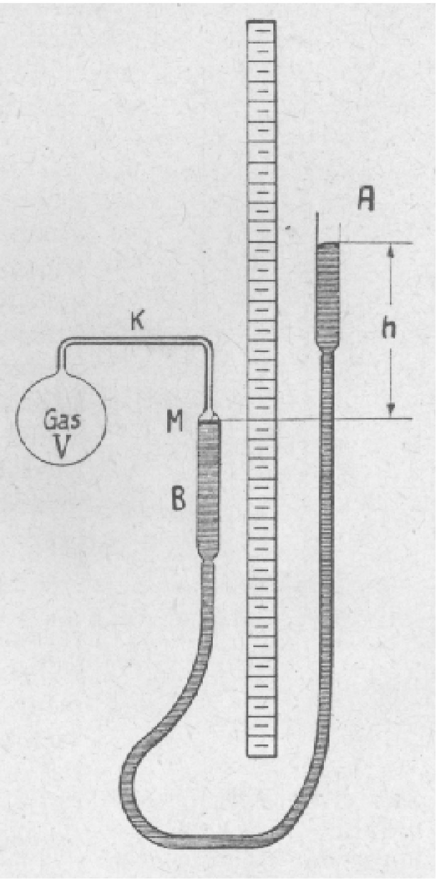
**Lightning:**

**30 000 °C**

# Thermometers

# Gas Thermometers



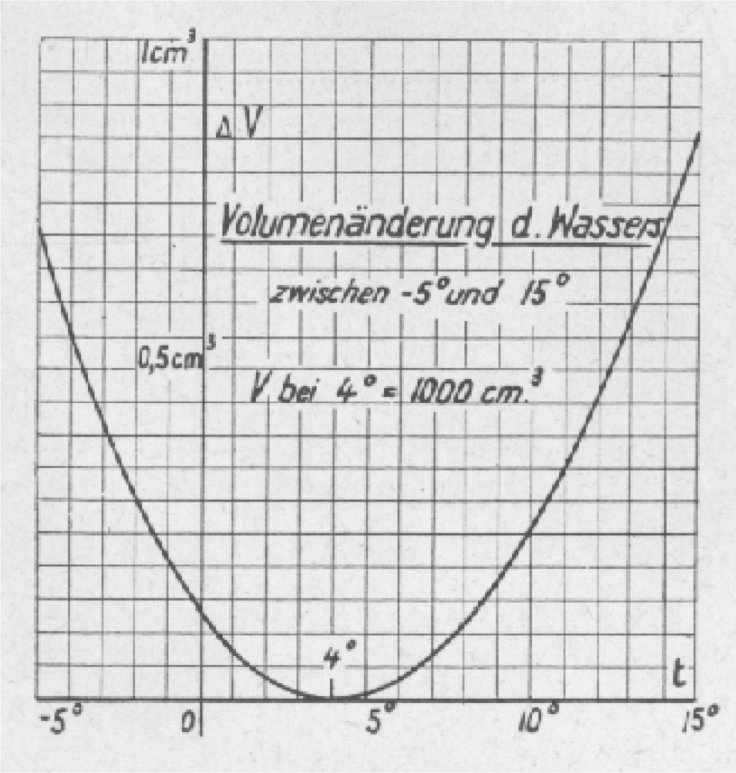



Ideal gas:

$$T = \left( \frac{V}{N\,k_{\mathrm{B}}} \right) p$$

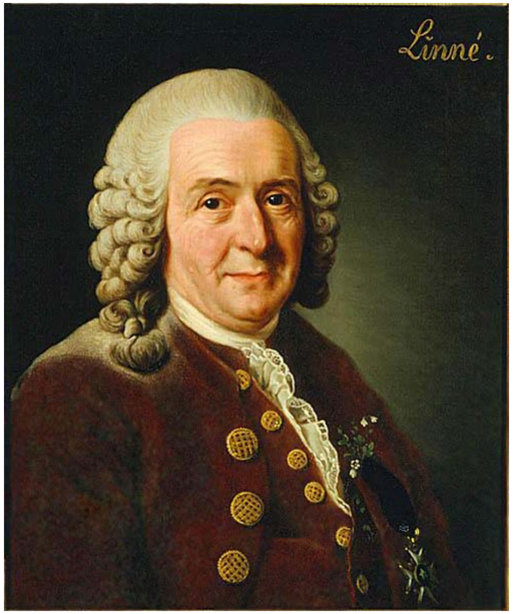
# Galilei Thermometer

Volume change of water




Upon increasing T → 4°C ⟶ Water volume shrinks

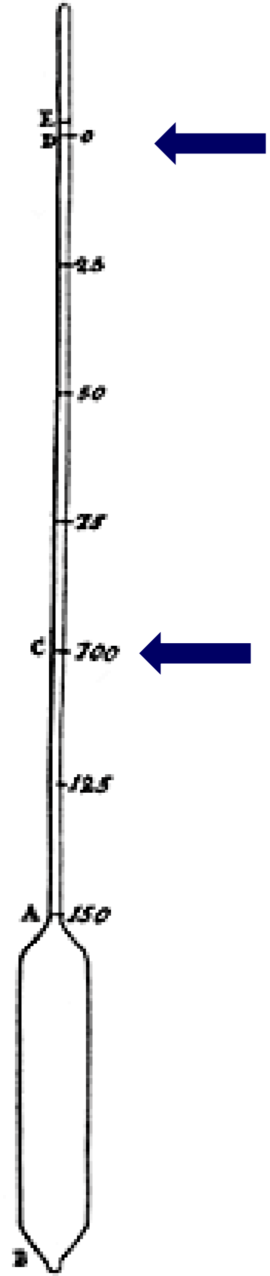
# Linneaus thermometer




**Carl von Linné**
(1707 – 1778)

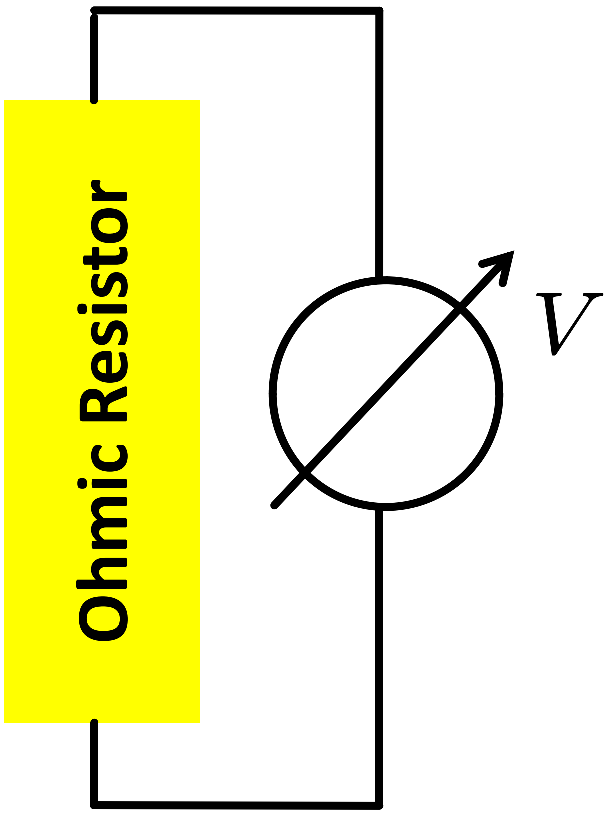
Reversed the Celsius scale

1744: broken on delivery
1745: botanical garden in Uppsala




Anders Celsius

# Noise Thermometer




Johnson – Nyquist noise

$$\mathrm{PSD}_V(\omega) = 2k_\mathrm{B}TR$$

**-- classical regime only --**

$\mathrm{PSD}_V$ : power spectral density of the voltage signal

$k_\mathrm{B}$ : Boltzmann constant

$R$ : resistance

# Quantum fluctuation-dissipation theorem

$$PSD_I(\omega) = \hbar\omega \coth\left(\frac{\hbar\omega}{2k_\mathrm{B}T}\right) \mathrm{Re}Y(\omega)$$

$$= 2\left(\frac{\hbar\omega}{2} + \frac{\hbar\omega}{\exp(\beta\hbar\omega) - 1}\right) \mathrm{Re}Y(\omega)$$

$k_\mathrm{B}T \gg \hbar\omega$

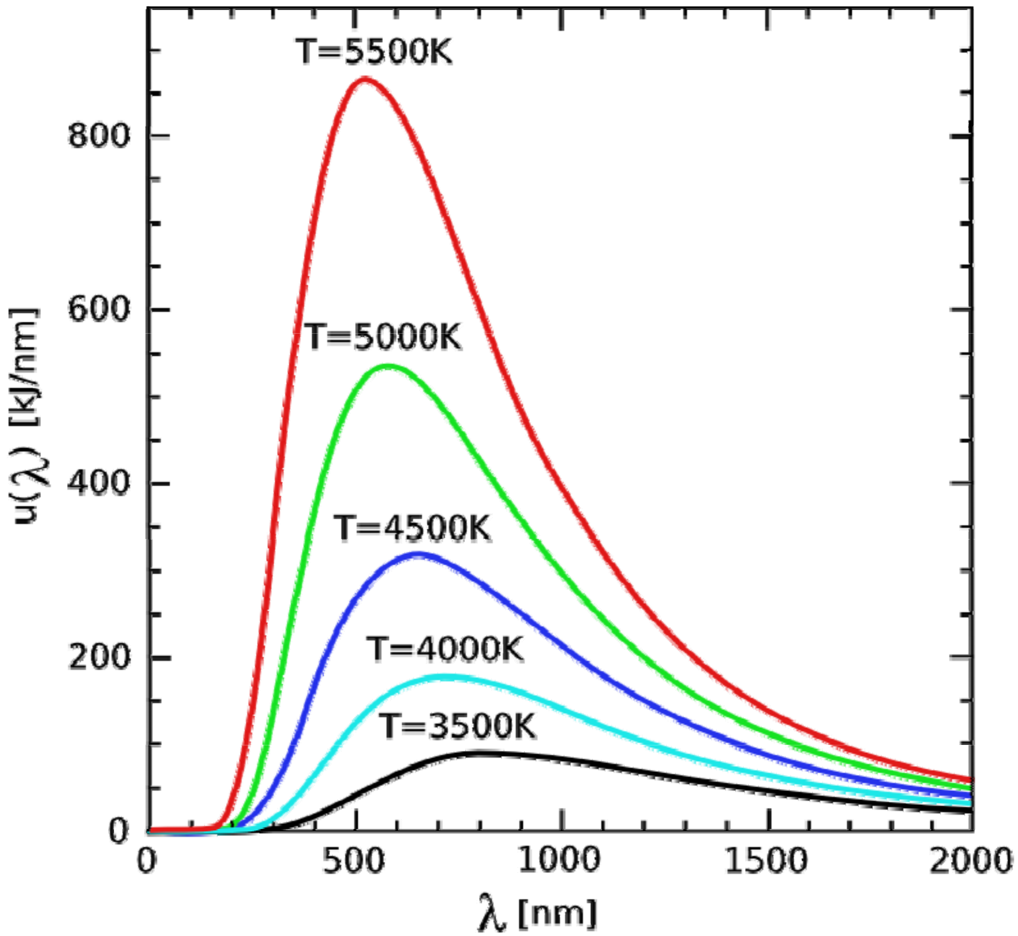
$k_\mathrm{B}T \ll \hbar\omega$

$$PSD_I(\omega) = 2k_\mathrm{B}T\,\mathrm{Re}Y(\omega)$$

$$PSD_I(\omega) = \hbar\omega\,\mathrm{Re}Y(\omega)$$

# Black body radiation

Planck's law [1901]: $u(\lambda, T) = \dfrac{8\pi hc}{\lambda^5} \dfrac{1}{\exp(\frac{hc}{\lambda k_\mathrm{B} T}) - 1}$




$u(\lambda, T)$ : spectral energy density

$\lambda$ : wavelength

$h$ : Planck constant

$c$ : speed of light

$k_\mathrm{B}$ : Boltzmann constant

Stefan – Boltzmann law:

