Brownian motion

Gypsum cristals in a closterium moniliferum

Movie

source: IWF Wissen und Medien gGmbH, 1949

Brownian motion reigning manipulation and transport on the nanoscale: Stochastic Resonance and Brownian motors

Peter Hänggi

Institut für Physik
Universität Augsburg

Why you should not do Brownian motion

- You know nothing about the subject

- Many very good people worked on it

  (Einstein, Langevin, Smoluchowski, Ornstein, Uhlenbeck, Wiener, Onsager, Stratonovich, …)

- You don't have your own pet theory yet

Why you should do Brownian motion

- You know nothing about the subject
- Many very good people worked on it
- You still can do your own pet theory

Robert Brown (1773-1858)

Source: www.anbg.gov.au

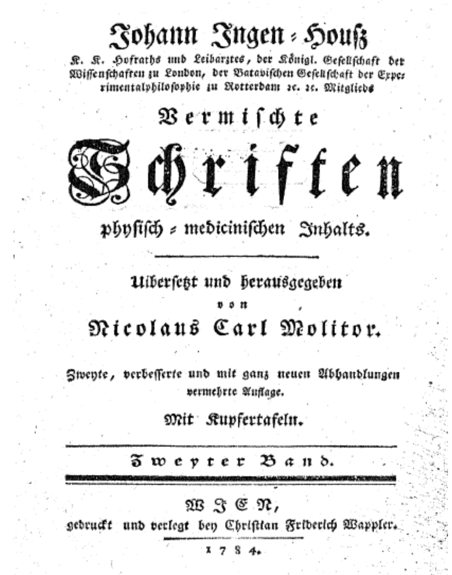
Source: permission kindly granted by Prof. Brian J. Ford
http://www.brianjford.com/wbbrowna.htm

1827 – irregular motion of granules of pollen in liquids

- Brown, Phil. Mag. **4**, 161 (1928)
- Deutsch: *Did Robert Brown observe Brownian Motion: probably not*, Sci. Am. **256**, 20 (1991)
- Ford: "*Brownian movement in clarkia pollen: a reprise of the first observations*", The Microscope **39**, 161 (1991)

Jan Ingen-Housz (1730-1799)

Source: www.americanchemistry.com



Johann Ingen-Housz
K. K. Hofraths und Leibarztes, der Königl. Gesellschaft der Wissenschaften zu London, der Batavischen Gesellschaft der Experimentalphilosophie zu Rotterdam ꝛc. ꝛc. Mitglieds

Vermischte
Schriften
physisch-medicinischen Inhalts.

Uibersetzt und herausgegeben
von
Nicolaus Carl Molitor.

Zweyte, verbesserte und mit ganz neuen Abhandlungen vermehrte Auflage.

Mit Kupfertafeln.

Zweyter Band.

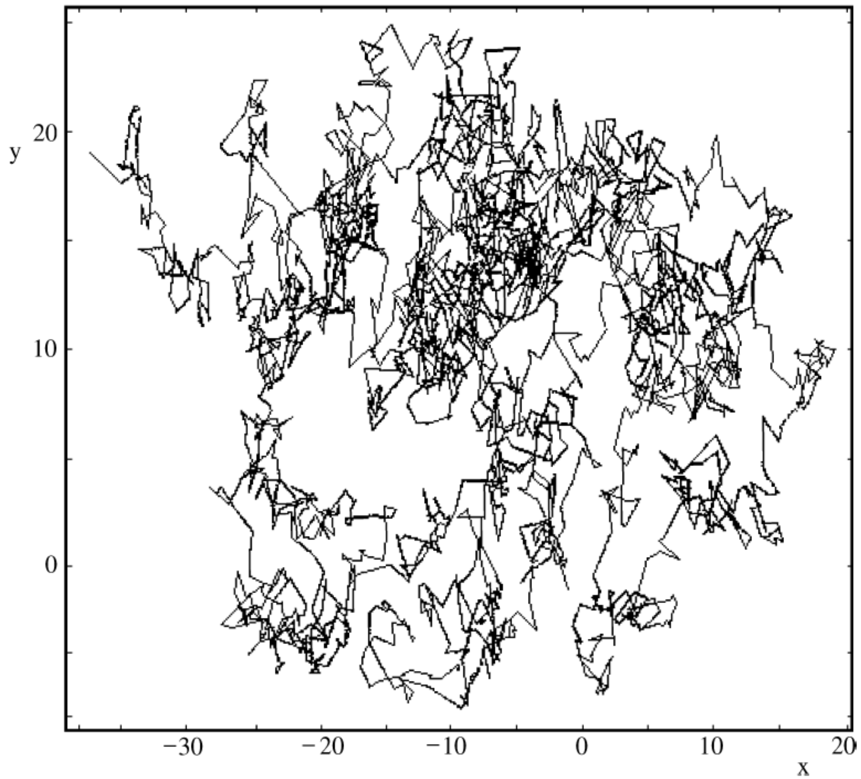
WIEN,
gedruckt und verlegt bey Christian Friderich Wappler.
1784.

To see clearly how one can deceive one's mind on this point if one is not careful, one has only to place a drop of alcohol in the focal point of a microscope and introduce a little finely ground charcoal therein, and one will see these corpuscules in a confused, continuous and violent motion, as if they were animalcules which move rapidly around.

Mean squared displacement

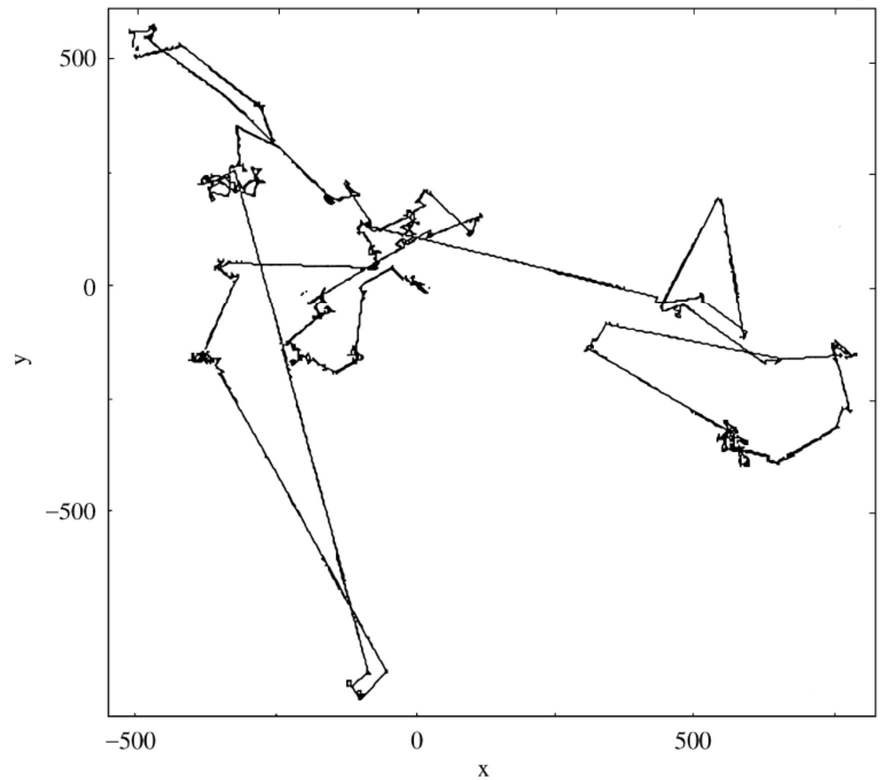
$$\langle x^2(t) \rangle \propto t^\alpha$$

Brownian movement $\alpha = 1$



Source: Physica A **282**, 13 (2000)

Lévy-Brownian movement $\alpha = \frac{4}{3}$



Source: Physica A **282**, 13 (2000)

Theory of Brownian motion

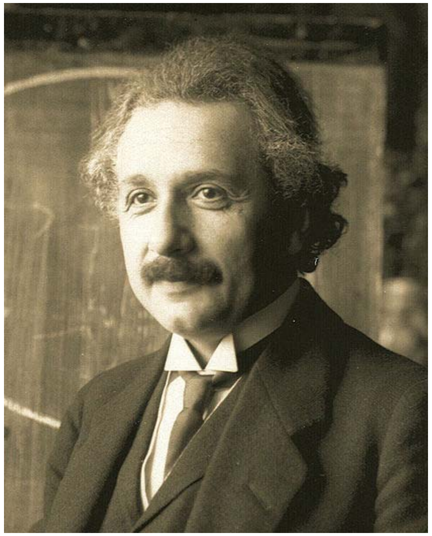
W. Sutherland (1858-1911)

Source: www.theage.com.au

$$D = \frac{RT}{6\pi\eta a C}$$

Phil. Mag. **9**, 781 (1905)

A. Einstein (1879-1955)



Source: wikipedia.org

$$\langle x^2(t) \rangle = 2Dt$$

$$D = \frac{RT}{N} \frac{1}{6\pi k P}$$

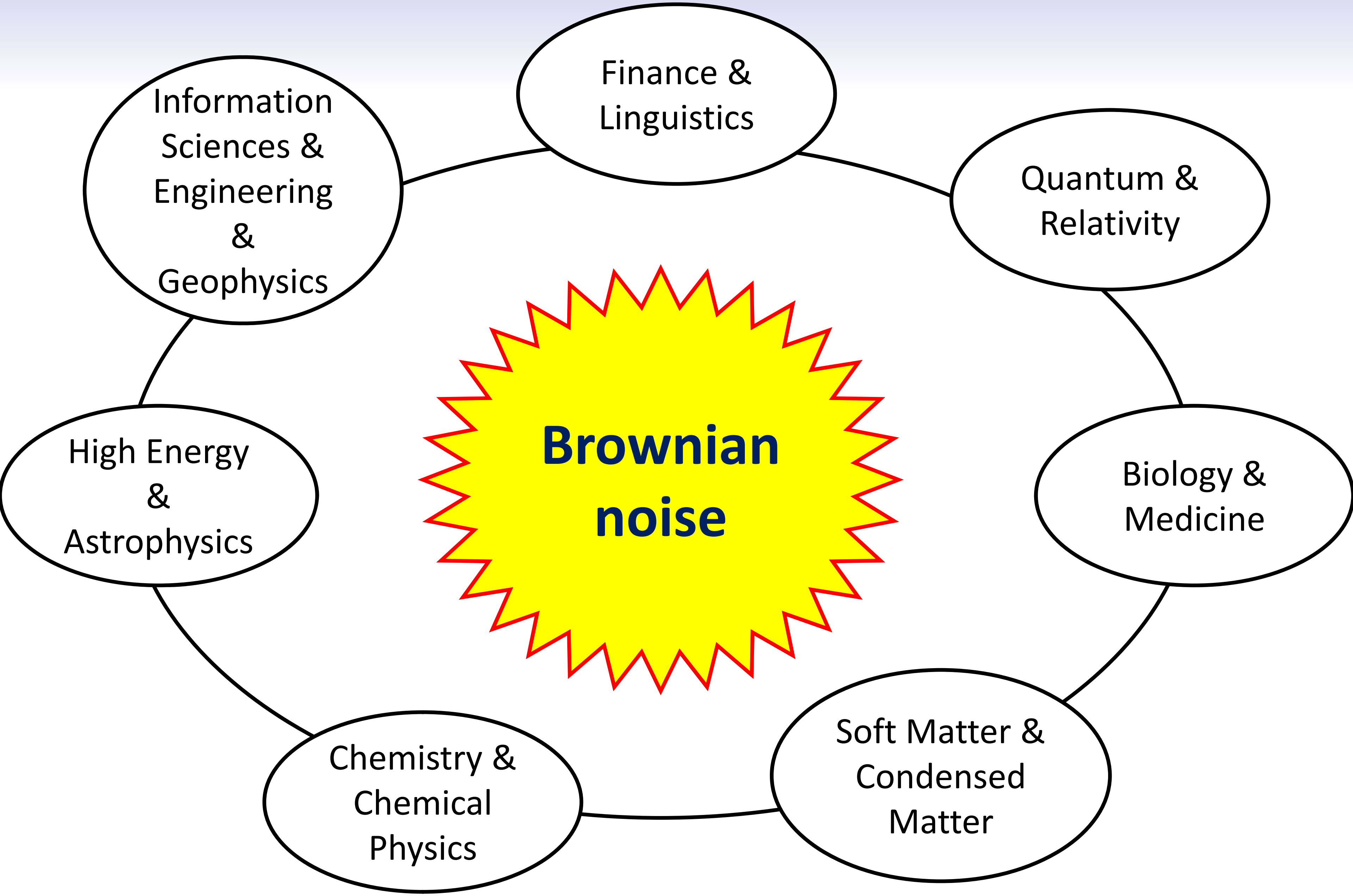
Ann. Phys. **17**, 549 (1905)

M. Smoluchowski (1872-1917)

Source: wikipedia.org

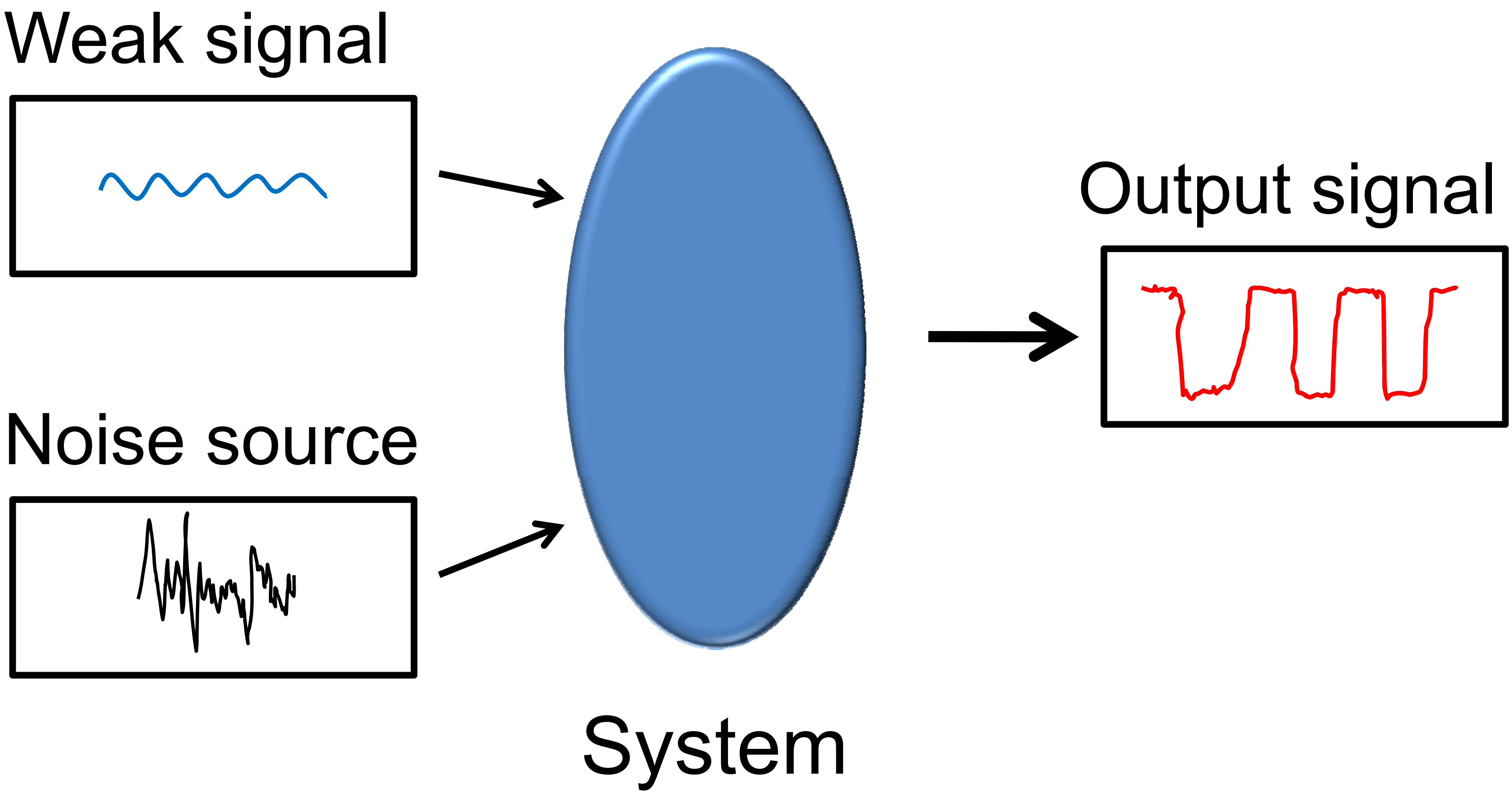
$$D = \frac{32}{243} \frac{mc^2}{\pi\mu R}$$

Ann. Phys. **21**, 756 (1906)


Brownian noise
Finance & Linguistics
Quantum & Relativity
Biology & Medicine
Soft Matter & Condensed Matter
Chemistry & Chemical Physics
High Energy & Astrophysics
Information Sciences & Engineering & Geophysics

Stochastic Resonance

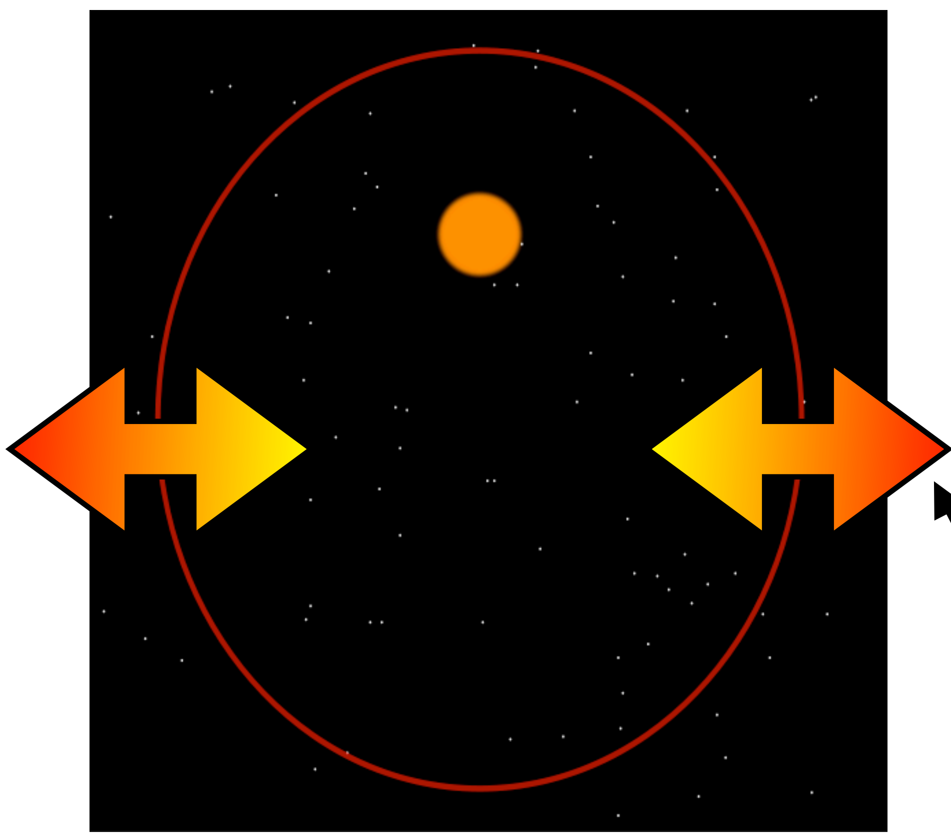
(in a nutshell)

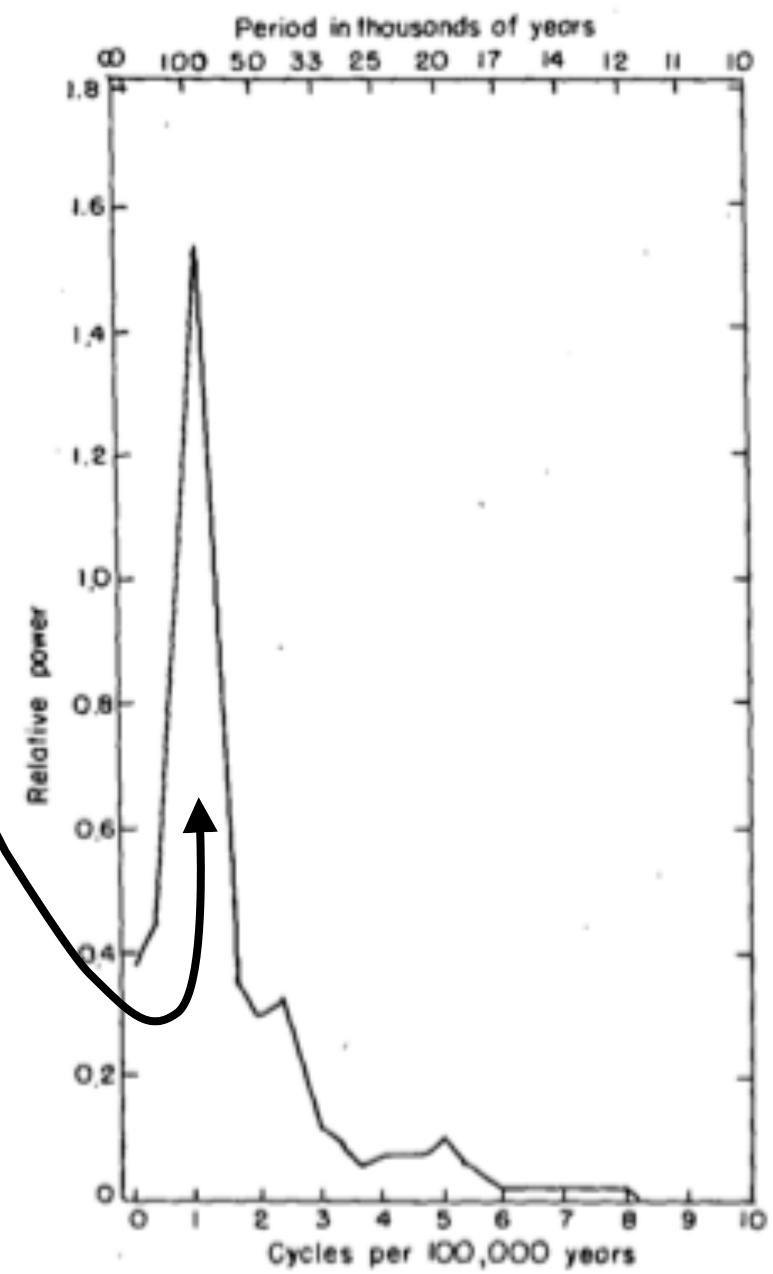


Why are the ice-ages so periodic ?

