Quantum Dissipation: A Primer

P. Hänggi

Institut für Physik
Universität Augsburg

QUANTUM DISSIPATION

$$L = \frac{1}{2} m_0 e^{\gamma t} \dot{x}^2 - \frac{1}{2} m_0 e^{\gamma t} \omega_0^2 x^2$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{x}} = \frac{d}{dt} m_0 e^{\gamma t} \dot{x} = m_0 e^{\gamma t} \ddot{x} + m_0 e^{\gamma t} \gamma \dot{x}$$

$$-\frac{\partial L}{\partial x} = m_0 e^{\gamma t} \omega_0^2 x$$

$$\Rightarrow \cancel{e^{\gamma t}} [m_0 \ddot{x} + m_0 \gamma \dot{x} + m_0 \omega_0^2 x] = 0$$

$$\text{QM: } L \to H = \frac{p^2}{2m_0} e^{-\gamma t} + \frac{1}{2} m_0 e^{\gamma t} \omega_0^2 x^2$$

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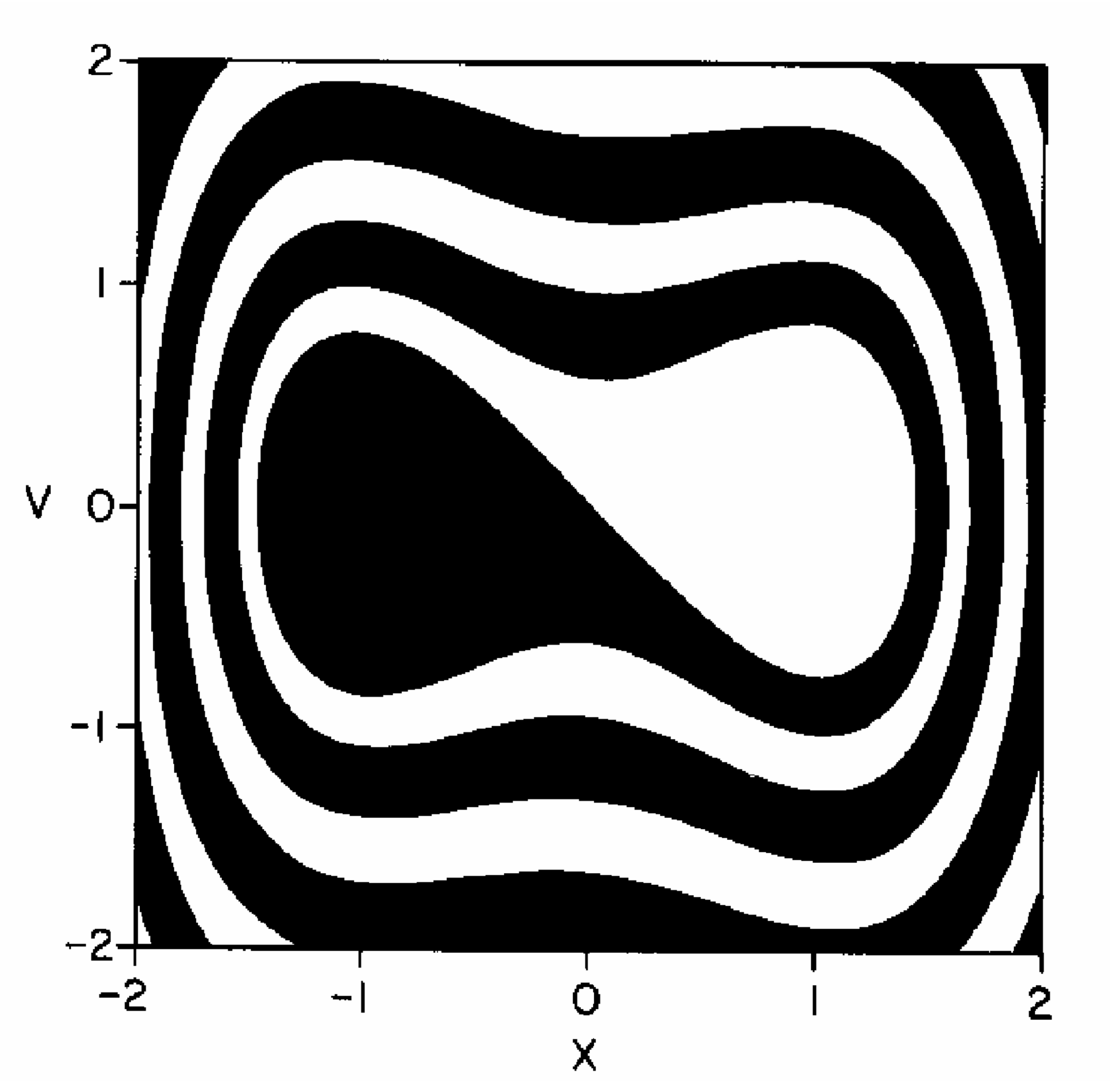
$$¡\quad [q, p] = +i\hbar e^{-\gamma t} \quad !$$