$$E \propto T^4$$

**Thermometer !**

# Small system – single realizations

**1-st law:** $\Delta E_\omega = \triangle Q_\omega + \triangle W_\omega$ **! random quantities ¡**

$\langle ... \rangle \Rightarrow \Delta U = \triangle Q + \triangle W$

Note: $\triangle Q = \Delta U - \triangle W$ $\Delta U = ?\ \&\ \triangle W = ?$

weak coupling: $\triangle Q = -\triangle Q_{\mathrm{bath}}$

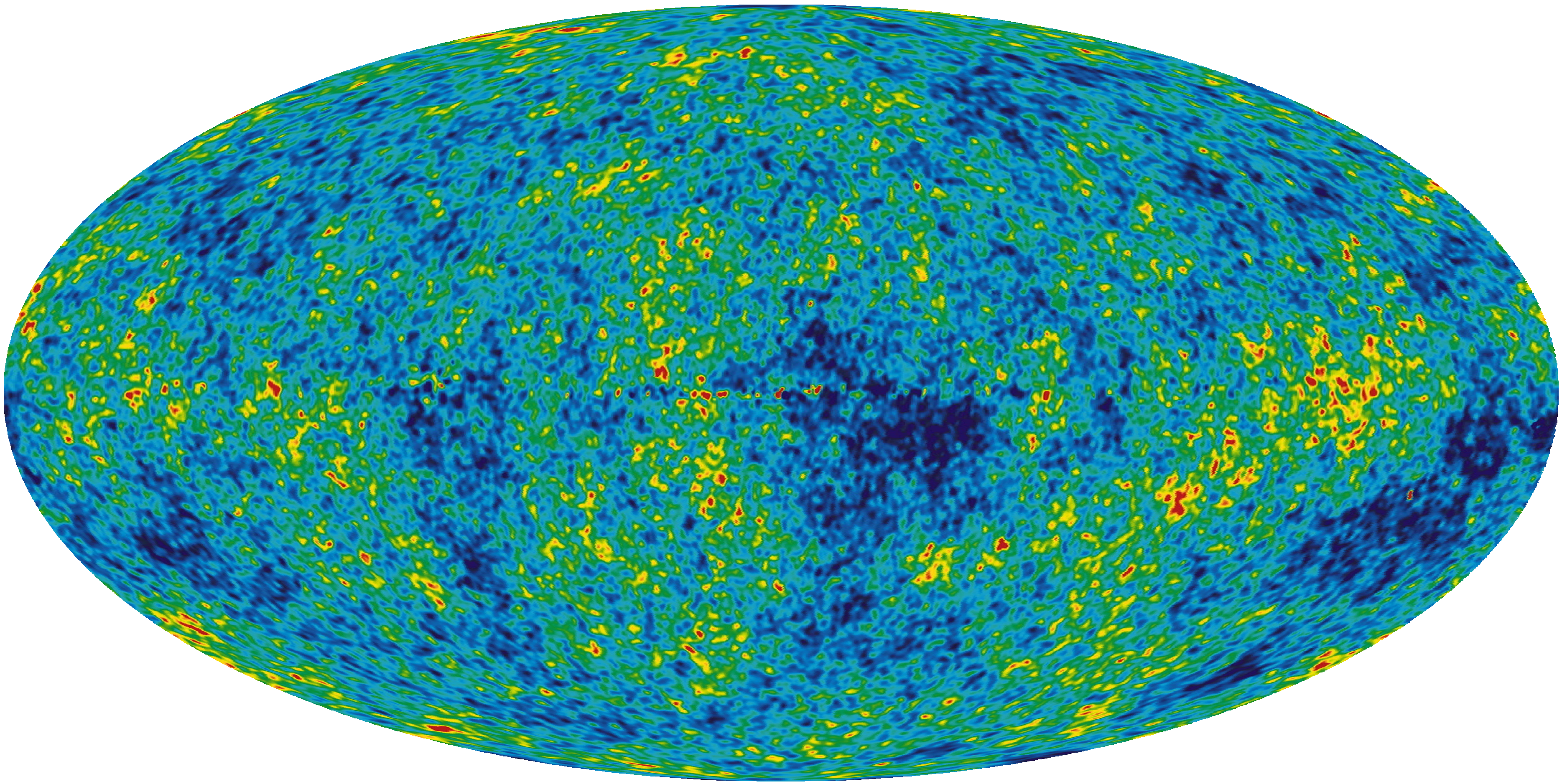
**2-nd law:** $\Delta S_{A\to B}^{\mathrm{REV}} \geq \frac{\triangle Q}{T}$

$$\Delta S_{A\to B}^{\mathrm{REV}} = \frac{\triangle Q}{T} + \sum^{\mathrm{IRREV}} (\geq 0)$$

$$= \frac{\Delta U - \triangle W}{T} + \sum^{\mathrm{IRREV}}$$

$$= \frac{-\triangle Q_{\mathrm{bath}}}{T} + \sum^{\mathrm{IRREV}}$$ ; for weak coupling

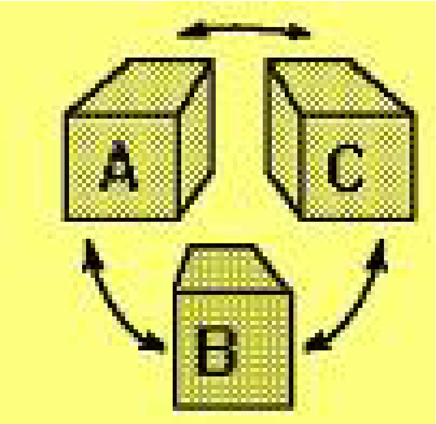
# Cosmic background temperature



$T = 2.725 \pm .... K$

# Four Grand Laws of Thermodynamics

# Zeroth Law




Transitivity !

$$\text{A in equilibrium with B:} \quad f_{\mathrm{AB}}(p_{\mathrm{A}}, V_{\mathrm{A}}; p_{\mathrm{B}}, V_{\mathrm{B}}, \ldots) = 0$$

$$\text{B in equilibrium with C:} \quad f_{\mathrm{BC}}(p_{\mathrm{B}}, V_{\mathrm{B}}; p_{\mathrm{C}}, V_{\mathrm{C}}, \ldots) = 0$$

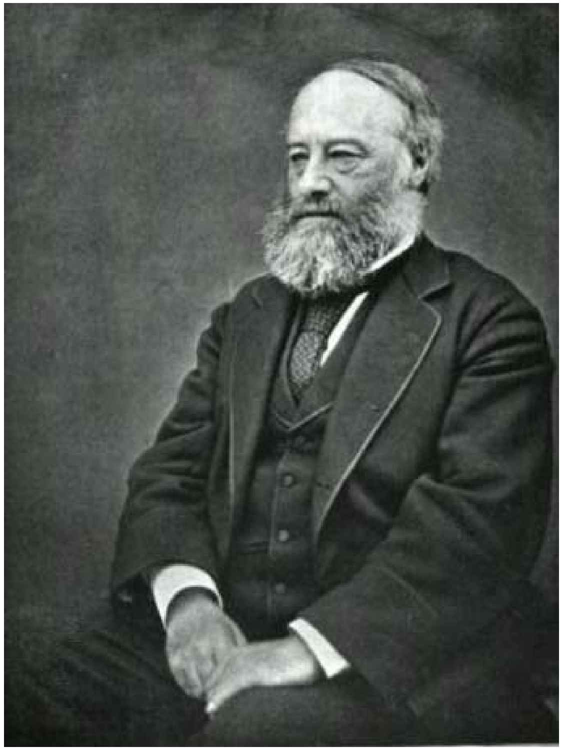
$$\Rightarrow \text{A in equilibrium with C} \Leftrightarrow f_{\mathrm{AC}}(p_{\mathrm{A}}, V_{\mathrm{A}}; p_{\mathrm{C}}, V_{\mathrm{C}}, \ldots) = 0$$

Allows the formal introduction of a temperature:

$$T = T_{\mathrm{A}}(p_{\mathrm{A}}, V_{\mathrm{A}}; \ldots) = T_{\mathrm{B}}(p_{\mathrm{B}}, V_{\mathrm{B}}; \ldots) = T_{\mathrm{C}}(p_{\mathrm{C}}, V_{\mathrm{C}}; \ldots)$$

# First Law

**Julius Robert von Mayer**
(1814 – 1878)



**James Prescott Joule**
(1818 – 1889)

**Hermann von Helmholtz**
(1821 – 1894)

# First Law – Energy Conservation

$$\Delta U = \Delta Q + \Delta W$$

$\Delta U$ change in internal energy

$\Delta Q$ heat added on the system

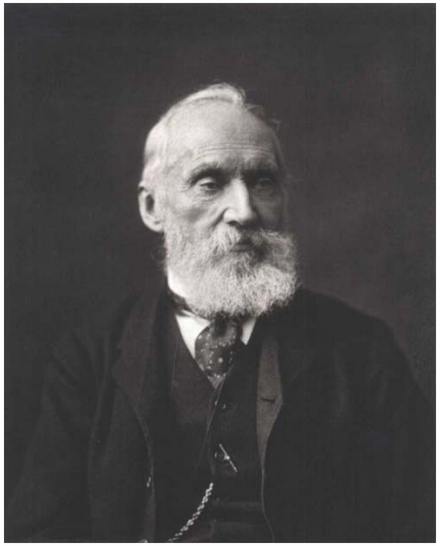
$\Delta W$ work done on the system

H. von Helmholtz: "Über die Erhaltung der Kraft" (1847)

$$\Delta U = (T\Delta S)_{\text{quasi-static}} - (p\Delta V)_{\text{quasi-static}}$$

# Second Law

**Rudolf Julius Emanuel Clausius**
(1822 – 1888)



**William Thomson alias Lord Kelvin**
(1824 – 1907)

Heat generally cannot spontaneously flow from a material at lower temperature to a material at higher temperature.

No cyclic process exists whose sole effect is to extract heat from a single heat bath at temperature T and convert it entirely to work.

$$\delta Q = T \mathrm{d}S \quad \text{(Zürich, 1865)}$$

# Thermodynamic Temperature

$$\delta Q^{\mathrm{rev}} = T\, dS \leftarrow \text{thermodynamic entropy}$$

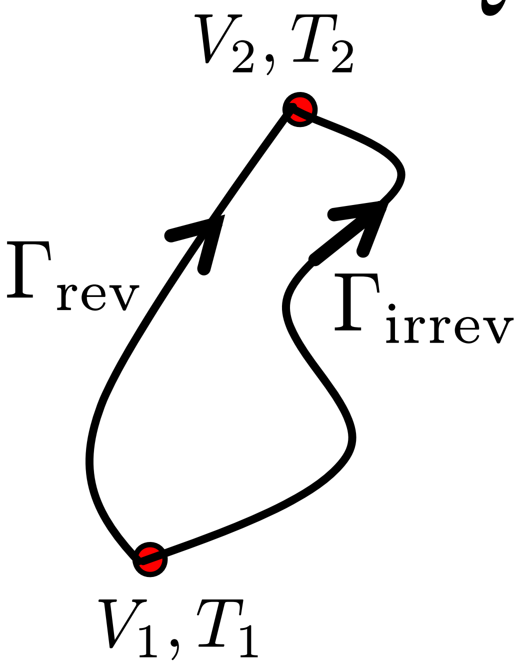
$$S = S(E, V, N_1, N_2, \ldots; M, P, \ldots)$$

$S(E, \ldots)$: (continuous) & differentiable and monotonic function of the internal energy $E$

$$\left(\frac{\partial S}{\partial E}\right)_{\ldots} = \frac{1}{T}$$

# Entropy S – content of *transformation*
# „Verwandlungswert“

$$dS = \delta Q^{\mathrm{rev}} / T\,; \quad \delta Q^{\mathrm{irrev}} < \delta Q^{\mathrm{rev}}$$



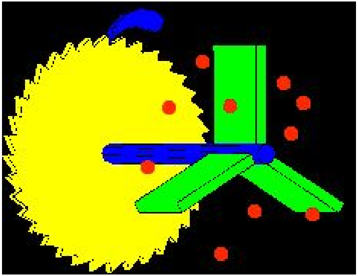

$$\oint_C \frac{\delta Q}{T} \leq 0$$

$$C = \Gamma_{\mathrm{rev}}^{-1} + \Gamma_{\mathrm{irrev}}$$

$$\left.\begin{aligned} S(V_2, T_2) - S(V_1, T_1) &\geq \int_{\Gamma_{\mathrm{irrev}}} \frac{\delta Q}{T} \\ S(V_2, T_2) - S(V_1, T_1) &= \int_{\Gamma_{\mathrm{rev}}} \frac{\delta Q}{T} \end{aligned}\right\} \quad \frac{\partial S}{\partial t} \geq 0 \quad \textbf{NO !}$$

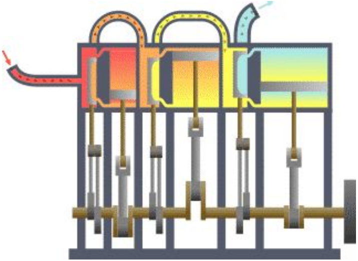
# Perpetuum mobile of the second kind ?

**One heat reservoir**



**NO !**

**Two heat reservoirs**



# Third Law

Walter Hermann NERNST
(1864 - 1941)

„mein Wärmesatz“
(during his lecture August 15, 1905)

$$\frac{\Delta H - \Delta G}{T} = \Delta S \longrightarrow 0 \quad \text{as} \quad T \longrightarrow 0$$

# Approaching zero temperature

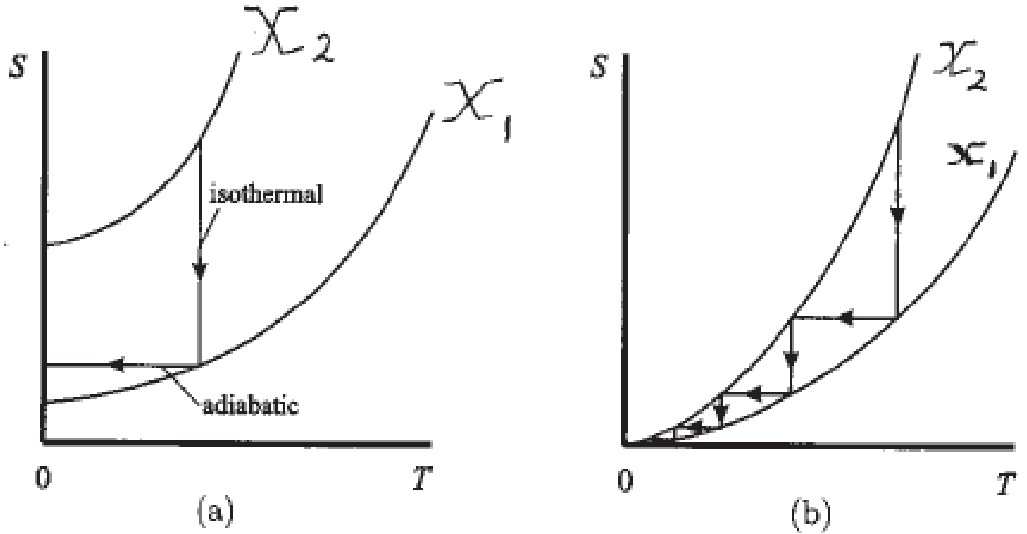



Fig. 1.13. Approaching absolute zero. System (a) not obeying the Third Law can get to $T = 0$ in two steps. System (b) obeying the Third Law cannot get to $T = 0$ in a finite number of steps.