Milankowitch cycles:
Small changes in earth orbit eccentricity with 100k year periodicity

Changes are small! (<0.1% of solar constant)



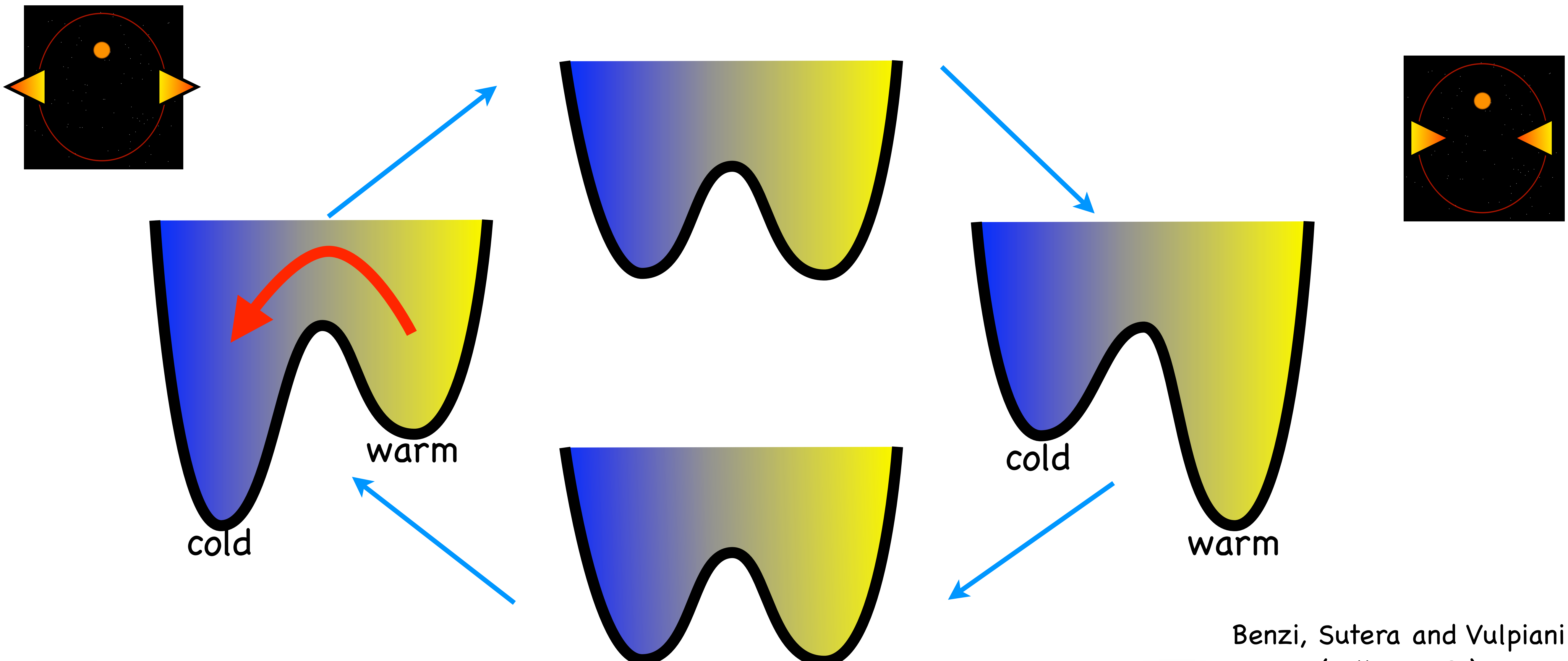


M. Milankowitch, Handbuch der Klimatologie I (1930)

What can amplify those small changes ?

Milankowitch Cycles and Bistability

Climate "landscape"



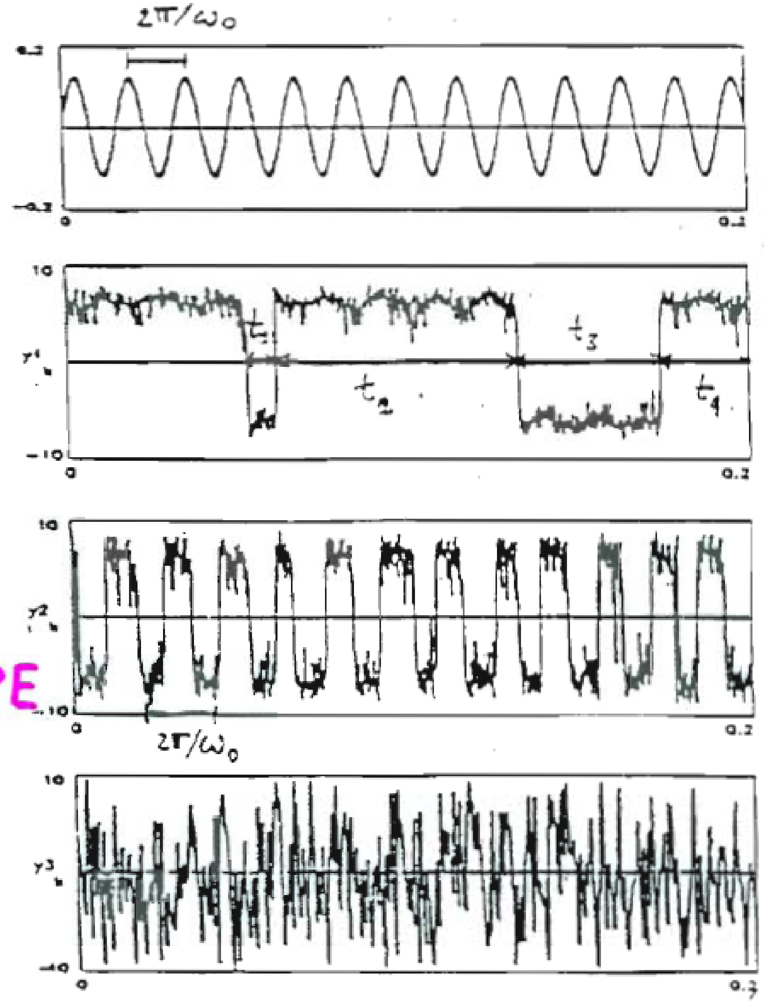
Benzi, Sutera and Vulpiani
(Tellus, 1981)
C. Nicolis and G. Nicoli
(Tellus, 1981)

- The 100ky cycles only bias the climate
- Fluctuations make climate switch
- small changes of conditions can have huge impact

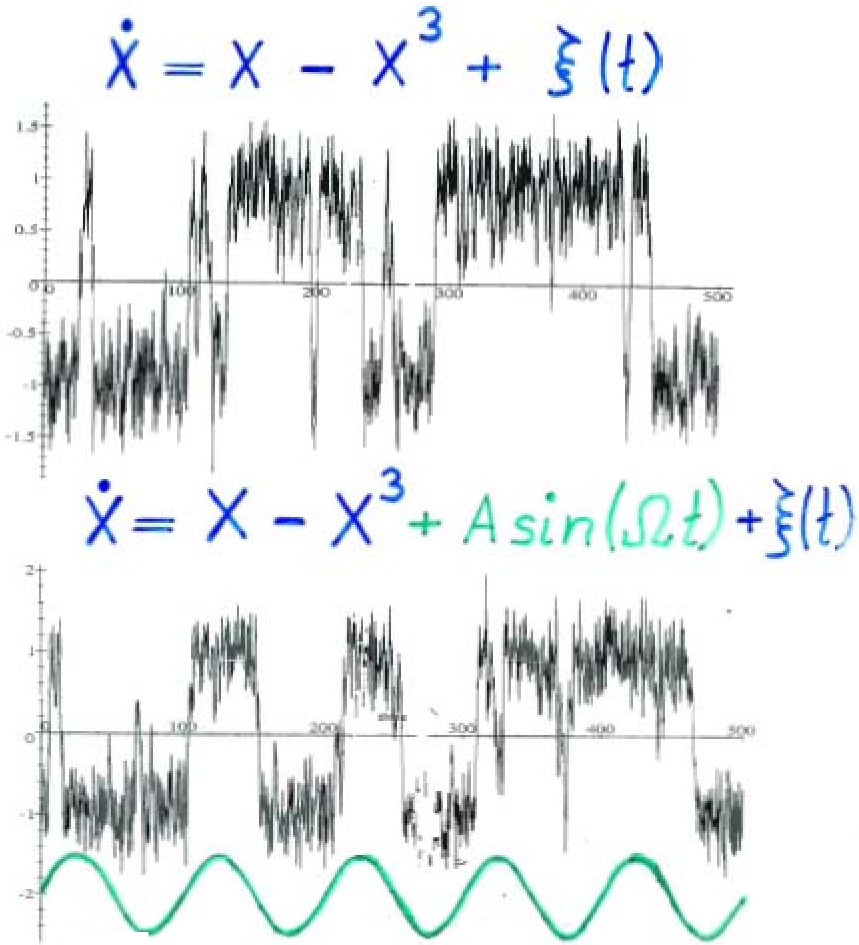
Synchronization

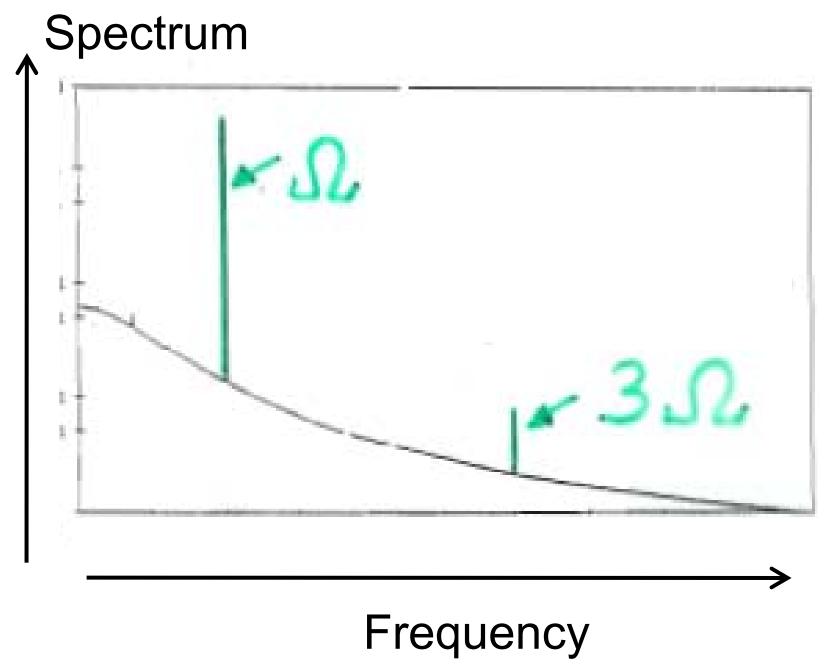
SIGNAL

$T_e \sim 2\Gamma^{-1}_{ESCAPE}$



Power spectral density





Measuring SR

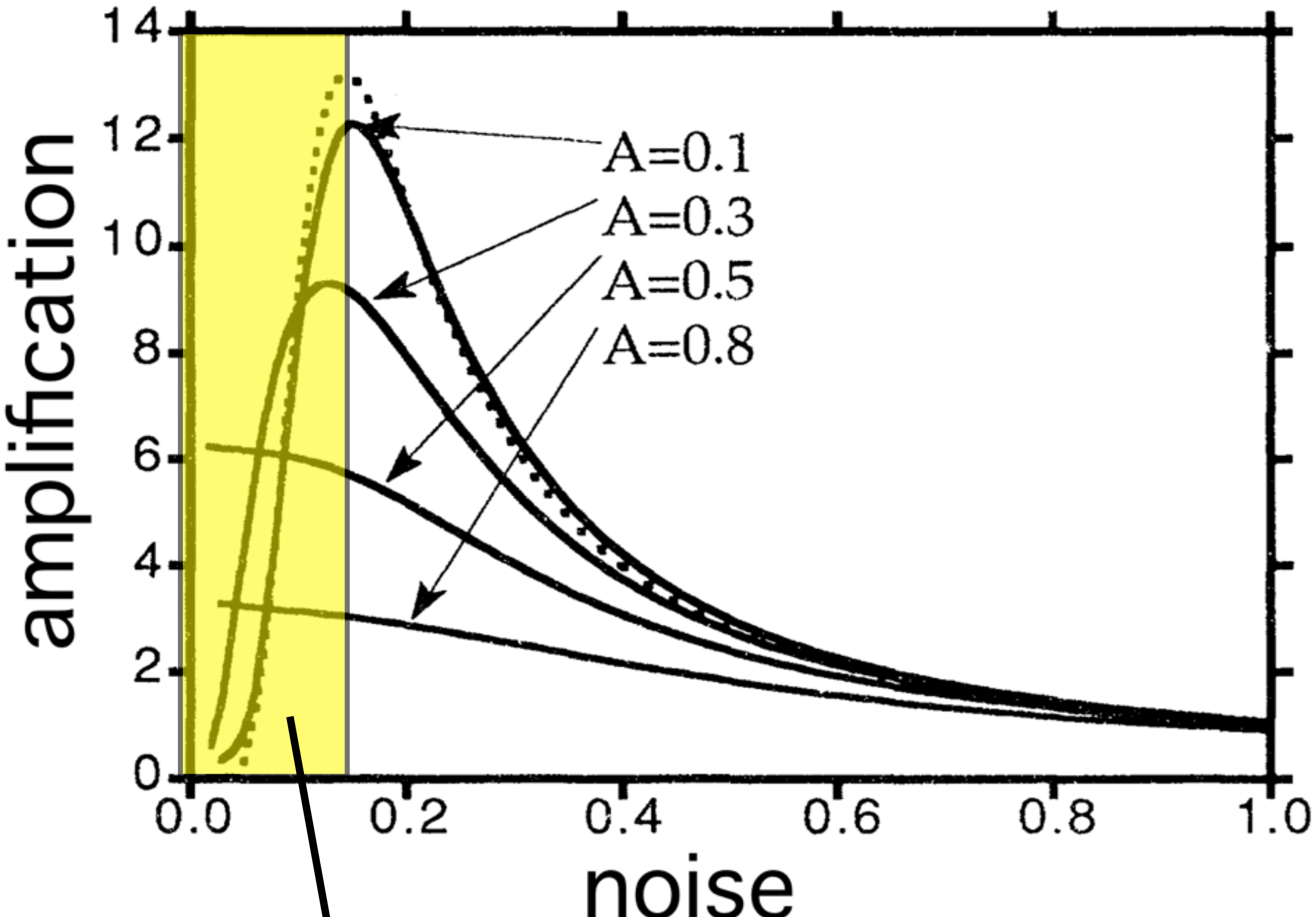
- Signal to noise ratio
- Spectral amplification
- mutual information
- cross-correlation: input ↔ output
- peak area, (phase-) synchronization, …

SR-reviews:

L. Gammaitoni, P. Hänggi, P. Jung, F. Marchesoni, Rev. Mod. Phys. **70**, 223 (1998)
P. Hänggi, ChemPhysChem **3**, 285 (2002)

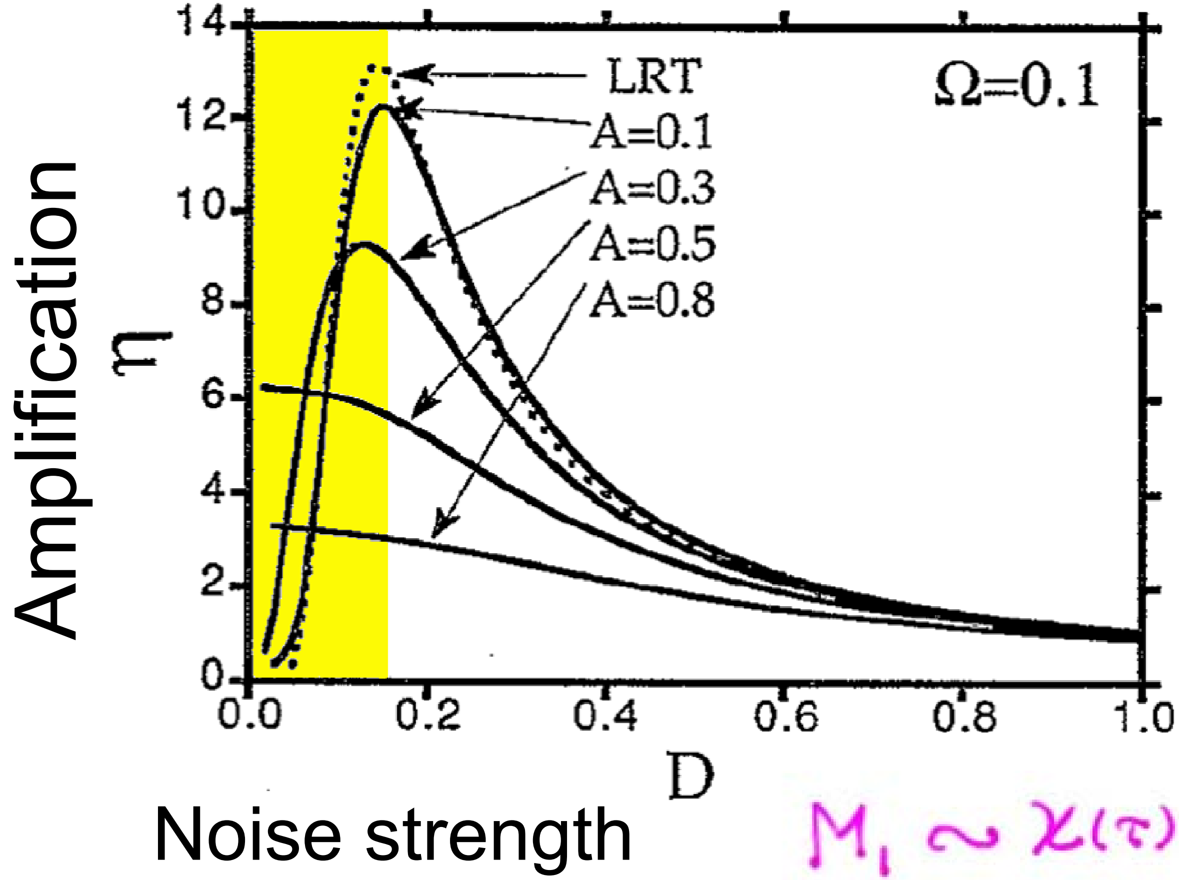
Amplification of small signals by noise

(P. Jung, P. Hänggi, Phys. Rev. A **44**, 8032 (1991))



More noise , more signal !!

