The ring of Brownian motion: the good, the bad, and the simply silly

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Acknowledgements

nim
nanosystems initiative munich

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Center for NanoScience
Ludwig-Maximilians-Universität

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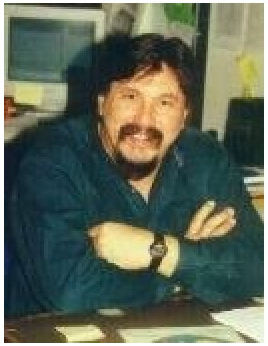
SFB 486

SFB 631

SFB 484

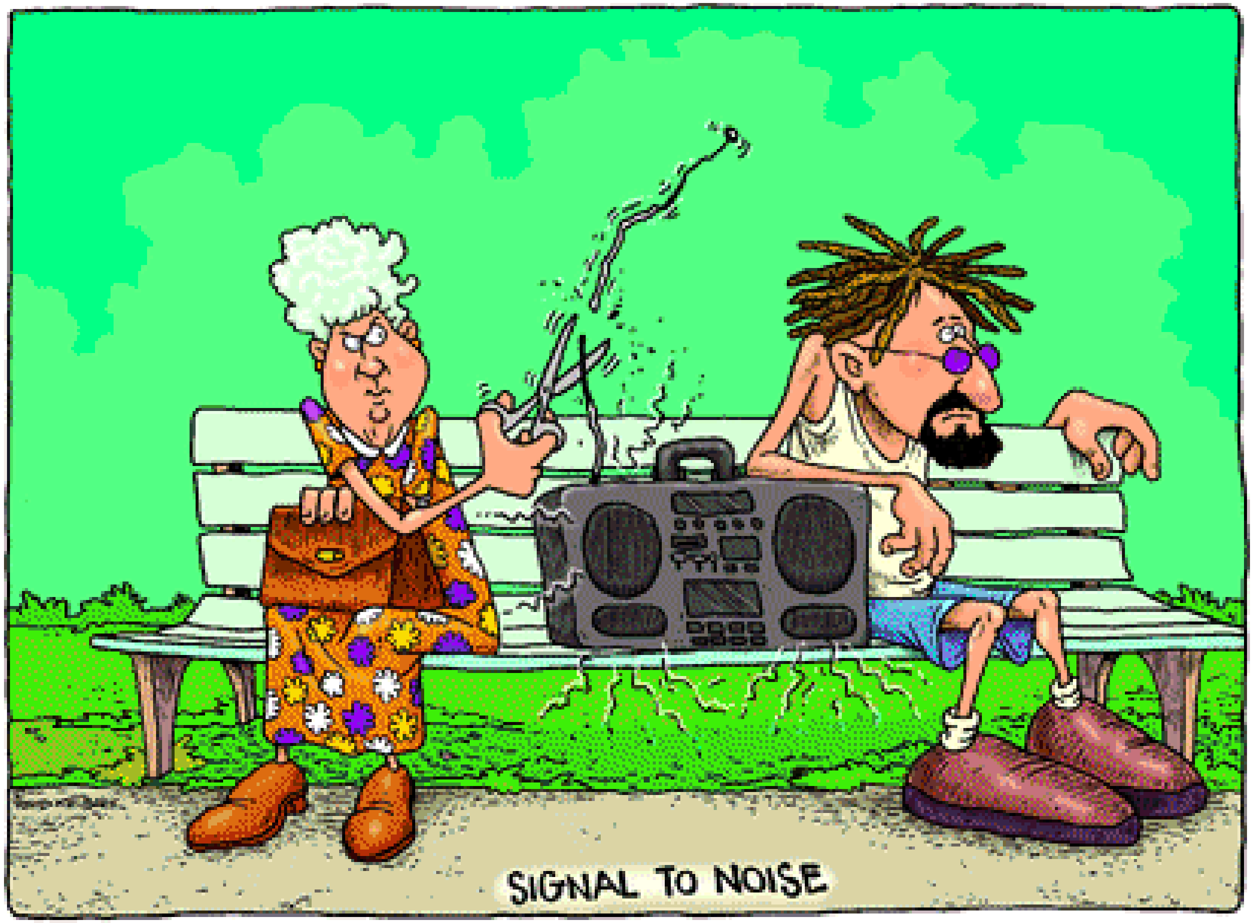
QUANTRANS



Prof. Dr.
Hermann E. Gaub

Prof. Dr.
Jörg P. Kotthaus

Noise – always bad ?



Source: Agilent Technologies

Why you should not do Brownian motion

- You know nothing about the subject
- Many very good people worked on it

  (Einstein, Langevin, Smoluchowski, Ornstein, Uhlenbeck, Wiener, Onsager, Stratonovich, …)
- You don’t have your own pet theory yet

Why you should do Brownian motion

- You know nothing about the subject
- Many very good people worked on it
- You still can do your own pet theory

History

Robert Brown (1773-1858)

Source: www.anbg.gov.au

Source: permission kindly granted by Prof. Brian J. Ford
http://www.brianjford.com/wbbrowna.htm

1827 – irregular motion of granules of pollen in liquids

- Brown, Phil. Mag. **4**, 161 (1928)
- Deutsch: *Did Robert Brown observe Brownian Motion: probably not*, Sci. Am. **256**, 20 (1991)
- Ford: "*Brownian movement in clarkia pollen: a reprise of the first observations*", The Microscope **39**, 161 (1991)

Jan Ingen-Housz (1730-1799)

Source: www.americanchemistry.com

Johann Ingen-Housz
K. K. Hofraths und Leibarztes, der Königl. Gesellschaft der Wissenschaften zu London, der Batavischen Gesellschaft der Experimentalphilosophie zu Rotterdam ꝛc. ꝛc. Mitglieds

Vermischte
Schriften
physisch-medicinischen Inhalts.

Uibersetzt und herausgegeben
von
Nicolaus Carl Molitor.

Zweyte, verbesserte und mit ganz neuen Abhandlungen vermehrte Auflage.

Mit Kupfertafeln.

Zweyter Band.

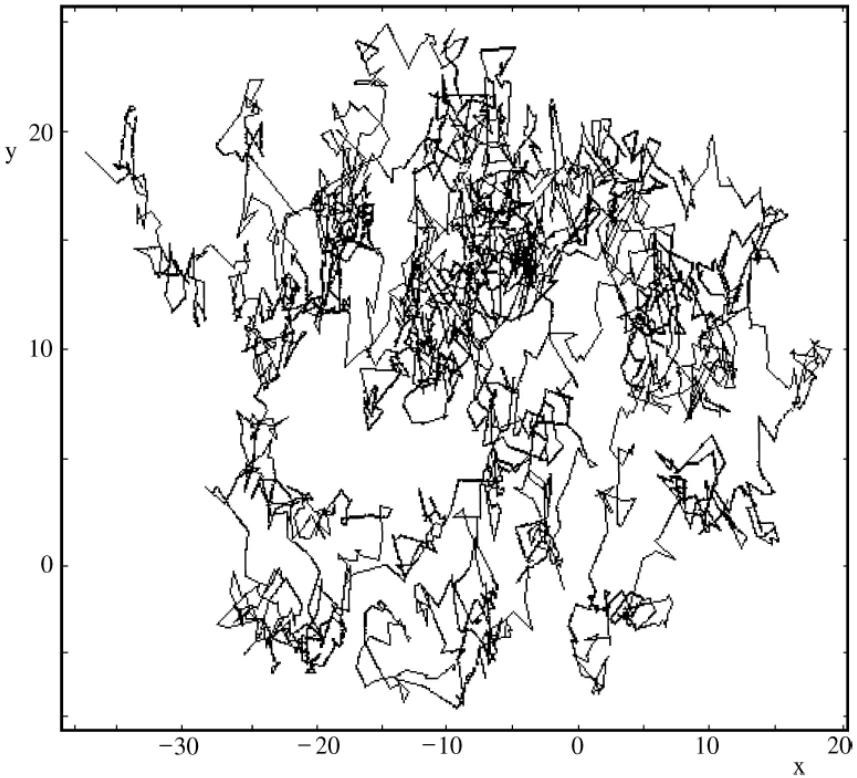
Wien,
gedruckt und verlegt bey Christian Friderich Wappler.
1784.

To see clearly how one can deceive one's mind on this point if one is not careful, one has only to place a drop of alcohol in the focal point of a microscope and introduce a little finely ground charcoal therein, and one will see these corpuscules in a confused, continuous and violent motion, as if they were animalcules which move rapidly around.

Mean squared displacement

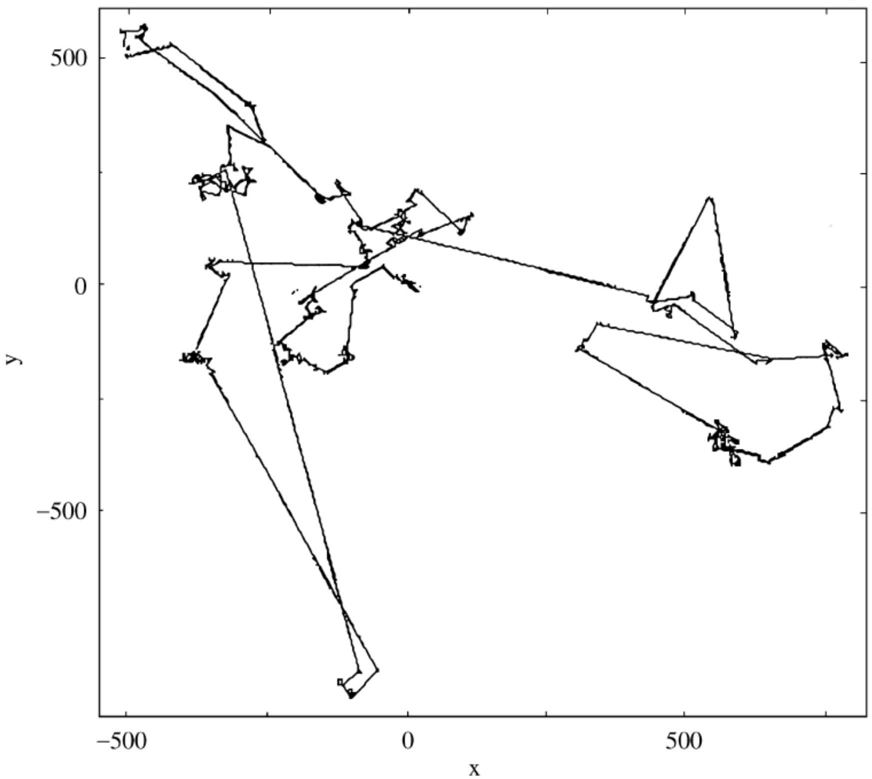
$$\langle x^2(t) \rangle \propto t^\alpha$$

Brownian movement $\alpha = 1$



Source: Physica A **282**, 13 (2000)

Lévy-Brownian movement $\alpha = \frac{4}{3}$



Source: Physica A **282**, 13 (2000)

Theory of Brownian motion

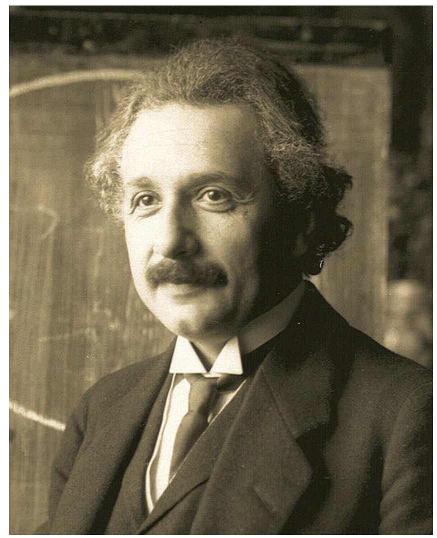
W. Sutherland (1858-1911)

Source: www.theage.com.au

$$D = \frac{RT}{6\pi\eta a C}$$

Phil. Mag. **9**, 781 (1905)

A. Einstein (1879-1955)



Source: wikipedia.org

$$\langle x^2(t) \rangle = 2Dt$$

$$D = \frac{RT}{N}\frac{1}{6\pi k P}$$

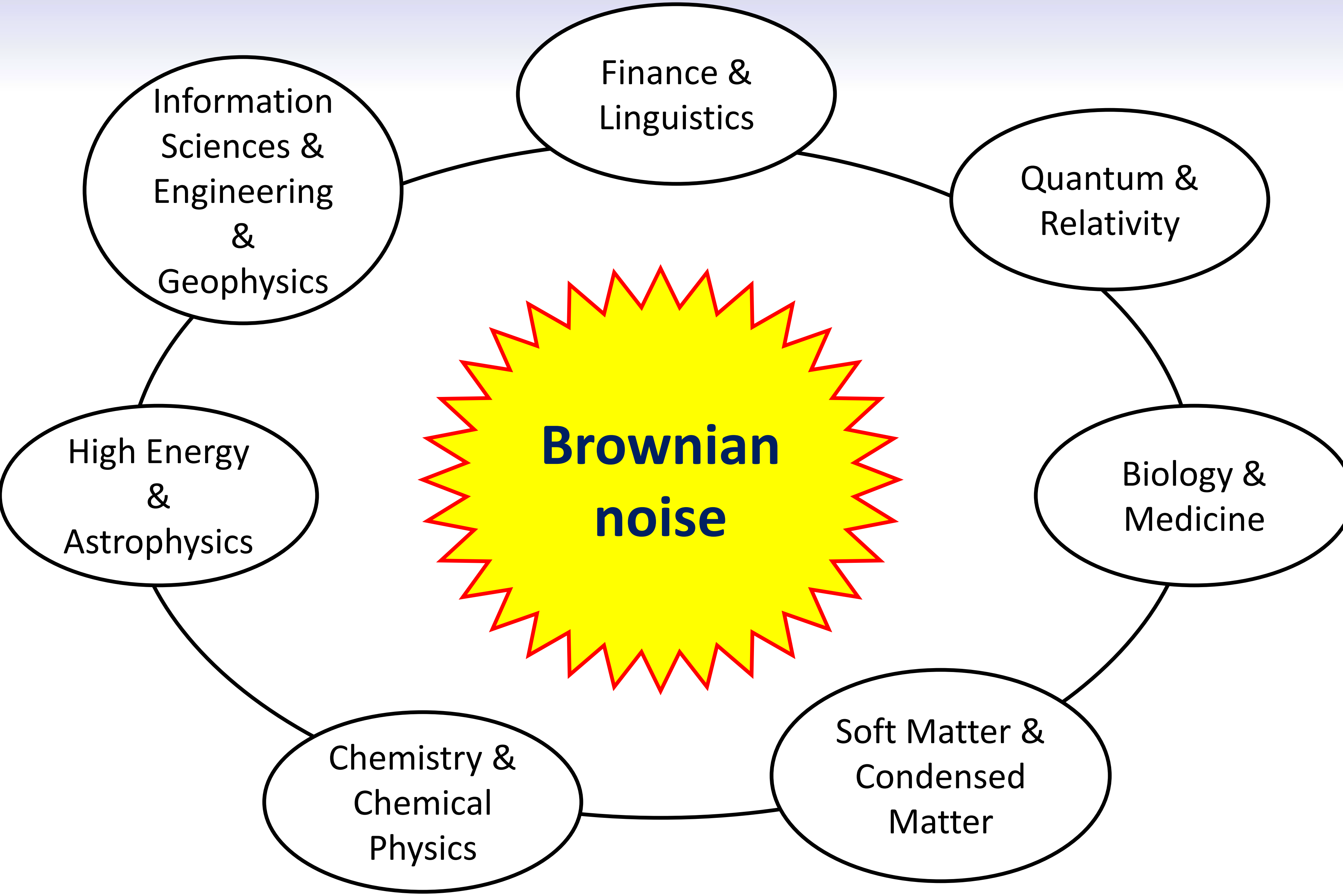
Ann. Phys. **17**, 549 (1905)