Nernst 1912 (after being criticized by Einstein in Brussels):
One cannot reach $T = 0$ with a finite number of (reversible) isothermal and adiabatic steps.

corollary: The $T = 0$ isotherm is also an isentrope. (H.B. Callen)

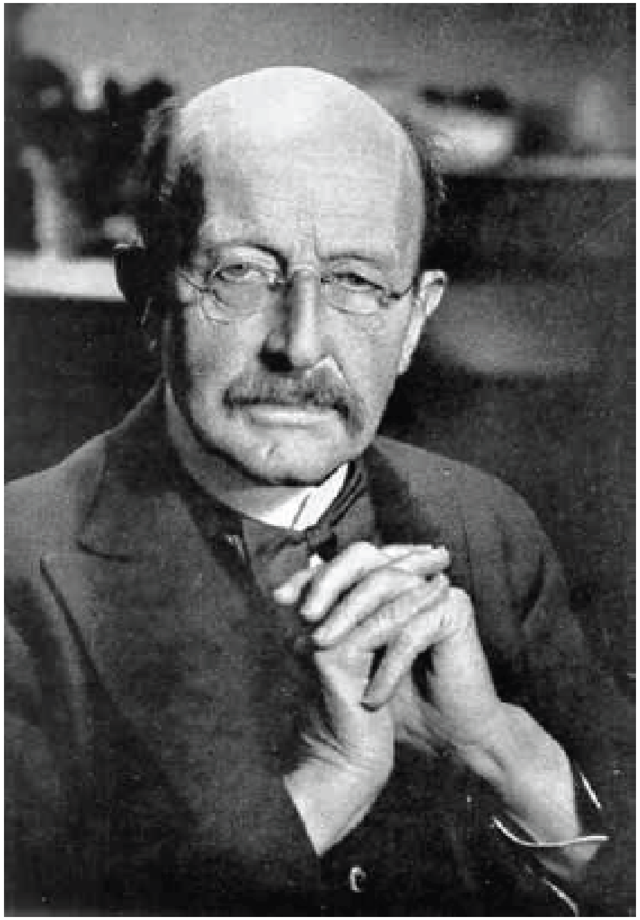
# Famous exception of the 3rd law

classical ideal gas

$$S = N\left[c_V \ln(T) + k_B \ln(V/N) + \sigma\right]$$

Moreover:
classical statistical mechanics: $n$-vector model with $n$-dimensional vectors > 1 violates third law.
(e.g. planar Heisenberg ($n = 2$) or the $n = 3$ Heisenberg model)

# … and Planck's version



Max PLANCK
(1858 - 1947)

The entropy $s = S/N$ per particle approaches at $T = 0$ a constant ($s_0 = k_B \ln g(N)/N$) value that possibly depends on the chemical composition of the system. This limiting value can generally be set to zero.

# THIRD LAW AND PLANCK

VORLESUNGEN

ÜBER

THERMODYNAMIK

VON

DR. MAX PLANCK,
PROFESSOR DER THEORETISCHEN PHYSIK
AN DER UNIVERSITÄT BERLIN.

**1913**

Zunächst ist leicht einzusehen, daß man, da der Wert der Entropie eine willkürliche additive Konstante enthält, jenen für $T = 0$ eintretenden Wert unbeschadet der Allgemeinheit gleich Null setzen kann, so daß das NERNSTsche Wärmetheorem nun lautet:

**Beim Nullpunkt der absoluten Temperatur besitzt die Entropie eines jeden chemisch homogenen festen oder flüssigen Körpers den Wert Null.**

[2] Diese Fassung des Theorems ist inhaltlich etwas weitergehend als die von NERNST a. a. O. selber gegebene, nach welcher für $T=0$ die Differenz der Entropien eines solchen Körpers in zwei verschiedenen Modifikationen gleich Null ist. Letzterer Satz läßt nämlich noch die Möglichkeit offen, daß die Entropie selber für $T = 0$ negativ unendlich wird.

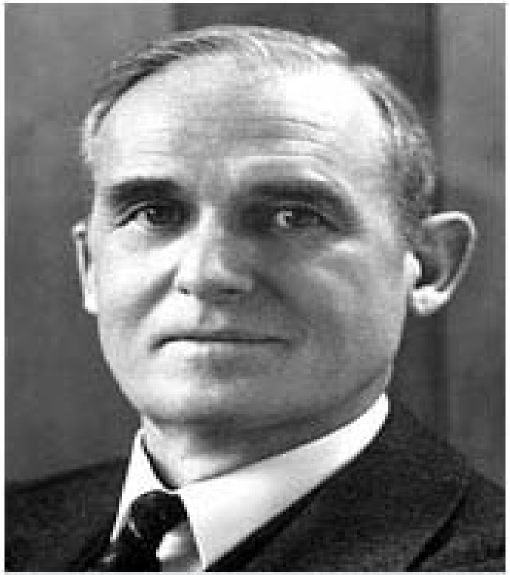
[1] Nach der von NERNST selber gegebenen Fassung (§ 282) tritt an die Stelle der Gleichung (256) die folgende:

$$S' - S = \int_0^T \frac{C_p' - C_p}{T} \, dT,$$

wobei $S' - S$ die Differenz der Entropien eines festen oder flüssigen Körpers in zwei verschiedenen Modifikationen bezeichnet. Dann kann man für $T = 0$ nur schließen, daß $C_p' = C_p$, aber nicht, daß $C_p' = C_p = 0$. Würden $C_p$ und $C_p'$ für $T = 0$ endlich bleiben, so wäre die Entropie $S$ für $T = 0$ nicht gleich Null, sondern negativ unendlich. Es ist daher von Wichtigkeit zu bemerken, daß, wenn die Folgerung $C_p = 0$ durch die Erfahrung nicht bestätigt werden sollte, die NERNSTsche Fassung des Theorems deshalb doch aufrecht erhalten werden könnte.

# The Nobel Prize in Chemistry 1949

"for his contributions in the field of chemical thermodynamics, particularly concerning the behaviour of substances at extremely low temperatures"



**William Francis Giauque**

USA

University of California
Berkeley, CA, USA

b. 1895
d. 1982

Specific heat: $$C^{\mathrm{E}} = \frac{\partial E}{\partial T}, \qquad E = \frac{\hbar\omega_0}{2} + \frac{\hbar\omega_0}{\exp\left(\hbar\omega_0\beta\right) - 1}$$

Quantum harmonic Oscillator: $$C^E = k_\mathrm{B}\left(\frac{\hbar\beta\omega_0}{2\sinh(\hbar\beta\omega_0/2)}\right)^2$$

low-temperature behavior

$$C^E = k_\mathrm{B}\left(\frac{\hbar\omega_0}{k_\mathrm{B}T}\right)^2 \exp\left(-\frac{\hbar\omega_0}{k_\mathrm{B}T}\right)$$

high-temperature behavior

$$C^E = k_\mathrm{B}\left[1 - \frac{1}{12}\left(\frac{\hbar\omega_0}{k_\mathrm{B}T}\right)^2 + O(T^{-4})\right]$$

the limit $\omega_0 \to 0$ does NOT yield the specific heat $C^{\mathrm{E}} = k_\mathrm{B}/2$ of the free particle

$$\Longrightarrow \quad E \neq \frac{1}{2}k_\mathrm{B}T$$

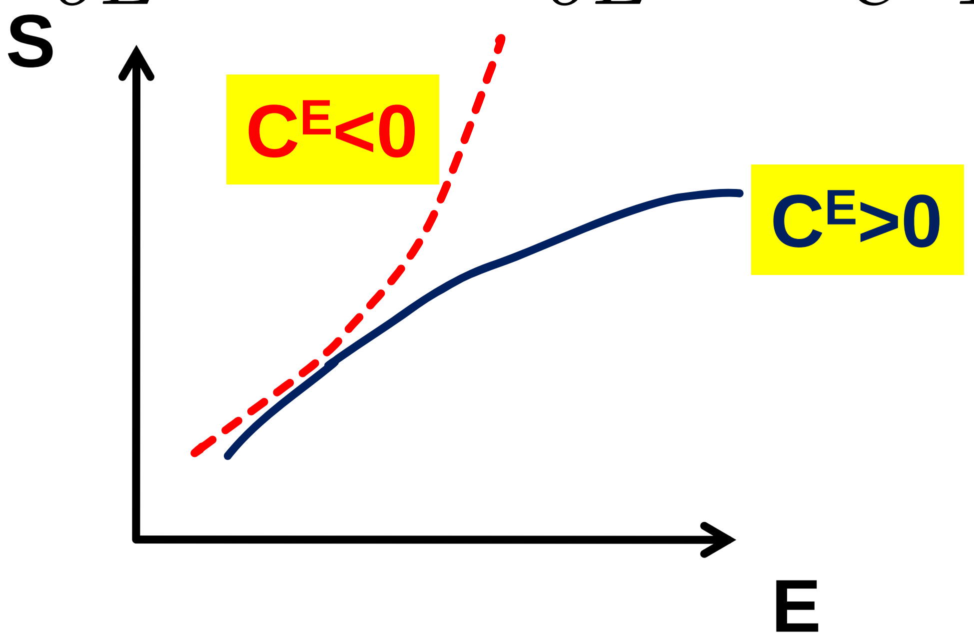
No virial theorem !
No equipartition th.!

# Specific Heat

$$E = \sum_i E_i \exp(-\beta E_i) \Big/ \sum_i \exp(-\beta E_i)$$

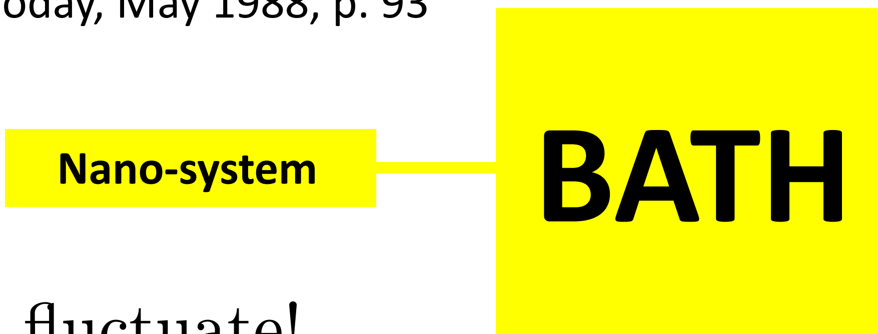
$$C^{\mathrm{E}} = \frac{\mathrm{d}E}{\mathrm{d}T} = (k_{\mathrm{B}}\, T^2)^{-1} \langle (E_i - \langle E_i \rangle)^2 \rangle > \mathbf{0}\,!$$

Note also: $\dfrac{\partial S}{\partial E} = 1/T\,, \qquad \dfrac{\partial^2 S}{\partial E^2} = -\dfrac{1}{C^{\mathrm{E}} T^2}$



# Temperature Fluctuations – An Oxymoron

Ch. Kittel, Physics Today, May 1988, p. 93




$$\beta = \frac{1}{k_{\mathrm{B}} T}$$

$T$ does NOT fluctuate!

$$|\Delta U|\,|\Delta \beta| = 1 \leftrightarrow |\Delta U|\,|\Delta T| = k_{\mathrm{B}} T^2 \quad \textbf{NO!}$$

"…Temperature is precisely defined only for a system in thermal equilibrium with a heat bath: The temperature of a system A, however small, is defined as equal to the temperature of a very large heat reservoir B with which the system is in equilibrium and in thermal contact. Thermal contact means that A and B can exchange energy, although insulated from the outer World. …"

# Molecular Dynamics

- Every quadratic term in the Hamiltonian contributes $\frac{1}{2}k_{\mathrm{B}}T$ to the internal energy
- Classical systems: **VIRIAL THEOREM**

$$[H_i \propto q_i^n \text{ or } p_i^n \Rightarrow \langle E_i \rangle = k_{\mathrm{B}}T/n]$$

- Classical harmonic Oscillator

$$\frac{\langle p_i^2 \rangle}{2m} = \frac{1}{2}m\langle v_i^2 \rangle = \frac{1}{2}k_{\mathrm{B}}T$$

$$\frac{1}{2}m\omega^2\langle x_i^2 \rangle = \frac{1}{2}k_{\mathrm{B}}T$$

**NOT SO FOR A QUANTUM OSCILLATOR**

# Specific heat paradox (microcanonical)

R. Emden (1907); A.S. Eddington (1926);
D. Lynden-Bell & R. Wood (1968); W. Thirring (1970);
W. Thirring, H. Narnhofer & H. A. Posch (2003)

Coulomb: $\langle \Phi_i \rangle = -k_{\mathrm{B}} T$ ¡ CLASSICAL !

$$\Longrightarrow \quad T_{\mathrm{kin}} = -\frac{1}{2}\langle \Phi_{\mathrm{tot}} \rangle = 3Nk_{\mathrm{B}}T/2$$

$$2T_{\mathrm{kin}} + \langle \Phi_{\mathrm{tot}} \rangle = 0 = E + T_{\mathrm{kin}}$$

$$\Longrightarrow \quad C^E = \frac{\mathrm{d}E}{\mathrm{d}T} = -\frac{\mathrm{d}T_{\mathrm{kin}}}{\mathrm{d}T} = -3Nk_{\mathrm{B}}/2 < 0 \quad !$$