Signal amplification



MORE NOISE

MORE SIGNAL

$$M_1 \sim \chi(\tau) = -\frac{1}{D}\frac{d}{d\tau}\langle \delta x(\tau)\,\delta\xi(0)\rangle$$

$$|M_1|^2 \propto \quad 1/D^2 \quad \exp(-2\Delta U/D)$$

P. Jung, P. Hänggi, Phys. Rev. A **44**, 8032 (1991)

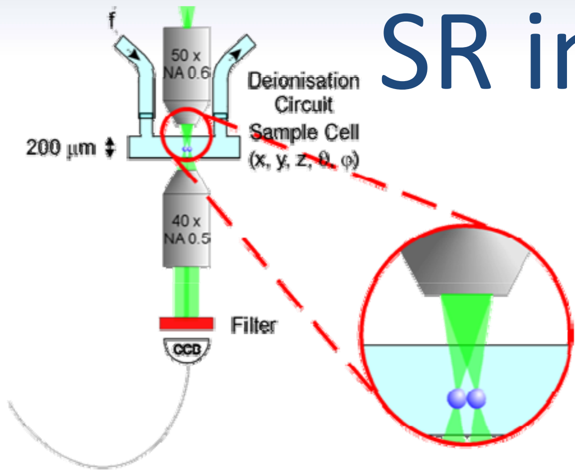
SR - Ingredients

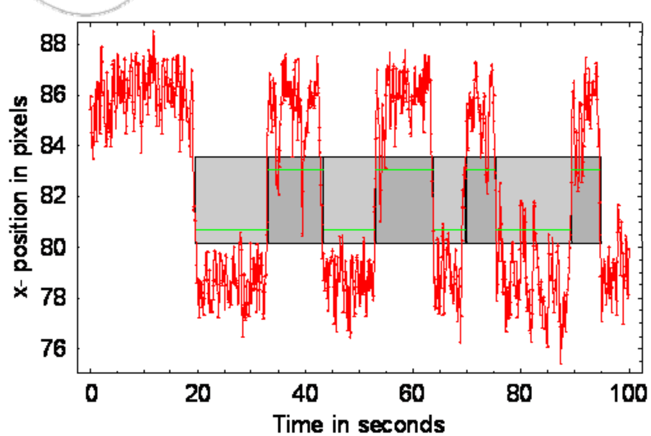
- ✓ Threshold system
- ✓ Weak (subthreshold) signal
- ✓ Noise

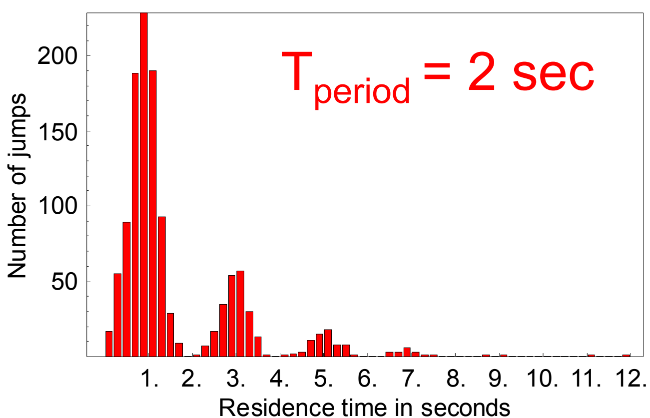
Anomalous amplification properties

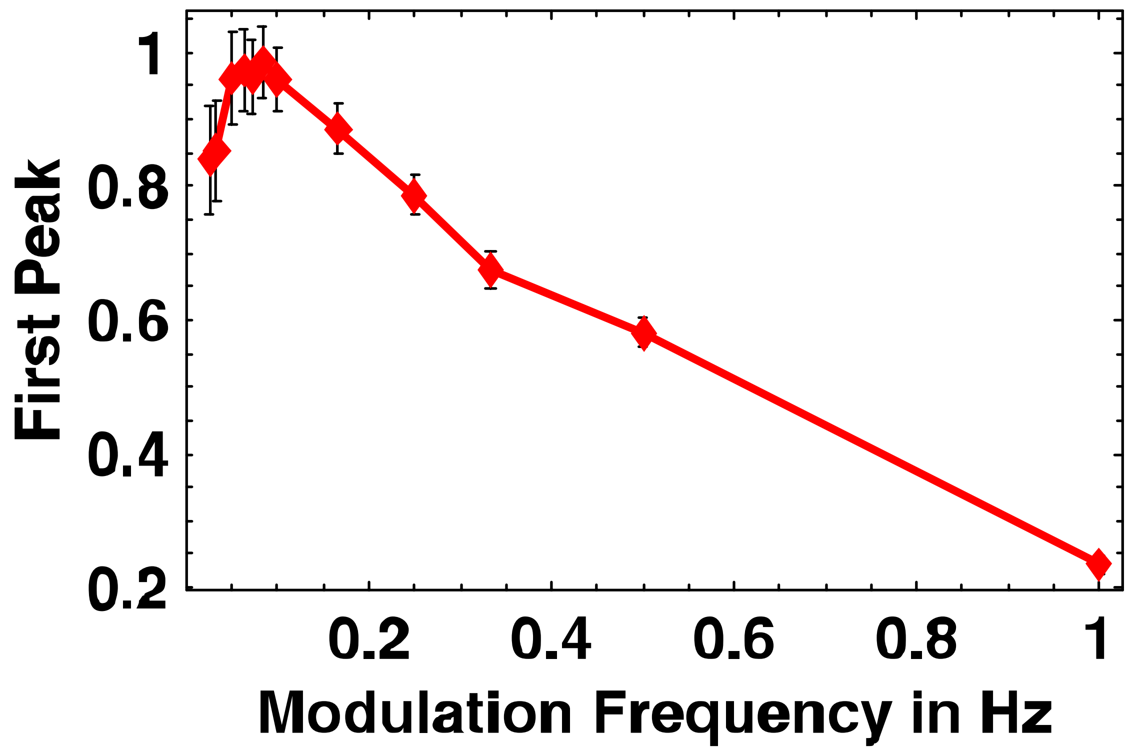
SR examples

SR in colloidal systems

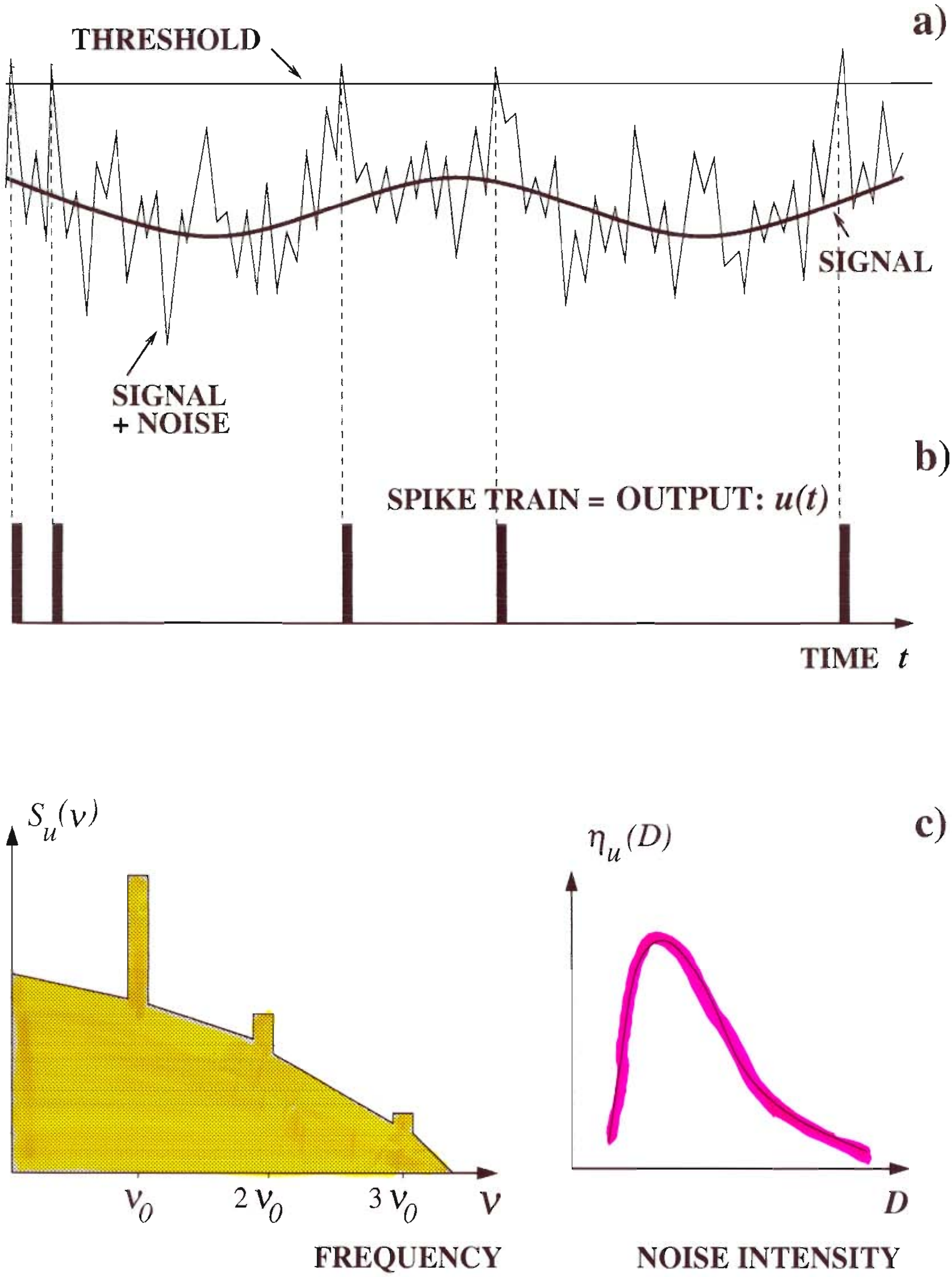




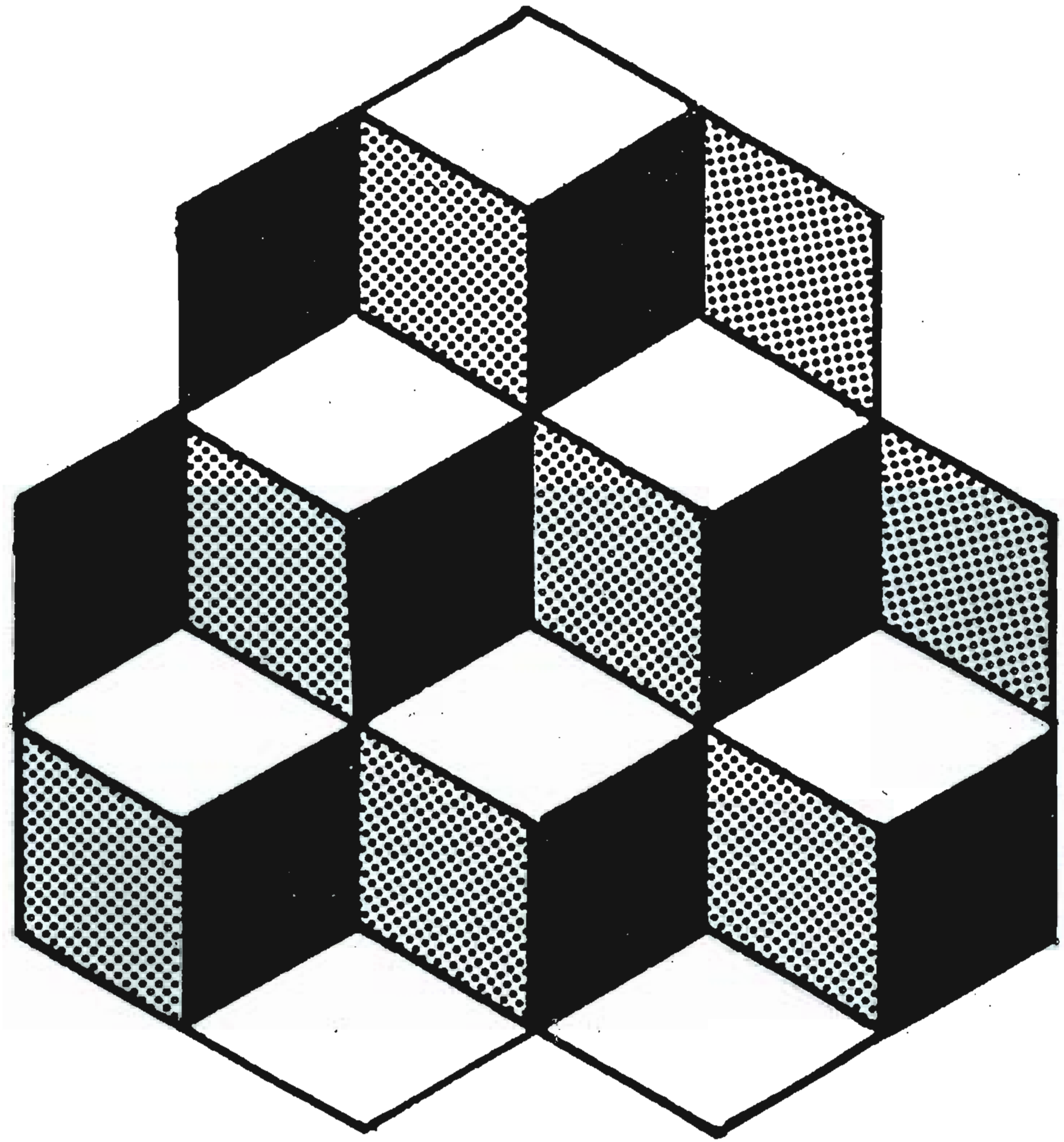




D. Babic, C. Schmitt, I. Poberaj, C. Bechinger, Europhys. Lett. **67**, 158 (2004)

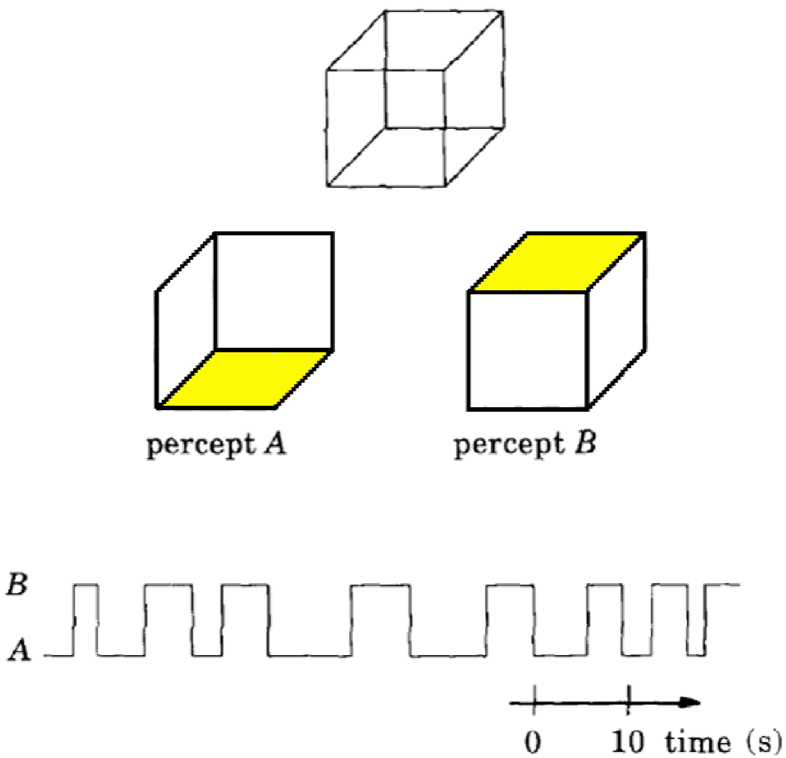


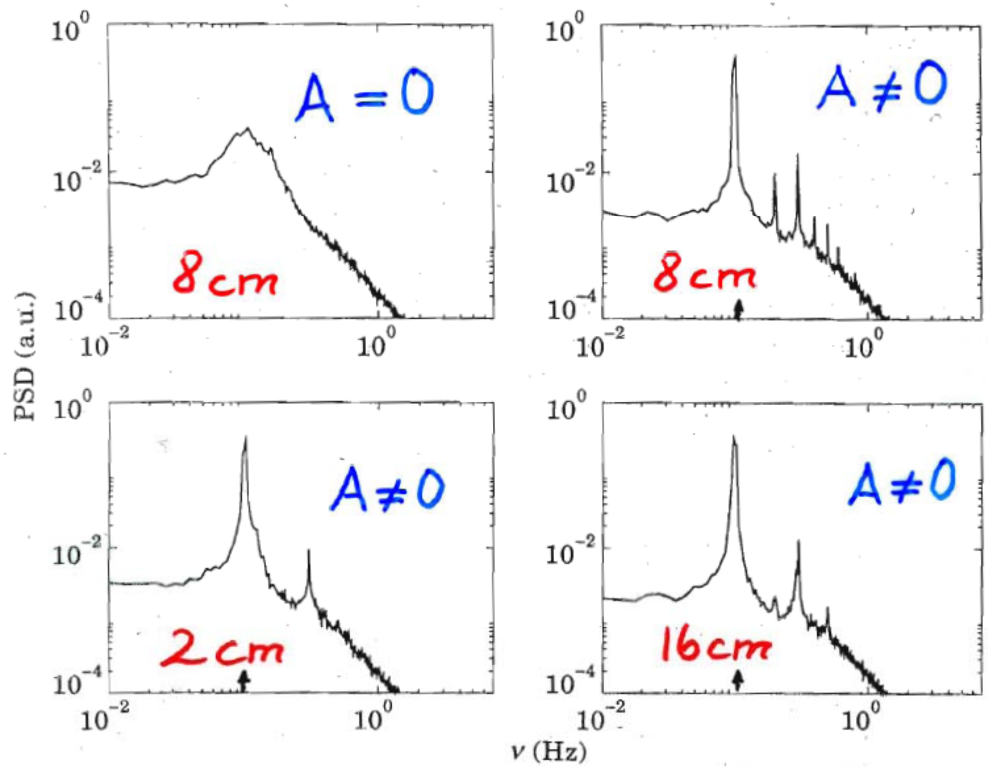
a)
THRESHOLD
SIGNAL
SIGNAL
+ NOISE
b)
SPIKE TRAIN = OUTPUT: u(t)
TIME t
c)
$S_u(\nu)$
ν_0
$2\nu_0$
$3\nu_0$
ν
FREQUENCY
$\eta_u(D)$
D
NOISE INTENSITY



SR in Visual Perception

Experiment:

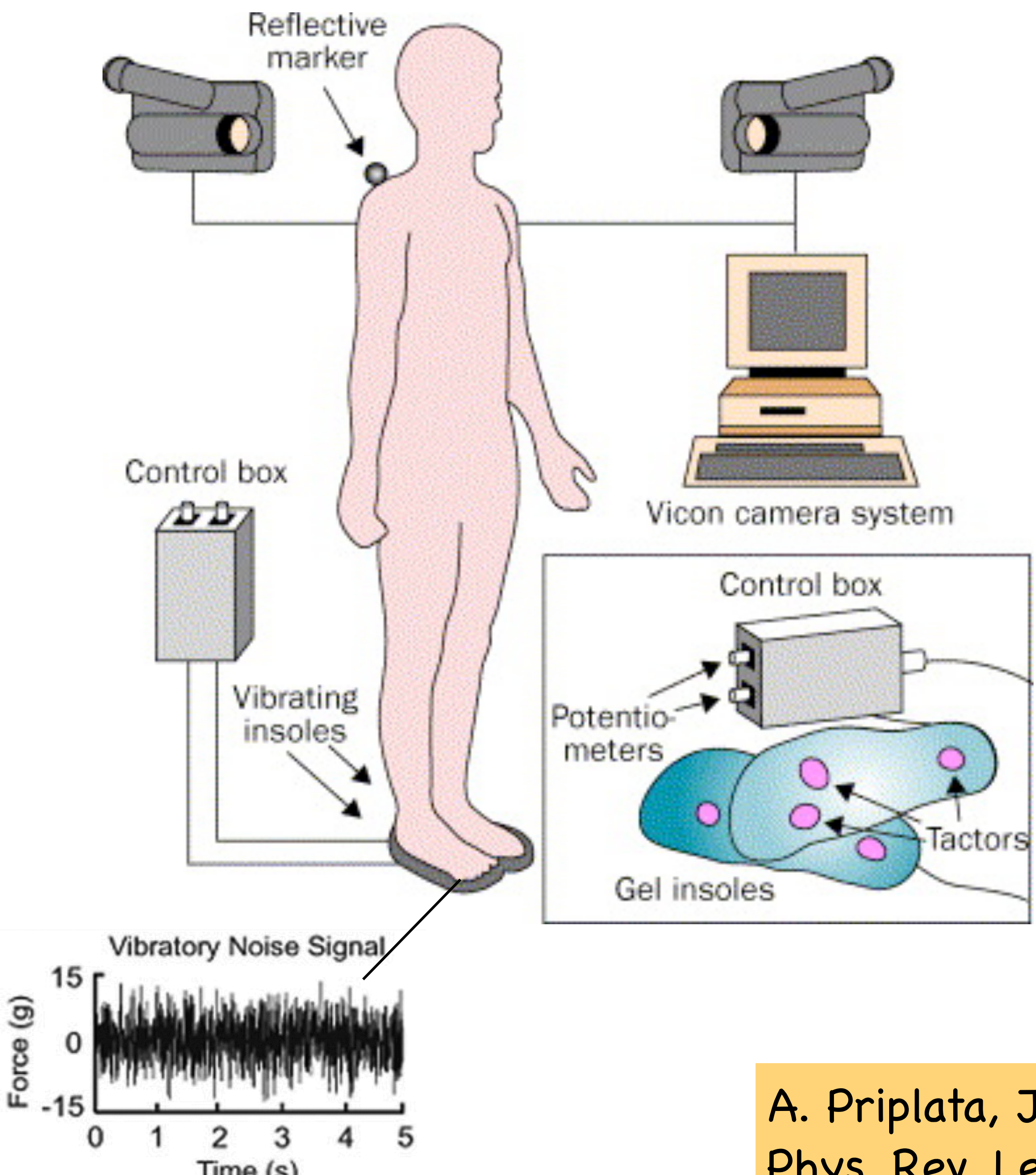


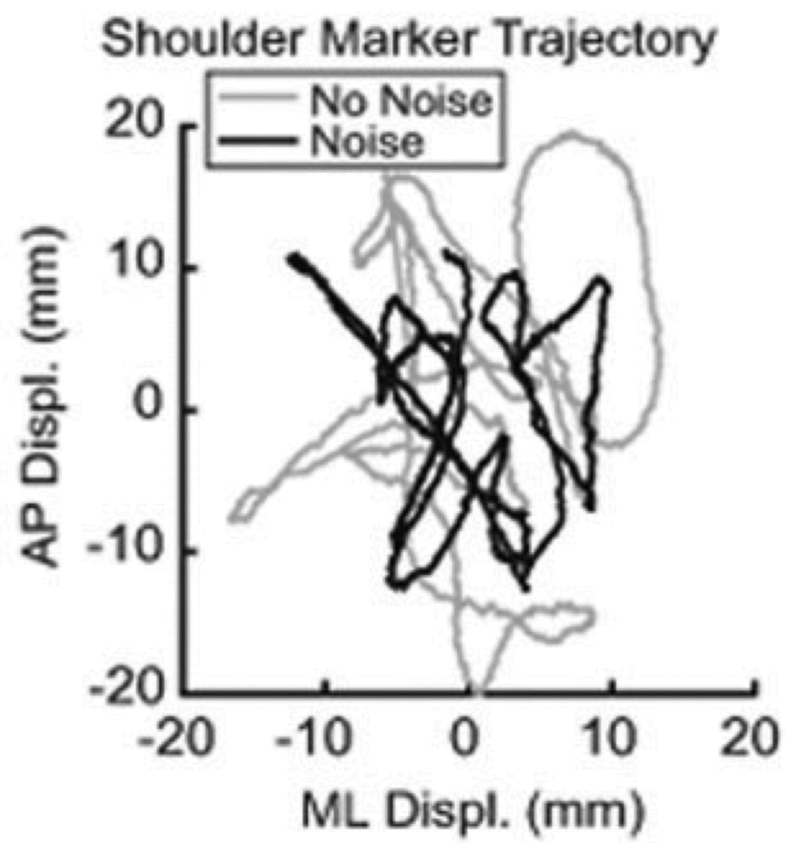


M. Riani, E. Simonotto, Nuovo Cimento D **17**, 903 (1995)

SR and human posture control

Somatosensory function declines with age and in diabetic patients. Can additional noise help restore function?