M. Smoluchowski (1872-1917)

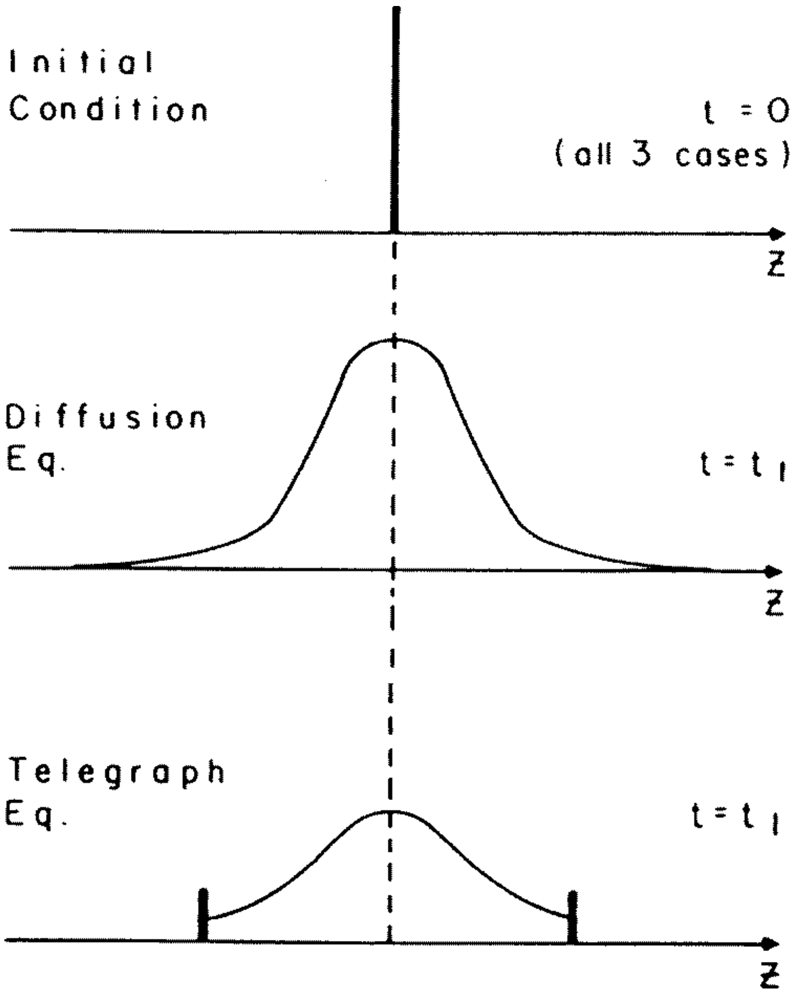
Source: wikipedia.org

$$D = \frac{32}{243}\frac{mc^2}{\pi\mu R}$$

Ann. Phys. **21**, 756 (1906)

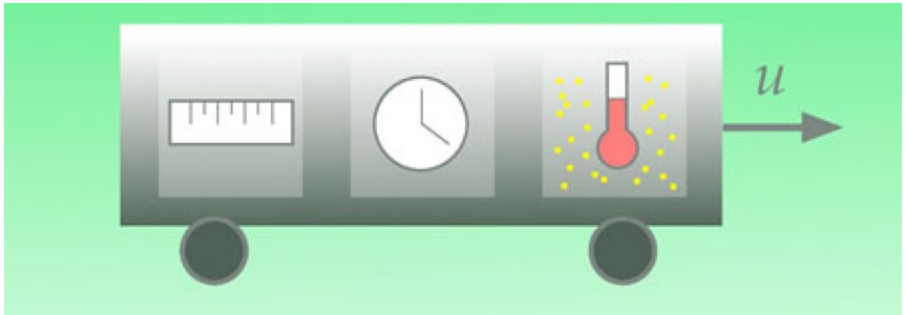

Brownian noise
Finance & Linguistics
Quantum & Relativity
Biology & Medicine
Soft Matter & Condensed Matter
Chemistry & Chemical Physics
High Energy & Astrophysics
Information Sciences & Engineering & Geophysics

Relativistic Brownian motion



Masoliver & Weiss,
Eur. J. Phys. **17**: 190 (1996)

J. Dunkel, P.H., Phys. Rep. **471**, 1 (2009)



$$\gamma = \left[1 - (u/c)^2\right]^{-1/2}$$

$$l(u) = l_0 \gamma^{\mathbf{-1}} \quad t(u) = t_0 \gamma^{\mathbf{+1}}$$

$$T(u) = T_0 \gamma^{\boldsymbol{\alpha}}$$
$$\alpha = ?$$

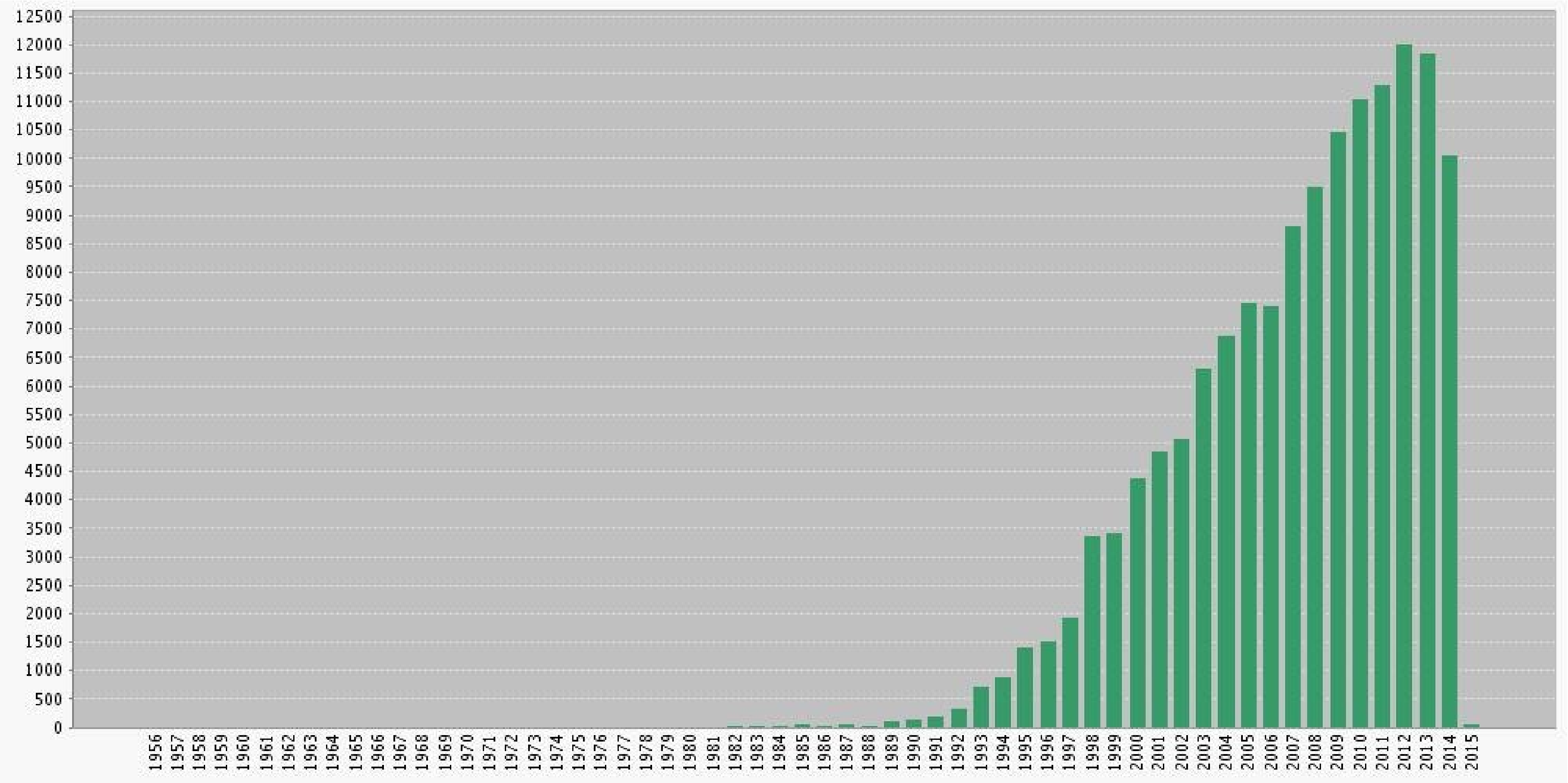
J. Dunkel, Mon 10:30 (BAR SCHÖ)

Two prominent examples

Stochastic Resonance

Brownian Motors

SR – Citations in Each Year



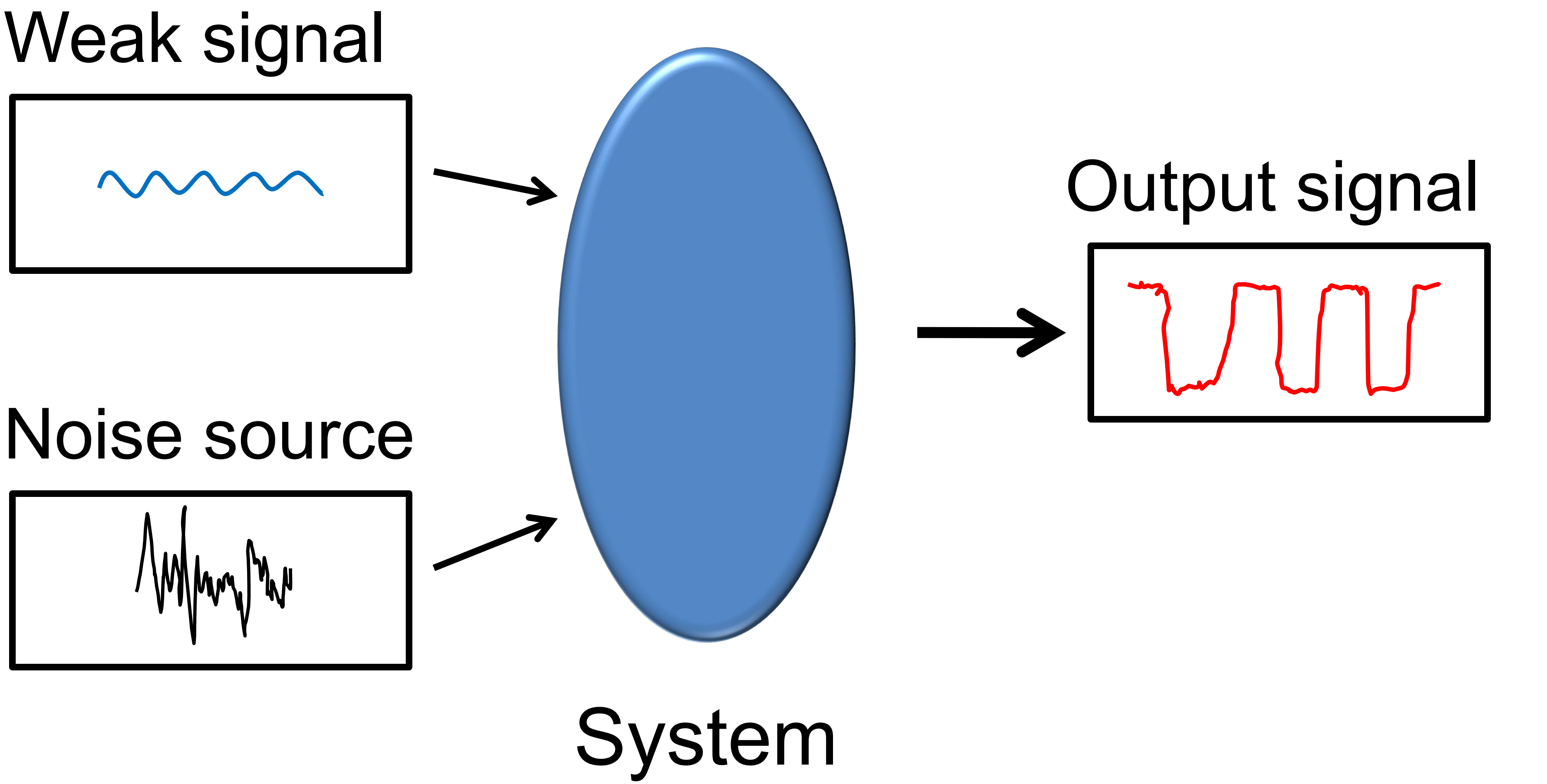
Source: www.isiknowledge.com

papers in 2013: ≈ 480

> 145.000 citations in total

Stochastic Resonance

(in a nutshell)