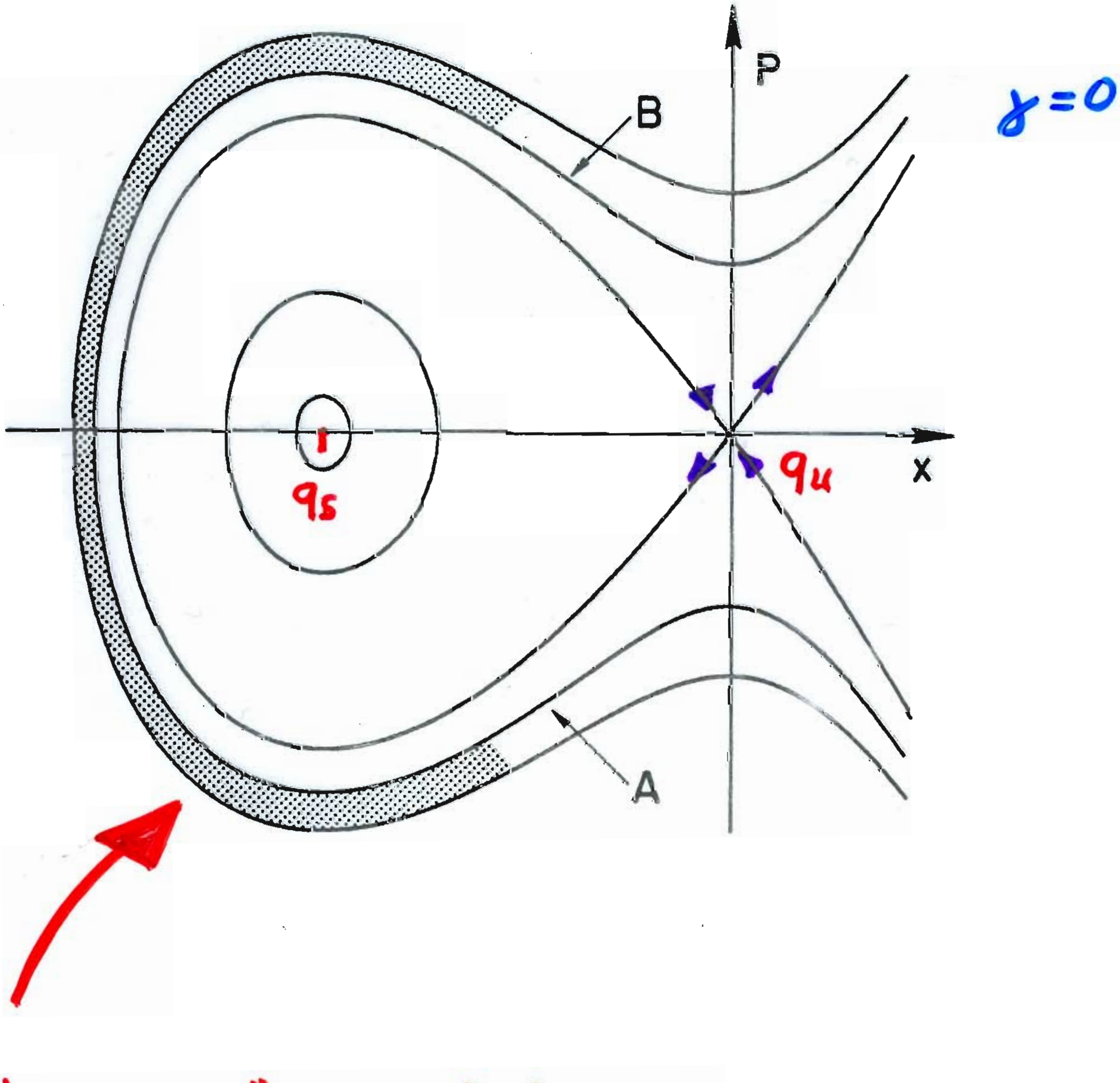
NOISE-INDUCED ESCAPE



$$\text{rate} = A(\gamma)\,\frac{\omega_0}{2\pi}\exp(-\Delta U/D)$$

RMP $\underline{62}$: 251 (90)

$$\Gamma = \kappa \cdot \underbrace{\frac{\omega_0}{2\pi} \exp(-E_b/kT)}_{\text{TST}}$$



thermal equilibrium

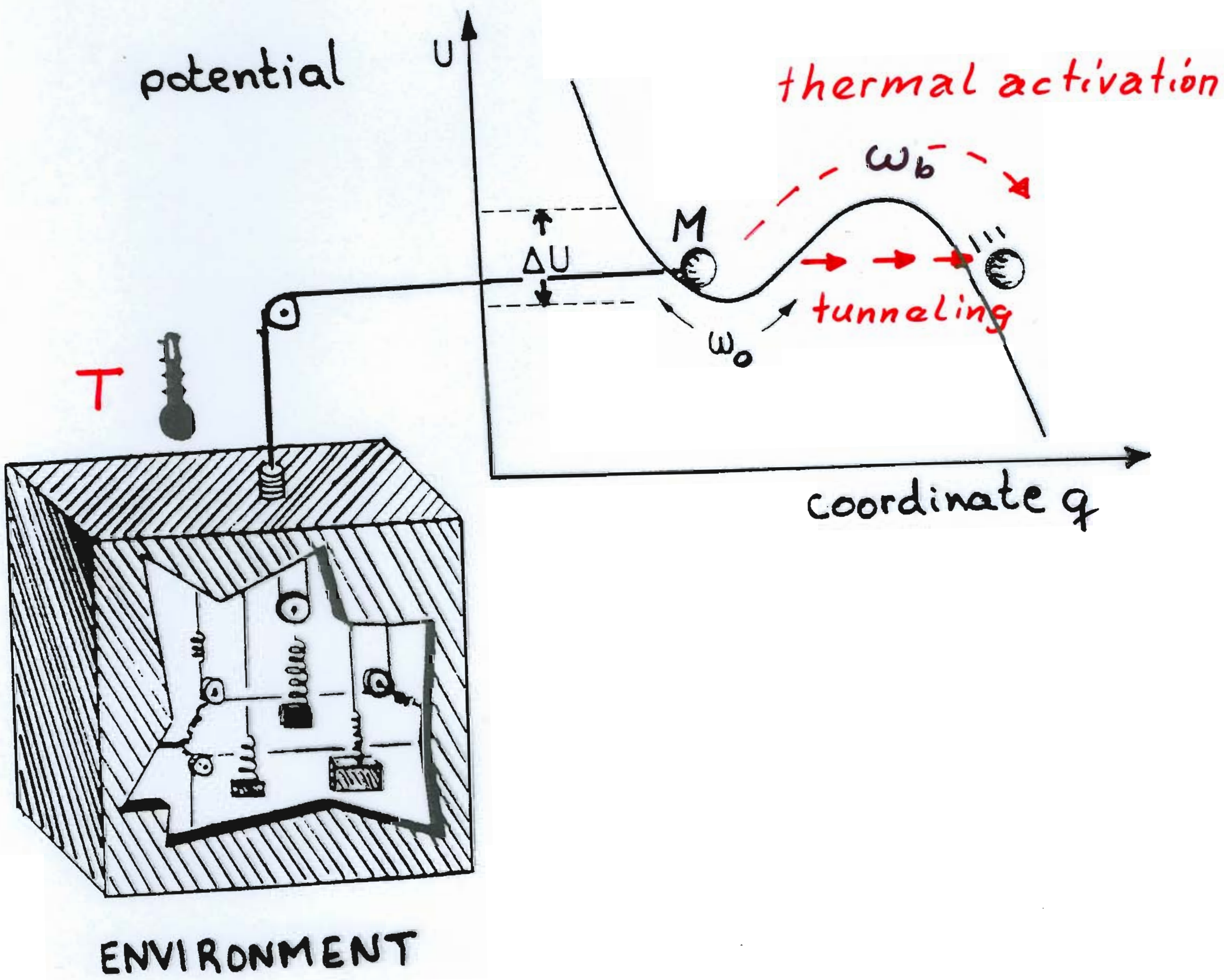
P. H., P. TALKNER, M. BORKOVEC
REV. MOD. PHYS. 62: 251 (1990)