Reduction in sway of person

A. Priplata, J. Niemi, M. Salen, J. Harry, L.A. Lipsitz and J.J. Collins
Phys. Rev. Lett. 89 (2002)

FREI
AKTIVE REHA-SYSTEME

THERAPIE.
PRÄVENTION.
TRAINING.

srt-zeptoring®
und
srt zeptor®

- Parkinson
- Multiple sclerosis (MS)
- Stroke / skull-brain-trauma
- Cross-section paralysis
- Depression
- Pain
- …

SRT Zeptor Training - Powerslide Team

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- Parkinson
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- Pain
- …

SRT Zeptor Training - Powerslide Team

Stochastic generalization of Hodgkin-Huxley model

$$I_{\text{ext}}(t) = g_{\text{K}}^{\text{max}} n^4 \cdot (V - V_{\text{K}}) + g_{\text{Na}}^{\text{max}} m^3 h \cdot (V - V_{\text{Na}}) + g_l \cdot (V - V_l) + C \frac{\mathrm{d}}{\mathrm{d}t} V$$

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \alpha_n(V)(1-n) - \beta_n(V)n + \sqrt{\frac{1}{N_{\text{K}}} \frac{\alpha_n(V)(1-n) + \beta_n(V)n}{2}}\, \xi_n(t)$$

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \alpha_m(V)(1-m) - \beta_m(V)m + \sqrt{\frac{1}{N_{\text{Na}}} \frac{\alpha_m(V)(1-m) + \beta_m(V)m}{2}}\, \xi_m(t)$$

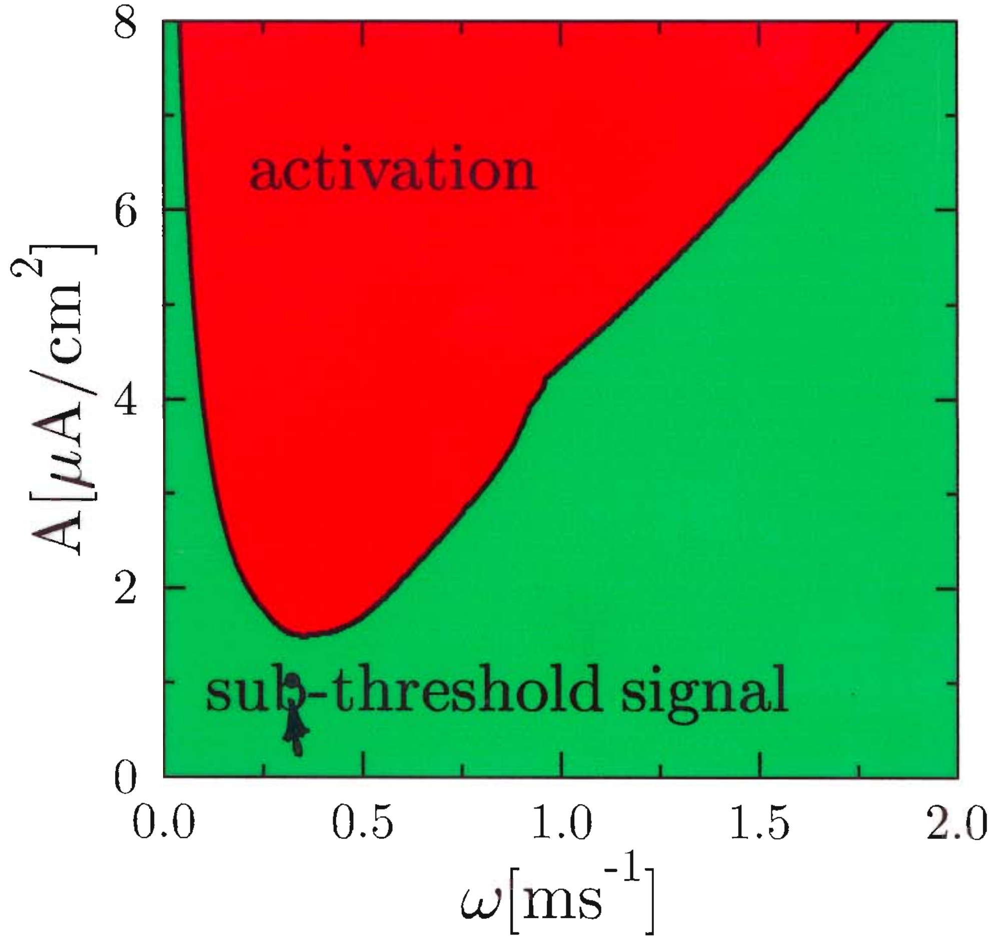
$$\frac{\mathrm{d}h}{\mathrm{d}t} = \alpha_h(V)(1-h) - \beta_h(V)h + \sqrt{\frac{1}{N_{\text{Na}}} \frac{\alpha_h(V)(1-h) + \beta_h(V)h}{2}}\, \xi_h(t)$$

$$\langle \xi_i(t) \rangle = 0\,, \langle \xi_i(t)\xi_j(t') \rangle = \delta_{ij}\delta(t-t') \quad \text{for} \quad i,j = n,m,h$$

R.F. Fox, Y. Lu, Phys. Rev. E (1994)

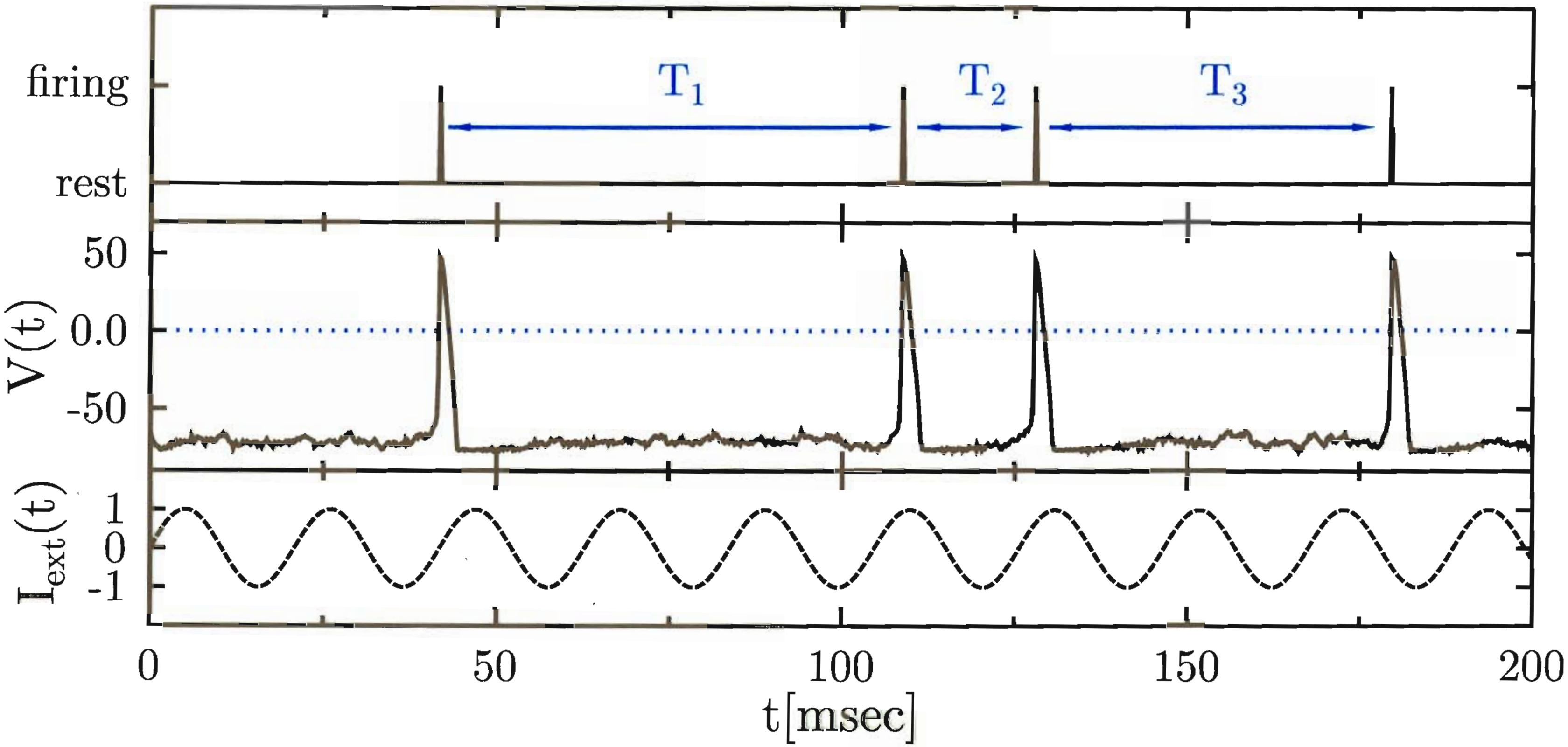
Activation in the deterministic Hodgkin-Huxley model

$$I_{\text{ext}}(t) = A\ \sin(\omega t) \quad [\mu\text{A/cm}^2]$$



Numerical Simulations

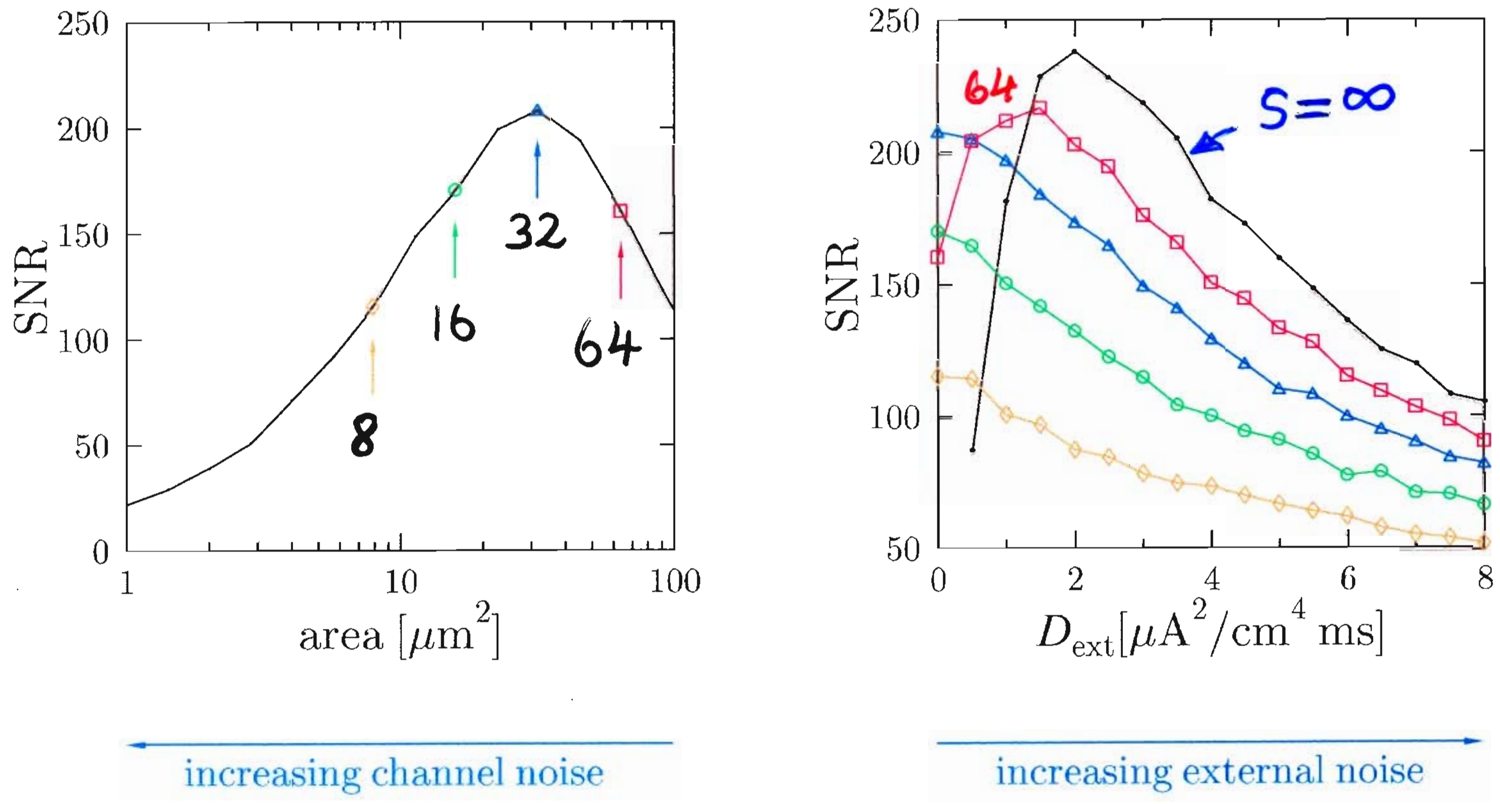
$$I_{ext}(t) = 1.0\ \sin(0.3 \cdot t) \quad [\mu A/cm^2]$$



Stochastic resonance

G.S., I.G. and P.H., Europhys. Lett. **56**, 22 (2001)

$$I_{\mathrm{ext}} = \sin(0.3 \cdot t) + \sqrt{2D_{\mathrm{ext}}}\,\xi_{\mathrm{GW}}(t) \quad [\mu\mathrm{A/cm}^2]$$



SR trends

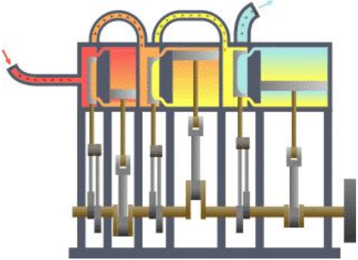
- Spatio – temporal SR
- Aperiodic SR
- Quantum SR

Brownian motors:

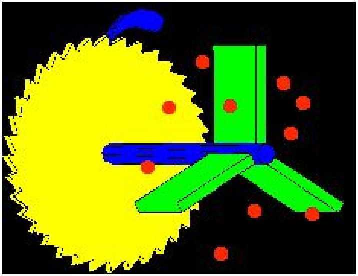
EX(E/O)RCISING DEMONS

Motors $\Longrightarrow$ Brownian motors

Two heat reservoirs



One heat reservoir

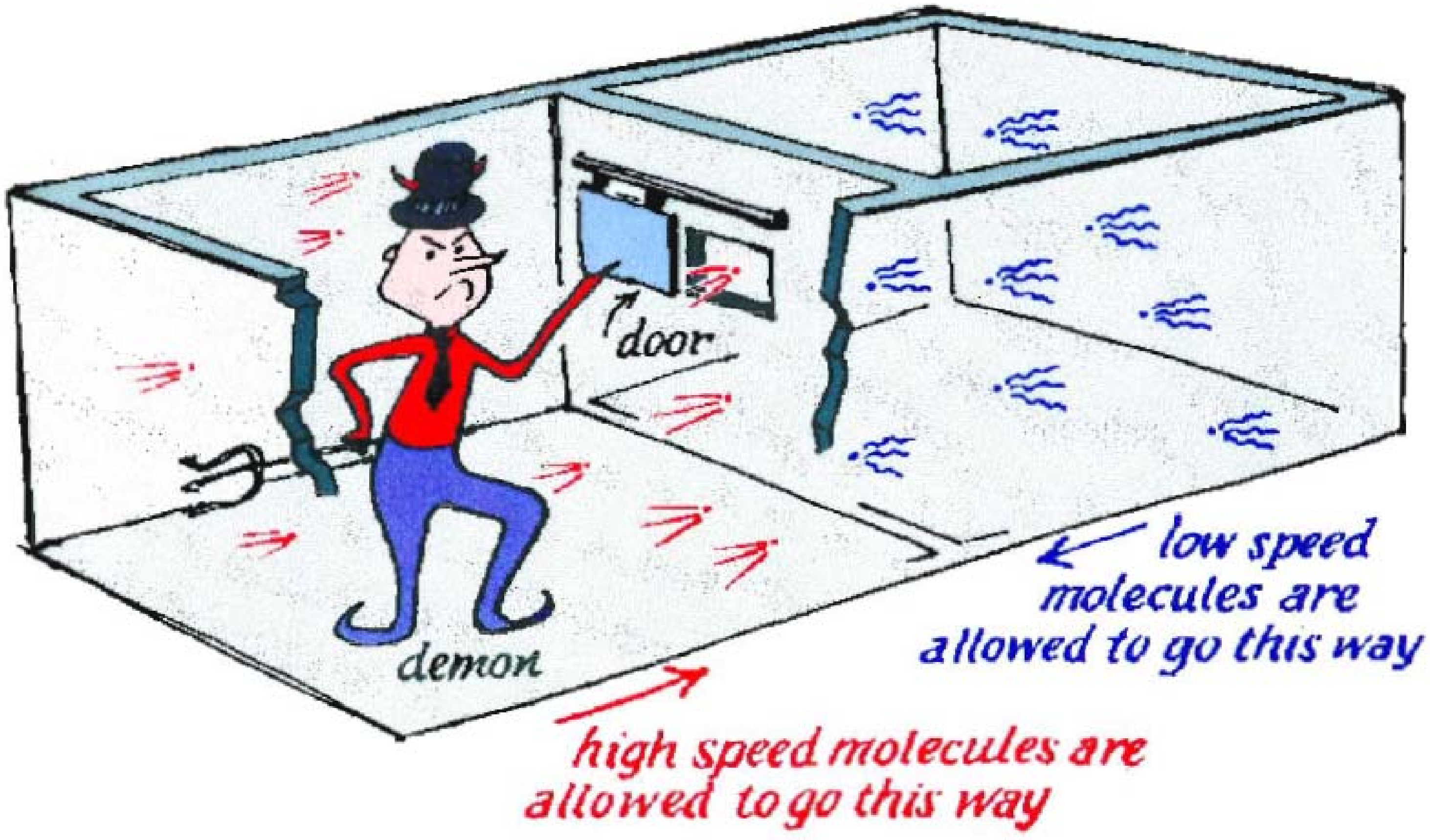


Perpetuum mobile of the second kind?