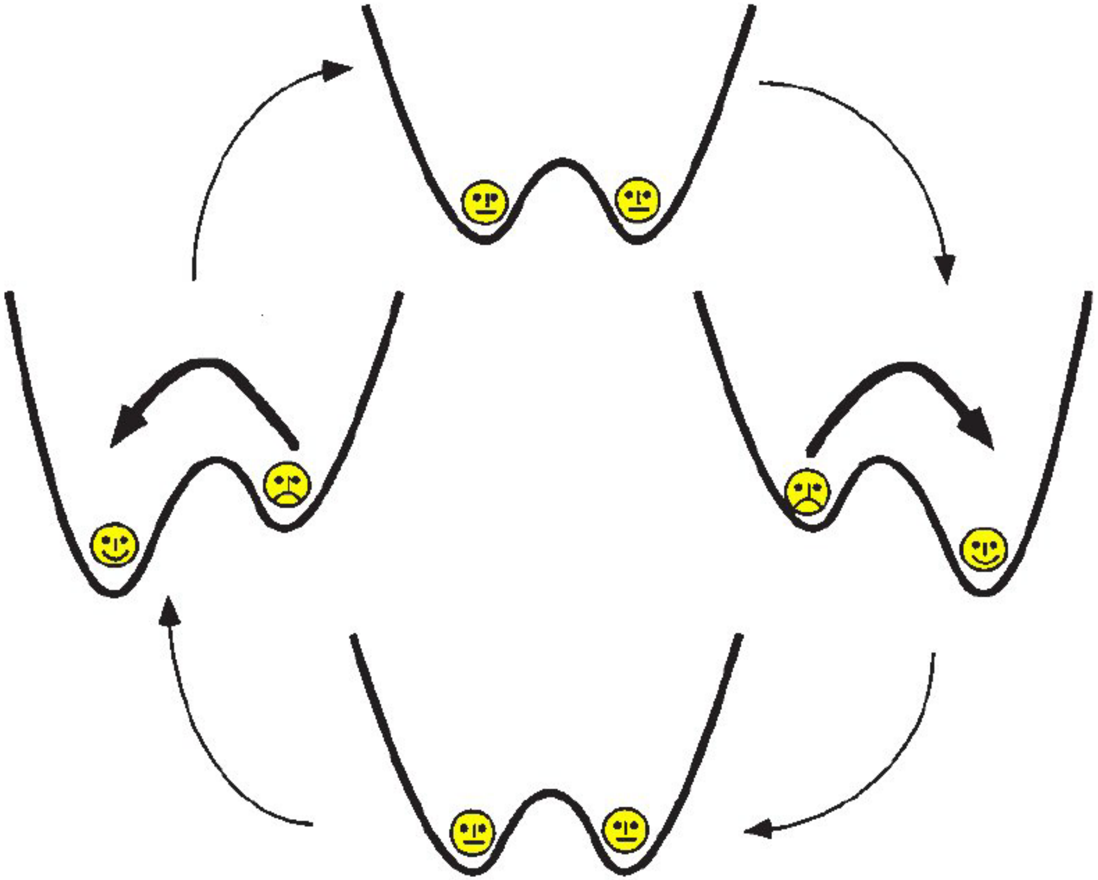
SR - Ingredients

- ✓ Threshold system
- ✓ Weak (subthreshold) signal
- ✓ Noise

⬇

Anomalous amplification properties

Noise-assisted synchronized hopping

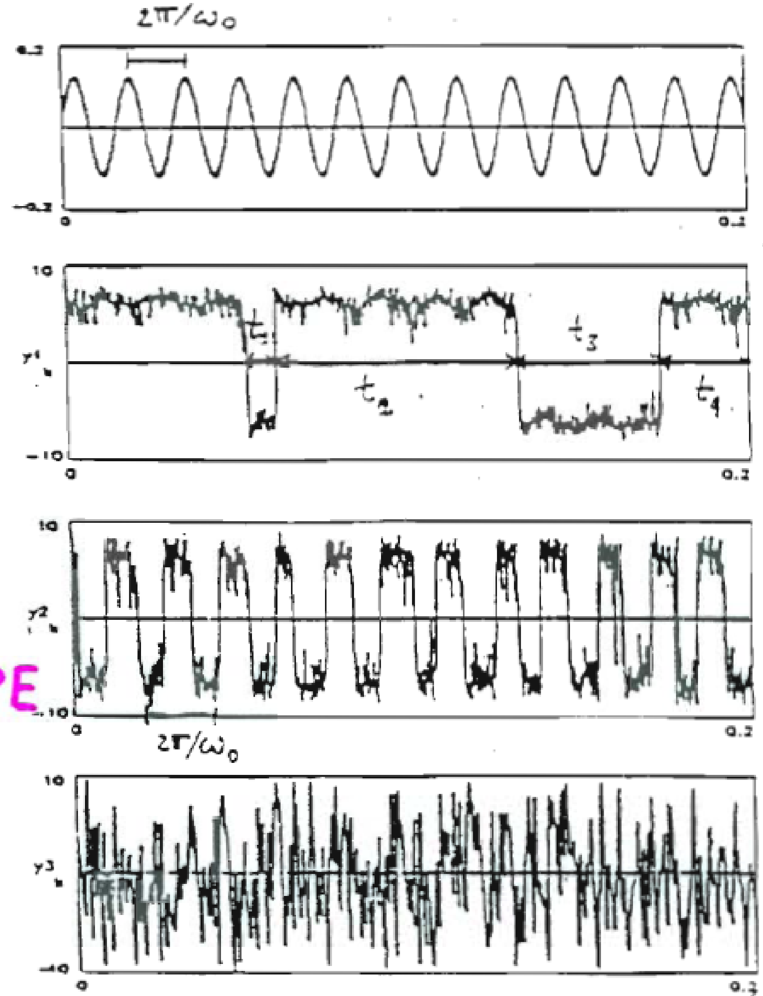


$$T_{\text{period}} \cong 2T_{\text{escape}}$$

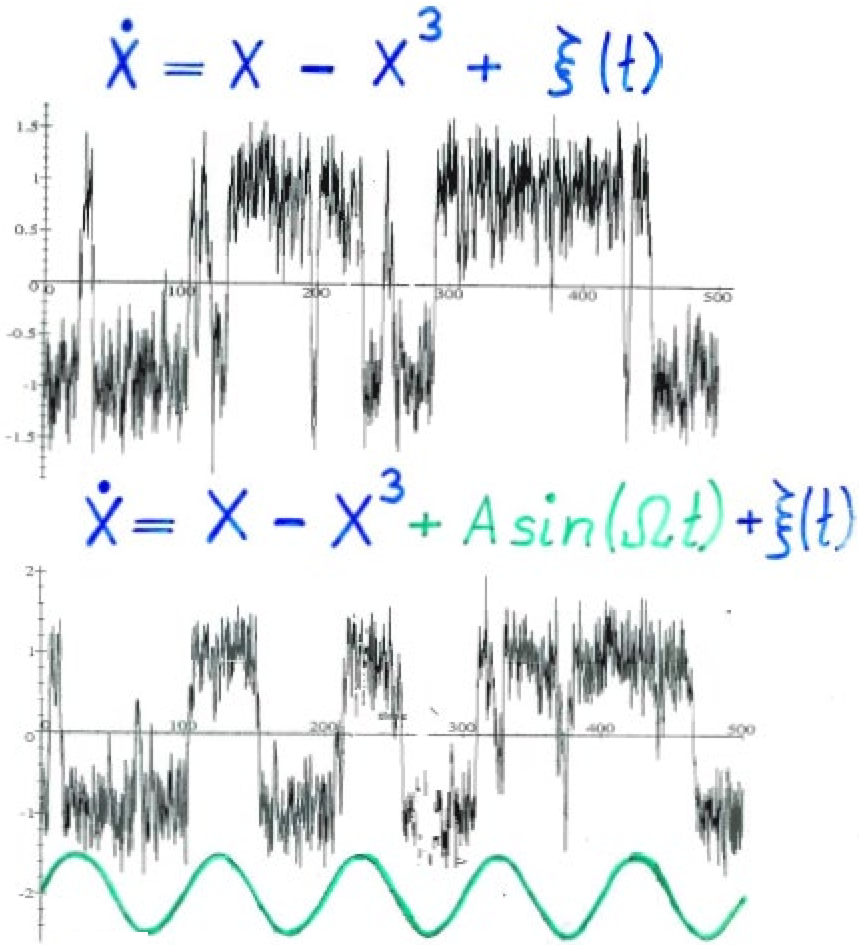
Synchronization

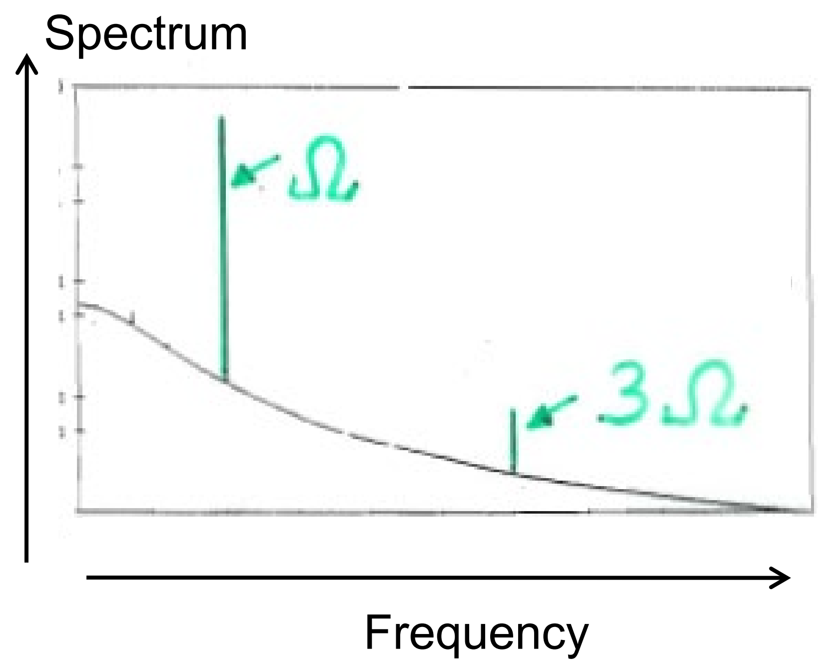
SIGNAL

$T_e \sim 2\Gamma^{-1}_{ESCAPE}$



Power spectral density





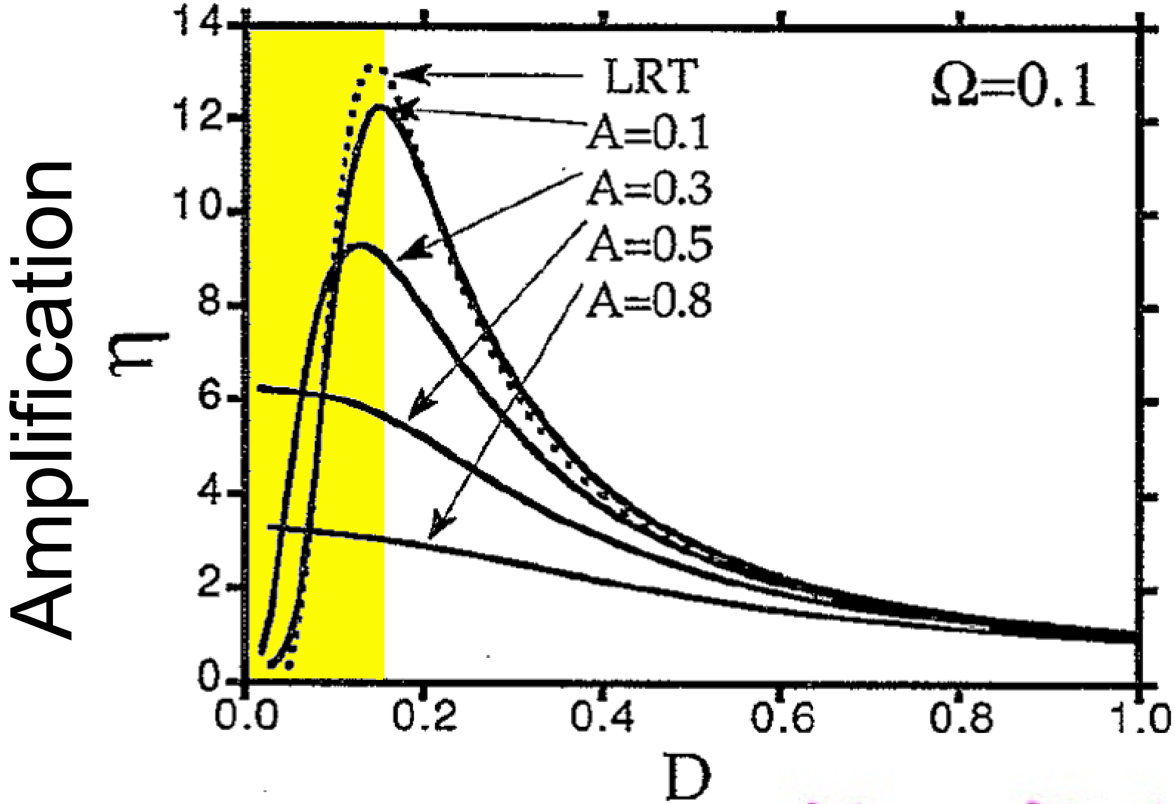
Measuring SR

- Signal to noise ratio
- Spectral amplification
- mutual information
- cross-correlation: input $\leftrightarrow$ output
- peak area, (phase-) synchronization, …

SR-reviews:

L. Gammaitoni, P. Hänggi, P. Jung, F. Marchesoni, Rev. Mod. Phys. **70**, 223 (1998)
P. Hänggi, ChemPhysChem **3**, 285 (2002)

Signal amplification



Noise strength

MORE NOISE

MORE SIGNAL

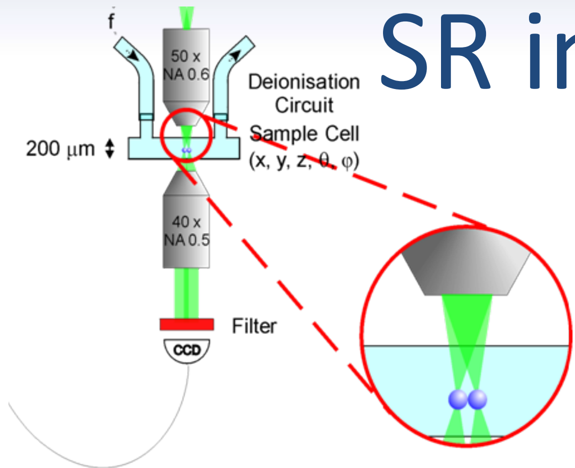
$$M_1 \sim \chi(\tau) = -\frac{1}{D}\frac{d}{d\tau}\langle \delta x(\tau)\,\delta\xi(0)\rangle$$

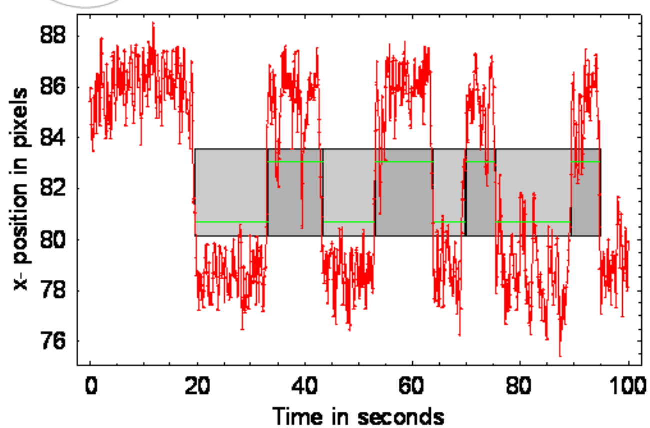
$$|M_1|^2 \propto 1/D^2 \qquad \exp(-2\Delta U/D)$$

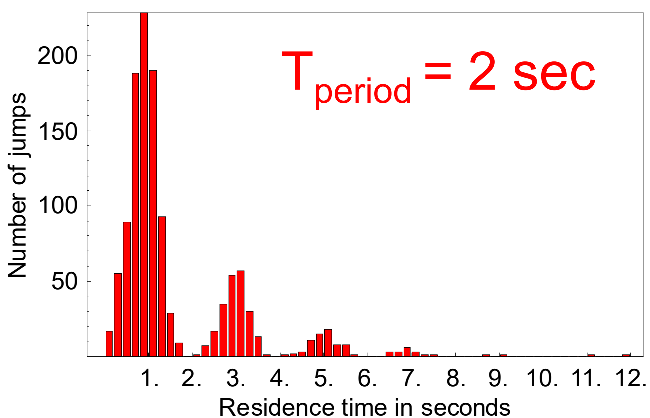
P. Jung, P. Hänggi, Phys. Rev. A **44**, 8032 (1991)