Reaction-rate theory: fifty years after Kramers

Peter Hänggi, Peter Talkner, Michal Borkovec

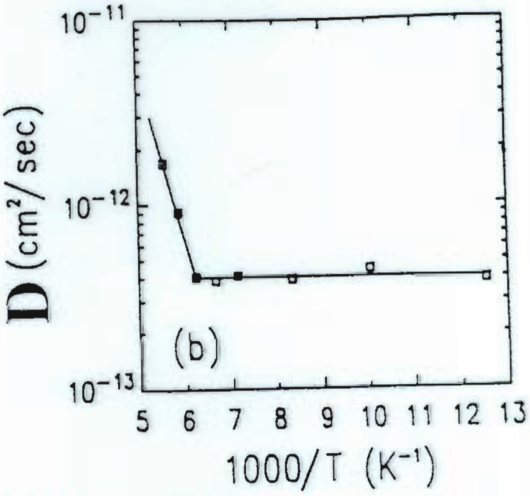
RMP 62: 251 (90)

THE PROBLEM

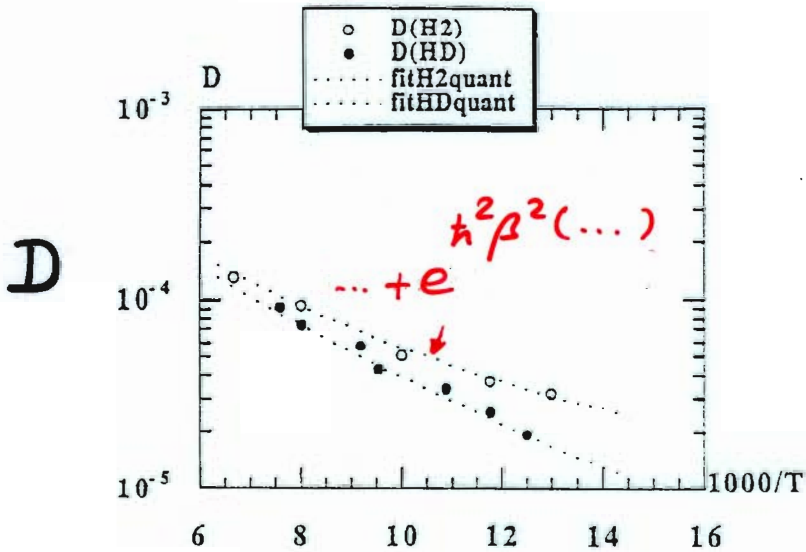


$$M\ddot{q} + \frac{dU}{dx} + \eta\dot{q} = 0$$

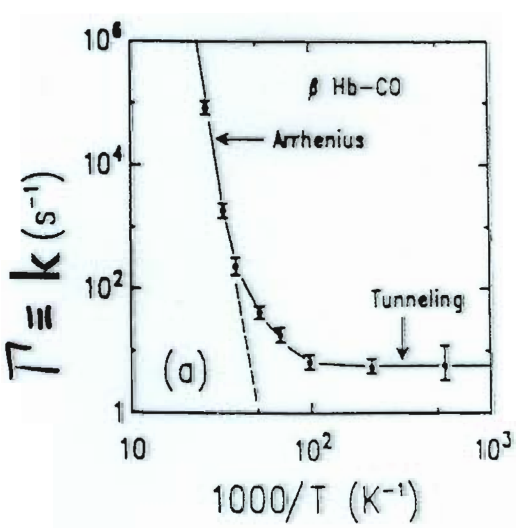
FACTS



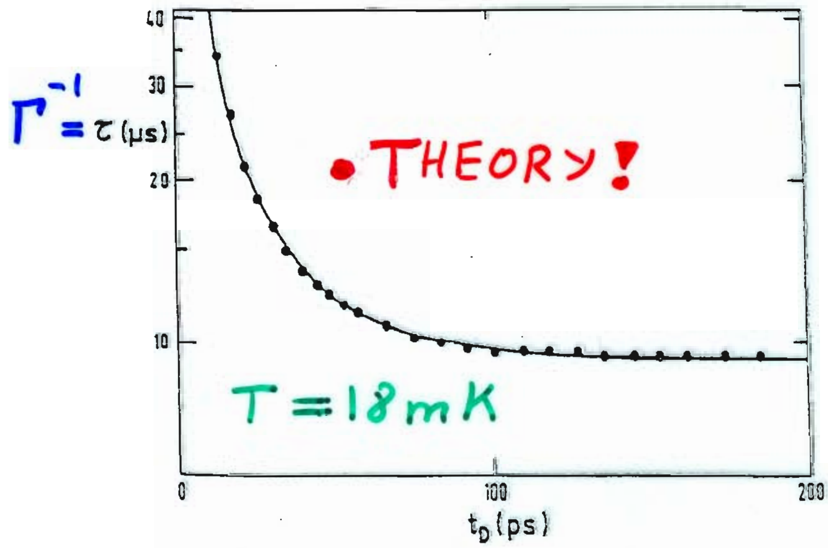
H on (110) tungsten [GOMER(82)]



H_2 & HD sorbed in zeolites [Bouchand et al.(92)]



CO-MIGRATION IN HEMOGLOBIN [Frauenfelder]



TUNNELING IN A JOSEPHSON JUNCTION SUBJECTED TO MEMORY FRICTION [Esteve et. al. (79)]

Results

T

Quantum Tunneling

$k = A \exp - B$

$B = S_B(T, \gamma)$

$B(T \neq 0) = B(T=0) - aT^2$!