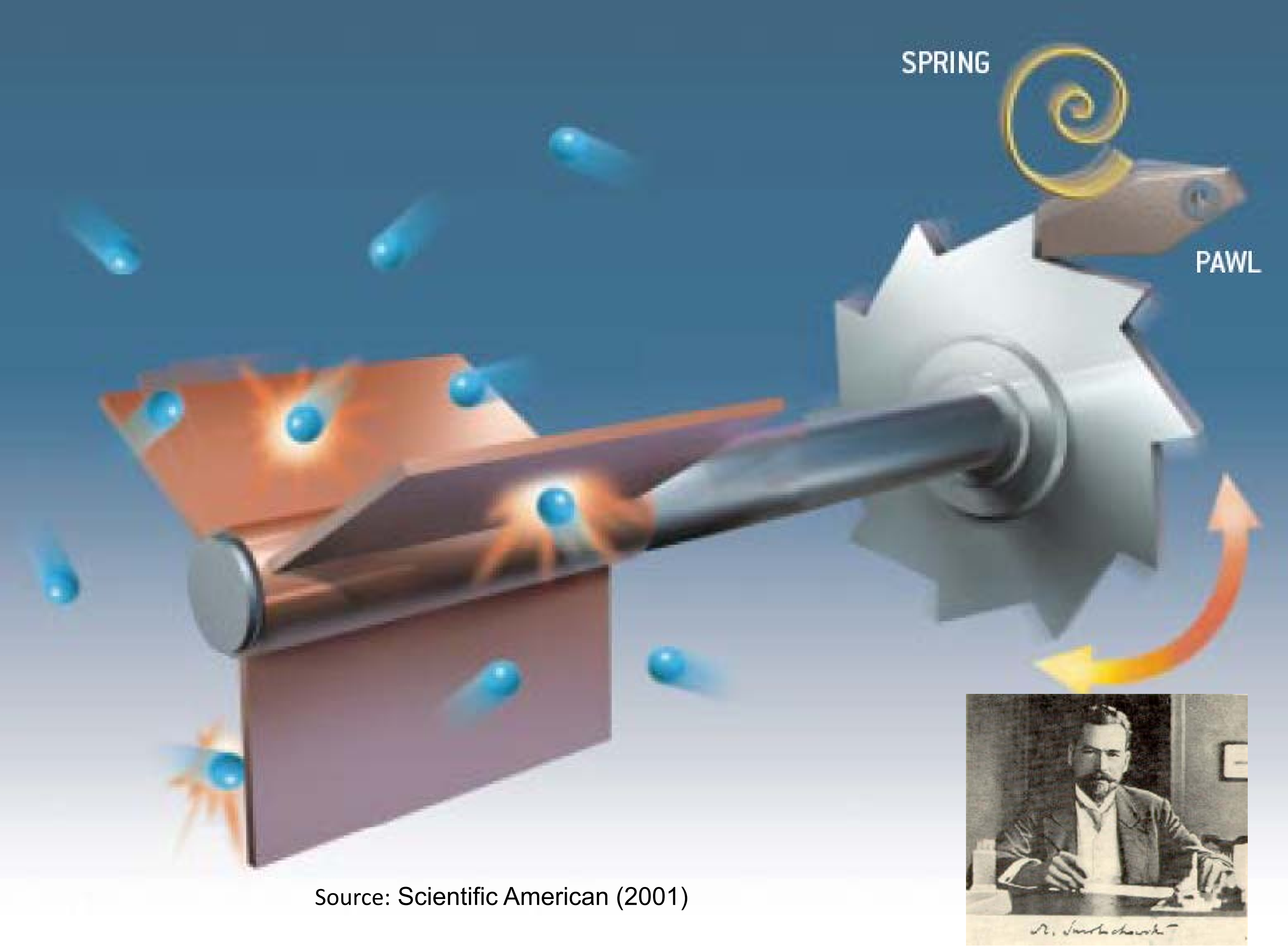
NO !

Brownian motors:

EX(E/O)RCISING DEMONS



Source: H.S. Leff, *Maxwell's Demon* (Adam Hilger, Bristol, 1990)

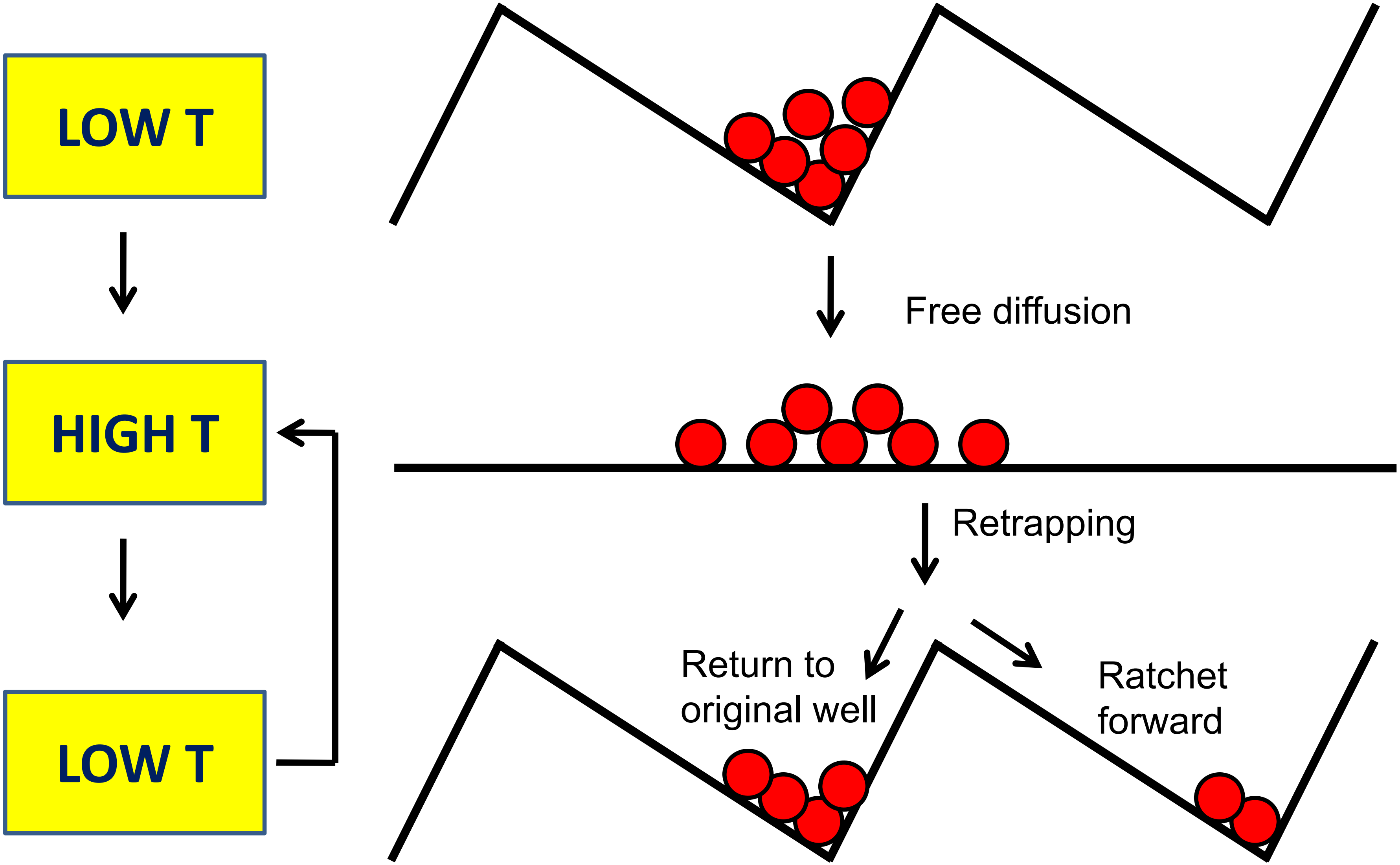


Source: Scientific American (2001)

Brownian motor

Movie

Temperature / Flashing Ratchet



Brownian motors - Characteristics

- Noise & AC-Input → **DC-Ouput**
- Non-equilibrium Noise → **Directed Transport**
- **Current reversals**
- **Applications:**
  - **Novel pumps and traps for charged or neutral particles**
  - **Brownian diodes & transistors**

Ask not what physics can do for biology, ask what biology can do for physics

REVIEWS OF MODERN PHYSICS, VOLUME 81, JANUARY–MARCH 2009

Artificial Brownian motors: Controlling transport on the nanoscale

P.H. and F. Marchesoni


SWISS TWENTY FIVE 25 JEWELS A/H 28'800
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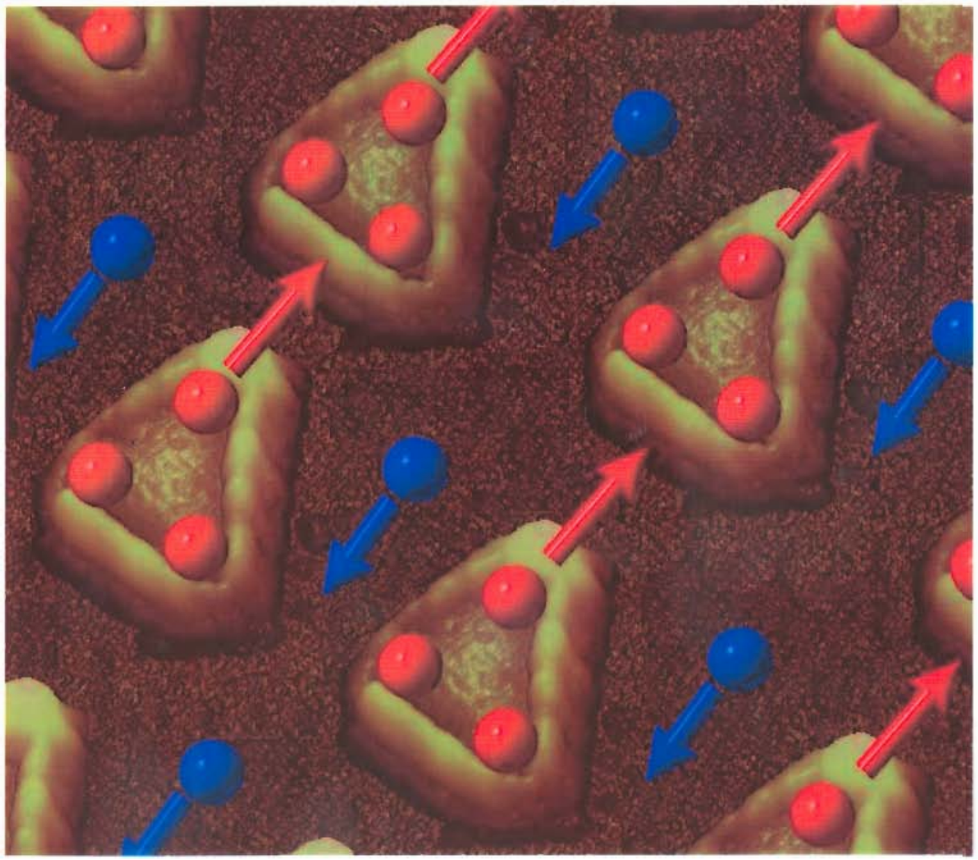
OCEAN STAR

Reversible Rectifier that Controls the Motion of Magnetic Flux Quanta in Superconductors

J.E. Villegas[1], Sergey Savel'ev[2], Franco Nori[2,3], E.M. Gonzalez[1], J.V. Anguita[4], R. García[4], J.L. Vicent[1]



[1] Universidad Complutense, Madrid, Spain.

[2] Frontier Research System, The Institute of Physical and Chemical Research (RIKEN), Saitama, Japan.

[3] The University of Michigan, Ann Arbor, MI, 48109-1120,USA.

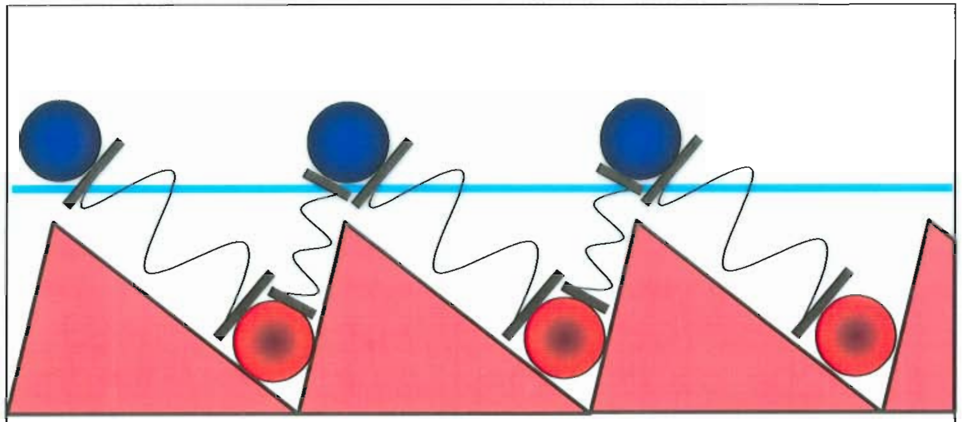
[4] Instituto de Microelectrónica de Madrid, CNM, CSIC, Madrid, Spain.

Science **302**, 1188

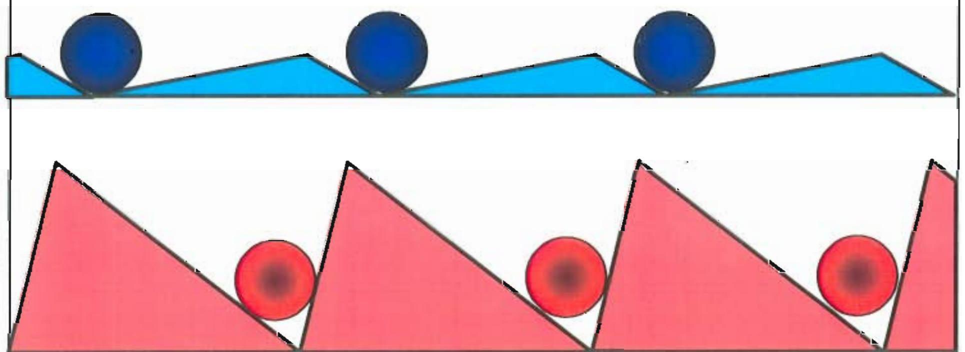
(Nov. 14, 2003)

Controlling Transport in Mixtures of Interacting Particles

Sergey Savel'ev,
Fabio Marchesoni,
and Franco Nori [1,3]



Pinned vortices (red) subject to an asymmetric substrate interacts with unpinned interstitial vortices (blue). The (red) pinned vortices create an effective substrate for the (blue) interstitials



Interstitial vortices (blue) feel an effective potential energy with opposite polarity from the potential felt by the pinned (red) vortices

Results published in

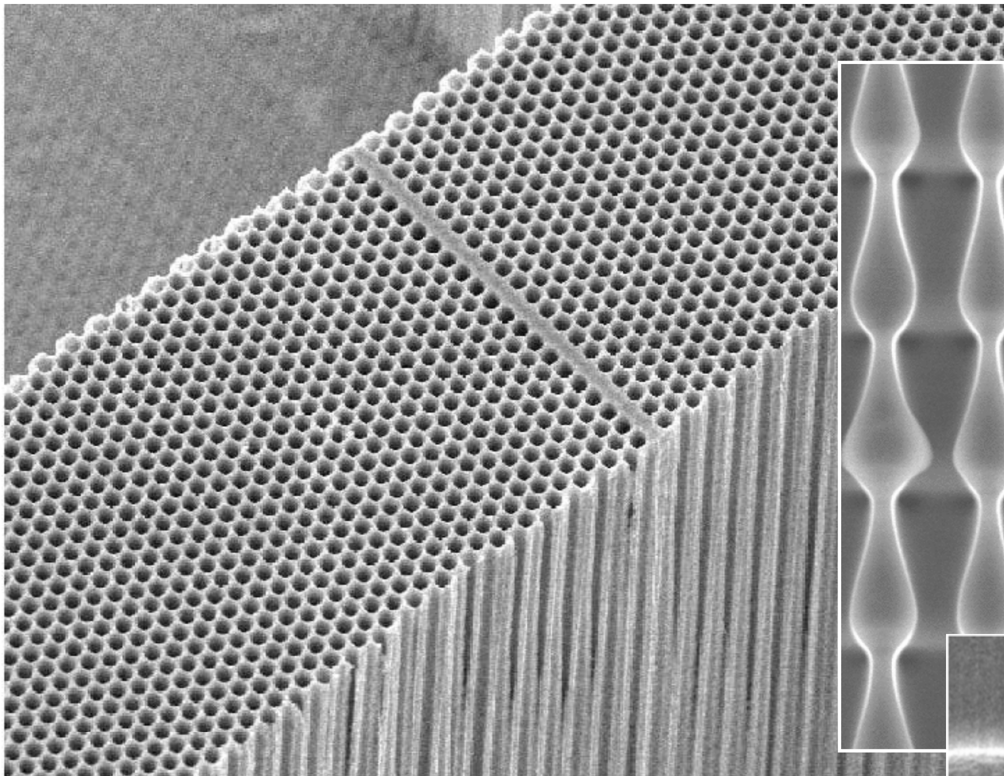
Physical Review Letters: **91**, 010601 (2003)

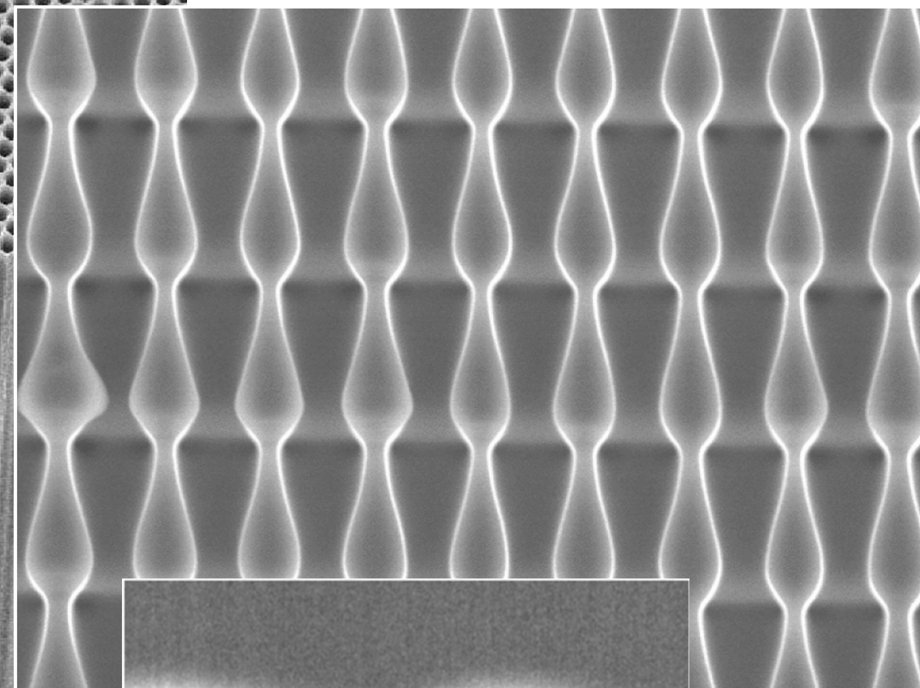
Physical Review Letters: **92**, 160602 (2004)

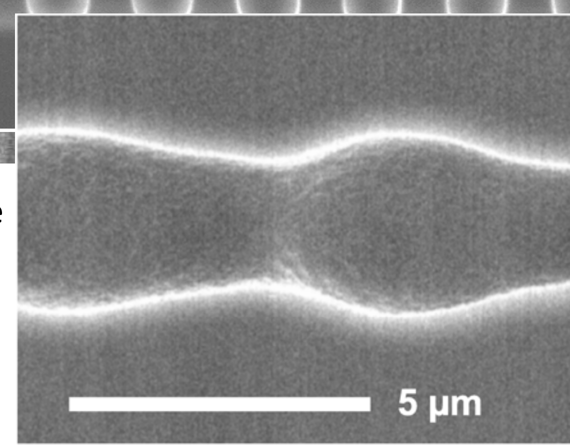
Phys. Rev. E, in press (2005).

Phys. Rev. E, in press (2005).