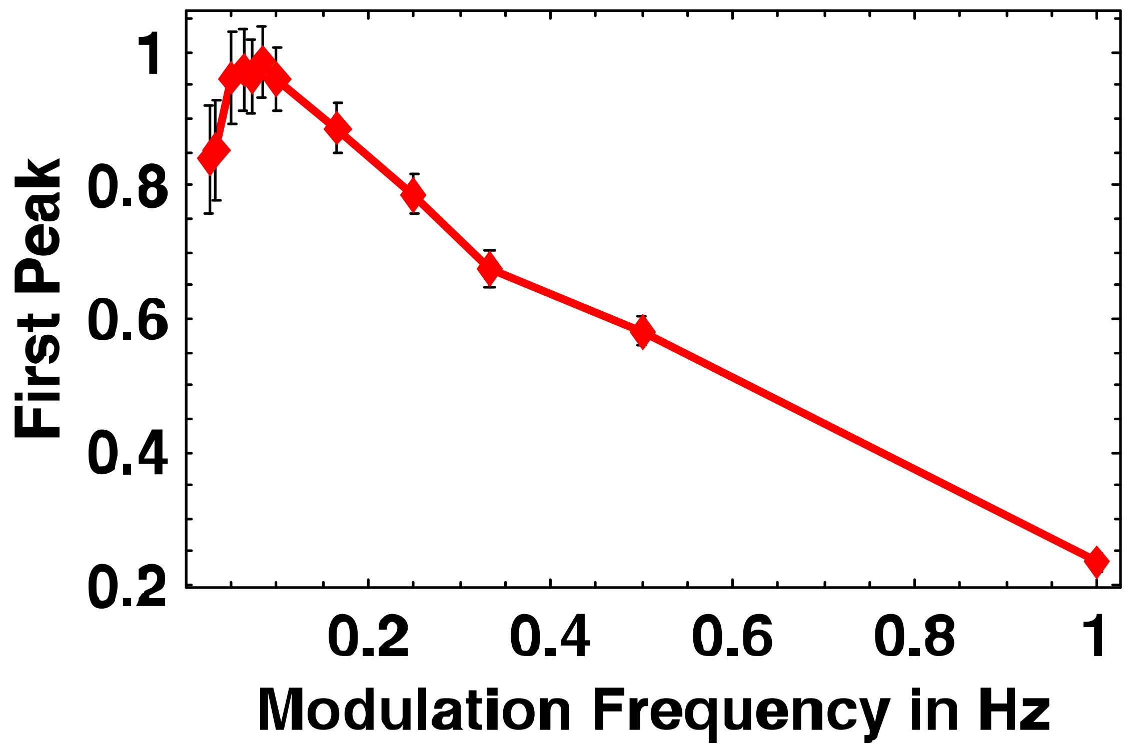
SR examples

SR in colloidal systems





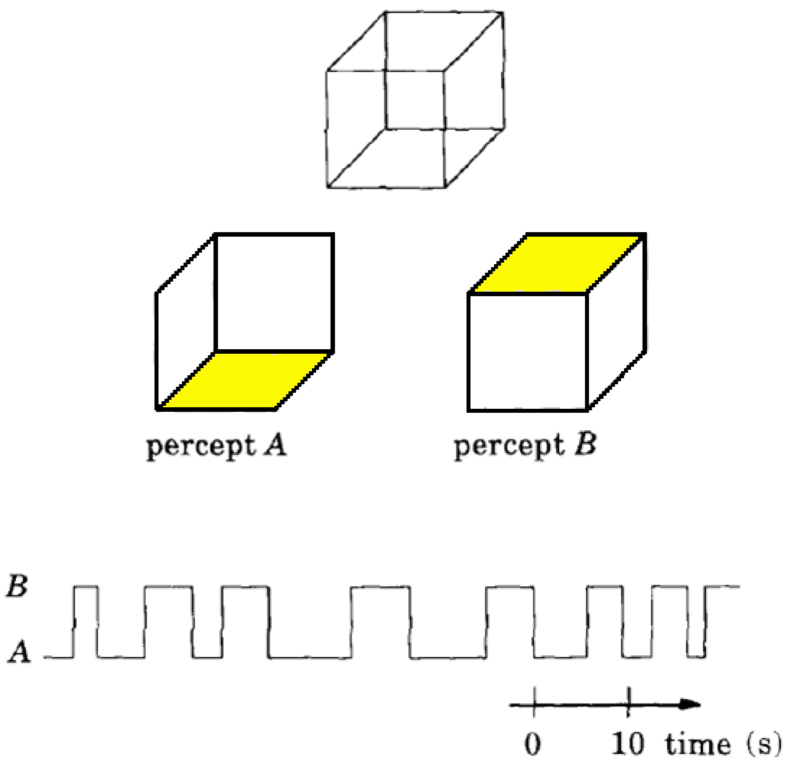


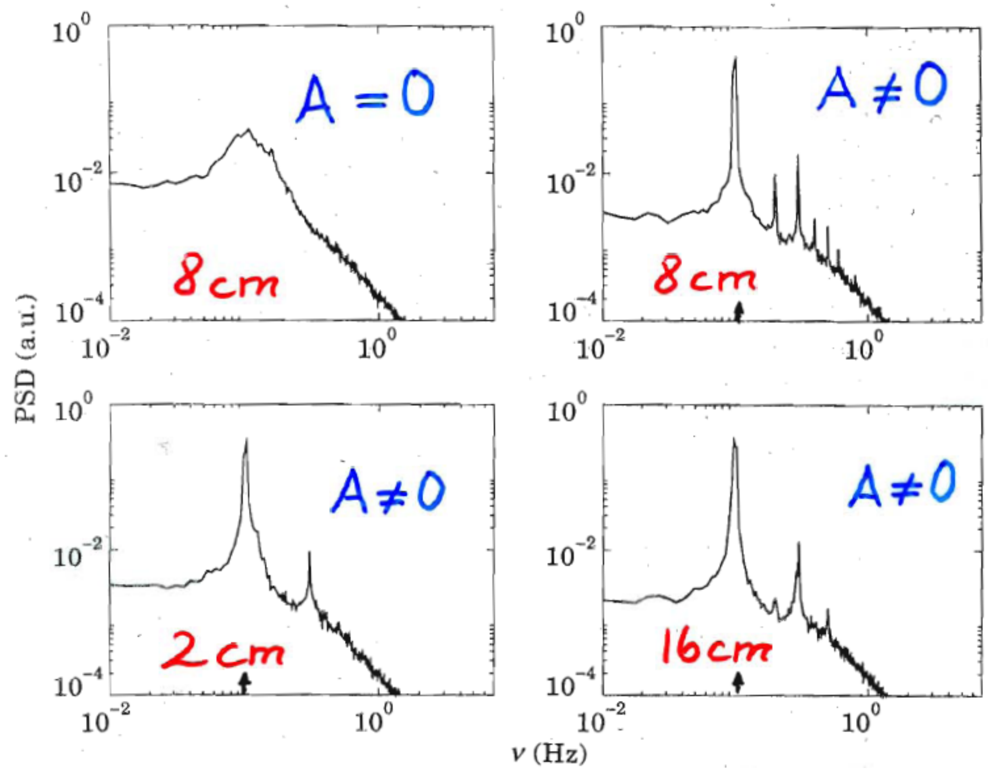


D. Babic, C. Schmitt, I. Poberaj, C. Bechinger, Europhys. Lett. **67**, 158 (2004)

SR in Visual Perception

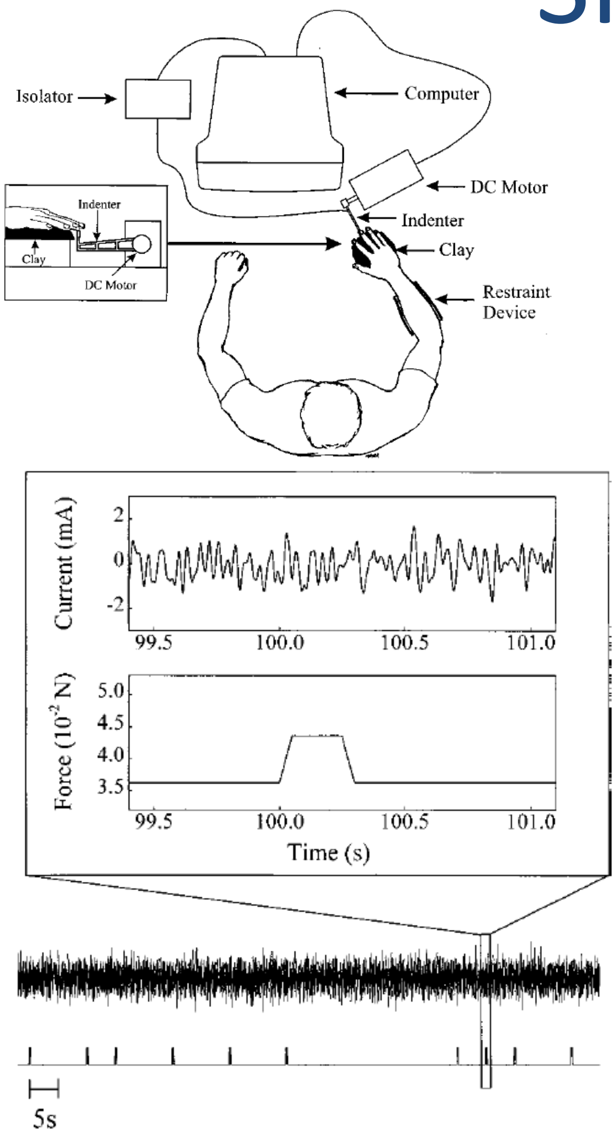
Experiment:





M. Riani, E. Simonotto, Nuovo Cimento D **17**, 903 (1995)

SR in tactile sensation – SRT - Zeptoring



J.J. Collins - group, Chaos **8**, 599 (1998)

- Parkinson
- Multiple sclerosis (MS)
- Stroke / skull-brain-trauma
- Cross-section paralysis
- Depression
- Pain
- …

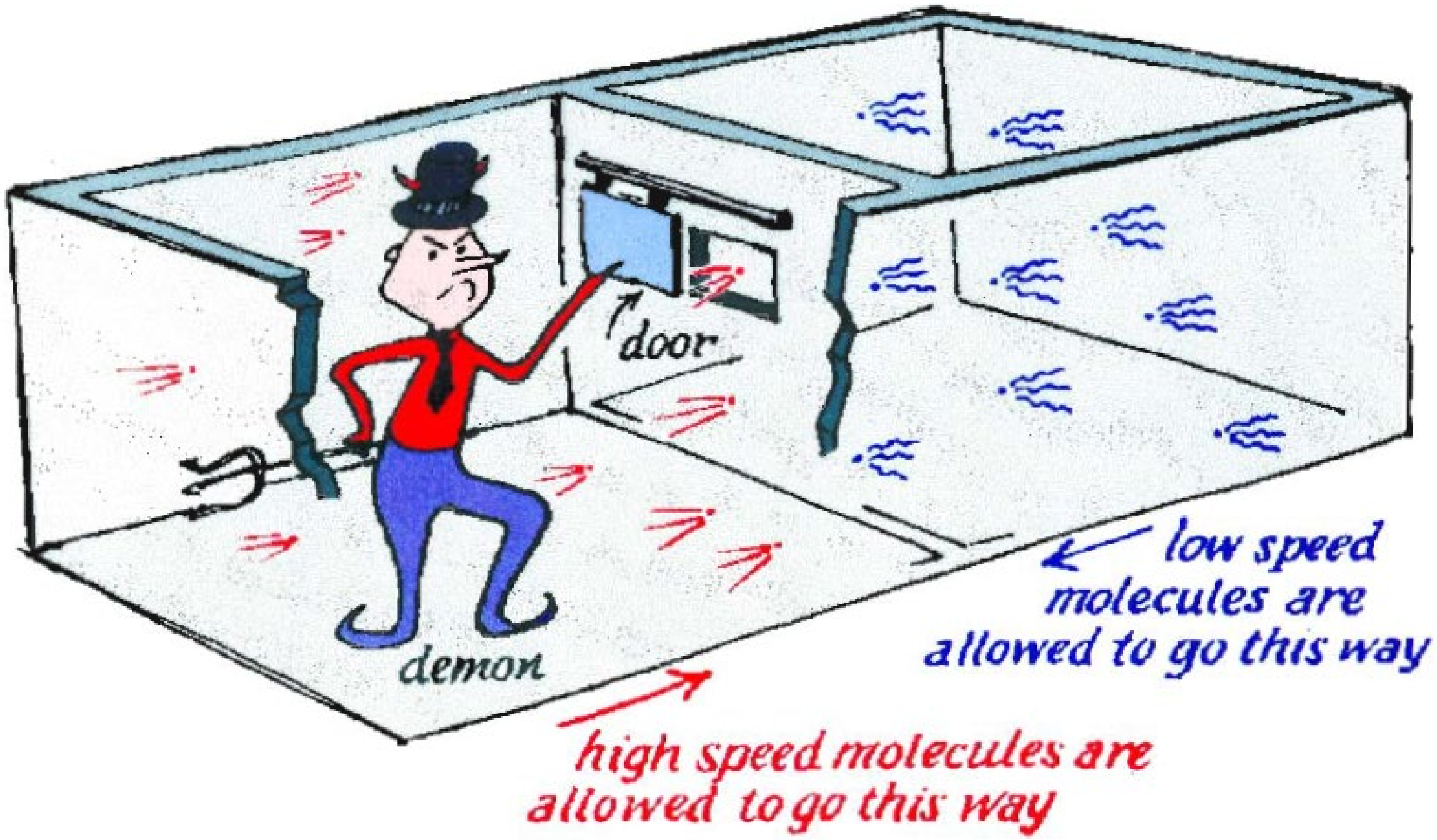
Source: www.sr-therapiesysteme.de

SR trends

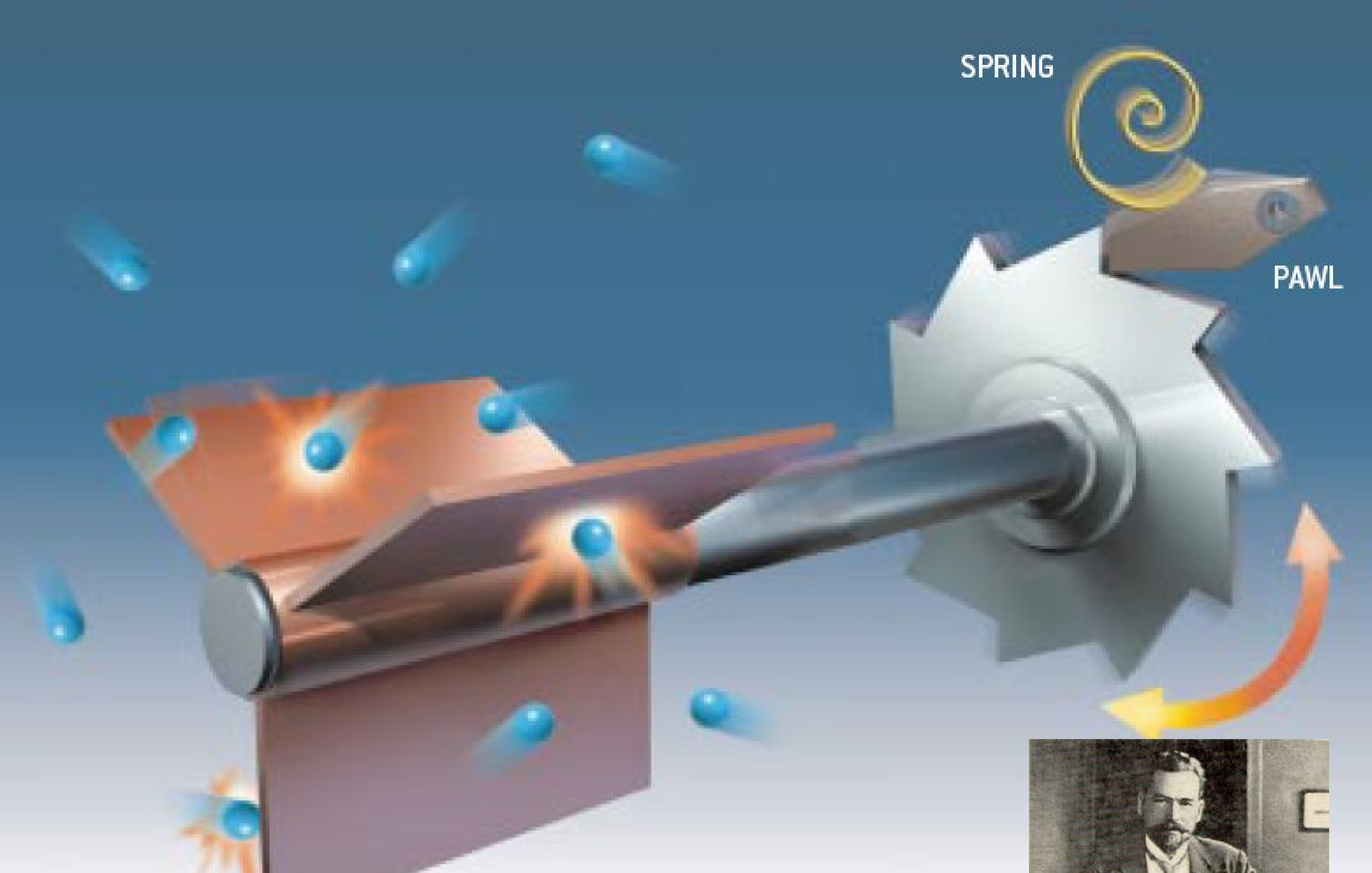
- Spatio – temporal SR
- Aperiodic SR
- Quantum SR

Brownian motors:

EX(E/O)RCISING DEMONS

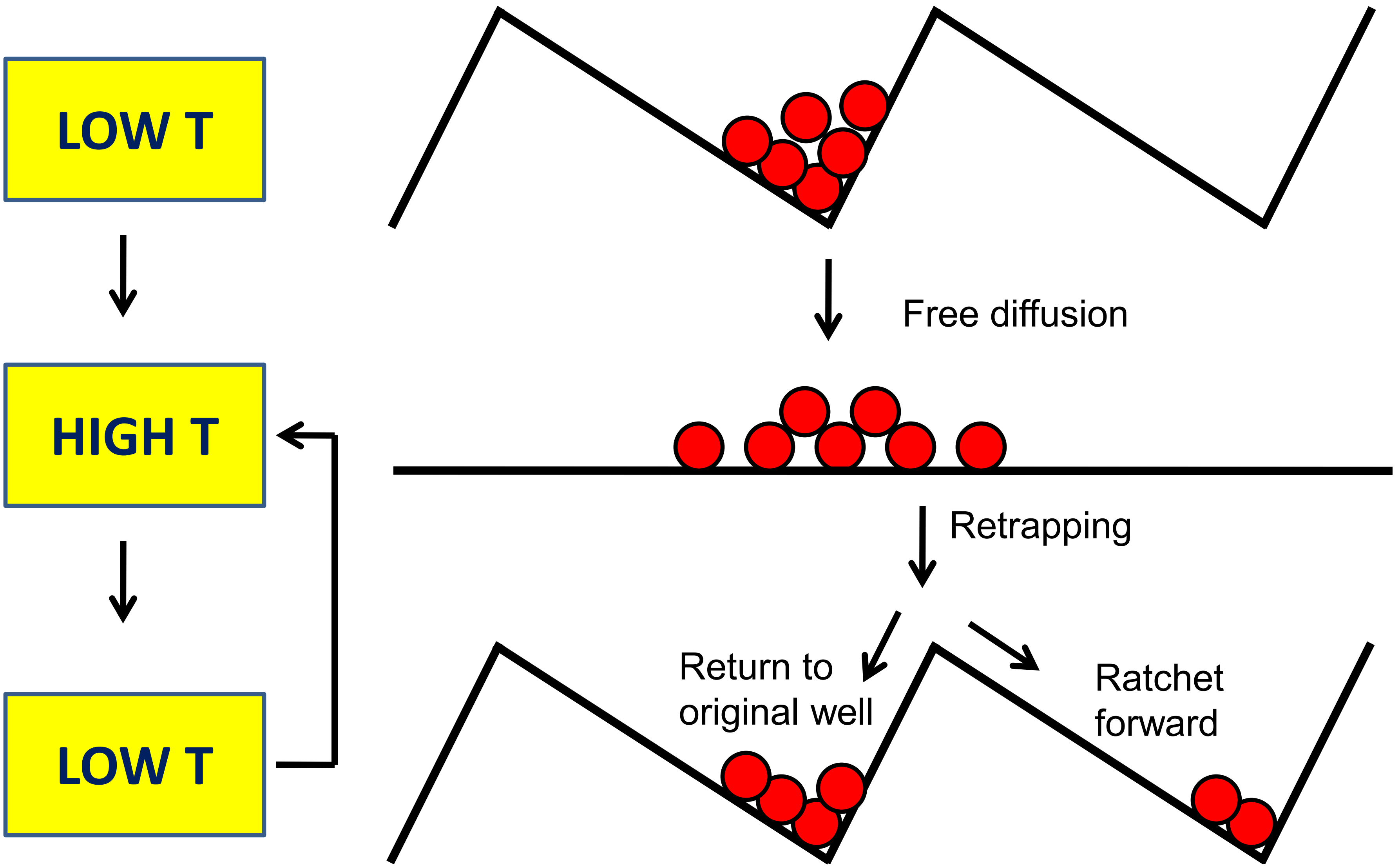


Source: H.S. Leff, *Maxwell's Demon* (Adam Hilger, Bristol, 1990)



Source: Scientific American (2001)

Temperature / Flashing Ratchet



On-Off Brownian motor – doing work

Brownian motors - Characteristics

- Noise & AC-Input → **DC-Ouput**
- Non-equilibrium Noise → **Directed Transport**
- **Current reversals**
- **Applications:**
  - **Novel pumps and traps for charged or neutral particles**
  - **Brownian diodes & transistors**

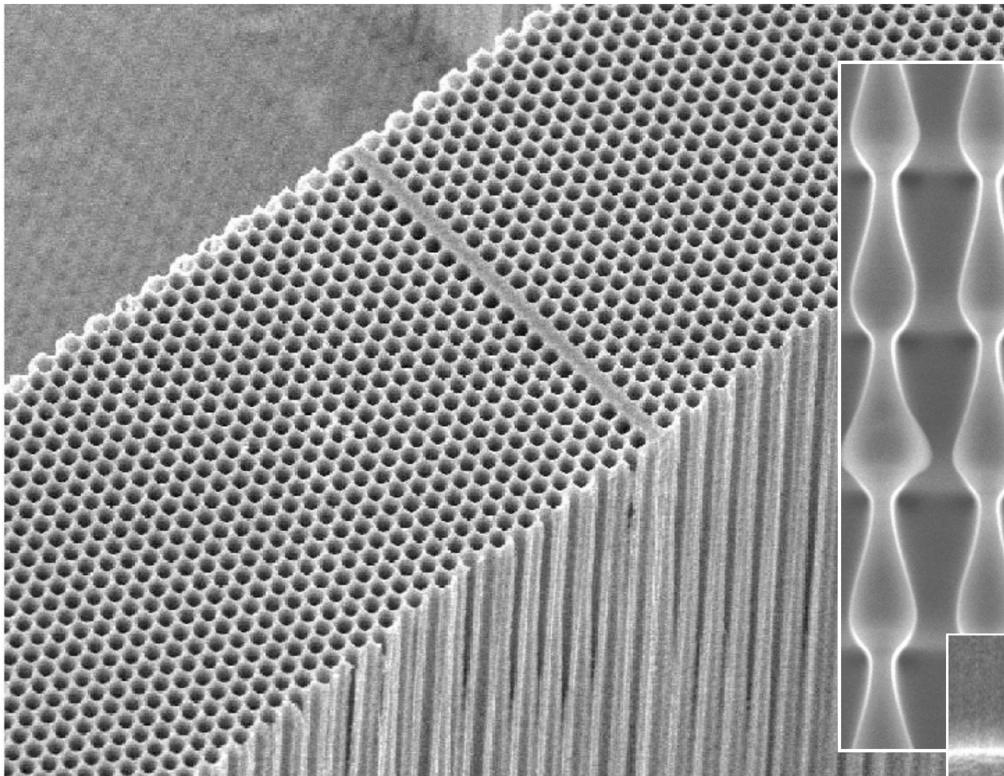
Ask not what physics can do for biology, ask what biology can do for physics

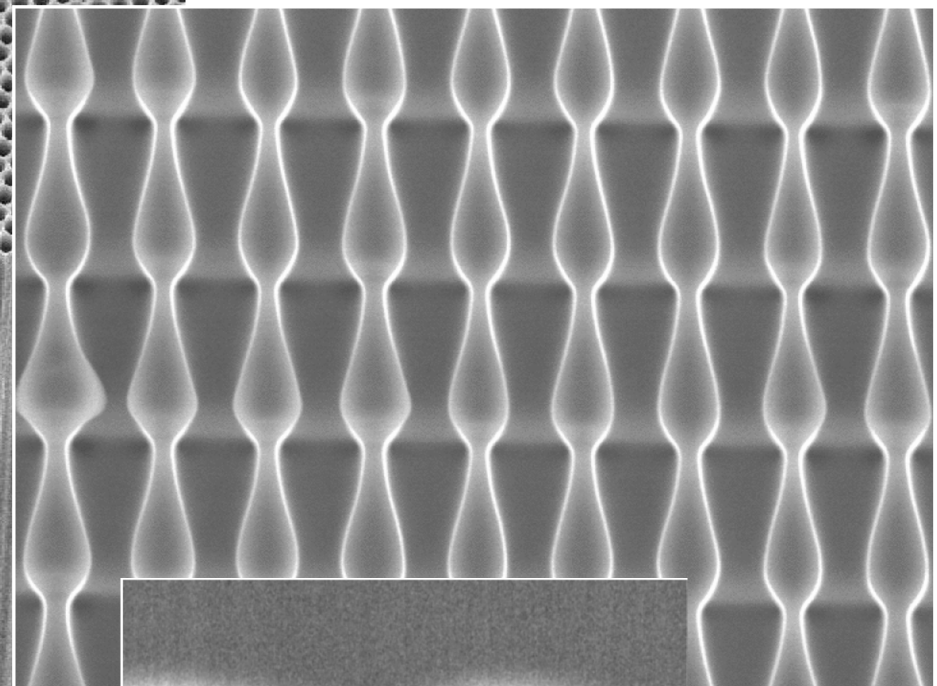
REVIEWS OF MODERN PHYSICS, VOLUME 81, JANUARY–MARCH 2009

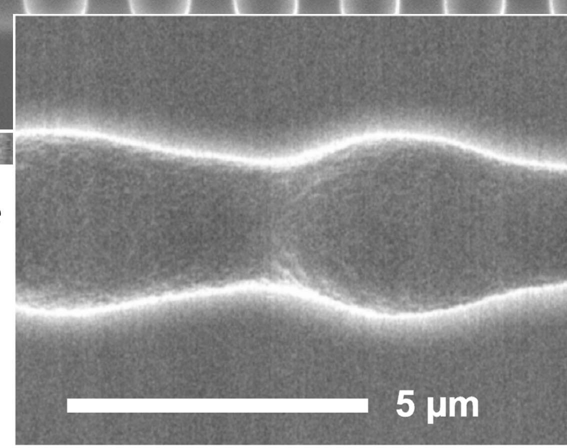
Artificial Brownian motors: Controlling transport on the nanoscale