$A(T,\gamma) \simeq A(\gamma)$

$\propto \gamma^{7/2}, \ \gamma \to \infty$

$\propto A_0\left(1 + \frac{2.80\gamma}{2M\omega_0}\right), \ \gamma \to 0$

Cross-over

2-0-modes

$\frac{S_B}{\hbar} = \frac{E_b}{kT_0}$

smooth!

Erfc-behavior

Quantum Corrections

$k = \Gamma_{cl}\, Q$

quantum enhancement

$Q \sim \exp \frac{\hbar^2(\omega_0^2 + \omega_b^2)}{(kT)^2}$

> 1

i.e.

$E_b \to E_b - \frac{c}{T}$

thermal activation

$k = A(\eta)\, e^{-E_b/kT}$

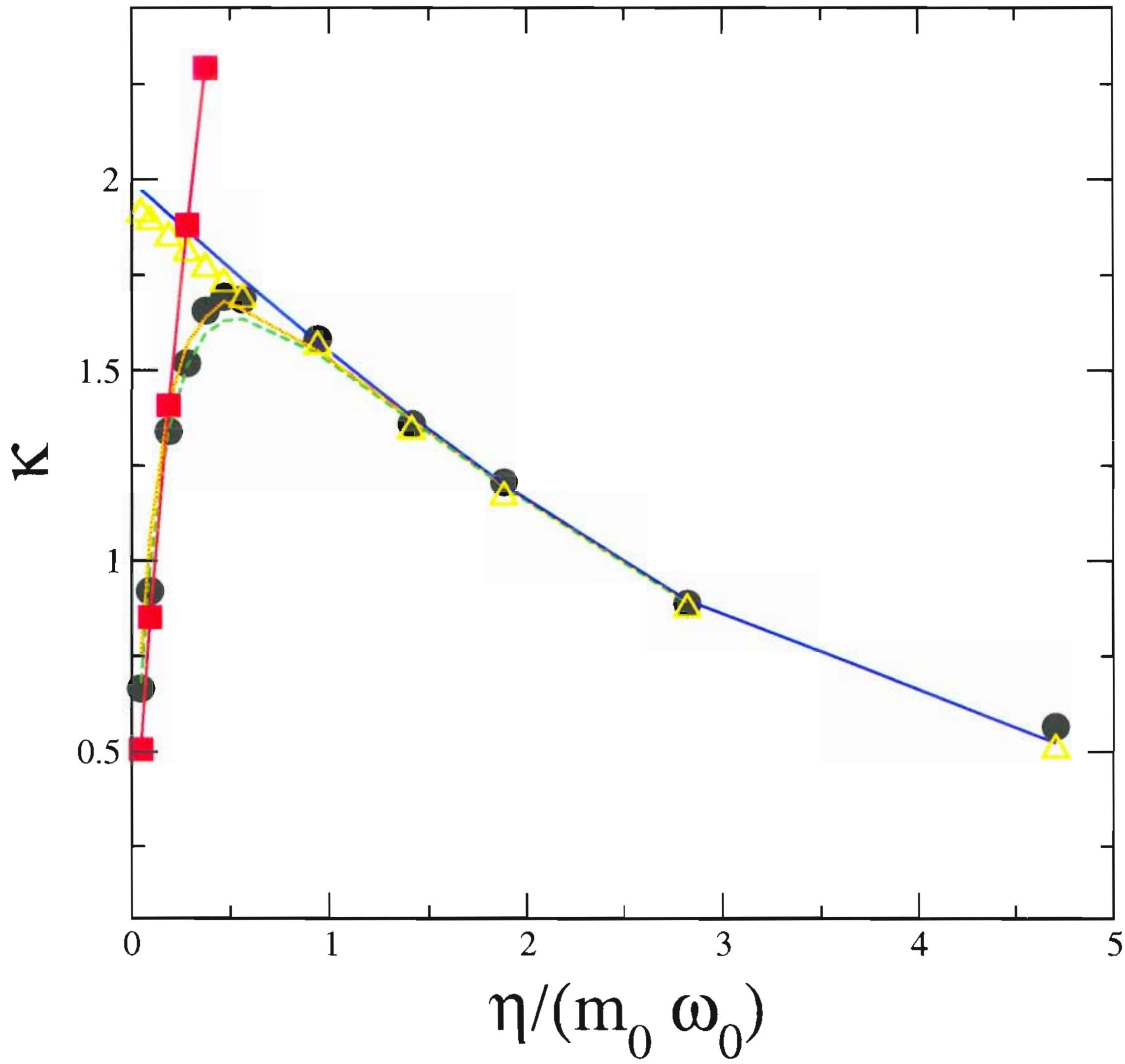
$\gamma = \eta/M$

Kramers

$\left\{\left(\frac{\gamma^2}{4} + \omega_b^2\right)^{1/2} - \frac{\gamma}{2}\right\} \cdot \frac{\omega_0}{2\pi}$