Drift Ratchet - Device



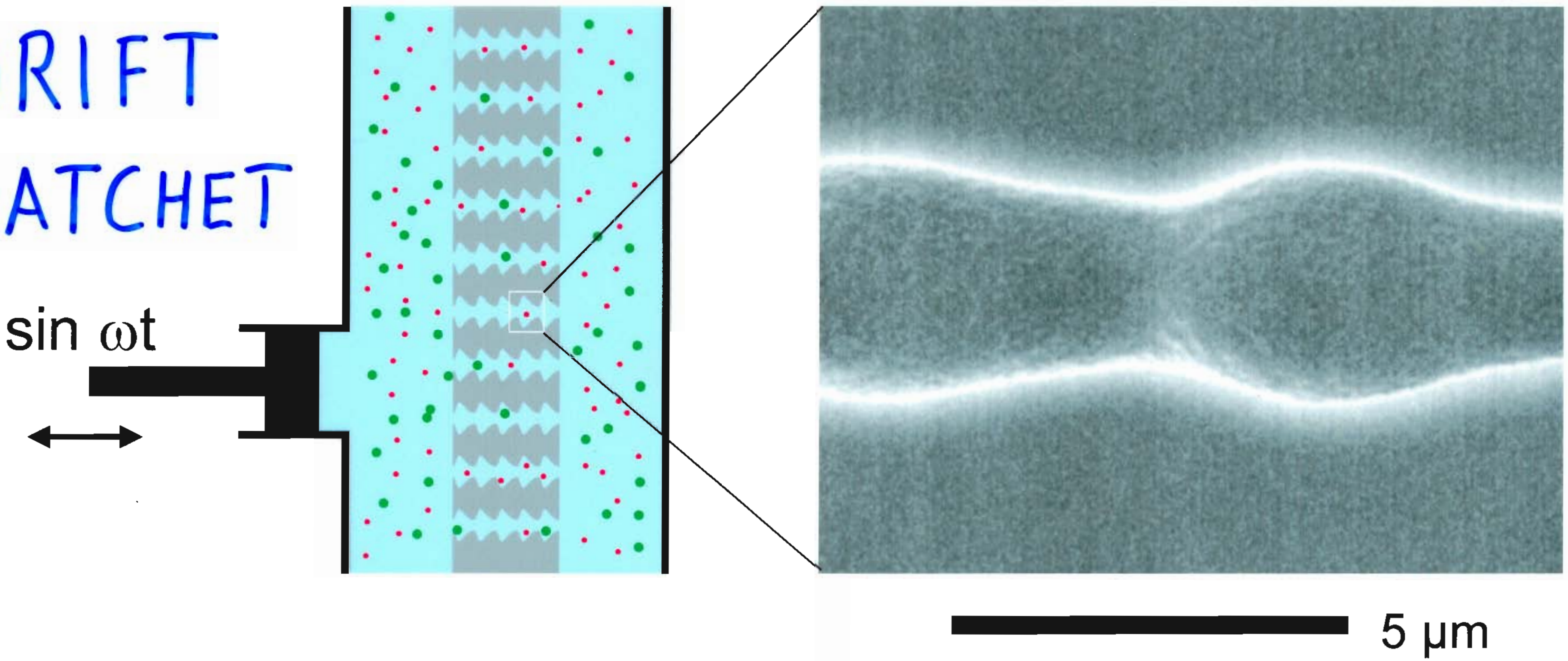




Source: F. Müller, MPI for microstructure physics, Halle

Micro Pump based on Macroporous Silicon

DRIFT RATCHET



CH. Kettner, P. Reimann, P. H., F. Müller, PHYS. REV. E 61: 312 (00)

F. Müller, A. Birner, U. Gösele
MPI of Microstructure Physics, Halle/Saale, Germany

LANGEVIN EQ. FOR BROWNIAN PARTICLES

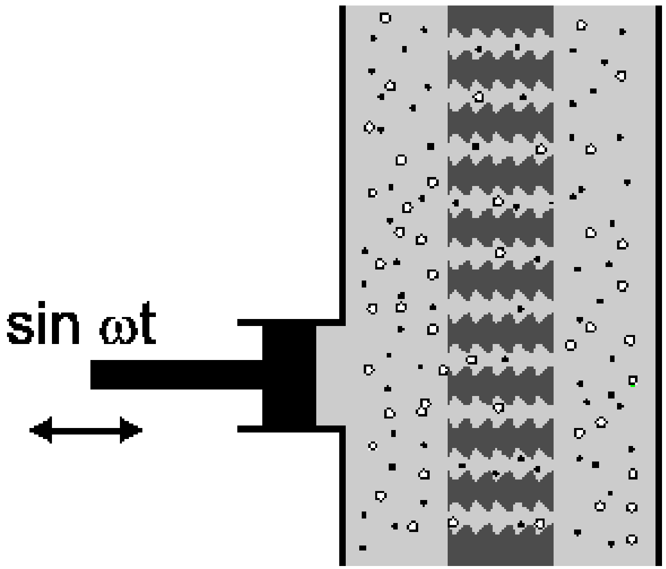
$$\dot{\vec{x}}(t) = \vec{v}(\vec{x}(t), t) + \sqrt{2D_{th}}\,\vec{\xi}(t)$$

$$D_{th} = \frac{k_B T}{\eta}$$

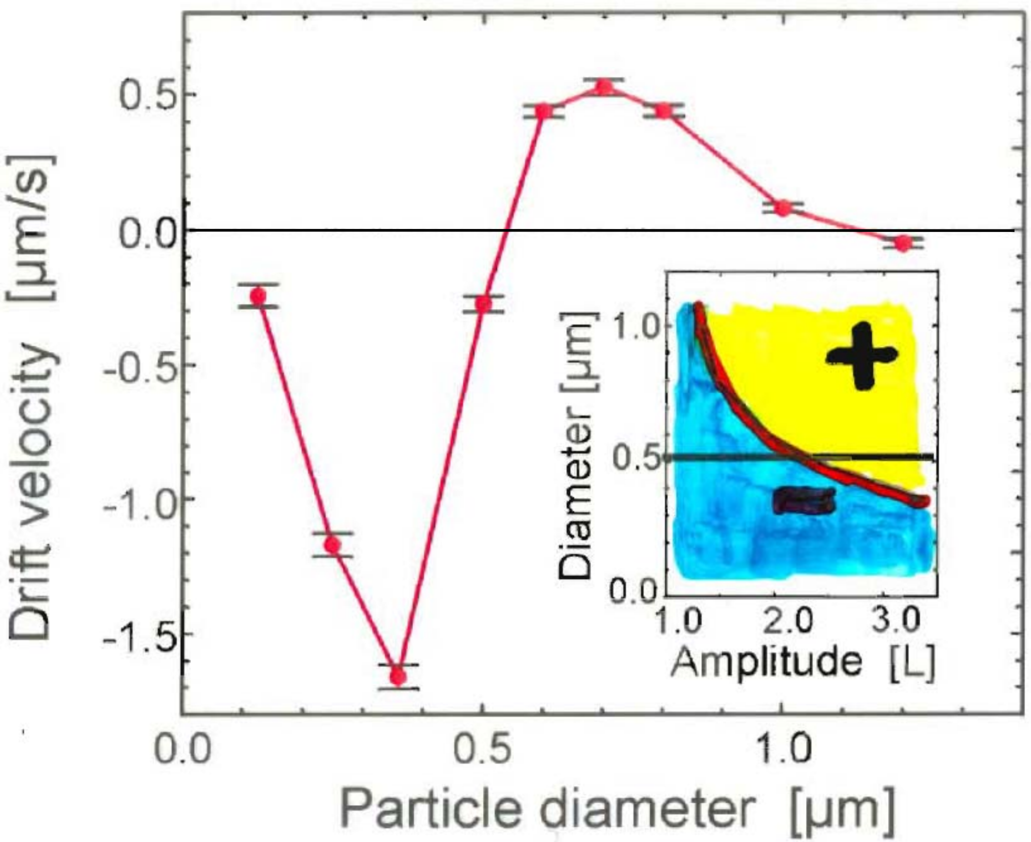
Drift Ratchet - Theory

C. Kettner, P. Reimann, P. H., F. Müller, Phys. Rev. E **61**, 312 (2000)

Setup

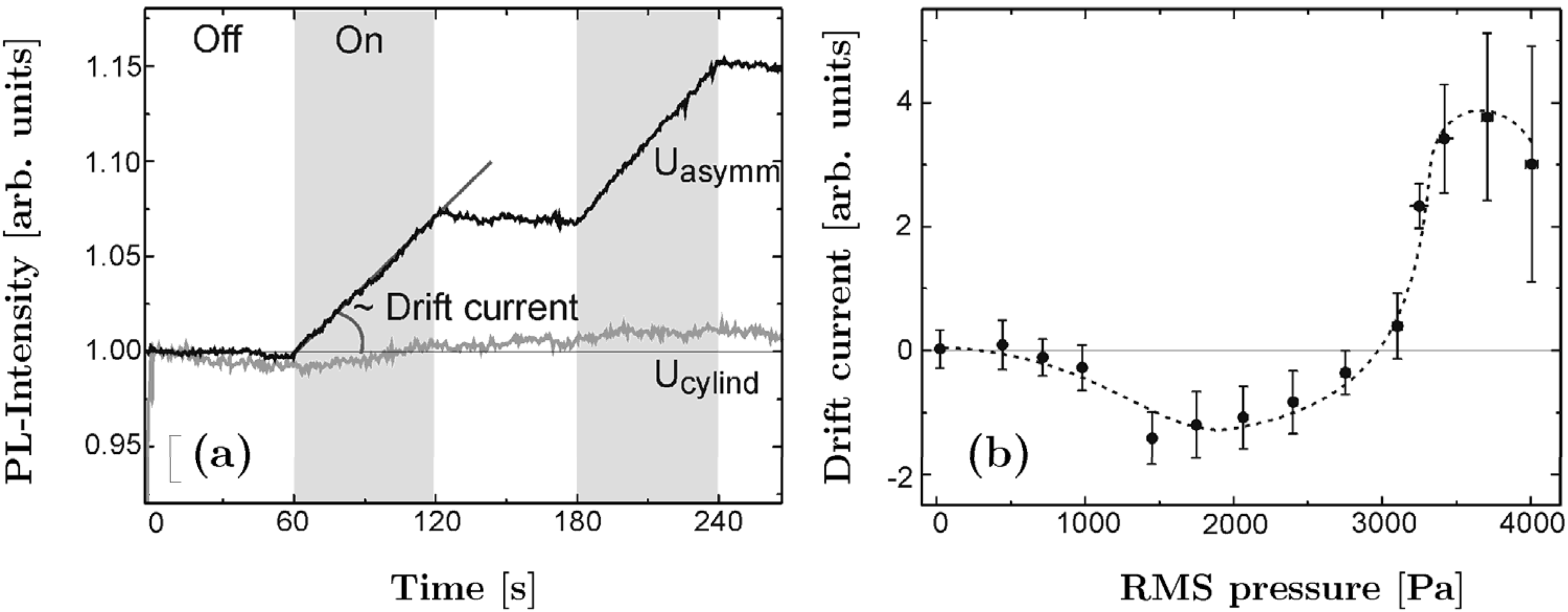


Particle Separation

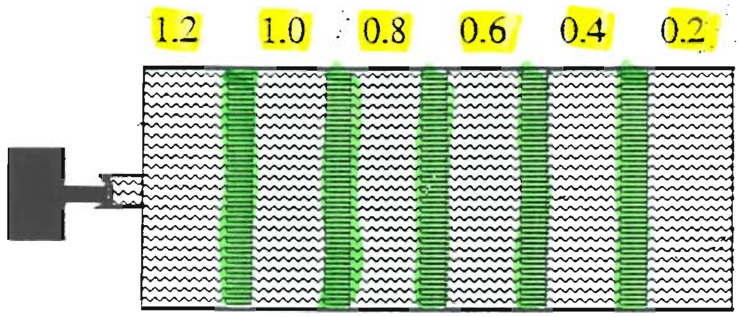


Drift Ratchet – Experiment

S. Matthias, F. Müller, Nature **424**, 53 (2003)



OUTLOOK



MEDICINE, BIOLOGY, CHEMISTRY

- NO NEED OF ELECTRIC OR DIELEKTRIC PROPERTIES OF PARTICLES

- ENORMOUS PARALLEL ARCHITECTURE

¡ 10^6 PORES / cm^2 !

Quantum clockwork

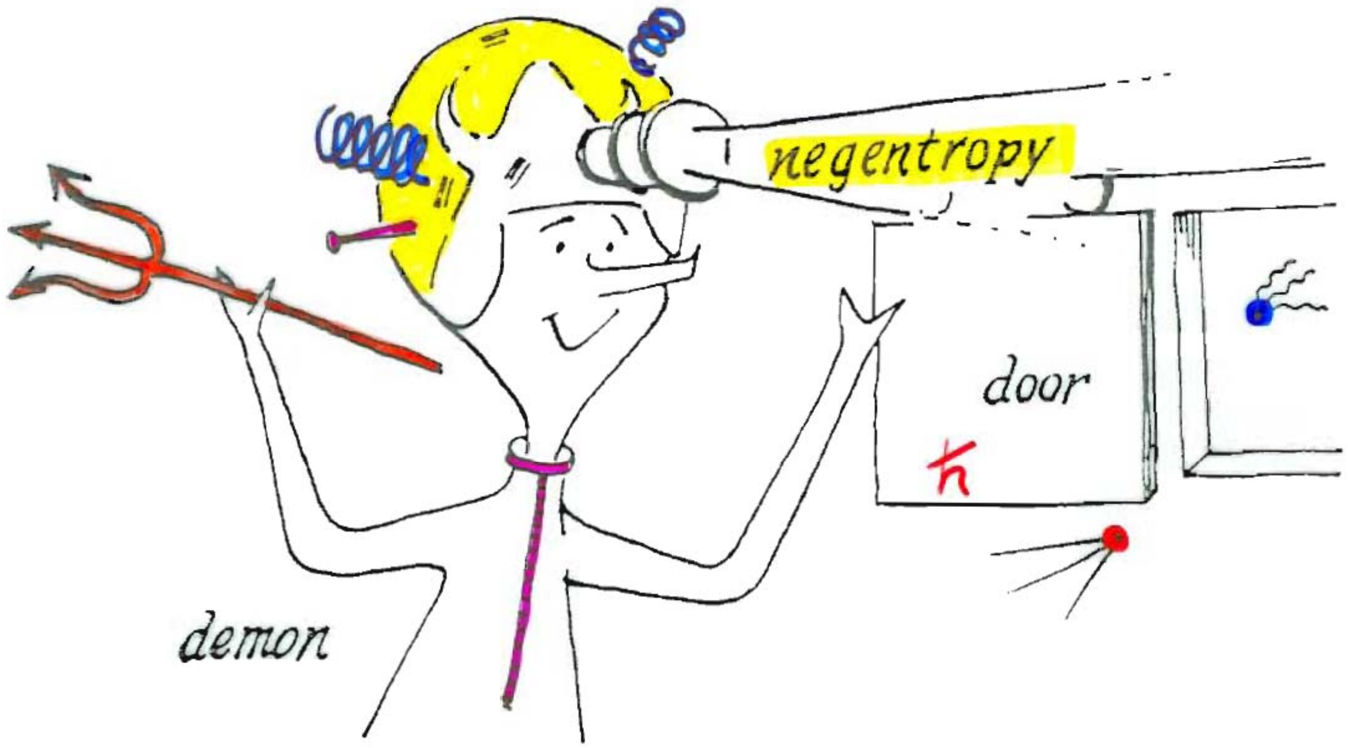
Goodbye
ratchet
mach
Mic

S

n
ne
wil
cial
a gene
ratchet
unfocus

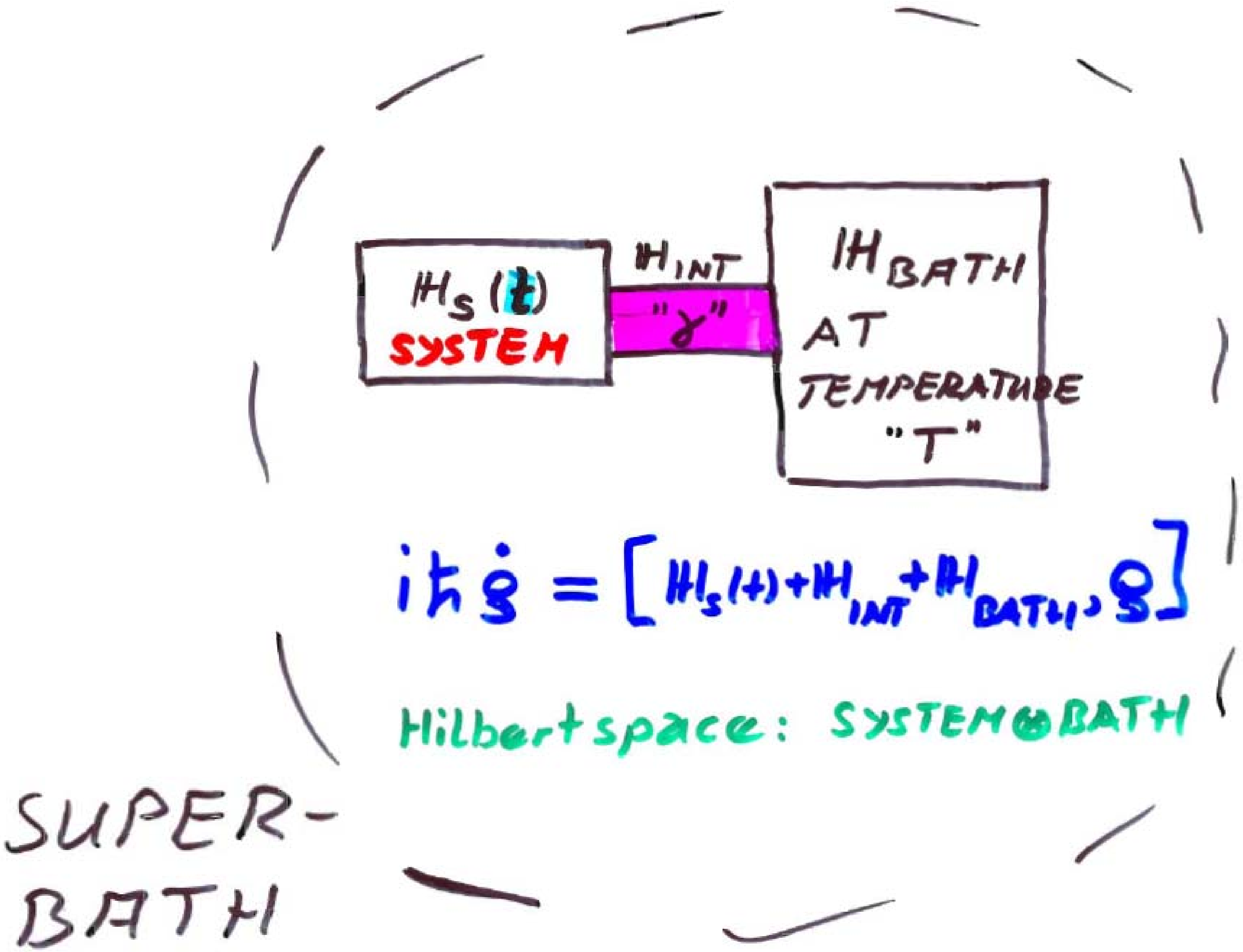
Quantum Demon ?

A measurement → Increase information → Reduction of entropy



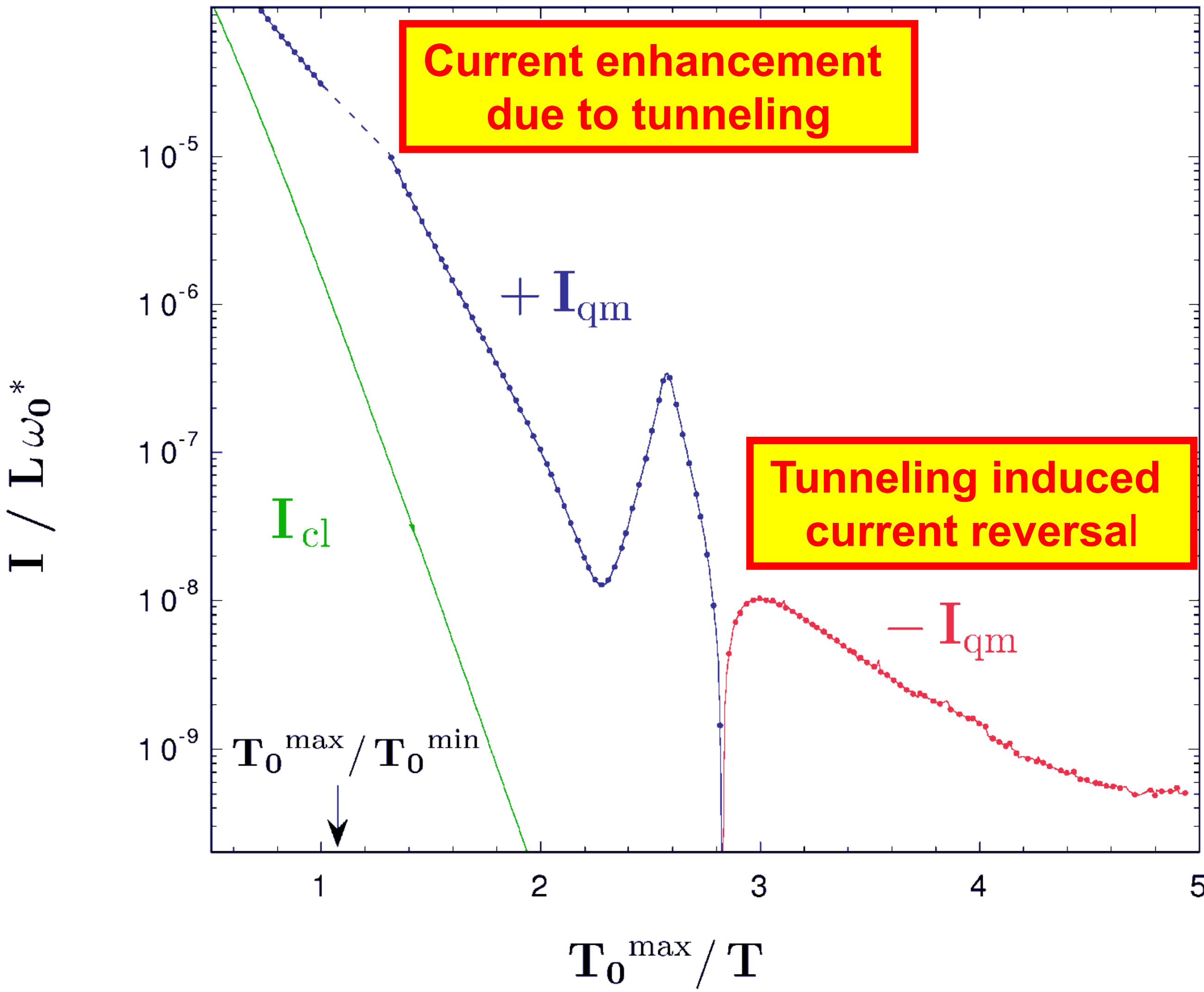
Source: H.S. Leff, *Maxwell's Demon* (Adam Hilger, Bristol, 1990)