# Relativistic Regime

$$\Sigma(u=0) \longrightarrow \Sigma'(u \neq 0)$$

$$x' = \gamma(u)\,(x - u \cdot t)$$

$$t' = \gamma(u)\,\left(t - ux/c^2\right)$$

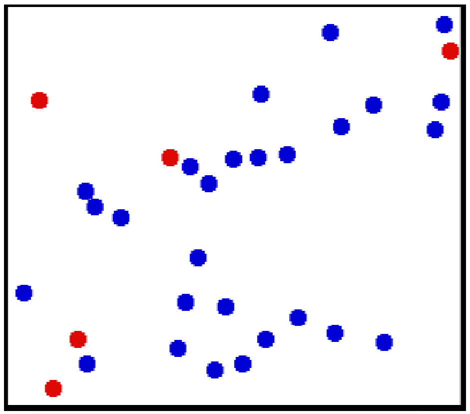
$$\text{with } \gamma(u) = \left(1 - \frac{u^2}{c^2}\right)^{-1/2}$$

$$x^2 - c^2 t^2 = (x')^2 - c^2 (t')^2$$

Note: thermodynamic observables are **NONLOCAL**

J. Dunkel & P.H., *Relativistic Brownian motion*, Phys. Rep. **471**, 1 - 73 (2009)

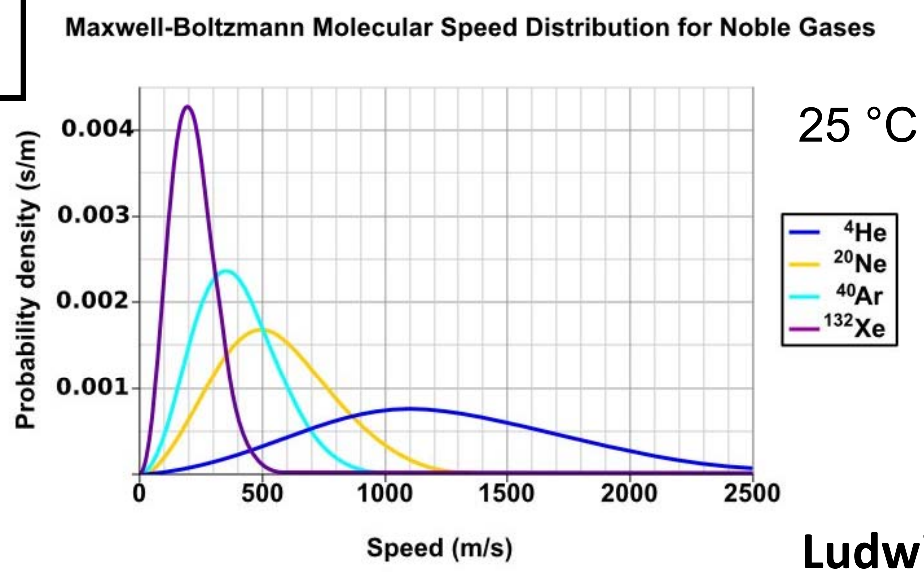
# Maxwell-Boltzmann distribution



velocity distribution:

$$f_{\vec{v}}(v_x, v_y, v_z) = \left(\frac{m}{2\pi\, k_{\rm B} T}\right)^{3/2} \exp\left(-\frac{v_x^2 + v_y^2 + v_z^2}{2\, k_{\rm B} T/m}\right)$$

James Clerk Maxwell
(1831 – 1879)



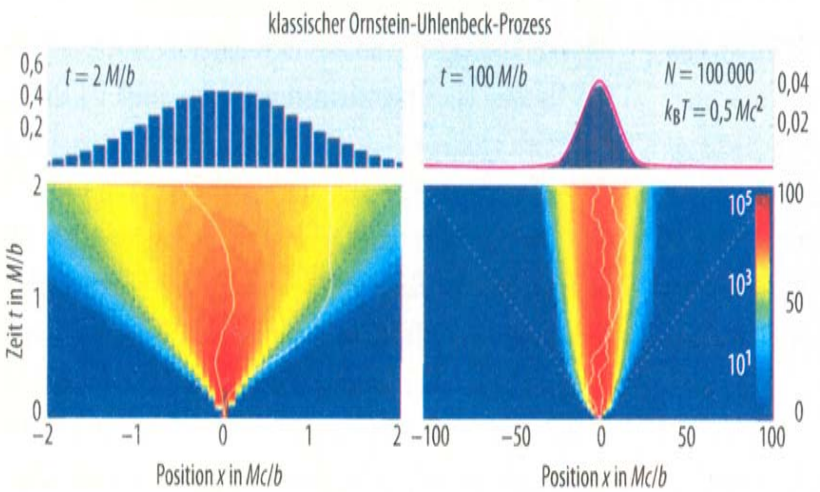

Ludwig Eduard Boltzmann
(1844 – 1906)

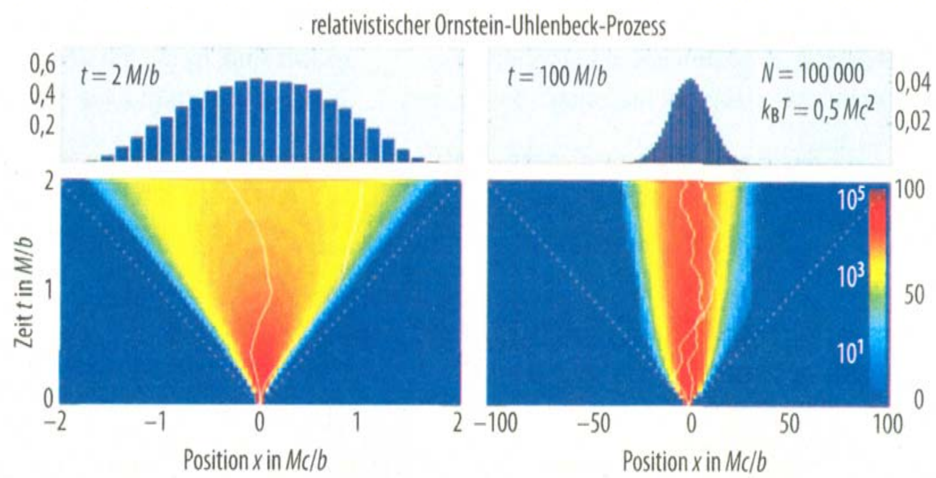
**! v < c violated !**

# Diffusion

**classical**

**relativistic**



# Diffusion: space-time only

classical diffusion (Markovian)

$$p(t,x|t_0,x_0) = \left[\frac{1}{4\pi\,\mathcal{D}(t-t_0)}\right]^{1/2} \exp\left[-\frac{(x-x_0)^2}{4\mathcal{D}(t-t_0)}\right].$$

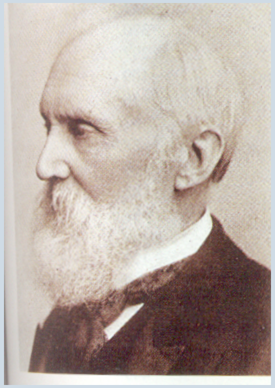
telegraph equation (non-Markovian)

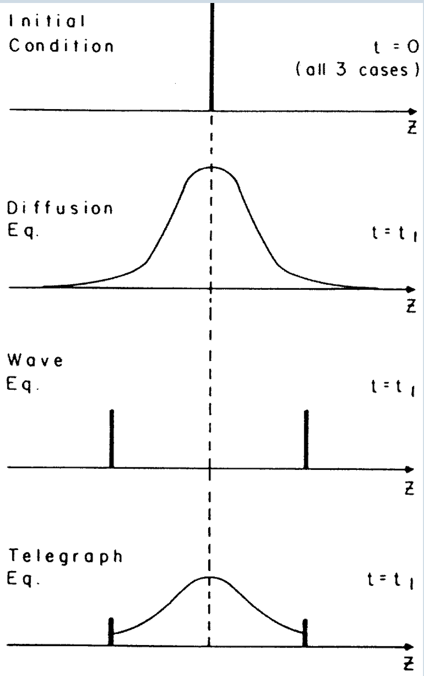
$$\tau_v \frac{\partial^2}{\partial t^2}\varrho + \frac{\partial}{\partial t}\varrho = \mathcal{D}\,\nabla^2\varrho,$$

alternative approach

$$\boxed{p(\bar{x}|\bar{x}_0) \propto \int_{a_-(\bar{x}|\bar{x}_0)}^{a_+(\bar{x}|\bar{x}_0)} \mathrm{d}a\ \exp\left(-\frac{a}{2\mathcal{D}}\right)}$$

*PRD **75**:043001 (2007)*



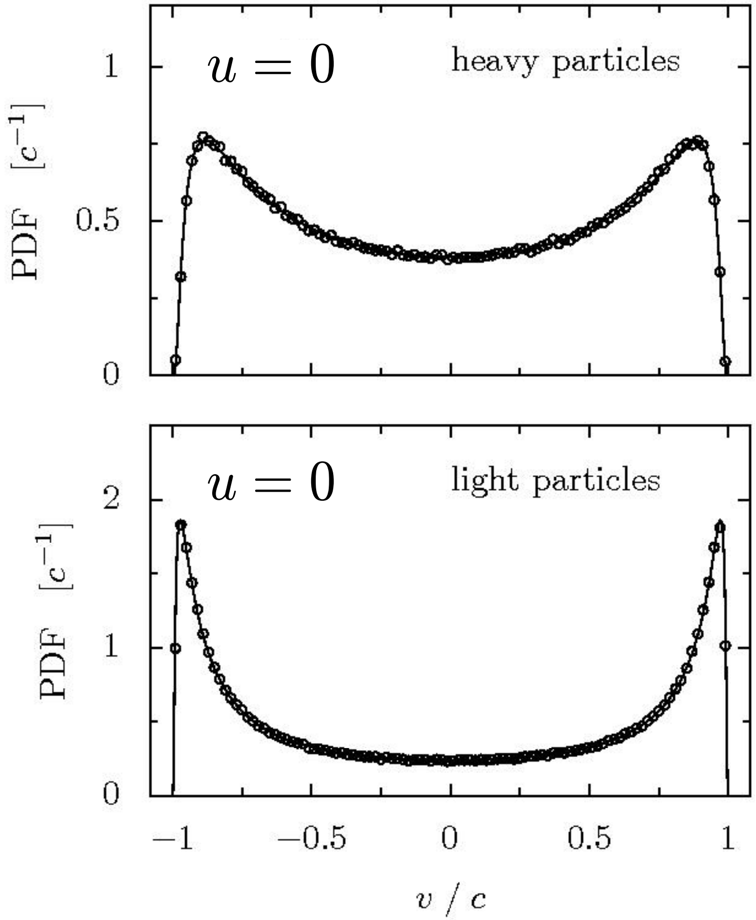



*J Masoliver & G H Weiss*
*Eur J Phys **17**:190 (1996)*

# Jüttner Gas

$$f_{\mathrm{Maxwell}}(\vec{p}) = [\beta/(2\pi m)]^{d/2} \exp\left(-\beta p^2/2m\right)$$

$$f_{\mathrm{Jüttner}}(\vec{p}) = Z_d^{-1} \exp\left[-\beta_{\mathrm{J}}(m^2c^4 + p^2c^2)^{1/2}\right]$$




$$\langle \vec{p} \cdot \vec{v} \rangle = d k_{\mathrm{B}} \mathcal{T} = d/\beta_{\mathrm{J}}$$

statistical realtivistic temperature

$$T = \mathcal{T} = (k_{\mathrm{B}}\beta_{\mathrm{J}})^{-1}$$

J. Dunkel & P.H., Phys. Rep. **471**, 1-73 (2009)

1908. № 6.

ANNALEN DER PHYSIK.

VIERTE FOLGE. BAND 26.

1. *Zur Dynamik bewegter Systeme;*

*von M. Planck.*

Aus den Sitzungsber. d. k. preuß. Akad. der Wissensch. vom 13. Juni 1907, mitgeteilt vom Verf.

$$p = p', \quad \frac{V}{\sqrt{1 - \frac{q^2}{c^2}}} = \frac{V'}{\sqrt{1 - \frac{q'^2}{c^2}}} \quad \text{und} \quad \frac{T}{\sqrt{1 - \frac{q^2}{c^2}}} = \frac{T'}{\sqrt{1 - \frac{q'^2}{c^2}}},$$

oder:

$$\frac{V'}{V} = \frac{T'}{T} = \sqrt{\frac{c^2 - q'^2}{c^2 - q^2}}, \quad p' = p, \quad S' = S \tag{17}$$

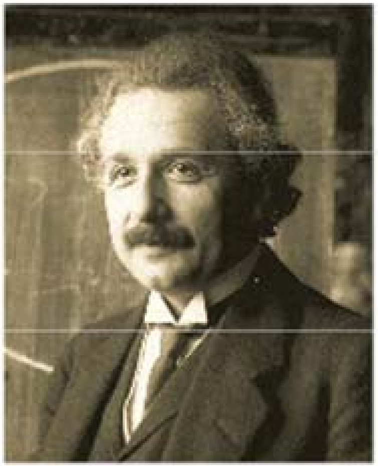
als *allgemein gültige Beziehung zwischen den gestrichenen und den ungestrichenen Variabeln.*

moving bodies appear cooler

**Über das Relativitätsprinzip und die aus demselben gezogenen Folgerungen.**

Von **A. Einstein.**

(Eingegangen 4. Dezember 1907.)



**IV. Zur Mechanik und Thermodynamik der Systeme.**

Einstein, Relativitätsprinzip u. die aus demselben gezog. Folgerungen. 453

$$\frac{T}{T_0} = \sqrt{1 - \frac{q^2}{c^2}}. \tag{27}$$

Die Temperatur eines bewegten Systems ist also in bezug auf ein relativ zu ihm bewegtes Bezugssystem stets kleiner als in bezug auf ein relativ zu ihm ruhendes Bezugssystem.

# THE MATHEMATICAL THEORY OF RELATIVITY

BY

A. S. EDDINGTON, M.A., M.Sc., F.R.S.

PLUMIAN PROFESSOR OF ASTRONOMY AND EXPERIMENTAL PHILOSOPHY IN THE UNIVERSITY OF CAMBRIDGE

CAMBRIDGE
AT THE UNIVERSITY PRESS
1923

$$\beta = (1 - u^2/c^2)^{-\frac{1}{2}},$$

$$T' = \beta T \quad \text{..............................} (14{\cdot}4).$$

In general it is unprofitable to apply the Lorentz transformation to the *constitutive equations* of a material medium and to coefficients occurring in them (permeability, specific inductive capacity, elasticity, velocity of sound). Such equations naturally take a simpler and more significant form for axes moving with the matter. The transformation to moving axes introduces great complications without any evident advantages, and is of little interest except as an analytical exercise.

moving bodies appear hotter

Zeitschrift für Physik 175, 70—104 (1963)

Aus dem Institut für Theoretische Physik der Universität Würzburg

# Lorentz-Transformation der Wärme und der Temperatur

Von
H. Ott*

Mit 1 Figur im Text

*(Eingegangen am 11. Januar 1963)*

In die übliche Herleitung der Lorentz-Transformation von Wärme und Temperatur haben sich zwei grundsätzliche Fehler eingeschlichen, nämlich

1. die Verwendung einer unvollkommenen Bewegungsgleichung für Körper mit veränderlicher Ruhemasse,

2. die irrtümliche Ansicht, die Leistung $(\mathfrak{K}\mathfrak{v})$ der an einem bewegten Leiter angreifenden Lorentz-Kraft $\mathfrak{K}$ trage nur zur Erhöhung der mechanischen Energie des Leiters und nichts zur Stromwärme bei.