Quantum transmission coefficient

$$\kappa = k/k_{cl,TST}$$
$$k_{cl,TST} = \frac{1}{2\pi\hbar}\frac{k_b T}{Z} e^{-\beta E_b}$$



- red - Redfield
- black - path integral
- blue - quantum Grote-Hynes theory
- brown - quantum turnover theory by Hänggi, Pollak, Grabert
- green - quantum turnover theory by Rips and Pollak
- triangles - centroid method

Reaction-rate theory: fifty years after Kramers

Peter Hänggi

Lehrstuhl für Theoretische Physik, University of Augsburg, D-8900 Augsburg, Federal Republic of Germany

Peter Talkner*

Department of Physics, University of Basel, CH-4056 Basel, Switzerland

Michal Borkovec

Institut für Lebensmittelwissenschaft, ETH-Zentrum, CH-8092 Zürich, Switzerland

The calculation of rate coefficients is a discipline of nonlinear science of importance to much of physics, chemistry, engineering, and biology. Fifty years after Kramers' seminal paper on thermally activated barrier crossing, the authors report, extend, and interpret much of our current understanding relating to theories of noise-activated escape, for which many of the notable contributions are originating from the communities both of physics and of physical chemistry. Theoretical as well as numerical approaches are discussed for single- and many-dimensional metastable systems (including fields) in gases and condensed phases. The role of many-dimensional transition-state theory is contrasted with Kramers' reaction-rate theory for moderate-to-strong friction; the authors emphasize the physical situation and the close connection between unimolecular rate theory and Kramers' work for weakly damped systems. The rate theory accounting for memory friction is presented, together with a unifying theoretical approach which covers the whole regime of weak-to-moderate-to-strong friction on the same basis (turnover theory). The peculiarities of noise-activated escape in a variety of physically different metastable potential configurations is elucidated in terms of the mean-first-passage-time technique. Moreover, the role and the complexity of escape in driven systems exhibiting possibly multiple, metastable stationary nonequilibrium states is identified. At lower temperatures, quantum tunneling effects start to dominate the rate mechanism. The early quantum approaches as well as the latest quantum versions of Kramers' theory are discussed, thereby providing a description of dissipative escape events at all temperatures. In addition, an attempt is made to discuss prominent experimental work as it relates to Kramers' reaction-rate theory and to indicate the most important areas for future research in theory and experiment.

CONTENTS

*Present address: Paul Scherrer Institut, CH-5232 Villigen, Switzerland.

LIST OF SYMBOLS

$A(T)$	temperature-dependent quantum rate prefactor
$C(t)$	correlation function
D	diffusion coefficient
E	energy function
E_b	activation energy (=barrier energy with the energy at the metastable state set equal to zero)
$E^{(A)}$	Hessian matrix of the energy function at the stable state
$E^{(S)}$	Hessian matrix of the energy function around the saddle-point configuration
I	action variable of the reaction coordinate
J	Jacobian
$K(x,x')$	transition probability kernel
M	mass of reactive particle
$P(E)$	period of oscillation in the classically allowed region
$P(E,E')$	classical conditional probability of finding the energy E, given initially the energy E'
Q	quantum correction to the classical prefactor
S_b	dissipative bounce action
T	temperature
T_0	crossover temperature
$T(E)$	period in the classically forbidden regime
$U(x)$	metastable potential function for the reaction coordinate
V	volume of a reacting system
Z	partition function, inverse normalization
Z_0, Z_A	partition function of the locally stable state (A)
$Z^{\neq}$	partition function of the transition rate
$\mathcal{H}$	Hamiltonian function of the metastable system
$\mathcal{F}$	complex-valued free energy of a metastable state
$\mathcal{L}$	Fokker-Planck operator
$\mathcal{L}^{\dagger}$	backward operator of a Fokker-Planck process
j	total probability flux of the reaction coordinate
h	Planck's constant
$\hbar$	$h(2\pi)^{-1}$
k_B	Boltzmann constant
k	reaction rate
k^+	forward rate
k^-	backward rate
k_{TST}	transition-state rate
$k(E)$	microcanonical transition-state rate, semiclassical cumulative reaction probability
k_S	spatial-diffusion-limited Smoluchowski rate
m_i	mass of ith degree of freedom
$p(x,t)$	probability density
$p_0(x)$	stationary nonequilibrium probability density for the reaction coordinate
p_i	momentum degree of freedom
q_i	configurational degree of freedom
$r(E)$	quantum reflection coefficient
$s(x)$	density of sources and sinks
$t(E)$	quantum transmission coefficient
$t_\Omega(x)$	mean first-passage time to leave the domain Ω, with the starting point at x
t_{MFPT}	constant part of the mean first-passage time to leave a metastable domain of attraction
$v=\dot{x}$	velocity of the reaction coordinate
x	reaction coordinate
x_0, x_a	location of well minimum or potential minimum of state A, respectively
x_b	barrier location
x_T	location of the transition state
β	inversion temperature $(k_B T)^{-1}$