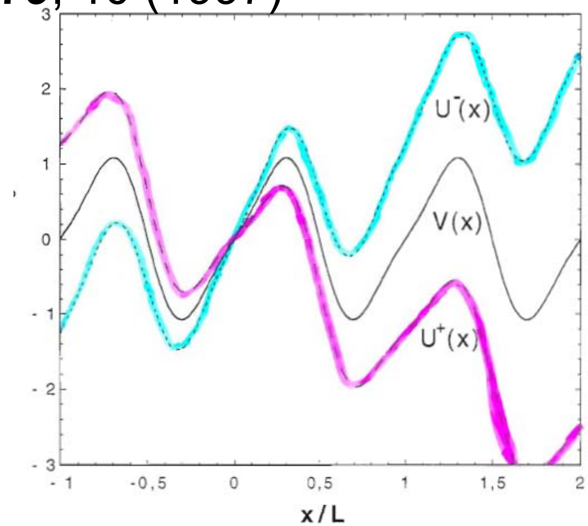
Quantum Brownian Motors



Rocking Ratchet - Theory

P. Reimann, M. Grifoni, P. H., Phys. Rev. Lett. **79**, 10 (1997)

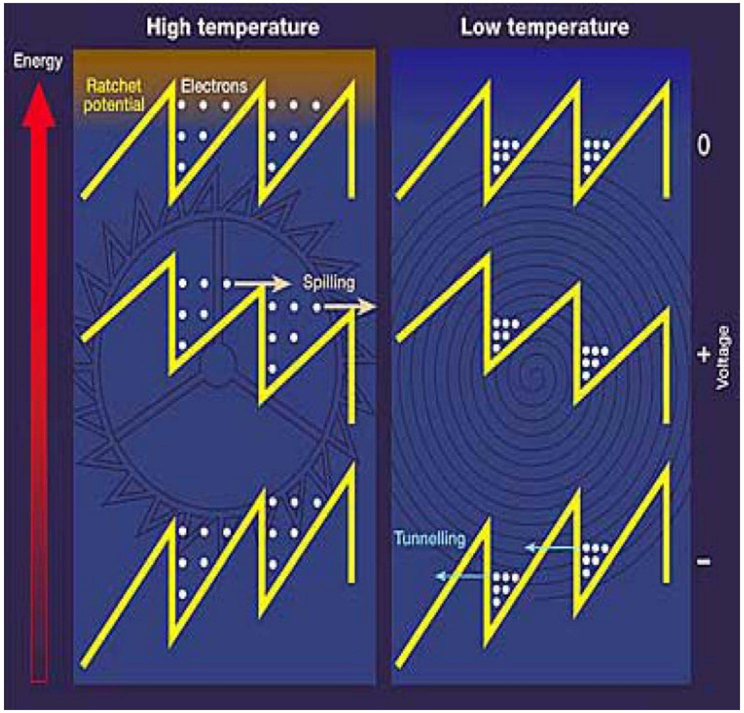


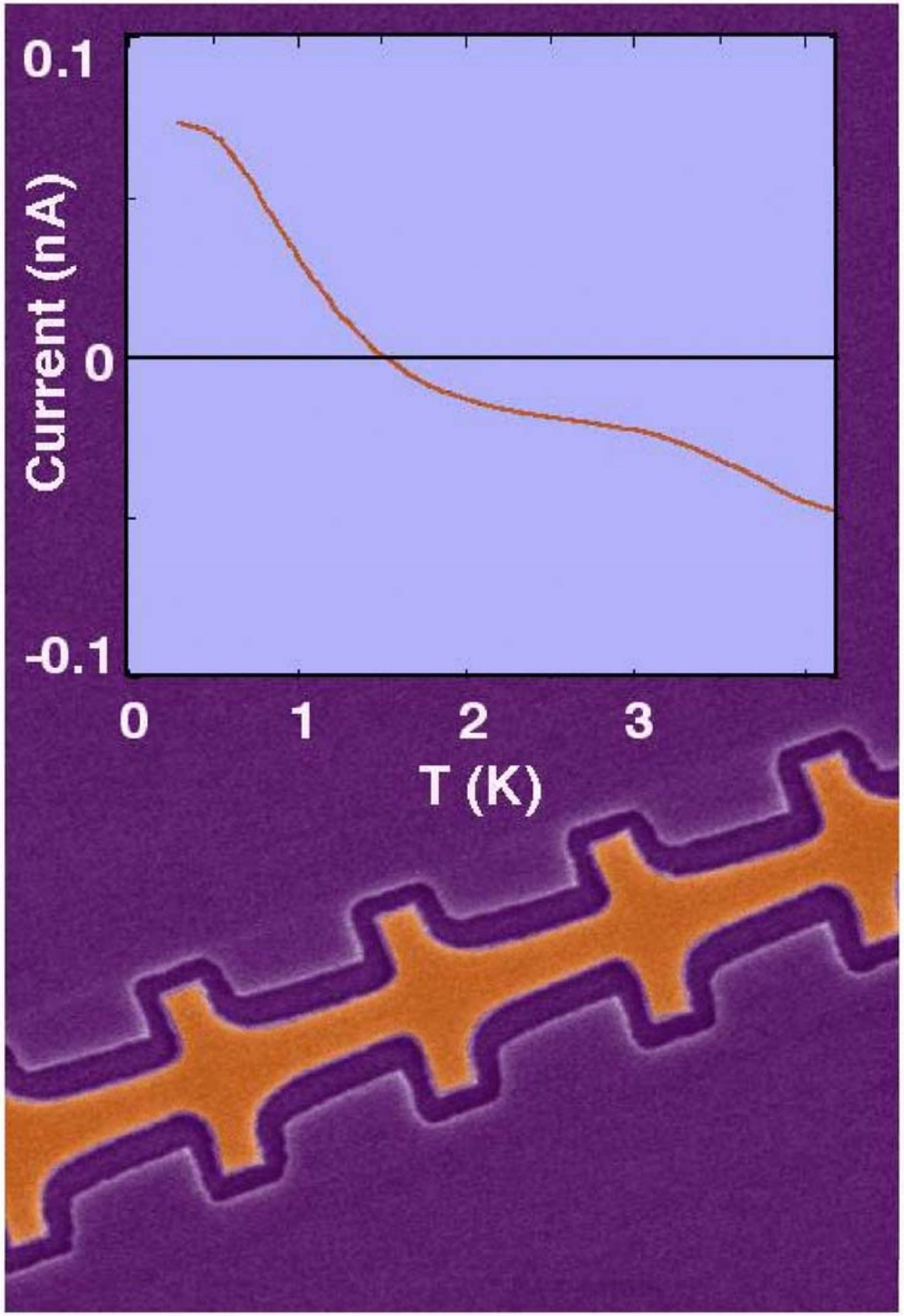


Finite current for T→0

Rocking QM Ratchet – Experiment

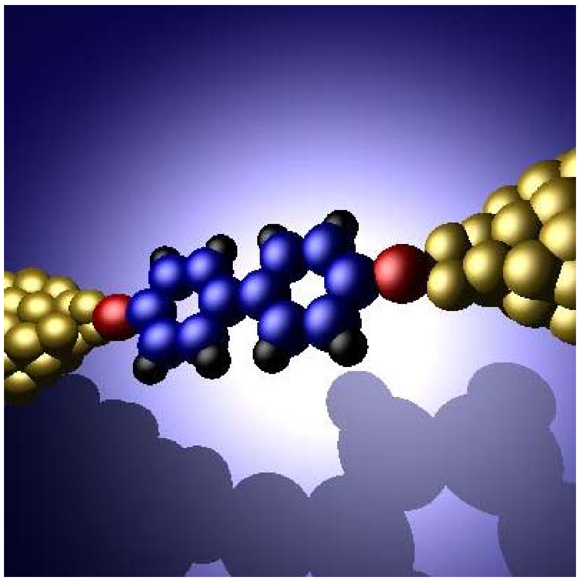
H. Linke, *et al.*,
SCIENCE **286**, 2314 (1999)

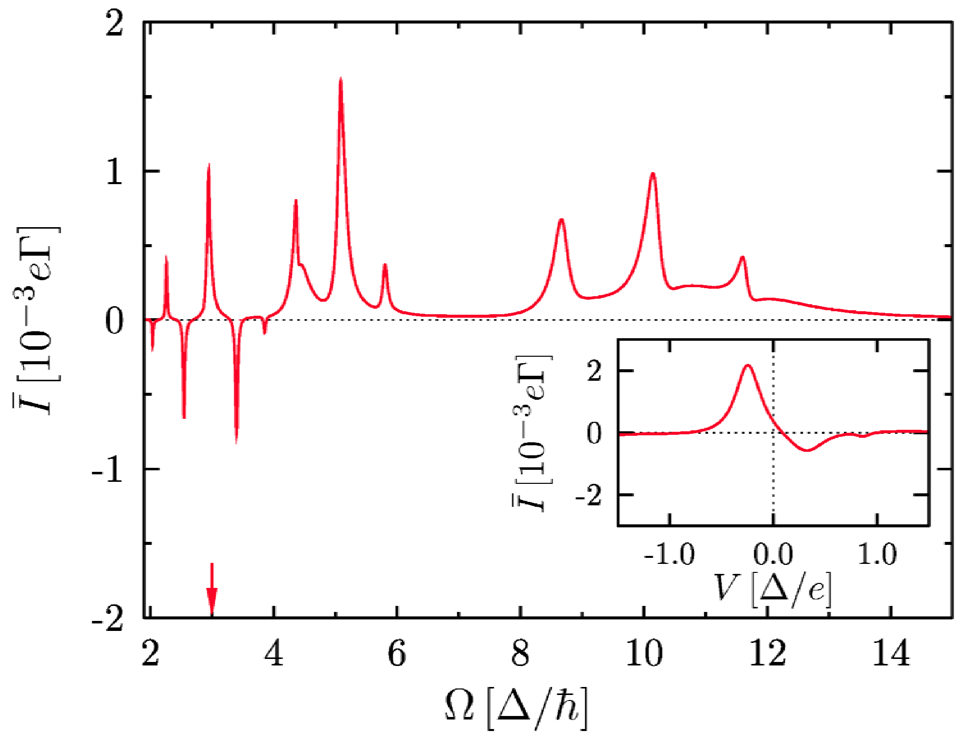




Molecular wires

J. Lehmann, S. Kohler, P. H., A. Nitzan, Phys. Rev. Lett. **88**, 228305 (2002)

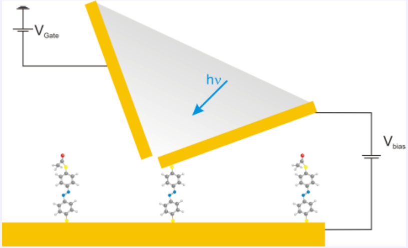




- Field strength E=106 V/cm
- Ω=3Δ corresponds to 4µm wavelength
- typical current: some nA

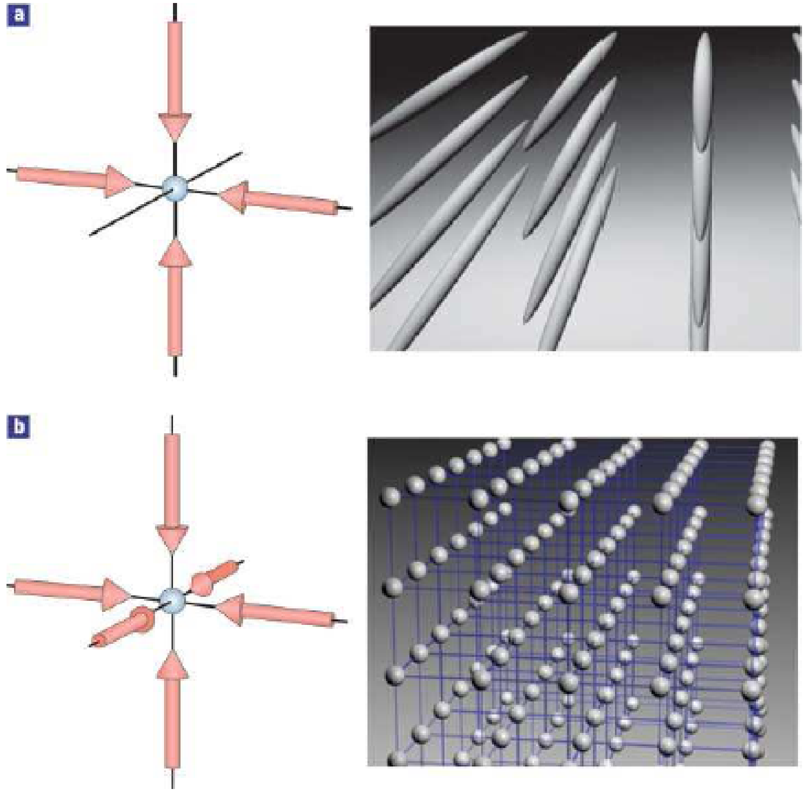
excitations of molecules

- direct exposure to light
  problem: affects contacts
- solution: coated SNOM tip
  (*scanning near-field optical microscope*)

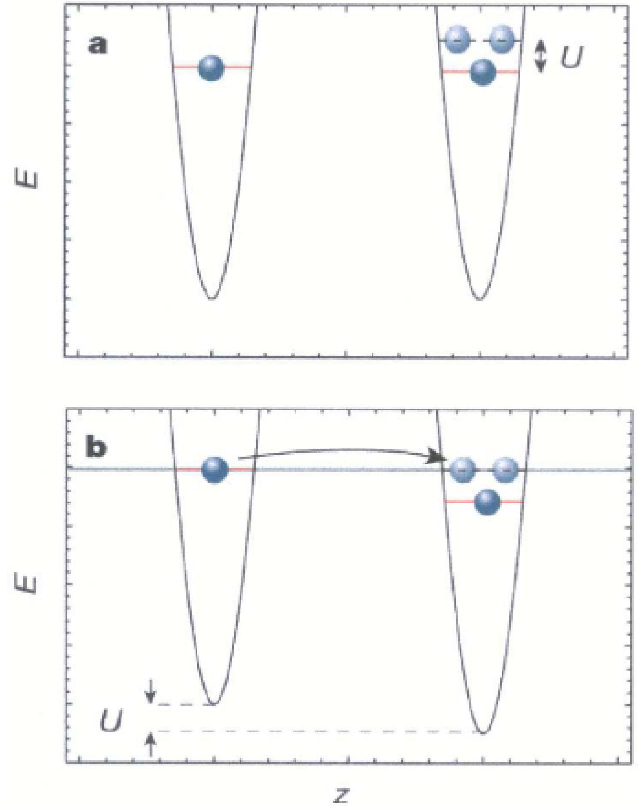


- focussing on molecule
- TERS (*tip-enhanced Raman spectroscopy*):
  local field enhancement by several orders of magnitude

Optical lattices: 1D-3D geometries



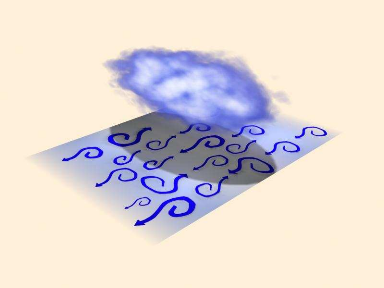
[I. Bloch, Nature Physics 1, (2005)]



[M. Greiner et al., 2002]

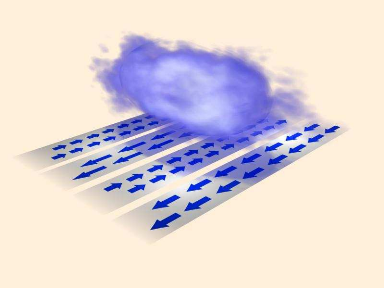
Quantum Ratchets

Classical ratchet



$J = J_{chaotic}$

Quantum ratchet



$J = \sum_\alpha C_\alpha(t_0) \cdot v_\alpha$

P. Hänggi and S. Denisov, Physik Journal 9, 18 (2010)

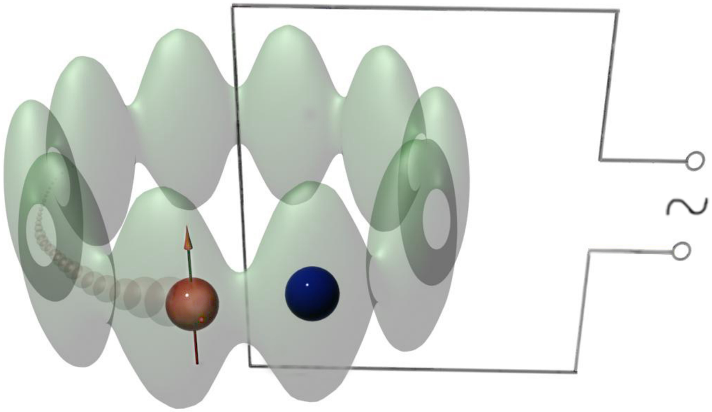
Atomic quantum motor

$$H_{\text{tot}} = H_c(t) + H_s + H_{\text{int}}$$

$$H_c(t) = -\frac{J_c}{2}\left(\sum_{l_c=1}^{L} e^{i\tilde{A}(t)}|l_c+1\rangle\langle l_c| + \text{H.c.}\right)\otimes \mathbf{1}_s$$

$$H_s = -\frac{J_s}{2}\left(\sum_{l_s=1}^{L} |l_s+1\rangle\langle l_s| + \text{H.c.}\right)\otimes \mathbf{1}_c$$

$$H_{\text{int}} = W\sum_{l_c,l_s=1}^{L} \delta_{l_c,l_s}|l_c\rangle\langle l_c| \otimes |l_s\rangle\langle l_s|,$$



short-range interaction *1) induced dipole-dipole (between neutral atoms)*

2) charge - induced dipole (between ion and neutral atom)

Motor speed vs. phase shift

(i) Direct time evolution (t=200T)

$$v_c(t_0) := \lim_{t\to\infty} \frac{1}{t} \int_0^t v_c(t';t_0)dt'$$

(ii) Floquet theory prediction

$$v_c(t_0) = \sum_{n=1}^{\mathcal{N}} \bar{v}_n |c_n(t_0)|^2$$

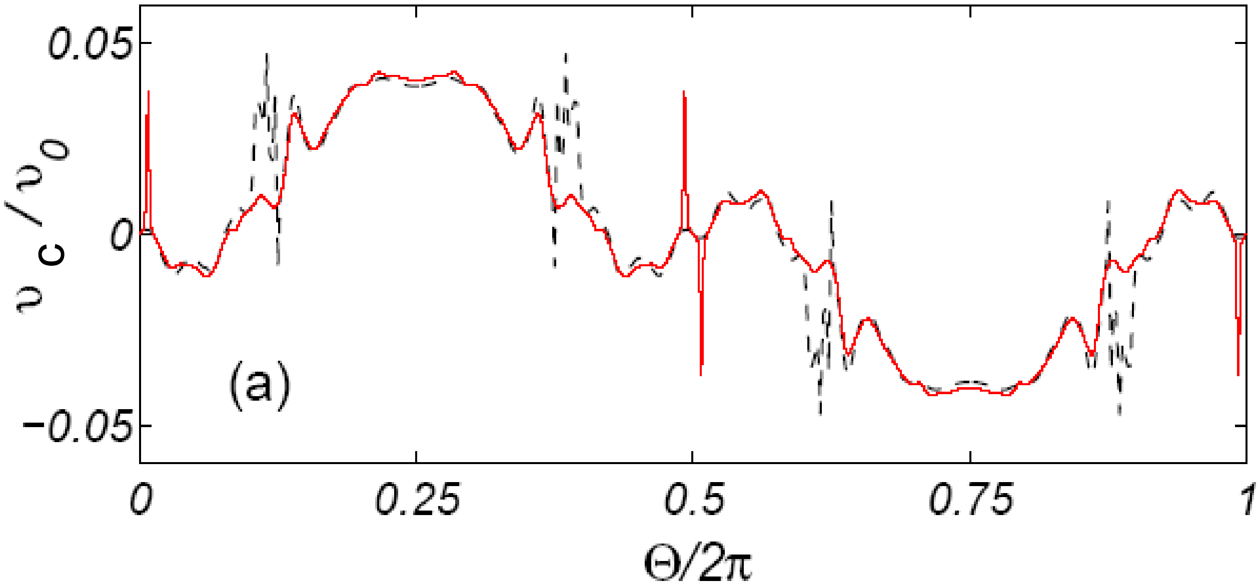
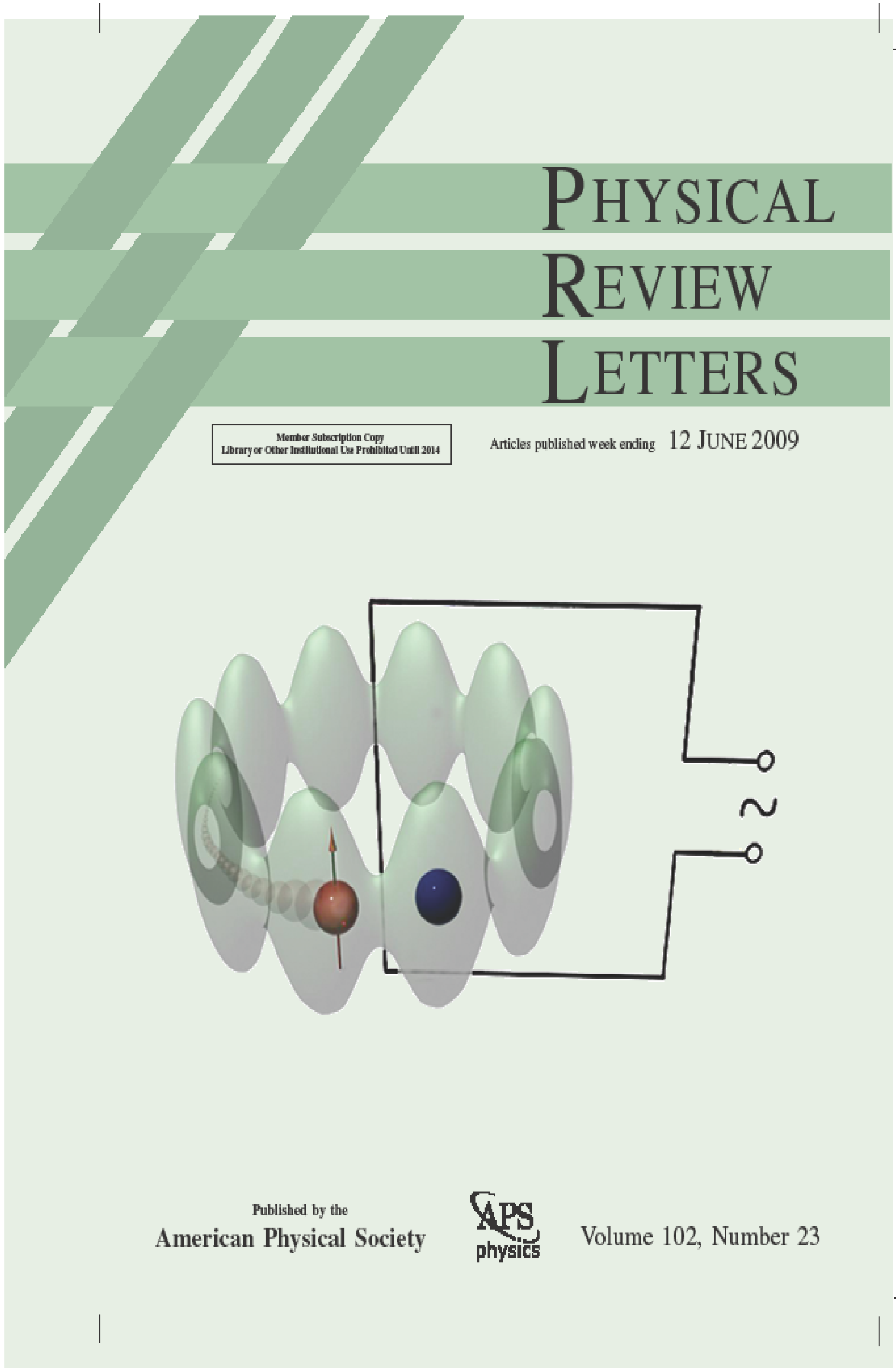


Figure. Carrier velocity for localized initial conditions.