Nach Richtigstellung ergeben sich für eine Wärmemenge $dQ$ und für die Temperatur die Transformationen

$$dQ = \frac{dQ^0}{\sqrt{1-\beta^2}}, \quad T = \frac{T^0}{\sqrt{1-\beta^2}}$$

($dQ^0$, $T^0$ beziehen sich auf das Eigensystem), wenn man den 1. und 2. Hauptsatz in der üblichen Form zugrunde legt.

Heinrich Ott (1894-1962)
photo provided by
Oscar Bandtlow (QMUL)

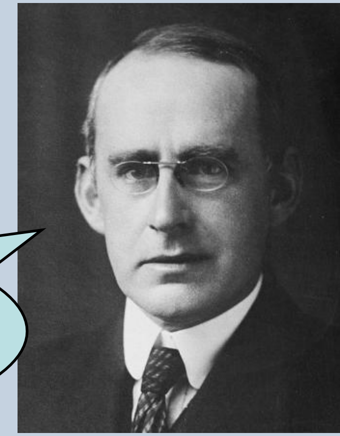
No. 5062 NOVEMBER 5, 1966 NATURE 571

# DOES A MOVING BODY APPEAR COOL?

By PROF. P. T. LANDSBERG
Department of Applied Mathematics and Mathematical Physics, University College, Cathays Park, Cardiff

We wish to ensure that the temperature is invariant, so that the equilibrium condition (8) becomes $T_1 = T_2$ even in a relativistic theory.

# “Temperature” problem in RTD?



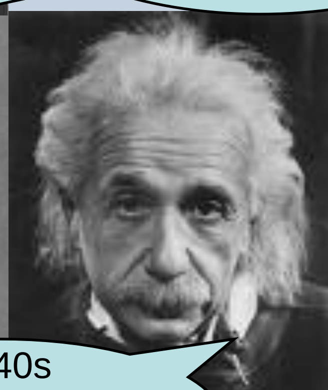

1923/1963

.. hotter!

1907/08

moving bodies appear cooler

$$T'(w) = T\,(1 - w^2)^{\alpha/2} \qquad \alpha = \begin{cases} +1 & \text{Planck, Einstein} \\ 0 & \text{Landsberg, van Kampen} \\ -1 & \text{Ott} \end{cases}$$

1940s

maybe ... not

T=T’ 1966-69



*CK Yuen, Amer. J. Phys. 38:246 (1970)*

# Heat, second law & temperature

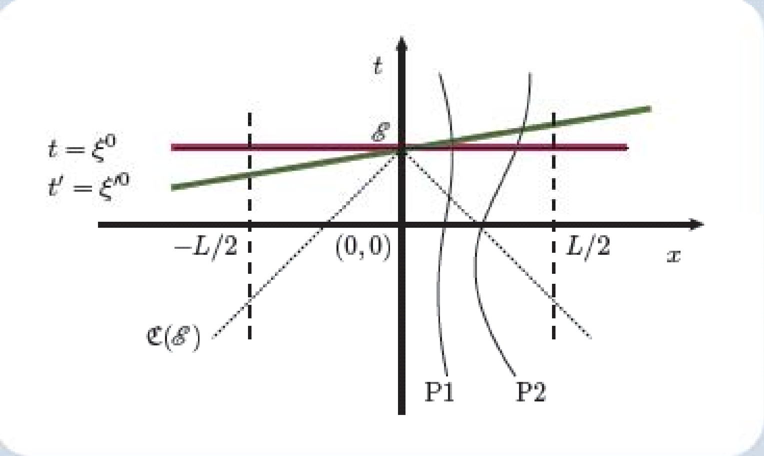
## Planck-Einstein

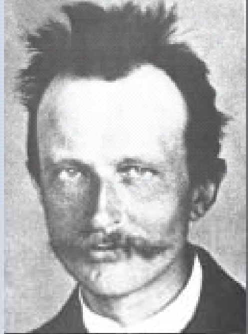
$$\text{đ}\mathcal{Q}'[\mathfrak{T}'] := \mathcal{T}'\mathrm{d}\mathcal{S}' := \mathrm{d}\mathcal{U}'^0[\mathfrak{T}'] - w'\mathrm{d}\mathcal{U}'^1[\mathfrak{T}'] + \mathcal{P}'\mathrm{d}V'$$

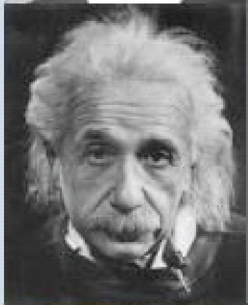
$$\gamma = (1 - w'^2)^{-1/2}$$

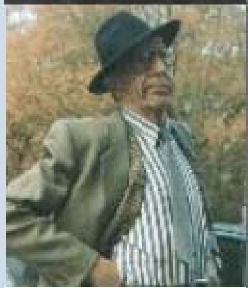
$$\mathcal{T}'\,\mathrm{d}\mathcal{S}' := \text{đ}\mathcal{Q}'^0[\mathfrak{T}'] = \gamma^{-1}\text{đ}\mathcal{Q}^0[\mathfrak{T}] = \gamma^{-1}\,\mathcal{T}\,\mathrm{d}\mathcal{S}$$

*cooler*







## Ott

$$\text{đ}\mathcal{Q}^\mu[\mathfrak{T}] := \mathrm{d}\mathcal{U}^\mu[\mathfrak{T}] - \text{đ}\mathcal{A}^\mu[\mathfrak{T}],$$

$$(\text{đ}\mathcal{A}^\mu[\mathfrak{T}]) := (-\mathcal{P}\mathrm{d}V, \mathbf{0}).$$

$$\mathcal{T}'\,\mathrm{d}\mathcal{S}' := \text{đ}\mathcal{Q}'^0 = \gamma\text{đ}\mathcal{Q}^0 = \gamma\,\mathcal{T}\,\mathrm{d}\mathcal{S}.$$

*hotter*

## van Kampen / Landsberg

$$\text{đ}\mathcal{Q}^\mu[\mathfrak{T}] := \mathrm{d}\mathcal{U}^\mu[\mathfrak{T}] - \text{đ}\mathcal{A}^\mu[\mathfrak{T}],$$

$$\text{đ}\mathcal{Q}' := -w'_\mu\text{đ}\mathcal{Q}'^\mu = -w_\mu\text{đ}\mathcal{Q}^\mu = \text{đ}\mathcal{Q} = \text{đ}\mathcal{Q}^0$$

$$\mathcal{T}'\mathrm{d}\mathcal{S}' := \text{đ}\mathcal{Q}' = \text{đ}\mathcal{Q} = \mathcal{T}\mathrm{d}\mathcal{S},$$

*T'=T*

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# Starting points in RTD?

"BAD" macroscopic variables $\mathcal{N}, \mathcal{U}^0, \boldsymbol{\mathcal{U}}$

- ambiguous definition
- hide non-local character

"GOOD" mesoscopic tensor densities $j^\mu(t,x),\ \theta^{\mu\nu}(t,x),\ \ldots$

- uniquely defined, e.g., can be derived from Lagrangians
- encode symmetries $\Leftrightarrow$ conserved Noether currents

# Thermodynamic variables

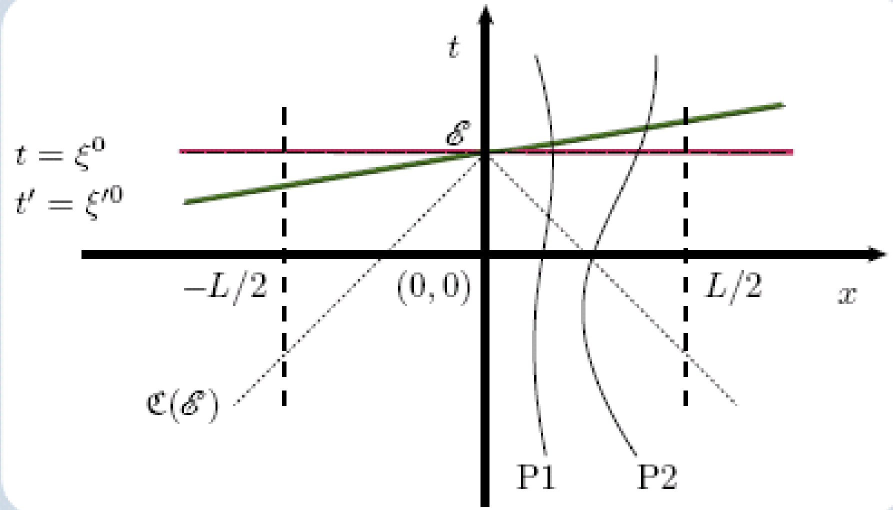
$$\partial_\mu j^\mu \equiv 0 \qquad \partial_\nu s^\nu \equiv 0 \qquad \boxed{\partial_\mu \theta^{\mu i} \not\equiv 0}$$

non-local TD variables $\Leftrightarrow$ surface integrals in space-time

$$\mathcal{N}[\mathfrak{H}] := \int_{\mathfrak{H}} \mathrm{d}\sigma_\nu \, j^\nu = \mathcal{N}[\mathfrak{H}'] \qquad \text{scalar}$$

$$\mathcal{S}[\mathfrak{H}] := \int_{\mathfrak{H}} \mathrm{d}\sigma_\nu \, s^\nu = \mathcal{S}[\mathfrak{H}'] \qquad \text{scalar}$$

$$\mathcal{U}^\nu[\mathfrak{H}] := \int_{\mathfrak{H}} \mathrm{d}\sigma_\mu \, \theta^{\mu\nu} \boxed{\neq \mathcal{U}^\nu[\mathfrak{H}']} \qquad \text{4-vector}$$




$$\mathcal{N}[\mathfrak{H}] = \mathcal{N}'[\mathfrak{H}] = \mathcal{N}'[\mathfrak{H}'] = \mathcal{N}[\mathfrak{H}'] \qquad \mathcal{S}[\mathfrak{H}] = \mathcal{S}'[\mathfrak{H}] = \mathcal{S}'[\mathfrak{H}'] = \mathcal{S}[\mathfrak{H}'] \qquad \Lambda^\nu{}_\mu \mathcal{U}^\mu[\mathfrak{H}] = \boxed{\mathcal{U}'^\nu[\mathfrak{H}] \neq \mathcal{U}'^\nu[\mathfrak{H}']} = \Lambda^\nu{}_\mu \mathcal{U}^\mu[\mathfrak{H}']$$

# “Photographic” thermodynamics

light-cone averaging

$$\mathfrak{C}(\mathscr{E}) \;:=\; \{\, (t, \boldsymbol{x}) \mid t = \xi^0 - |\boldsymbol{x} - \boldsymbol{\xi}| \,\}$$

photographic state variables

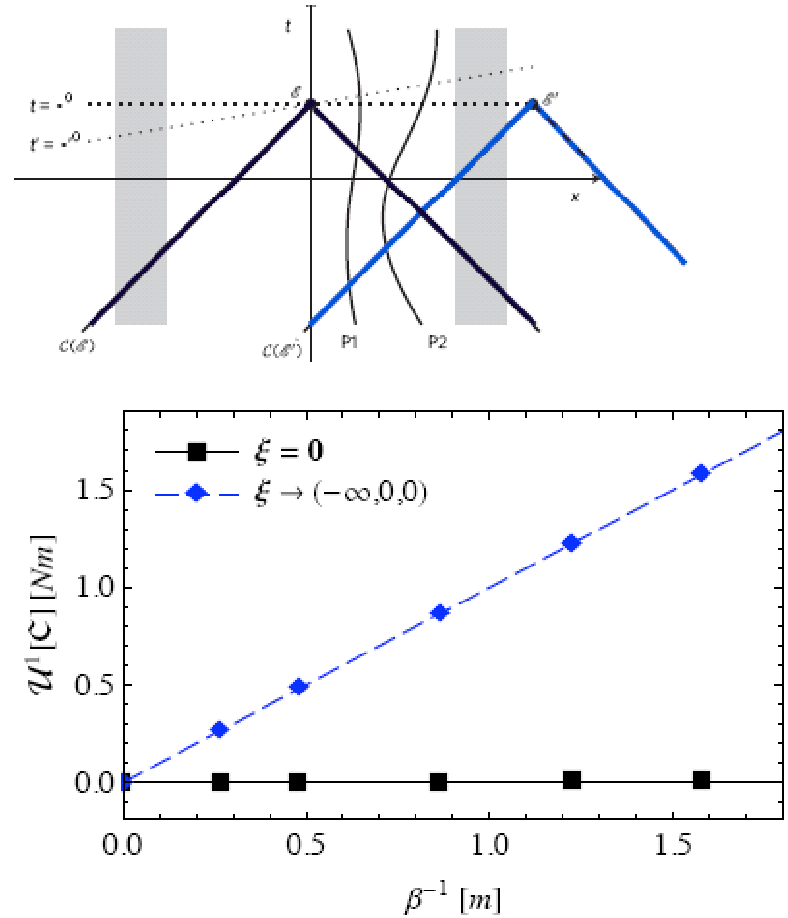
$$\mathcal{U}^0[\mathfrak{C}] \;=\; N\,\langle p^0 \rangle$$

$$\mathcal{U}^i[\mathfrak{C}] \;=\; \frac{N}{\beta} \int \mathrm{d}^3x \, \frac{x^i - \xi^i}{|\boldsymbol{x} - \boldsymbol{\xi}|} \, \varrho(\boldsymbol{x})$$

distant observer (at rest !)

$$\mathcal{U}^i[\mathfrak{C}] = -\frac{\xi^i}{|\boldsymbol{\xi}|} N k_{\mathrm{B}} \mathcal{T}$$

apparent drift !