microscopic approach

$$H^{total} = \underbrace{\tfrac{1}{2} M \dot{q}^2 + U(q)}_{\text{system}}$$

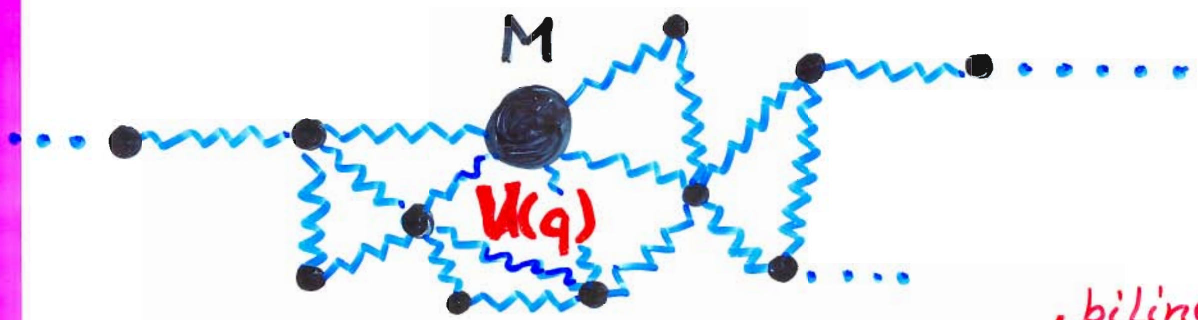
$$+ \underbrace{\tfrac{1}{2} \sum_\alpha m_\alpha \dot{q}_\alpha^2 + \sum_\alpha m_\alpha \omega_\alpha^2 q_\alpha^2}_{\text{(harmonic) bath}}$$

$$+ \underbrace{q \sum_\alpha c_\alpha q_\alpha}_{\text{linear coupling}}$$

$$+ \underbrace{q^2 \sum_\alpha \frac{c_\alpha^2}{2 m_\alpha \omega_\alpha^2}}_{\text{compensation of frequency shift}}$$

- path integral approach to density matrix at temperature T
- trace out environment

dissipation



$$H^{T} = H_{system} + H_{bath} + H_{Int}$$

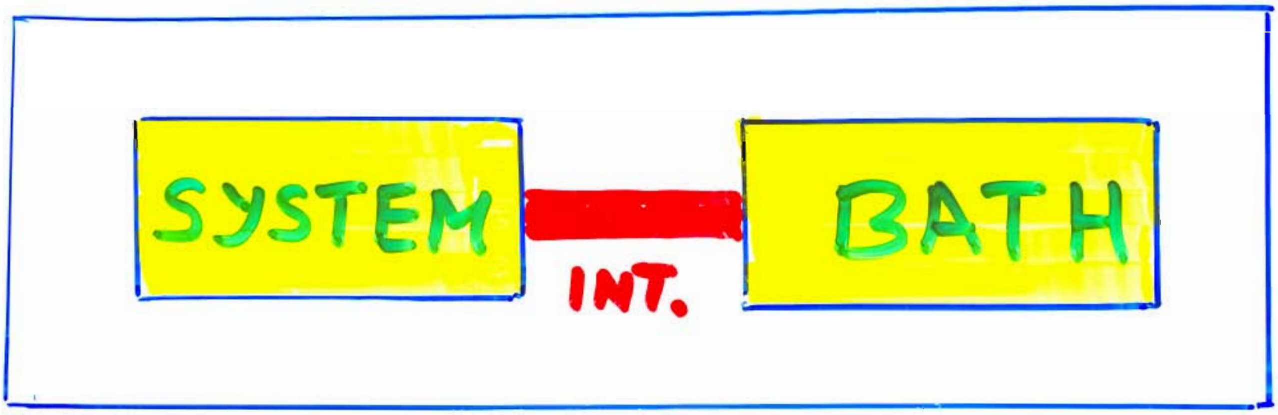
bilinear

$$\ddot{q} = -\frac{1}{M}\frac{\partial U}{\partial q} - \int_0^t \gamma(t-s)\,\dot{q}(s)\,ds$$

$$S_E = S_{rev.\ motion} + S_{dissipation}\ \text{(nonlocal)}$$

QUANTUM NOISE

QUANTUM L.-EQ.



$$|0\rangle_{S+B} \neq |0\rangle_S \, |0\rangle_B$$

↳ DECOHERENCE AT $T = 0$

$$\mathbb{H}_{S+B} = \mathbb{H}_S + \mathbb{H}_{S-B} + \mathbb{H}_B$$

$$= \frac{p^2}{2m} + V(x) + \sum_\alpha \left[\frac{p_\alpha^2}{2m_\alpha} + \frac{m_\alpha \omega_\alpha^2}{2} \left(q_\alpha - \frac{c_\alpha}{m_\alpha \omega_\alpha^2} x \right)^2 \right]$$

$$¡ \; \rho_S \neq Z^{-1} \exp\left(- \frac{\mathbb{H}_S}{kT} \right) \; !$$

$$\rho_{Total} = \rho_{S+B} = Z^{-1} \exp\left(- \frac{\mathbb{H}_{S+B}}{kT} \right)$$

QLE

$$i\hbar \dot{O} = [O, H_T]$$

$$m\ddot{x} + m\int_0^t ds\,\gamma(t-s)\,\dot{x}(s) + \frac{\partial V(x)}{\partial x}$$

$$= \eta(t) - \underbrace{m\gamma(t-0)\,x(0)}_{\text{INITIAL SLIP}}$$

$$!\ \gamma(t-s) = \frac{1}{m}\sum_\alpha \frac{c_\alpha^2}{m_\alpha \omega_\alpha^2}\cos(\omega_\alpha(t-s))$$

$$= \gamma(s-t)$$

$$\eta(t) = \sum_\alpha c_\alpha \Big[q_\alpha(0)\cos(\omega_\alpha t)$$

$$+ \frac{p_\alpha}{m_\alpha \omega_\alpha}\sin(\omega_\alpha t)\Big]$$

- $[\eta(t), \eta(s)] = -i\hbar \sum_\alpha \frac{c_\alpha^2}{m_\alpha \omega_\alpha} \sin(\omega_\alpha (t-s))$