P.H. and F. Marchesoni

Drift Ratchet - Device





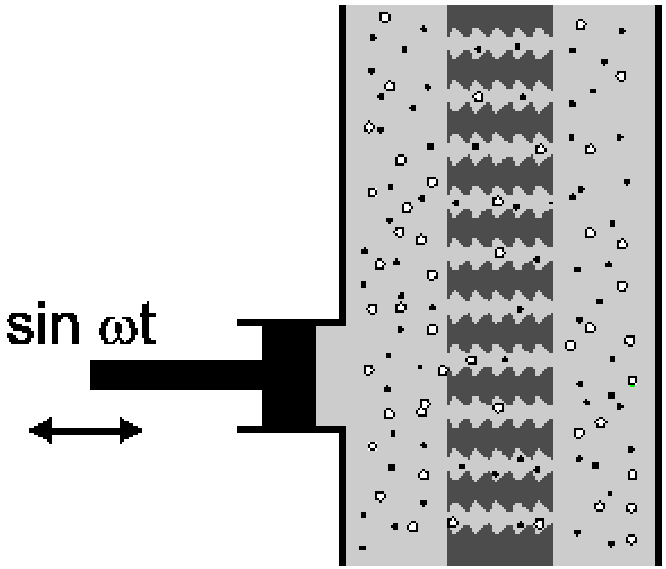


Source: F. Müller, MPI for microstructure physics, Halle

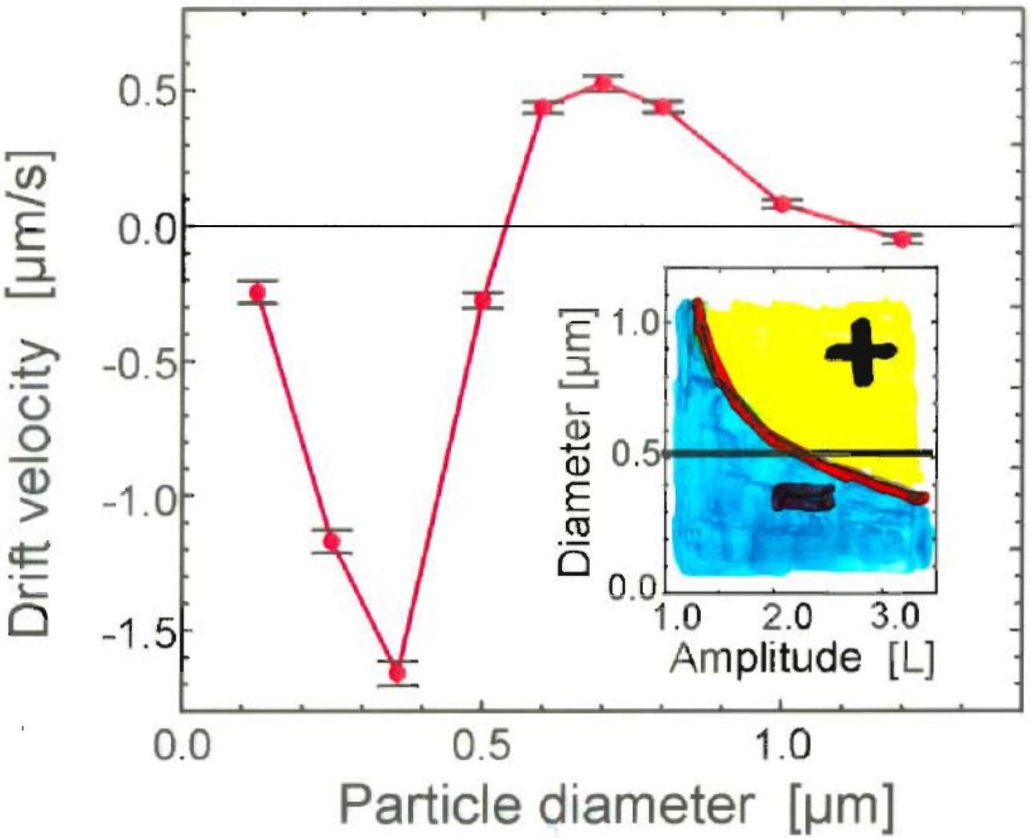
Drift Ratchet - Theory

C. Kettner, P. Reimann, P. H., F. Müller, Phys. Rev. E **61**, 312 (2000)

Setup

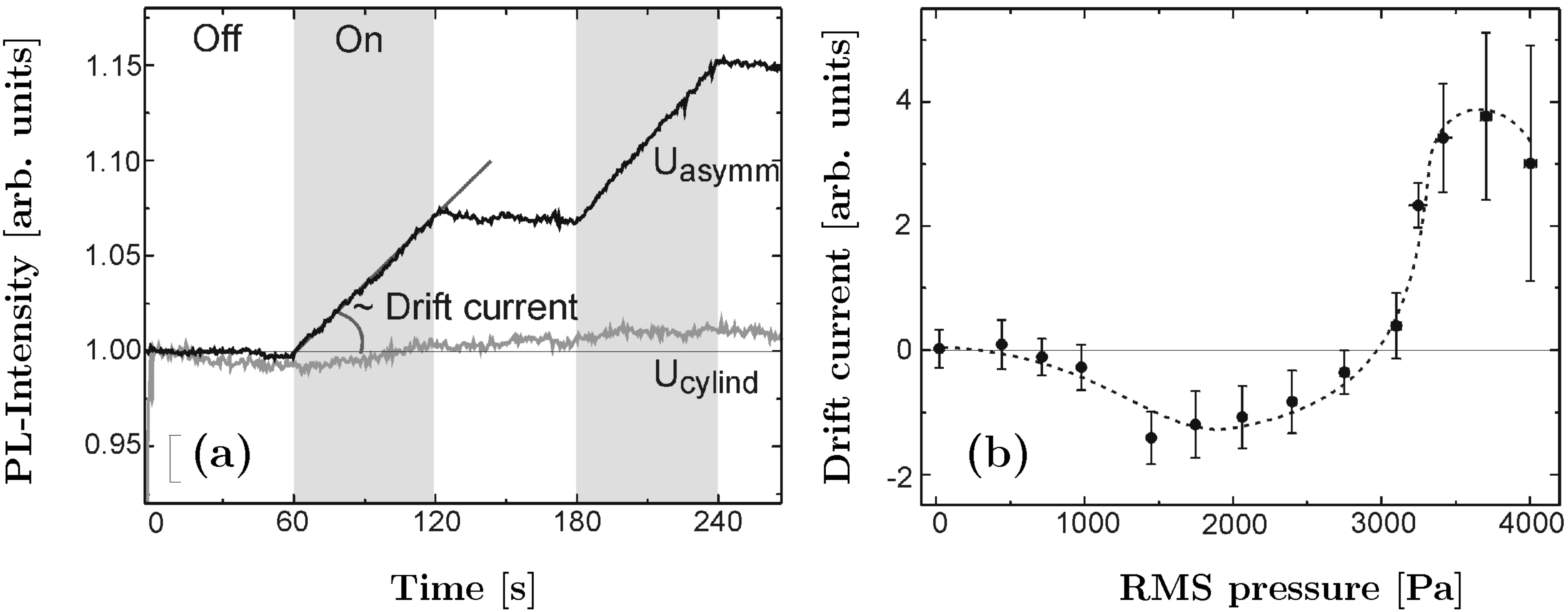


Particle Separation



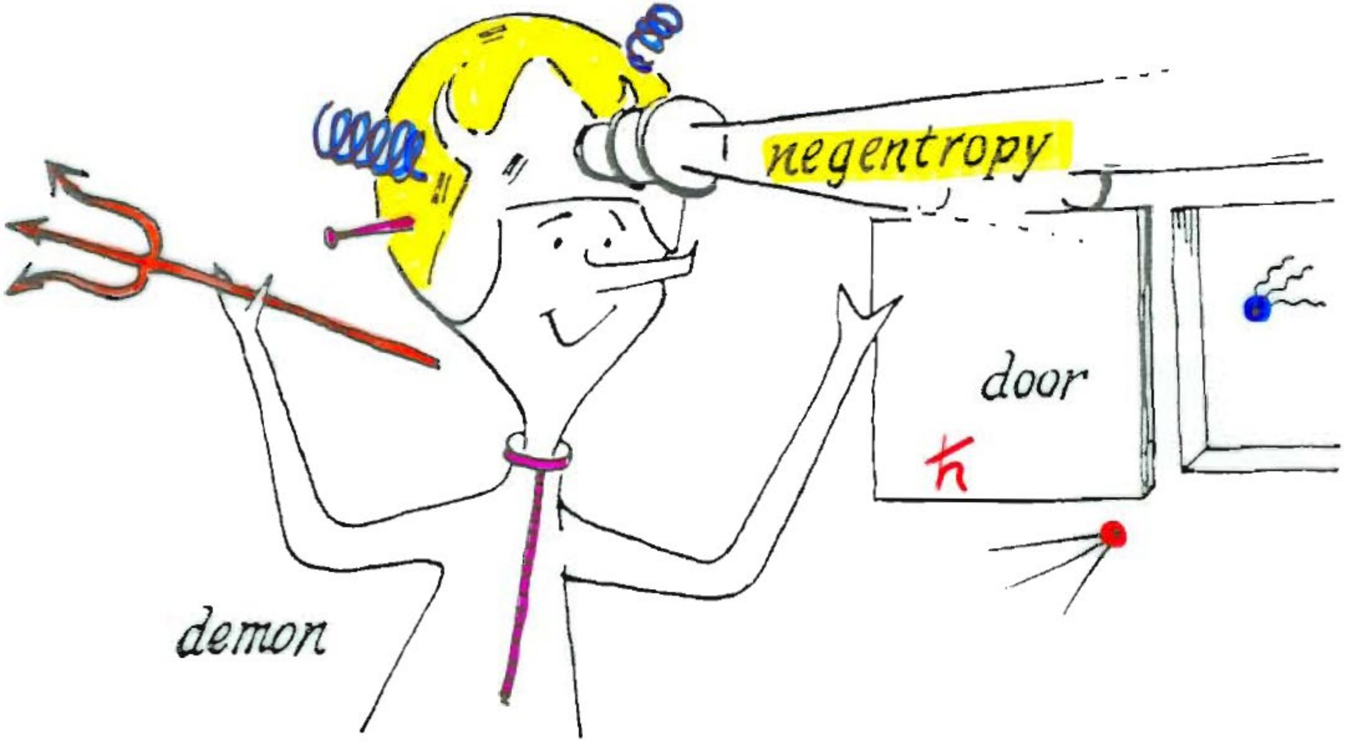
Drift Ratchet – Experiment

S. Matthias, F. Müller, Nature **424**, 53 (2003)



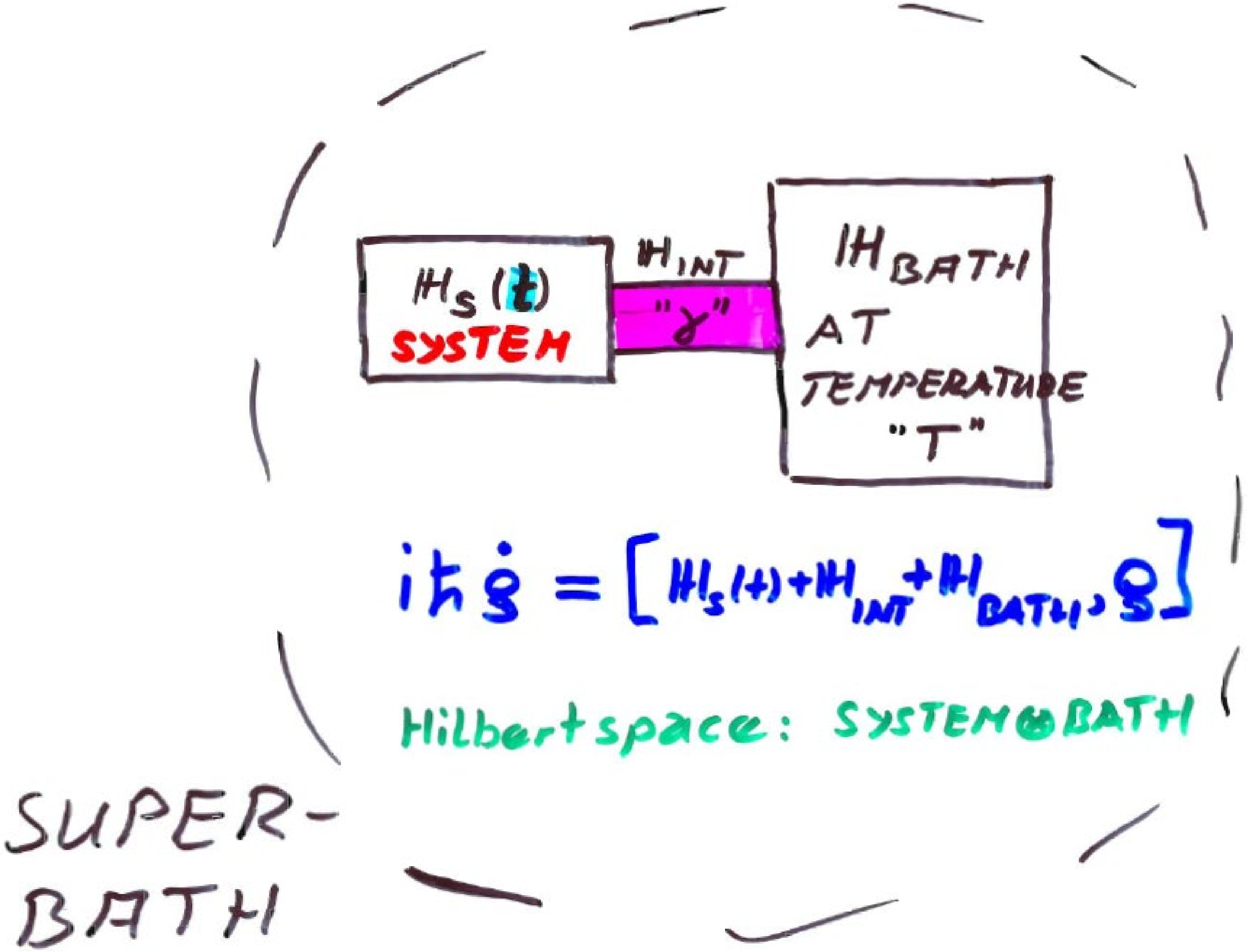
Quantum Demon ?

A measurement → Increase information → Reduction of entropy



Source: H.S. Leff, *Maxwell's Demon* (Adam Hilger, Bristol, 1990)

Quantum Brownian Motors



Quantum-Langevin-equation

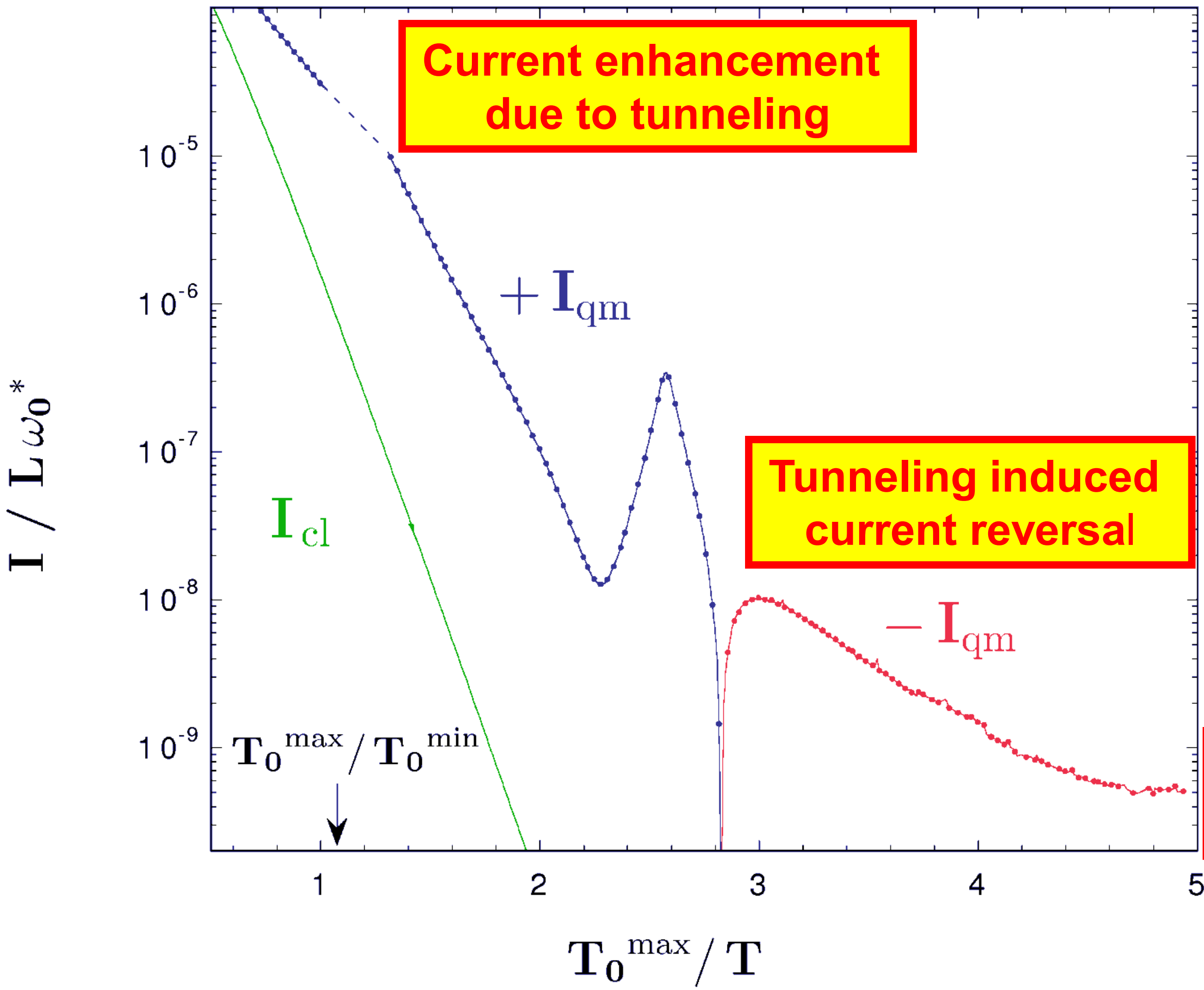
$$m\ddot{\mathbf{x}}(t) + \int_{-\infty}^{t} \gamma(t-t')\,\dot{\mathbf{x}}(t')\,\mathrm{d}t' + V'(\mathbf{x};t) = \boldsymbol{\xi}(t)$$

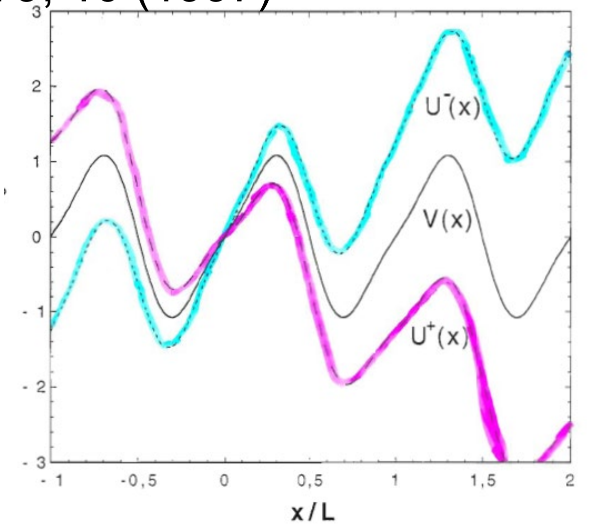
$$\frac{1}{2}\langle \boldsymbol{\xi}(t)\boldsymbol{\xi}(s) + \boldsymbol{\xi}(s)\boldsymbol{\xi}(t)\rangle_{\mathrm{bath}} = \frac{m}{\pi}\int_0^\infty \mathrm{Re}\hat{\gamma}(-i\omega+0^+)\,\hbar\omega\coth\left(\frac{\hbar}{2k_{\mathrm{B}}T}\right)\cos\left[\omega\,(t-s)\right]\,\mathrm{d}\omega$$