JD, Peter Hanggi & Stefan Hilbert, Nature Physics 5:741, 2009

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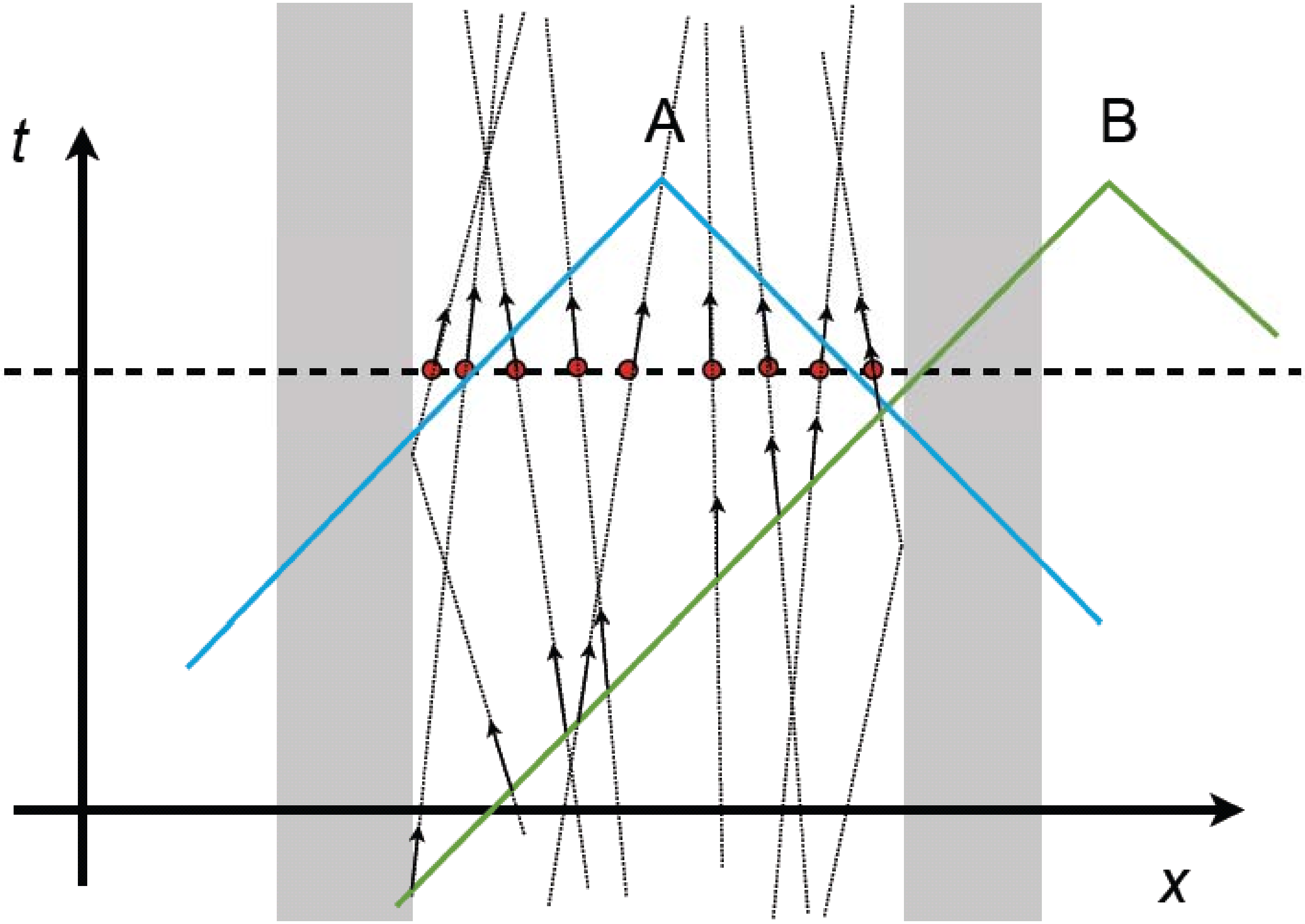
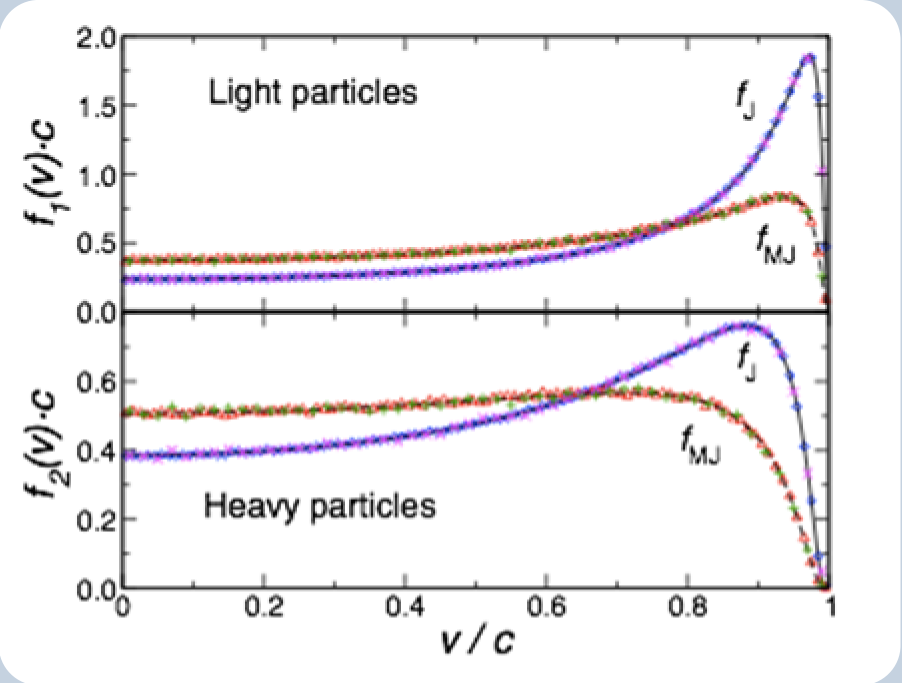

t
A
B
x

# Time parameters and (relative) entropy

$$f_t(\boldsymbol{v}) = \frac{1}{t}\int_0^t dt'\,\delta[\boldsymbol{v} - \boldsymbol{V}(t')],$$

$$\hat{f}_\tau(\boldsymbol{v}) = \frac{1}{\tau}\int_0^\tau d\tau'\,\delta[\boldsymbol{v} - \hat{\boldsymbol{V}}(\tau')]$$




$$S[\phi|\rho] = \int d^d p\,\phi(\boldsymbol{p})\ln\left[\frac{\phi(\boldsymbol{p})}{\rho(\boldsymbol{p})}\right]$$

$$\epsilon_\rho = \int d^d p\,\phi(\boldsymbol{p})\,p^0$$

$$1 = \int d^d p\,\phi(\boldsymbol{p})$$

$$\rho = \rho_0 : \quad \phi_J \propto \exp(-\beta p^0) \quad \Leftrightarrow \quad f(t\to\infty)$$

$$\rho \propto \frac{1}{p^0} : \quad \phi_{MJ} \propto \frac{\exp(-\beta p^0)}{p^0} \quad \Leftrightarrow \quad \hat{f}(\tau\to\infty)$$

EPL 87: 30005 (2009)

# ? Negative Temperature ?

**Spin system:** $|\vec{S}| = 1/2\,; \quad \vec{\mu} = \gamma \vec{S}\,; \quad H = -\sum \vec{\mu}_i \cdot \vec{B}$

$$\vec{S} \parallel \vec{B} \Rightarrow \text{ Two-State-System:} \quad \epsilon_g = -\frac{1}{2}\gamma B \, < \epsilon_e = +\frac{1}{2}\gamma B = \mu B$$

$$N = n_g + n_e \quad \&\ E = \mu B(n_e - n_g)\,,\ \text{typically}\ \ E < 0$$

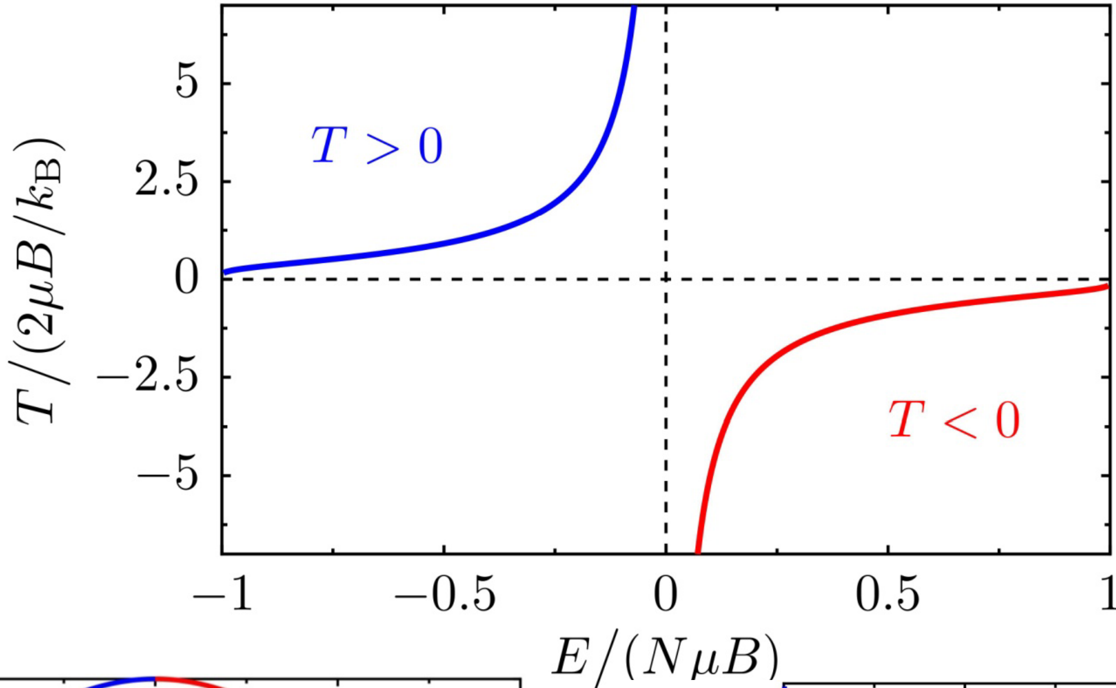
$$\Omega = \frac{N!}{n_g!\, n_e!} \Rightarrow S = k_\mathrm{B} \ln \Omega$$

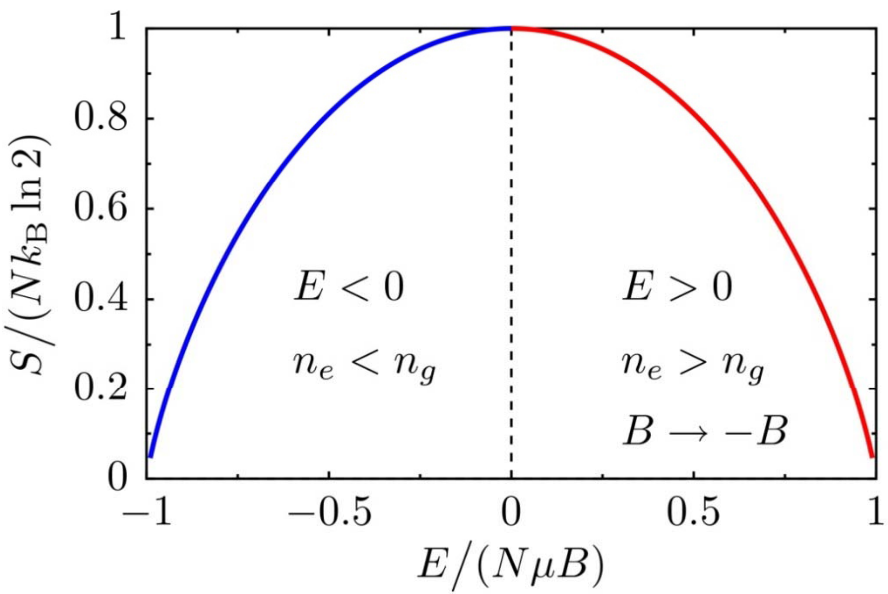
$$n_g = \frac{1}{2}\left(N - \frac{E}{\mu B}\right)$$

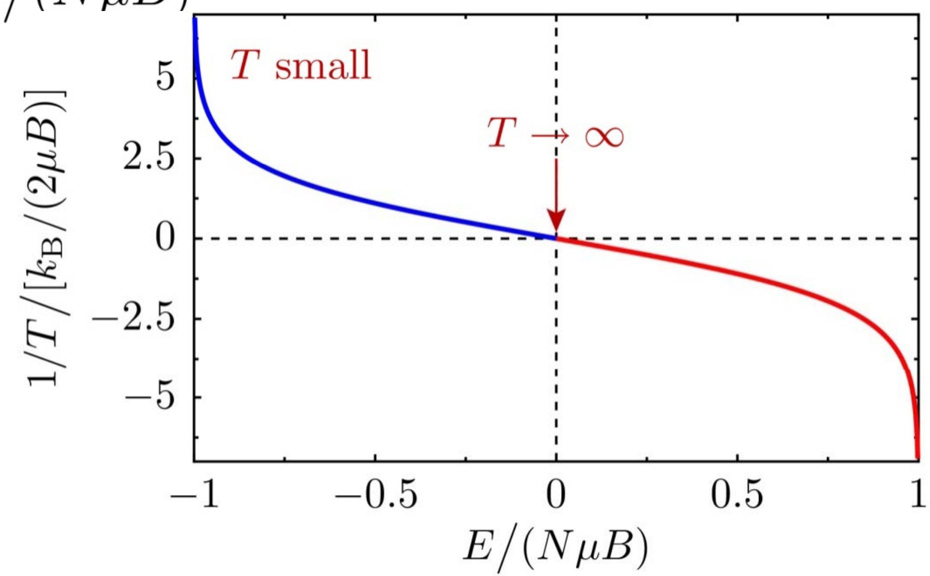
$$n_e = \frac{1}{2}\left(N + \frac{E}{\mu B}\right)$$

$$\Rightarrow \frac{1}{T} = \frac{\partial S}{\partial E}$$

# Negative (Spin)-Temperature!

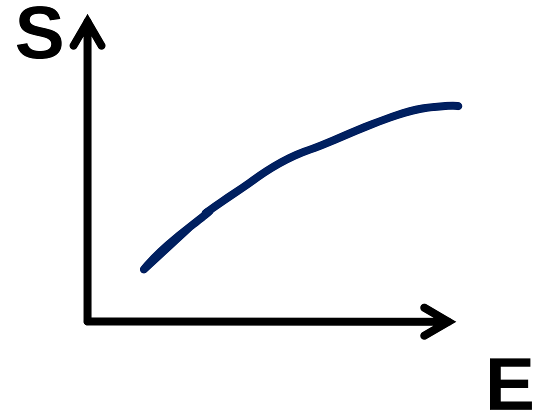


# Take Home Messages

- $\exists$ absolute temperature $T \geq 0$




- $T = \frac{\partial E}{\partial S} \ldots$

  with $S(E)$ a concave function of $E$ !