  $\neq 0$

  $$\rho_B = Z^{-1} \exp\left\{-\beta\left[\sum_\alpha \left(\frac{p_\alpha^2}{2m_\alpha} + \frac{m\omega_\alpha^2}{2} q_\alpha^2\right)\right]\right\}$$

- $$\langle \eta(t) \rangle_{\rho_B} = 0$$

- $$\frac{1}{2}\langle \eta(t)\eta(s) + \eta(s)\eta(t) \rangle = C(t-s)$$

  $$= C(\tau) = \frac{\hbar}{2} \sum_\alpha \frac{c_\alpha^2}{m_\alpha \omega_\alpha} \coth\left(\frac{\hbar\omega_\alpha}{2kT}\right) \cos(\omega_\alpha \tau)$$

  $$\xrightarrow{kT \gg \hbar\omega_\alpha} kT\gamma(\tau)$$

$$\hat{\gamma}(z) = \int_0^\infty \exp(-zt)\,\gamma(t)\,dt$$

$$\tilde{\gamma}(\omega) = \hat{\gamma}(z = -i\omega)$$

OHMIC DISSIPATION

$$J(\omega) = \gamma\,\omega\,\exp(-\omega/\omega_c)$$

cut-off frequency
$\omega_c \gg \omega_0,\ \omega_b$

KONDO-PARAMETER

$$= \underbrace{(2\pi\hbar/a^2)\,\boxed{\alpha}}_{=\gamma}\,\omega\,\exp(-\omega/\omega_c)$$

$a = 2q_a$: tunneling length

REMARKS

$\boxed{1.}$

QLE OPERATES IN FULL HILBERT SPACE OF $S \oplus B$

$\boxed{2}$

$$\hat{\gamma}(z) = \int_0^\infty e^{izt} \gamma(t) dt = \frac{i}{2m} \sum_\alpha \frac{c_\alpha^2}{m_\alpha \omega_\alpha^2} \left[\frac{1}{z-\omega_\alpha} + \frac{1}{z+\omega_\alpha} \right]$$

$$\frac{1}{x+i0^+} = P\left(\frac{1}{x}\right) - i\pi\delta(x) \qquad \mathrm{Im}\, z > 0$$

$$\mathrm{Re}\, \hat{\gamma}(z = \omega + i0^+) = \frac{\pi}{2m} \sum_\alpha \frac{c_\alpha^2}{m_\alpha \omega_\alpha^2} \left[\delta(\omega - \omega_\alpha) + \delta(\omega + \omega_\alpha) \right]$$

$$\boxed{C(\tau) = \frac{m}{\pi} \int_0^\infty d\omega \, \mathrm{Re}\, \hat{\gamma}(\omega + i0^+) \cos(\omega\tau) \times \coth\left(\frac{\hbar\omega}{2kT}\right)}$$

$\boxed{3}$ with $\xi(t) = \eta(t) - m\gamma(t) x(0)$

$$\hat{\rho}_B = Z^{-1} \exp -\beta \left[\sum_\alpha \left(\frac{p_\alpha^2}{2m_\alpha} + \frac{m_\alpha \omega_\alpha^2}{2} \left(q_\alpha - \frac{c_\alpha}{m_\alpha \omega_\alpha^2} x \right)^2 \right) \right]$$

$$\Rightarrow \langle \xi(t) \rangle_{\hat{\rho}} = 0$$

$$\frac{1}{2} \langle \xi(\tau)\xi(0) + \xi(0)\xi(\tau) \rangle_{\hat{\rho}} = C(\tau)$$

$\boxed{4.}$ ¡DEPHASING AT $T=0$!

$$\langle x(0)\,\xi(t)\rangle_{\hat\rho} \neq 0$$

$$\langle \mathbb{H}_{INT}\rangle_{\hat\rho} \neq 0$$

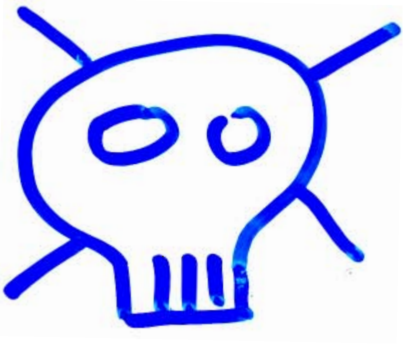
$\boxed{5.}$

$\xi(t) \longrightarrow$ C-NOISE $\xi(t)$

WITH CORRELATION

$C(\tau)$

IS INCONSISTENT



SYNOPSIS

LINEAR RESPONSE THEORY & QUANTUM-FDT

$$\hat{H}(t) = \hat{H}_0 - \mathcal{F}(t)\hat{A} \; ; \; \varrho_\beta = Z^{-1}\exp(-\beta\hat{H}_0)$$

$$\langle\hat{B}(t)\rangle - \langle\hat{B}(t)\rangle_\beta = \langle\delta\hat{B}(t)\rangle = \int_{t_0}^{t} \chi_{BA}(t-s)\,\mathcal{F}(s)\,ds$$

KUBO: $$\chi_{BA}(\tau) = \Theta(\tau)\frac{i}{\hbar}\langle[\hat{B}(\tau), \hat{A}(0)]\rangle_\beta$$

$$= -\Theta(\tau)\int_0^\beta \langle\hat{A}(-i\hbar\lambda)\dot{\hat{B}}(\tau)\rangle_\beta\, d\lambda$$

classical limit $\longrightarrow -\Theta(\tau)\,\beta\,\langle\dot{B}(\tau)A(0)\rangle_\beta$