And: $[\boldsymbol{\xi}(t), \boldsymbol{\xi}(s)] = -i\hbar\cdots \neq 0$ **!**

Rocking Ratchet - Theory

P. Reimann, M. Grifoni, P. H., Phys. Rev. Lett. **79**, 10 (1997)

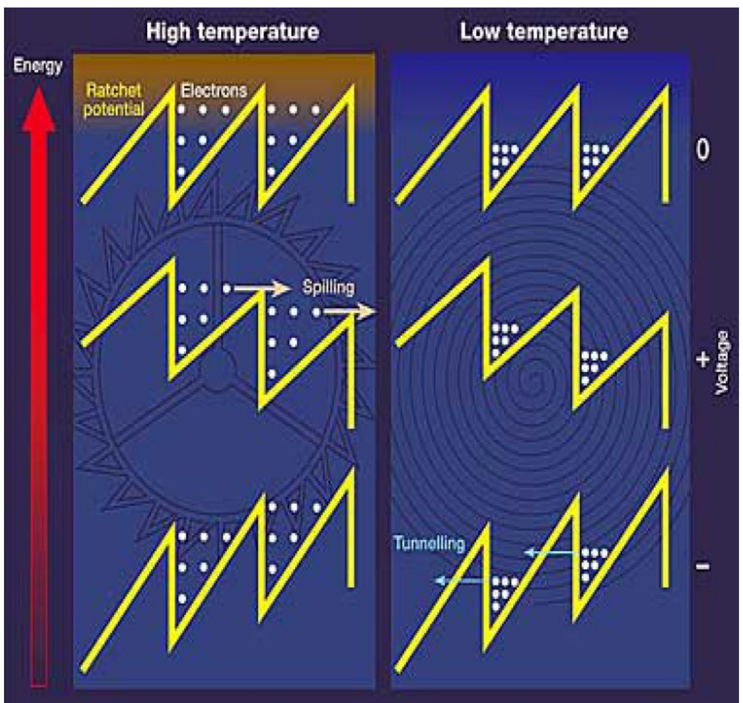


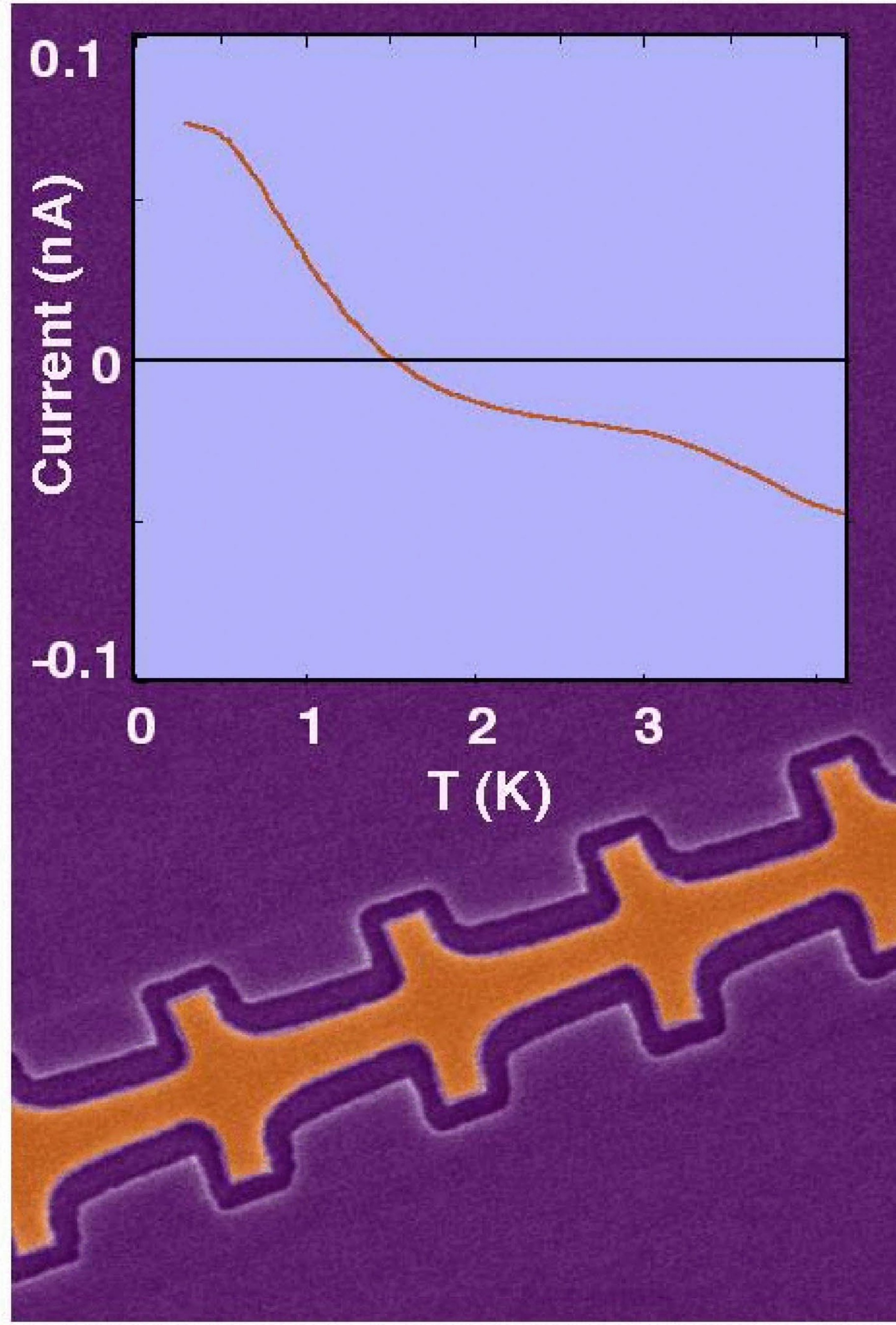


Finite current for T→0

Rocking QM Ratchet – Experiment

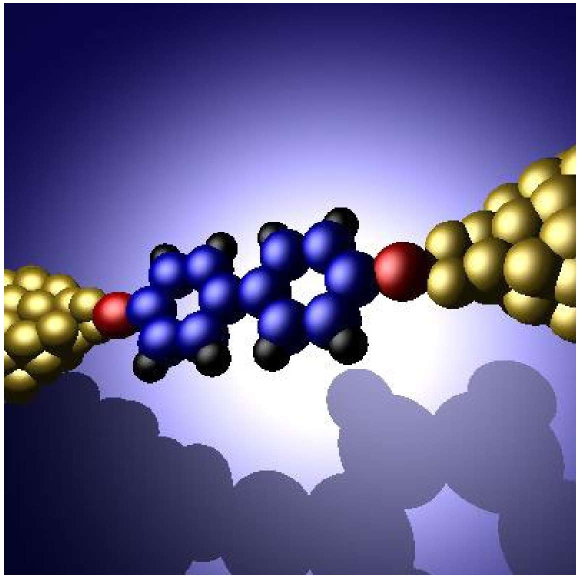
H. Linke, *et al.*,
SCIENCE **286**, 2314 (1999)

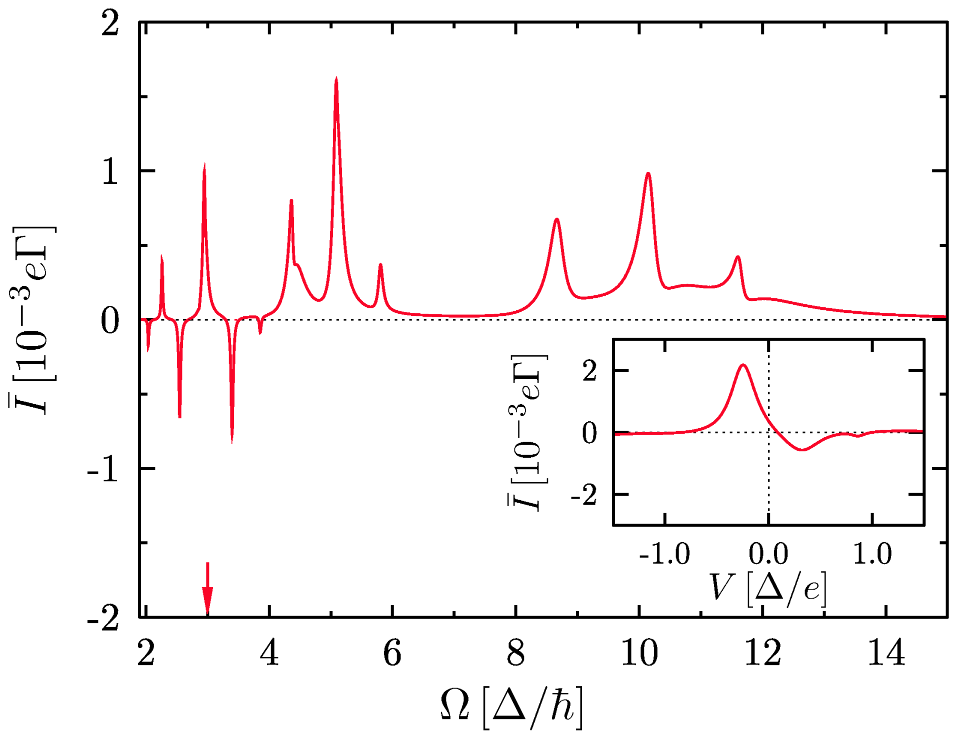




Molecular wires

J. Lehmann, S. Kohler, P. H., A. Nitzan, Phys. Rev. Lett. **88**, 228305 (2002)

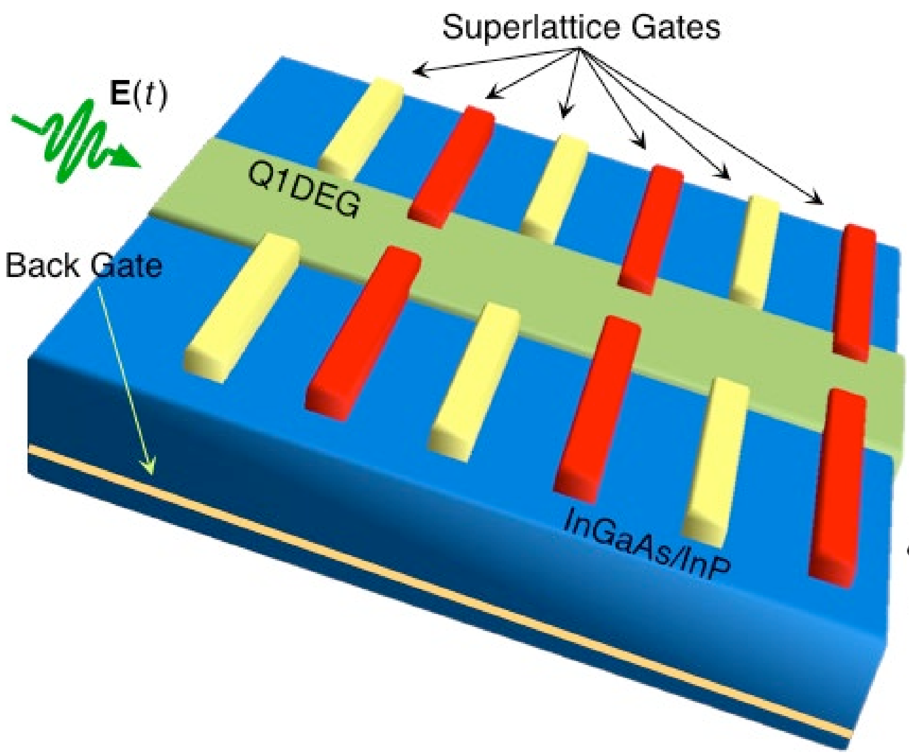




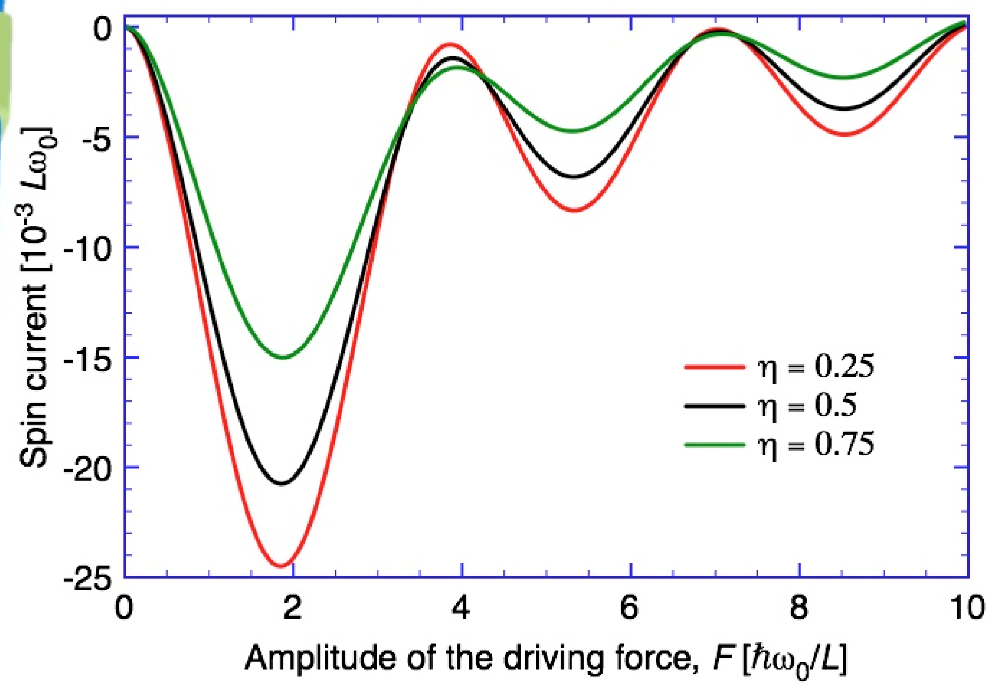
- Field strength E=106 V/cm
- Ω=3Δ corresponds to 4µm wavelength
- typical current: some nA

Spin Ratchet

S. Smirnov, D. Bercioux, M. Grifoni, K. Richter, Phys. Rev. Lett. **100**, 230601 (2008)



M. Grifoni, Thu 11:30 (HSZ 105)



Generalizations of Brownian Motion

Brownian motion: Generalized Langevin-equation

Hamiltonian: $H_{\text{tot}} = H_{\text{sys}} + H_{\text{env}} + H_{\text{WW}}$

$$\Rightarrow \quad m\ddot{\mathbf{x}}(t) + \int_{-\infty}^{t} \gamma(t-t')\,\dot{\mathbf{x}}(t')\,\mathrm{d}t' + V'(\mathbf{x};t) = \boldsymbol{\xi}(t)$$

Asymptotically normal, anomalously fast, or anomalously slow
– via fractional Brownian motion –

$$\int_0^\infty \gamma(t)\,\mathrm{d}t = \begin{cases} \text{const} & \Rightarrow \text{normal} \\ 0 & \Rightarrow \text{superfast} \\ \infty & \Rightarrow \text{superslow} \end{cases}$$

Connection to the fractional Fokker-Planck-equation

Fractional Fokker-Planck equation

Subdiffusion (α<1):

$$\frac{\partial P(x,t)}{\partial t} = \left[\frac{\partial}{\partial x}\frac{V'(x,t)}{\gamma_\alpha} + K_\alpha \frac{\partial^2}{\partial x^2}\right] {}_0D_t^{1-\alpha} P(x,t)$$

Fat tails in the distribution of the residence times

Riemann-Liouville Operator

Superdiffusion (α>1):

$$\frac{\partial P(x,t)}{\partial t} = \left[\frac{\partial}{\partial x}\frac{V'(x,t)}{\gamma_\alpha} + K_\alpha \frac{\partial^{2/\alpha}}{\partial |x|^{2/\alpha}}\right] P(x,t)$$

Fat tails in the distribution of the jump lengths

Riesz-derivative

Quantum-Mechanics
= Brownian Motion ?
= Stochastic Mechanics ?