Here, (i) and (ii) averaged over t_0: $v_c = \langle v_c(t_0) \rangle_{t_0}$

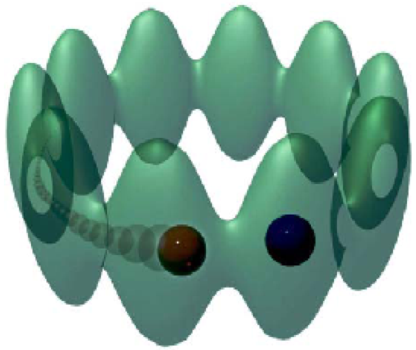
Two-atom Quantum Motor

A. Ponomarev, S. Denisov, & P. Hänggi, PRL 102, 230601 (2009)



To Fix This Engine, You'll Need a Quantum Mechanic

By Adrian Cho
*Science*NOW Daily News
9 June 2009



Crank it! If it could be built, this quantum-mechanical contraption consisting of two atoms in a ring of light would be the smallest electrical motor.

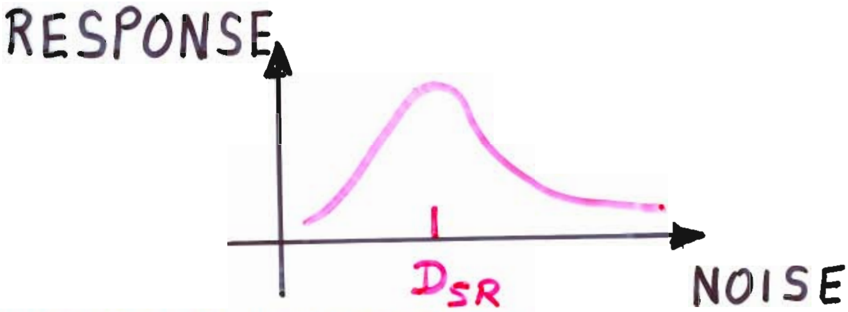
Credit: Ivan Ponomarev

The first electric motor whirred to life nearly 2 centuries ago, and in recent decades scientists and engineers have worked to build ever-smaller ones. Now, a team of theoretical physicists has proposed a fully quantum-mechanical version of the classic spinning electric motor that consists of just two atoms trapped in a ring of light. Experimenters might be able to construct the thing now, the researchers say, even though they themselves don't have an intuitive explanation of exactly how it works.

An electric motor transforms the energy in an electric or magnetic field into…

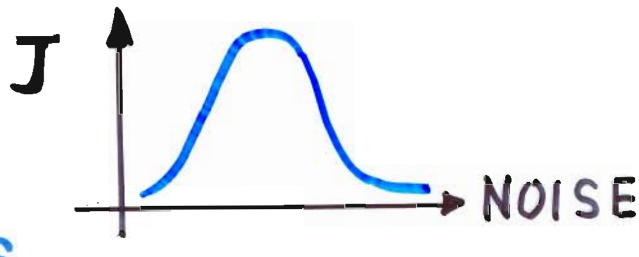
CONCLUSIONS

* SR:



* BROWNIAN MOTORS

DIRECTED CURRENT



NEW PARTICLE PUMPS

BROWNIAN RECTIFIERS

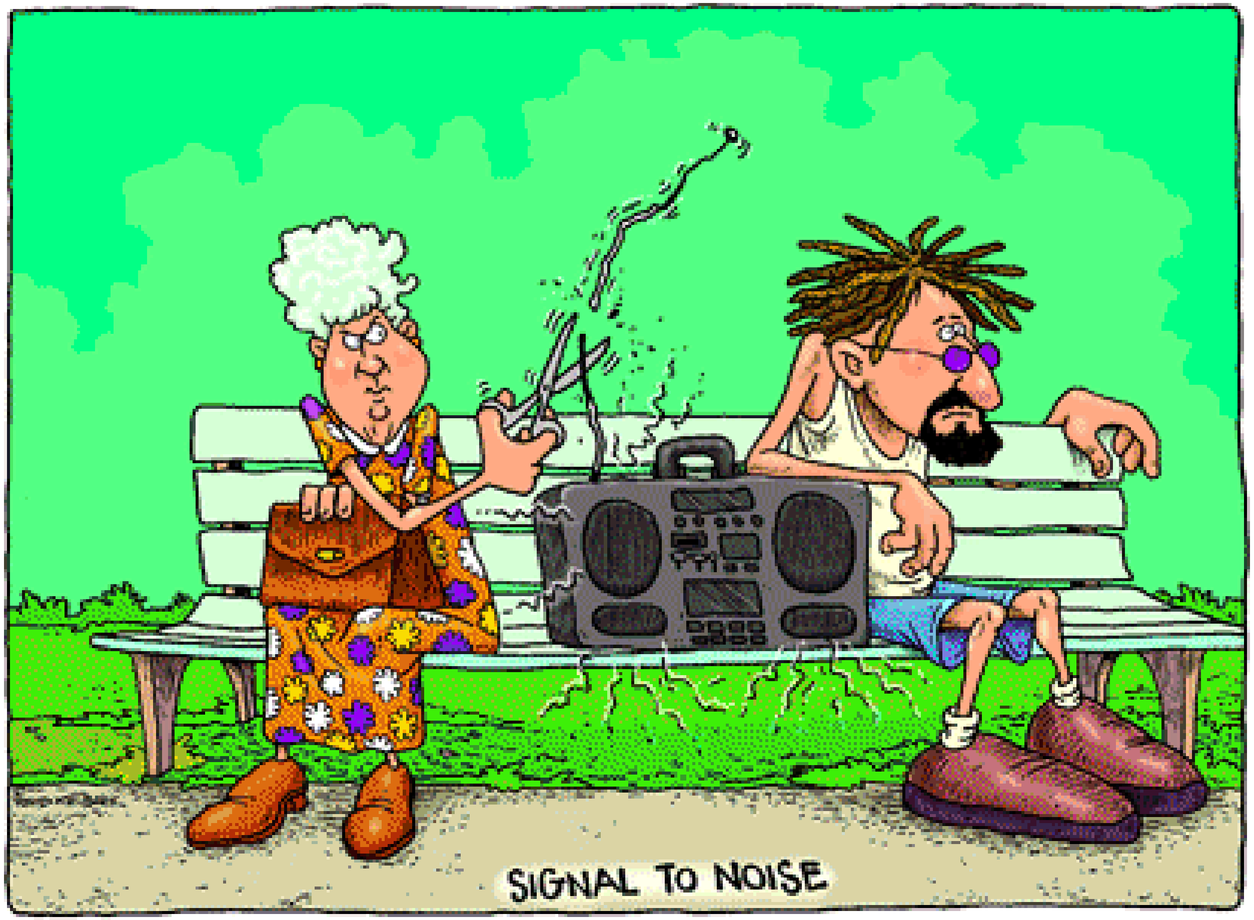
PARTICLE SELECTION - DRIFT RATCHET

* QUANTUM DEMONS

$\hbar \neq 0$

FINITE CURRENT AT T = 0

Noise – always bad ?



Source: Agilent Technologies

CONCLUSION

DO

WORK IN BROWNIAN MOTION

ENDE

FIN

THAT'S IT

Acknowledgements

nanosystems initiative munich

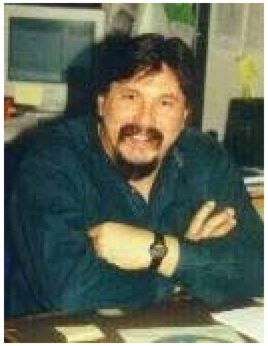
CeNS
Center for NanoScience
Ludwig-Maximilians-Universität

nano
SFB 486

SFB 631

SFB 484

QUANTRANS

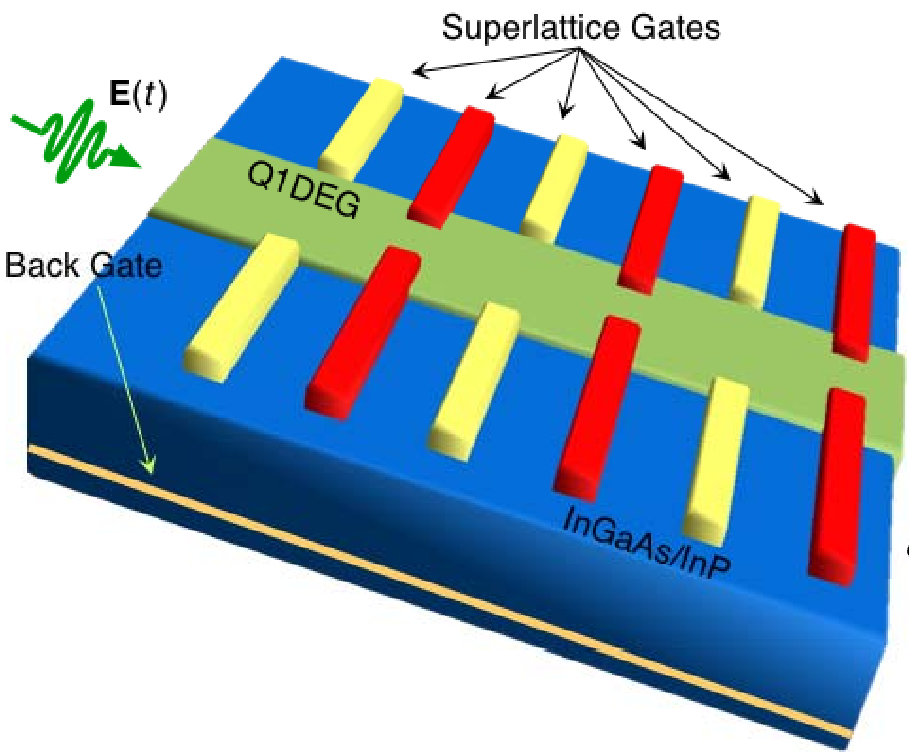


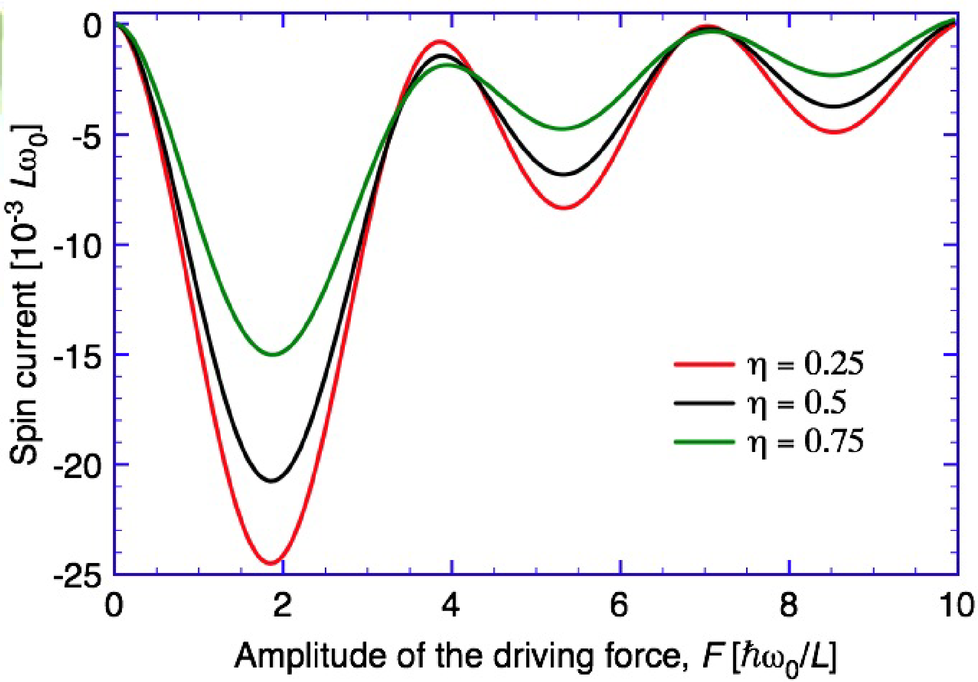
Prof. Dr.
Hermann E. Gaub

Prof. Dr.
Jörg P. Kotthaus

Spin Ratchet

S. Smirnov, D. Bercioux, M. Grifoni, K. Richter, Phys. Rev. Lett. **100**, 230601 (2008)





BRAIN AND COMPUTER – BOTH WORK ELECTRICALLY

↓ IONS

↓ ELECTRONS

I BET ON BRAIN

Jan Ingen-Housz (1730-1799)

William Sutherland (1859-1911)

Johann Ingen-Housz

K. K. Hofraths und Leibarztes, der Königl. Gesellschaft der Wissenschaften zu London, der Batavischen Gesellschaft der Experimentalphilosophie zu Rotterdam ꝛc. ꝛc. Mitglieds

Vermischte Schriften physisch-medicinischen Inhalts.

Uibersetzt und herausgegeben
von
Nicolaus Carl Molitor.

Zweyte, verbesserte und mit ganz neuen Abhandlungen vermehrte Auflage.

Mit Kupfertafeln.

Zweyter Band.

Wien,
gedruckt und verlegt bey Christian Friderich Wappler.
1784.

To see clearly how one can deceive one's mind on this point if one is not careful, one only has to place a drop of alcohol in the focal point of a microscope and introduce a little finely ground charcoal therein, and one will see these corpuscules in a confused, continuous and violent motion, as if they were animalcules which move rapidly around.

Sutherland's papers
The *Phil Mag* paper

LXXV. *A Dynamical Theory of Diffusion for Non-Electrolytes and the Molecular Mass of Albumin.* By WILLIAM SUTHERLAND ‡.

IN a paper communicated to the Australian Association for the Advancement of Science at Dunedin, 1904, on the Measurement of Large Molecular Masses, a purely dynamical theory of diffusion was outlined, with the aim of getting a formula for calculating from the data of diffusion those large molecular masses for which the ordinary methods fail. The formula obtained made the velocity of diffusion of

A dynamical theory of diffusion for non-electrolytes and the molecular mass of albumin. *Philosophical Magazine*, S.6, 9 (1905), 781-785.

Mailed to London in March 1905, and published in June 1905

Now use the gas law

Write the gas law as

$$p = n\frac{R}{N_0}T = \frac{N}{V} \cdot k_B \cdot T$$

so

$$\frac{\Delta p}{\Delta x} = \frac{R}{N_0}T\frac{\Delta n}{\Delta x} = k_B \cdot T \frac{\Delta n}{\Delta x}$$

$$k_B \cdot T \frac{\Delta n}{\Delta x} = 6\pi\eta a v n = 6\pi\eta a D \frac{\Delta n}{\Delta x}$$

$$v n = D \frac{\Delta n}{\Delta x}$$

$$D = \frac{k_B \cdot T}{6\pi\eta a}$$

where β is the coefficient of sliding friction if there is slip between the diffusing molecule and the solution. For N molecules of solute per c.c. of solution the total resistance will be N times this, and in the steady state of diffusion will equilibrate the driving force due to variation of the osmotic pressure of the solute, namely dp/dx, which by the osmotic laws is $\mathrm{RT}dc/dx$, if c is the concentration of the solute at x and R is the gas constant. Hence

$$\mathrm{RT}\frac{dc}{dx} = 6\pi \mathrm{V}\eta a \mathrm{N}\frac{1+2\eta/\beta a}{1+3\eta/\beta a}; \quad . \quad . \quad . \quad . \quad (2)$$

and the required formula for the coefficient of diffusion with C for the number of molecules in a gramme-molecule is

$$\mathrm{D} = \frac{\mathrm{RT}}{6\pi\eta a \mathrm{C}}\,\frac{1+3\eta/\beta a}{1+2\eta/\beta a}. \quad . \quad . \quad . \quad . \quad (3)$$

If $\beta=\infty$, that is, if there is no slipping of solution at surface of molecule, aD is the same for all molecules diffusing through a given solvent at a given temperature. Now for a large molecule of solute moving amongst smaller ones of solvent, we can see that the slipping is probably small. But in the other extreme case of a small molecule of solute moving amongst larger ones of solvent, an effect analogous to slipping will occur, since the small molecule will travel a good deal in the gaps which would be left if the molecules of solvent were forced almost into permanent contact. We have thus two extreme cases of the formula.

$$\left.\begin{array}{ll} \text{When } \beta=0, & \mathrm{D}=\dfrac{\mathrm{RT}}{4\pi\eta a \mathrm{C}} \\ \text{and when } \beta=\infty, & \mathrm{D}=\dfrac{\mathrm{RT}}{6\pi\eta a \mathrm{C}} \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Thus with increasing values of a we should have aD diminishing from the upper limit $\mathrm{RT}/4\pi\eta\mathrm{C}$, when a is small, to the lower limit $\mathrm{RT}/6\pi\eta\mathrm{C}$, when a is large. This is analogous to the actual behaviour of $\mathrm{B}^{\frac{1}{3}}\mathrm{D}$ obtained from experiment, B being the volume of the molecules in a gramme-molecule of solute. The first of the following tables contains the coefficients of diffusion for various gases through water determined by Hüfner *. I have reduced these all to a temperature of 16° C., and expressed them with the second as unit of time instead of the day. The values of B are taken mostly from "Further Studies on Molecular Force" (Phil. Mag. [6] xxxix.). In the second last row are given the values of

* Wied. *Ann.* 1897, vol. xl., and *Zeit. f. Phys. Chem.* xxvii.

Enter Sutherland

One paper discussed by Einstein and Besso was by William Sutherland (of Melbourne):

Ionization, ionic velocities, and atomic sizes. *Philosophical Magazine*, S.6, 3 (1902), 161-177.

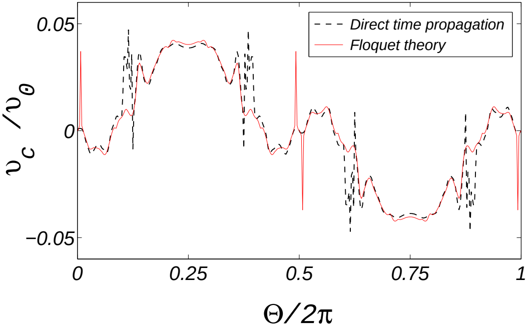
It contains the quote *"Now this simple theory must have been written down by many a physicist and found to be wanting"*

But it contains the important idea that one can use Stokes' Law to determine the size of large molecules

Two-atom Quantum Motor

Quantum current

$$v_c(t_0) := \lim_{t\to\infty} \frac{1}{t} \int_0^t v_c(t'; t_0)dt' \qquad v_c(t_0) = \sum_{n=1}^{\mathcal{N}} \overline{v}_n\, |c_n(t_0)|^2$$



A. Ponomarev, S. Denisov, & P. Hänggi, PRL 102, 230601 (2009)