- $S(E)$ : microcanonical NOT always $=$ $S(E)$ : canonical ( long range, non-ergodic parts )

- $T$ does **NOT** fluctuate

- quantum mechanics: no virial theorem; no equipartion

- relativistic statistical thermometer

$$\mathcal{T}_\Sigma = \mathcal{T}_{\Sigma'}$$

# Jüttner Gas

$$f_{\text{Maxwell}}(\vec{p}) = [\beta/(2\pi m)]^{d/2} \exp\left(-\beta p^2/2m\right)$$

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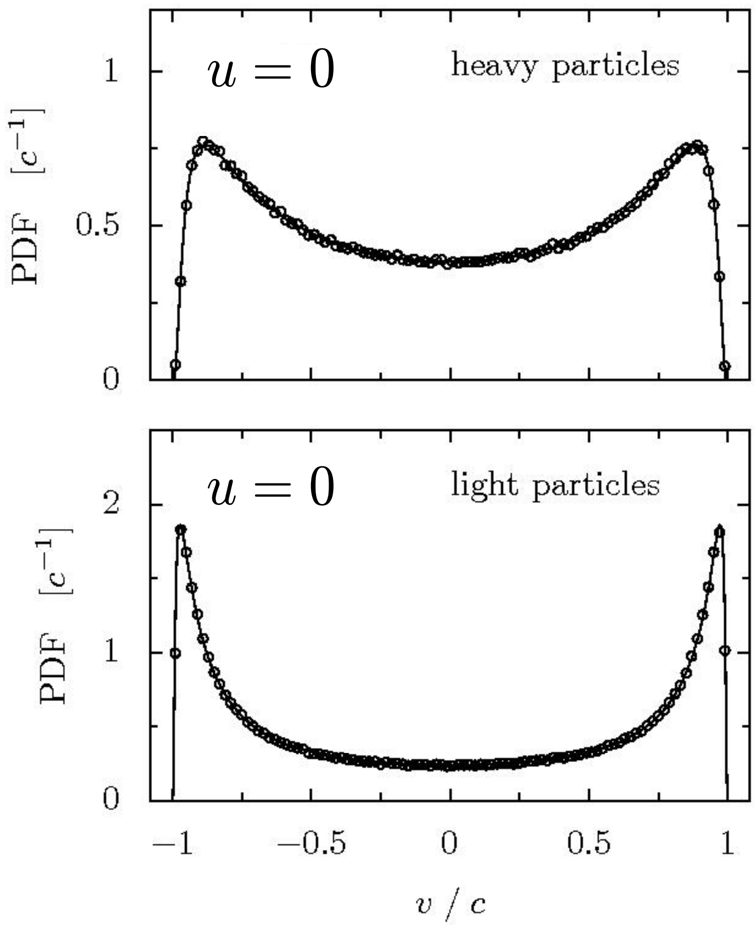


$$\langle \vec{p} \cdot \vec{v} \rangle = d k_{\text{B}} \mathcal{T} = d/\beta_{\text{J}}$$

statistical relativistic temperature

$$T = \mathcal{T} = (k_{\text{B}}\beta_{\text{J}})^{-1}$$

J. Dunkel & P.H., Phys. Rep. **471**, 1-73 (2009)

1908. № 6.

# ANNALEN DER PHYSIK.

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oder:

$$\frac{V'}{V} = \frac{T'}{T} = \sqrt{\frac{c^2 - q'^2}{c^2 - q^2}}, \quad p' = p, \quad S' = S \tag{17}$$

als *allgemein gültige Beziehung zwischen den gestrichenen und den ungestrichenen Variabeln.*

moving bodies appear cooler

# Comparision of temperature scales

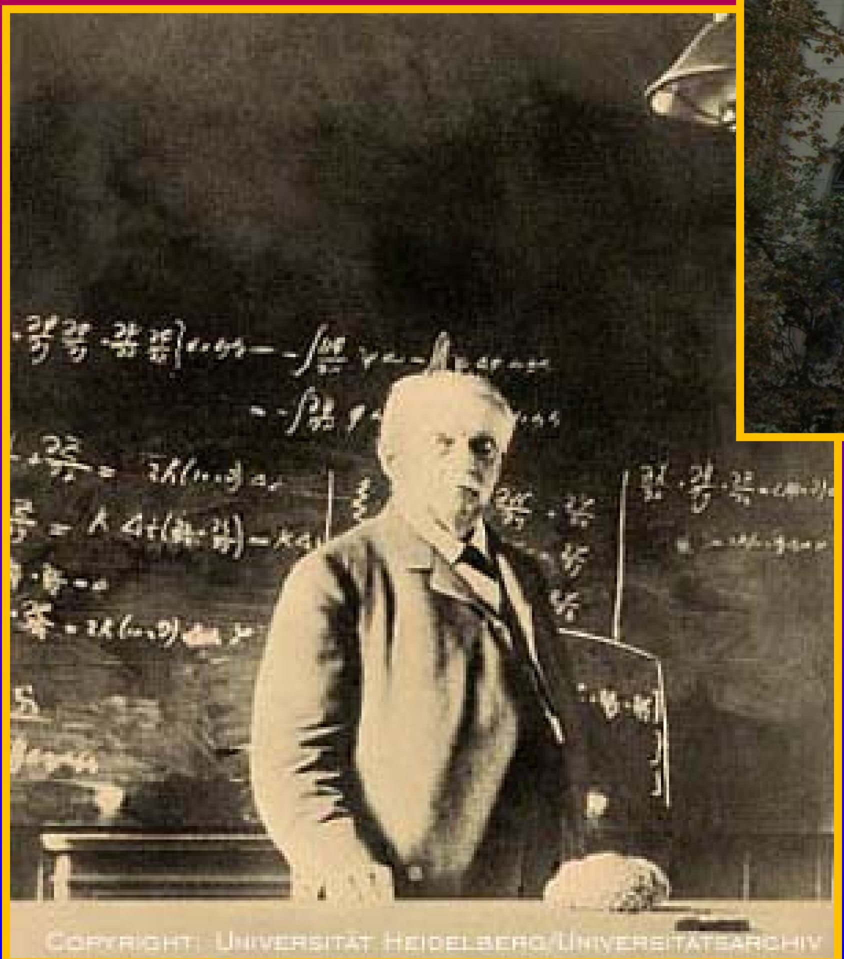
| | Kelvin [K] | Celsius [°C] | Fahrenheit [°F] |
|---|---|---|---|
| Absolute zero | 0.00 | -273.15 | -459.67 |
| Fahrenheit's ice/salt mixture | 255.37 | -17.78 | 0.00 |
| Ice melts (at standard pressure**) | 273.15 | (100) = 0.00 * | 32.00 |
| Average human body temperature | 309.95 | 36.80 | 98.24 |
| Water boils (at standard pressure) | 373.13 | (0) = 100 * | 211.97 |
| Titanium melts | 1941.00 | 1668.00 | 3034.00 |

source: wikipedia.org

Further temperature scales: Newton, Delisle, Réaumur, Rømer, M. Planck (linear scales)
Rudolf Plank (logarithmic scale)

* Celsius used a reversed temperature scale with "0" indicating the boiling point of water and "100" the melting point of ice

** 1 atm = 760 mmHg = 101325 Pa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# Temperature definitions

# Overview

I. History

II. Temperature scales

III. Thermometers

IV. Grand Laws of Thermodynamics

V. Temperature characteristics

# Temperature scales

# “Isochronous” RTD useful at all?

shortcomings

(i) not feasible experimentally

(ii) extension to General Relativity problematic

# “Isochronous” RTD useful at all?

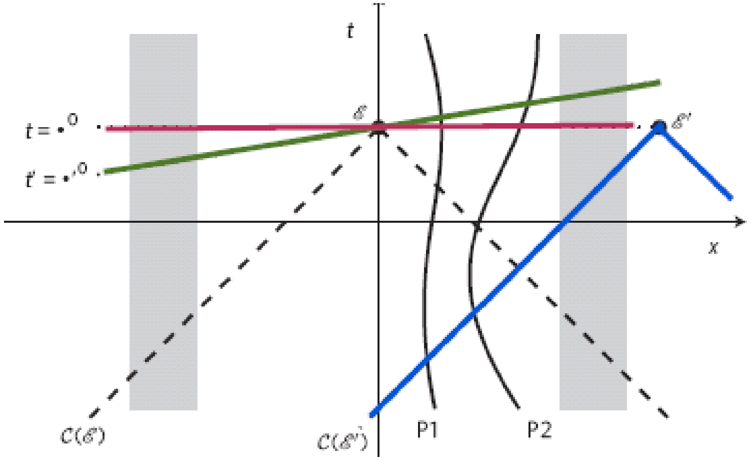
shortcomings

(i) not feasible experimentally
(ii) extension to General Relativity problematic

SOLUTION: replace isochronous hyperplanes by lightcones

Why?

✓ photographic measurements
✓ equivalent for moving observers
✓ GR extension
✓ nonrelativistic limit




UNIVERSITY OF OXFORD

# Temperatur:
# Was ist das eigentlich ?

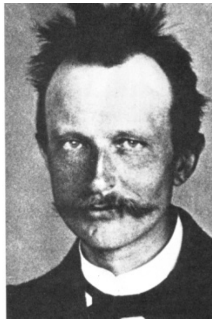

Peter Hänggi
Universität Augsburg

In Zusammenarbeit mit
Dr. Gerhard Schmid

# What is temperature?

Peter Hänggi
Universität Augsburg

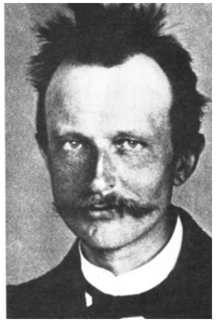
In collaboration with
Dr. Gerhard Schmid



# Planck units

| Constant | Symbol | Value in SI units |
|---|---|---|
| Speed of light | c | 299 792 458 m s$^{-1}$ |
| Gravitational constant | G | 6.67429 · 10$^{-11}$ m$^{3}$ kg$^{-1}$ s$^{-2}$ |
| reduced Planck's constant | $\hbar$ | 1.054571628 ·10$^{-34}$ J s |
| Coulomb force constant | $(4\pi\epsilon_0)^{-1}$ | 8 987 551 787.368 kg m$^{3}$ s$^{-2}$ C$^{-2}$ |
| Boltzmann constant | $k_{\mathrm{B}}$ | 1.3806504·10$^{-23}$ J K$^{-1}$ |

| Name | Expression | SI equivalents |
|---|---|---|
| **Planck temperature** | $T_{\mathrm{P}} = \sqrt{\hbar c^5 G^{-1} k^{-2}}$ | 1.41168· 10$^{32}$ K |
| Planck length | $l_{\mathrm{P}} = \sqrt{\hbar G c^{-3}}$ | 1.61625· 10$^{-35}$ m |
| Planck mass | $m_{\mathrm{P}} = \sqrt{\hbar c G^{-1}}$ | 2.17644· 10$^{-8}$ kg |
| Planck time | $t_{\mathrm{P}} = \sqrt{\hbar G c^{-5}}$ | 5.39124· 10$^{-44}$ s |



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# Thermodynamic potentials

**Internal energy:** $E(S, V, \{N_i\}) = T\,S - p\,V + \sum \mu_i\,N_i$

**Helmholtz free energy:** $F(T, V, \{N_i\}) = E - T\,S$

**Enthalpy:** $H(S, p, \{N_i\}) = E + p\,V$

**Gibbs free energy:** $G(T, p, \{N_i\}) = E + p\,V - T\,S = \sum \mu_i\,N_i$

**Planck potential:** $\Phi(T, p, \{N_i\}) = -G/T$

# Quantum Regime

$$h = 6.62606896 \cdot 10^{-34}\,\mathrm{J\ s} = \frac{4}{K_\mathrm{J}^2 R_\mathrm{K}}$$

$$\text{with } K_\mathrm{J} = 2e/h\,,\ R_\mathrm{K} = h/e^2$$

$$h\nu = 1\,\mathrm{kg}\,c^2$$

$$\rightarrow \nu = 135639273 \cdot 10^{42}\mathrm{Hz}$$

# Quantum Harmonic Oscillator

$$H_{\mathrm{S}} = \frac{p^2}{2M} + \frac{M}{2}\omega_0^2 x^2\,, \quad E_n = (n + \frac{1}{2})\hbar\omega_0\,, \quad \beta = \frac{1}{k_{\mathrm{B}}T}$$

partition function

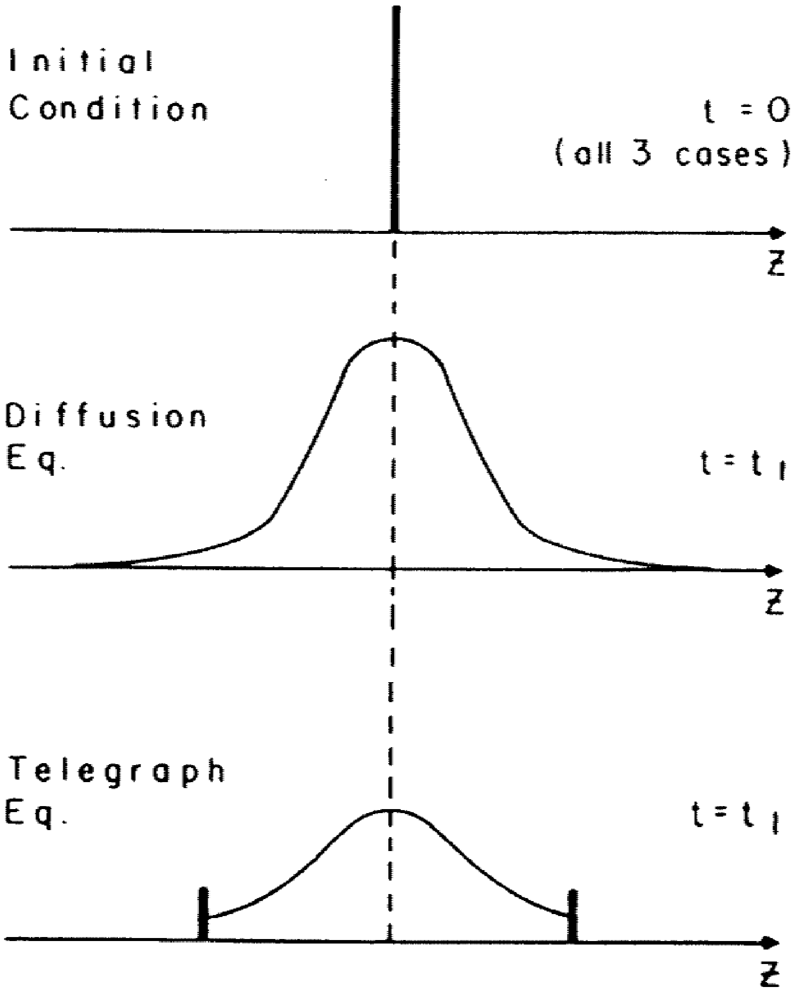
$$Z = \sum_{n=0}^{\infty} \mathrm{e}^{-\beta E_n} = \frac{1}{2\sinh[\hbar\beta\omega_0/2]}$$

entropy

$$S = k_{\mathrm{B}}\left[\ln(Z) - \beta\frac{\partial}{\partial\beta}\ln(Z)\right]$$