(E. Nelson; 1966, 1986)

$$p(x,t) = |\Psi(x,t)|^2$$

Schrödinger-gleichung

$$\Psi(x,t) = |\Psi(x,t)|\, e^{iS(x,t)}$$

$$\dot{p}(x,t) = -\frac{\hbar}{m}\nabla\left[\left(\nabla\ \ln|\Psi(x,t)| + \nabla S(x,t)\right)p(x,t)\right] + \frac{\hbar}{2m}\nabla^2 p(x,t)$$

$$\mathbf{f}_1 \geq 0\,,\ \mathbf{f}_2 \geq 0\ :\quad \frac{1}{2}\langle \mathbf{f}_1(t_1)\mathbf{f}_2(t_2) + \mathbf{f}_2(t_2)\mathbf{f}_1(t_1)\rangle \geq 0$$

QM: **NO !**

H. Grabert, P.H., P. Talkner, Phys. Rev. A **19**, 2440 (1979)

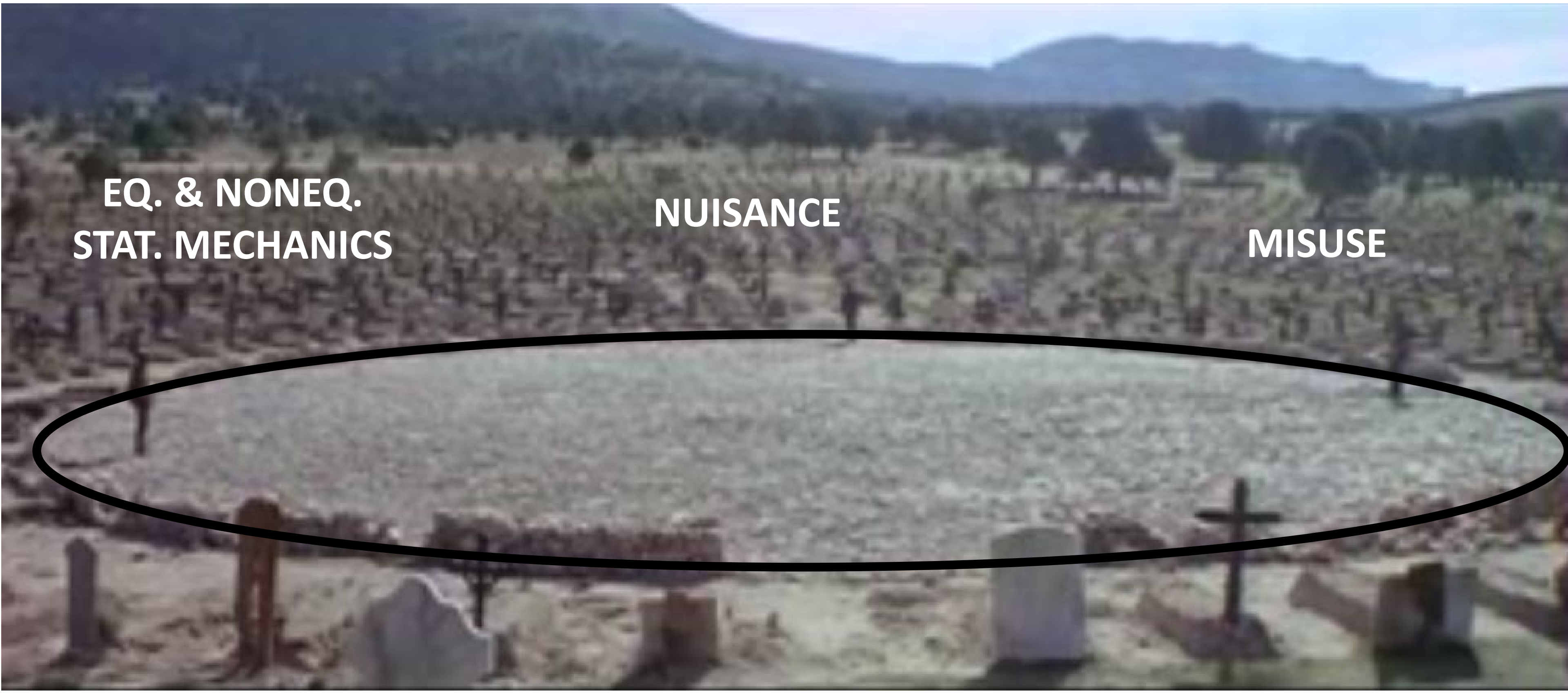
Brownian motion

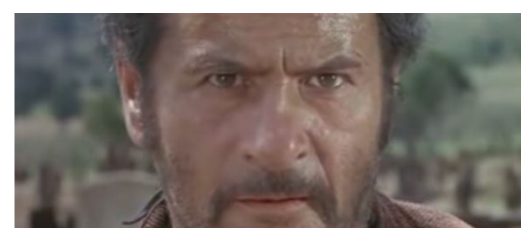
EQ. & NONEQ.
STAT. MECHANICS

NUISANCE

MISUSE

The good, the bad and the simply silly





Source: www.youtube.com

¡ B.-M. -IMPACT !

FDT & RESPONSE

EINSTEIN, NYQUIST
CALLEN-WELTON
GREEN-KUBO

$t \leftrightarrow -it$
$D \leftrightarrow \hbar/2M$

NON-EQ.

P.H., BOCHKOV & KUZOLEV
JARZYNSKI
GALLAVOTTI-COHEN-EVANS

NOISE

LEVY, CAUCHY

S.-D.-EQ's

ITO, STRATONOVICH

↔

Q.-M.

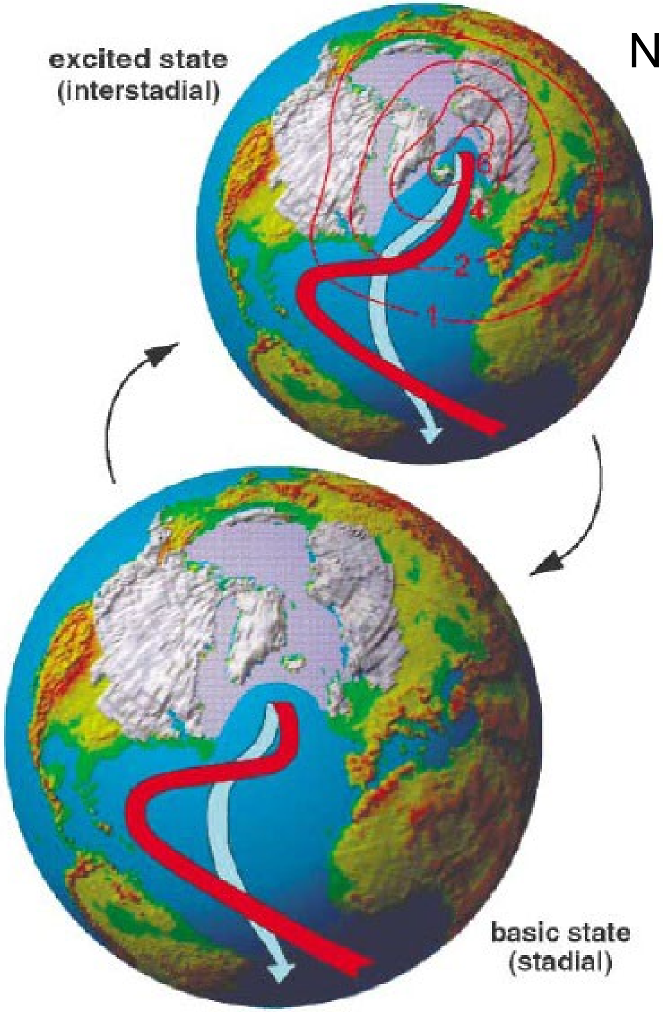
SUM OVER PATHS
FEYNMAN, KAC, ONSAGER, MACHLUP
STOCHASTIC QUANTIZATION
PARISI-WU
STOCHASTIC (Q)-E.-D.
STOCHASTIC "MECHANICS"
NELSON
DISSIPATIVE-Q.-M.
FEYMAN-VERNON

MICROSCOPIC ORIGIN

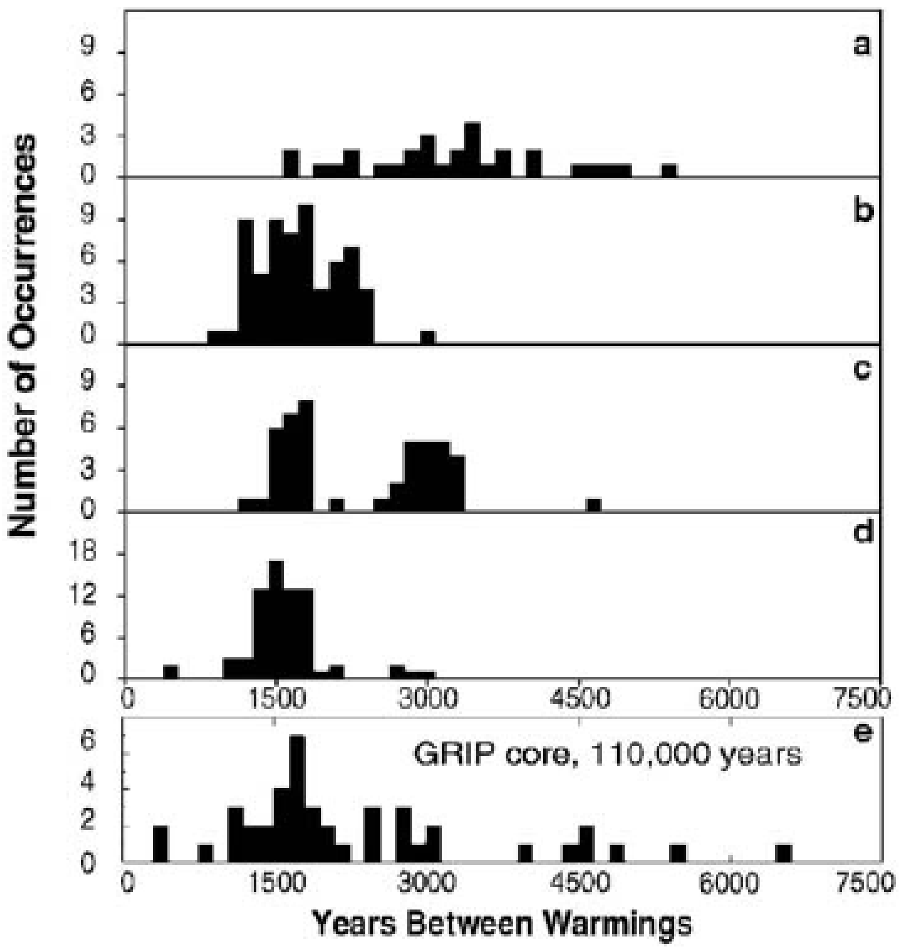
GLE, PROJECTOR METHODS
MAGALINSKII, MORI, KUBO, KAWASAKI

MONTE CARLO METH., MOL.-DYN., NOISE-IND.-TRANSITIONS
COLLOIDAL DYN, SOFTMATTER, S.R., B.-MOTORS, ...
RATE THEORY

Occurrence of ice ages



Noise and periodic of solar origin



A. Ganopolski, S. Rahmstorf, Phys. Rev. Lett. **88**, 038501 (2002)

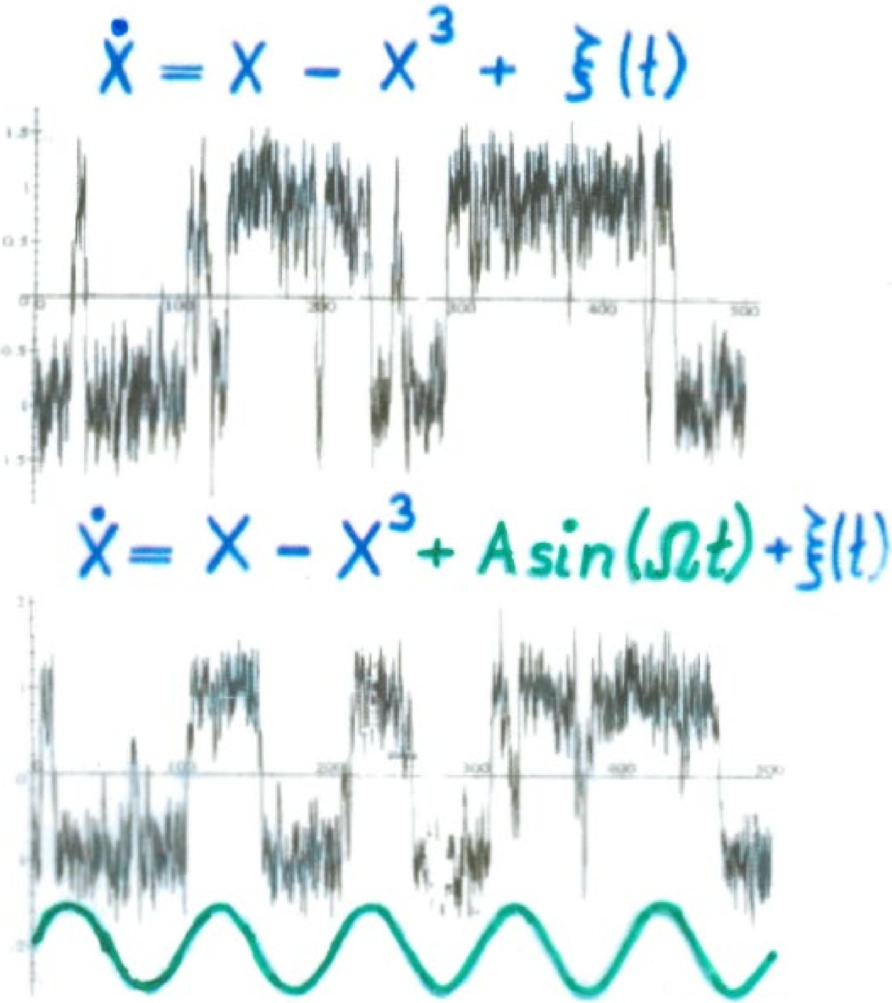
SR – Medical Purpose

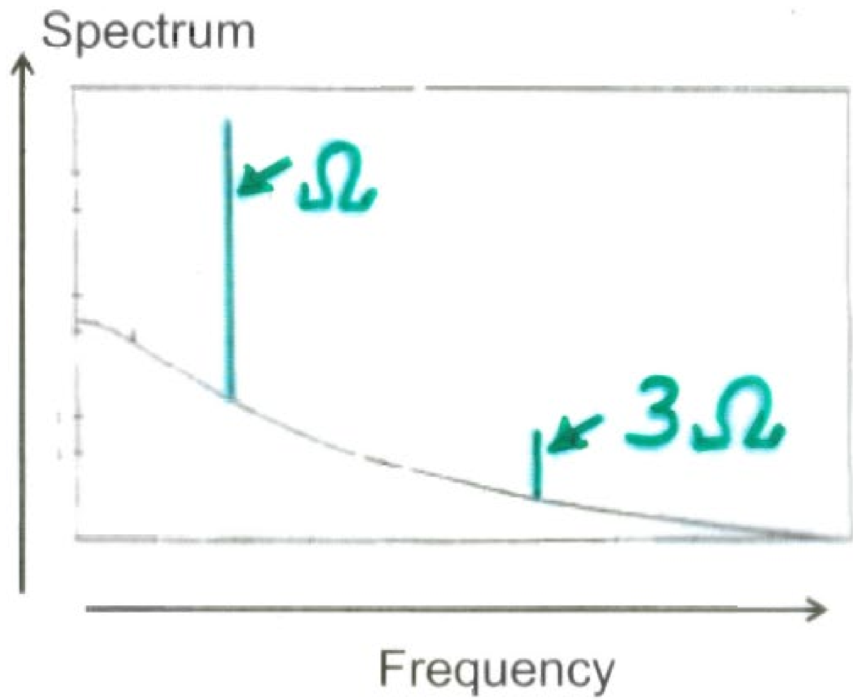
SRT - Zeptoring

- Prevention
- Ataxie
- M. Parkinson
- Multiple sclerosis (MS)
- Stroke / skull-brain-trauma
- Cross-section paralysis
- Depression
- Urine-incontinence
- Orthopedic Lesioning
- Osteporosis
- Neuropathie / diabetes
- Pain

Source: www.sr-therapiesysteme.de

Power spectral density





Fractional Fokker-Planck equation

subdiffusive (α<1)

$$\frac{\partial P(x,t)}{\partial t} = \left[\frac{\partial}{\partial x}\frac{V'(x,t)}{\eta_\alpha} + K_\alpha \frac{\partial^2}{\partial x^2}\right] {}_0D_t^{1-\alpha} P(x,t)$$

Riemann-Liouville Operator

$${}_0D_t^{1-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \frac{d}{dt} \int_0^t \frac{f(t')}{(t-t')^{1-\alpha}} dt'$$