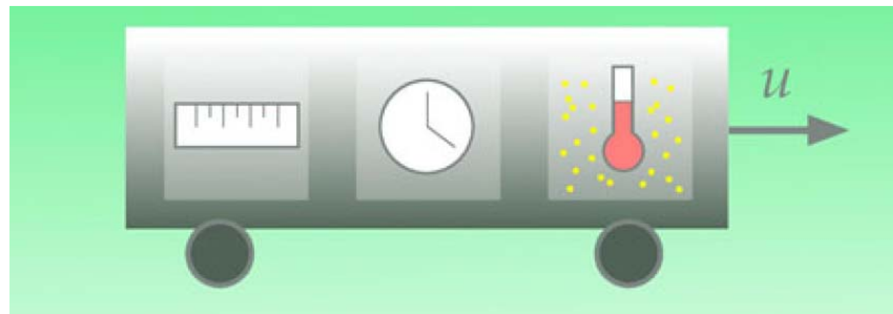
$$= k_{\mathrm{B}}\left[\frac{\hbar\beta\omega_0}{\exp(\hbar\beta\omega_0) - 1} - \ln\big(1 - \exp(-\hbar\beta\omega_0)\big)\right]$$

# Relativistic Brownian motion




Masoliver & Weiss,
Eur. J. Phys. **17**: 190 (1996)

J. Dunkel, P.H., Phys. Rep. **471**, 1 (2009)




$$\gamma = \left[1 - (u/c)^2\right]^{-1/2}$$

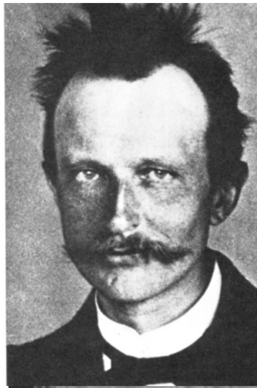
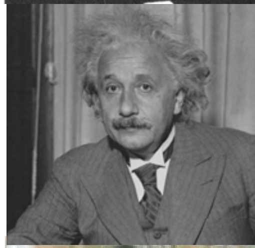
$$l'(u) = l_0 \gamma^{-1} \qquad t'(u) = t_0 \gamma^{+1}$$

$$\boxed{\begin{gathered} T'(u) = T_0 \gamma^{-\alpha} \\ \alpha = ? \end{gathered}}$$

# Relativistic thermodynamics

$$\boxed{T'(w) = T\,(1 - w^2)^{\alpha/2}}$$

$$\alpha = \begin{cases} +1 & \text{Planck, Einstein} \\ 0 & \text{Landsberg, van Kampen} \\ -1 & \text{Ott} \end{cases}$$

Why this confusion?

(i) R-TD non-local theory

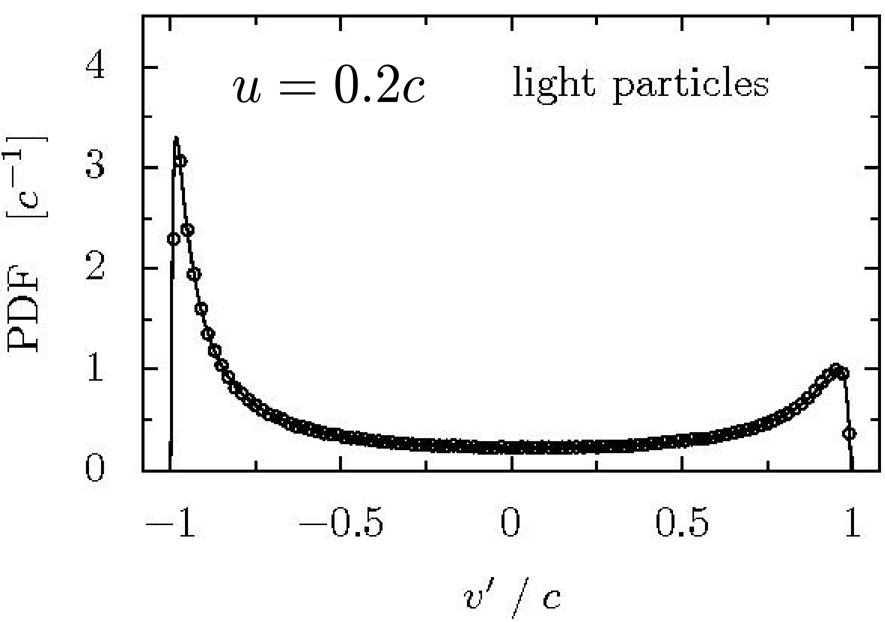
(ii) different definitions of work and heat

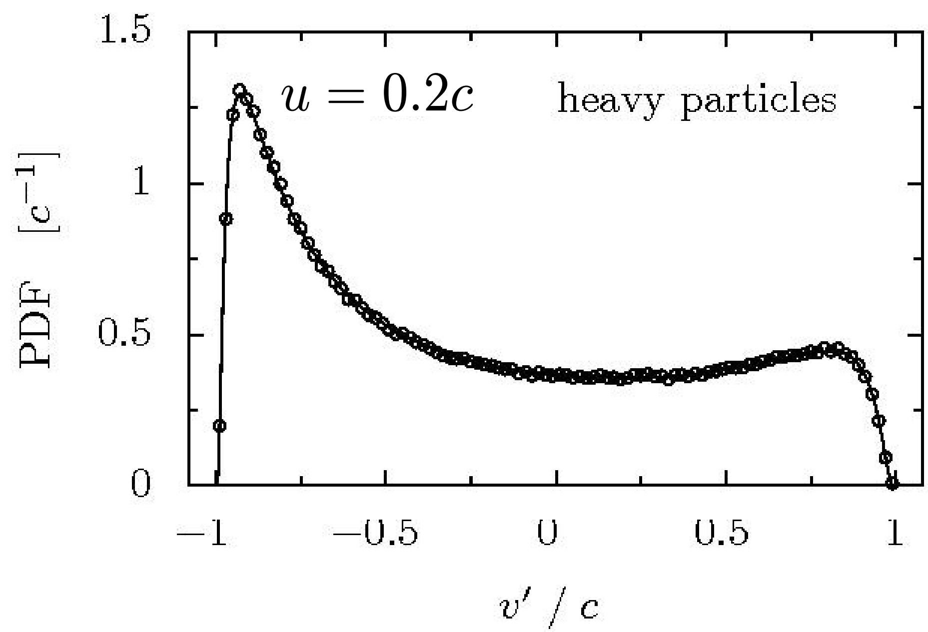
# Moving Observers

$$\Sigma(u=0) \longrightarrow \Sigma'(u \neq 0)$$

$$f'_{\text{Jüttner}}(v', u) = Z_1^{-1} \frac{m\gamma(v')^3}{\gamma(u)} \exp\left[-\beta_{\mathrm{J}}\gamma(u)m\gamma(v')(c^2 + uv')\right]$$

$v'$ : velocity in moving frame



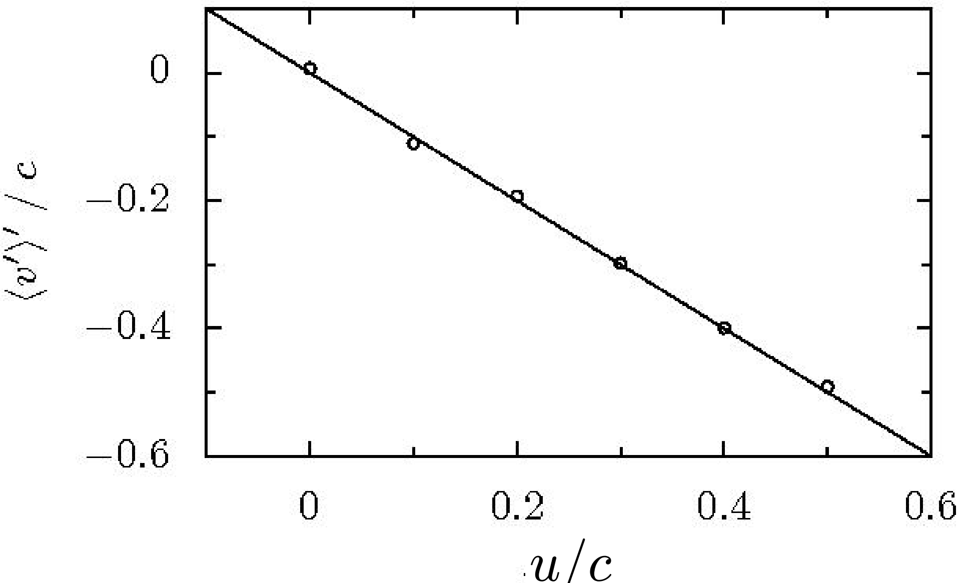

J. Dunkel & P.H., Phys. Rep. **471**, 1-73 (2009)

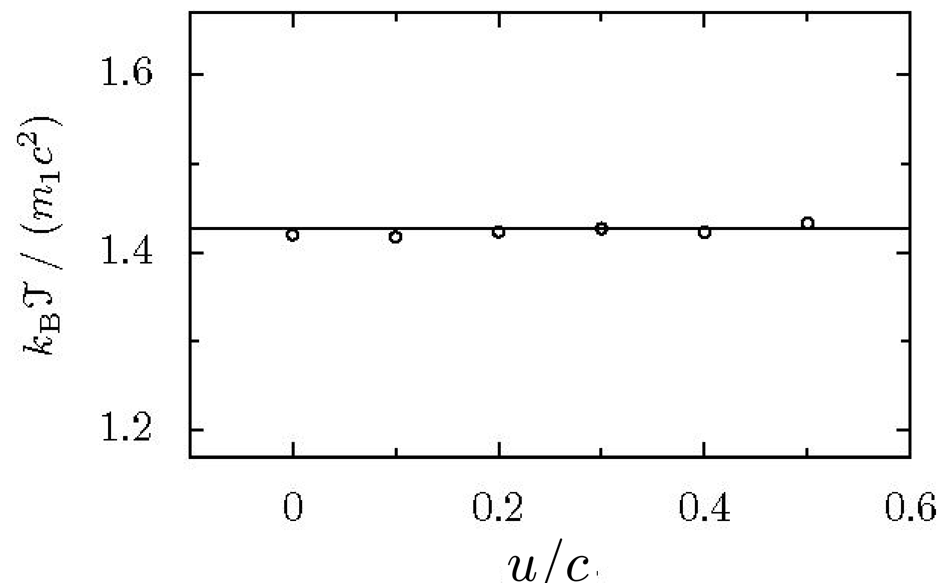
# Measuring temperature in Lorentz invariant way

Lorentz invariant equipartition theorem

$$k_{\mathrm{B}}\mathcal{T} = m\gamma(u)^3\langle\gamma(v')(v'+u)^2\rangle_{t'}$$

with $u = -\langle v'\rangle_{t'}$



# A statistical definition of Temperature

$\Omega(E,V,N)$ : number of microstates for a given macrostate $(E,V,N)$

$$\boxed{E_1,\ V_1,\ N_1}\ \boxed{E_2,\ V_2,\ N_2} \Rightarrow \boxed{E_1,\ V_1,\ N_1 \,\big|\, E_2,\ V_2,\ N_2}$$

$$\Omega_1(E_1,V_1,N_1) \quad \Omega_2(E_2,V_2,N_2) \ \Rightarrow \Omega = \Omega_1 \cdot \Omega_2$$

$$\text{in equilibrium: } \delta\Omega = \frac{\partial \Omega_1}{\partial E_1}\delta E_1\, \Omega_2 + \Omega_1\, \frac{\partial \Omega_2}{\partial E_2}\delta E_2 = 0$$

$$\text{energy conservation: } \delta E_1 = -\delta E_2 \Rightarrow \frac{1}{\Omega_1}\frac{\partial \Omega_1}{\partial E_1} = \frac{1}{\Omega_2}\frac{\partial \Omega_2}{\partial E_2}$$

Boltzmann (Planck): $\mathbf{S = k_B ln\Omega}$

$$\Rightarrow \frac{1}{T_1} = \frac{1}{T_2} \Rightarrow \mathbf{T_1 = T_2 = T}$$

# *Typical* temperature values [°C]

| | |
|---|---|
| Boiling point of Nitrogen | -195.79 |
| Lowest recorded surface temperature on Earth (Vostok, Antarctica – July 21, 1983) | -89 |
| Highest recorded surface temperature on Earth (Al' Aziziyah, Libya – September 13, 1922) | 58 |
| Temperature in the Earth's Thermosphere (80 - 650 km above the surface) | ~ 1500 |
| Melting point of diamond | 3547 |
| Surface temperature of the sun (photosphere) | ~ 5526 |
| Temperature in the interior of the sun | ~ $15 \cdot 10^6$ |

# Brief history of the temperature concept