SYNOPSIS

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classical limit $\longrightarrow -\Theta(\tau)\beta \langle \dot{B}(\tau) A(0) \rangle_\beta$

$$\hat{B} = \hat{A} = \hat{q} \quad ; \quad \mathcal{F}(t) = A\cos\Omega t$$

$$\langle \delta\hat{q}(t) \rangle = P_1 e^{-i\Omega t} + P_{-1} e^{-i\Omega t}$$

$$P_{1,-1} = \frac{A}{2} e^{\mp i\Omega t} \chi(\pm\Omega)$$

QUANTUM-FDT

$$S_{BA}(\tau) = \frac{1}{2}\Big\langle \big(\hat{B}(t) - \langle \hat{B}\rangle_\beta\big)\big(\hat{A}(0) - \langle \hat{A}\rangle_\beta\big) + \big(\hat{A}(0) - \langle \hat{A}\rangle_\beta\big)\big(\hat{B}(\tau) - \langle \hat{B}\rangle_\beta\big)\Big\rangle_\beta$$

$$\chi_{BA}(\tau) = \underbrace{\chi'_{BA}(\tau)}_{\frac{1}{2}[\chi_{BA}(t) + \chi_{AB}(-t)]} + i\,\underbrace{\chi''_{BA}(\tau)}_{-\frac{i}{2}[\chi_{BA}(t) - \chi_{AB}(-t)]}$$

$$\chi_{BA}(\omega) = \int_{\cancel{-\infty}\;0}^{\infty} \chi_{BA}(t)\, e^{i\omega t}\, dt$$

$$\Rightarrow \quad \chi''_{BA}(\omega) = \frac{1}{\hbar}\tanh(\hbar\omega\beta/2)\, S_{BA}(\omega)$$

$$S_{BA}(\omega) = \hbar\coth(\hbar\omega\beta/2)\,\chi''_{BA}(\omega)$$

$$\xrightarrow{\hbar\omega\beta \ll 1} 2\,\chi''_{BA}(\omega)/(\beta\Omega)$$

__NOTE:__ $\chi''_{BA}(\omega) = \frac{i}{2}[\chi^*_{AB}(\omega) - \chi_{BA}(\omega)]$

$\neq \mathrm{Im}\,\chi_{BA}(\omega)$; except $\hat{A} = \hat{B}$

$\hat{A} = \hat{B} = \hat{q}$: $\underline{S_{qq}(\Omega) = \hbar\coth(\hbar\Omega\beta/2)\,\mathrm{Im}\,\chi_{qq}(\Omega)}$

EQ.-CURRENT NOISE

$\mathbb{A} = \mathbb{B}$ $\qquad \mathbb{I} := \frac{d\mathbb{B}}{dt}$; $\langle \delta I(t) \rangle = \int_{-\infty}^{t} Z(t-s) F(s) ds$

$$Z(t-s=\tau) = \frac{d\chi(\tau)}{d\tau}$$

$$\chi''_{AA}(\omega) = \frac{1}{\omega} \operatorname{Im}\left(\frac{Z(\omega)}{i}\right) = -\frac{1}{\omega} \operatorname{Re} Z(\omega)$$

$$S_{II}(\omega) = -\omega^2 S_{BB}(\omega)$$

$$S_{II}(\omega) = (\hbar\omega) \coth\left(\frac{\hbar\omega}{2kT}\right) \operatorname{Re} Z(\omega)$$

$kT \gg \hbar\omega$: $S_{II}(\omega) \to 2kT \operatorname{Re} Z(\omega)$

$$\xrightarrow{\text{MARKOV}} 2kT/R$$

JOHNSON-NYQUIST (1928)

$kT \ll \hbar\omega \longrightarrow \hbar\omega \operatorname{Re} Z(\omega)$

quantum-zero point fluct.

$S_{II}(\omega=0) = 0$ at $\omega = 0$

1900-1951

J.B. Johnson

Thermal agitation of electricity in conductors.

Phys. Rev. (1928) 32 (July) 97-109

H. Nyquist

Thermal agitation of electric charge in conductors.

Phys. Rev. (1928) 32 (July) 110-113

L. Onsager

Reciprocal relation in irreversible process.

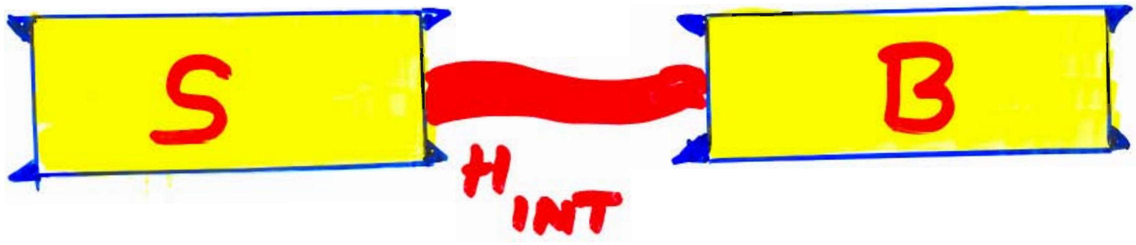
Phys. Rev. (1931) 32 (February) 405-426

H.B. Callen, T.A. Welton

Irreversibility and Generalized Noise.

Phys. Rev. (1951) 83 (1) 34-40

QUANTUM NOISE



NO QUANTUM EQ.-PARTITION-TH.

FEYNMAN-PATH-INT.

⇕

QUANTUM LANG.-EQ.

⇕

GME

⇕

STOCH.-L.-v.N.-EQ.

$$\rho := |\psi_1(t)\rangle\langle\psi_2(t)| \; ; \; \mu := \frac{2}{\pi}\int_0^\infty d\omega \, \frac{J(\omega)}{\omega}$$

$$i\hbar\dot{\rho} = [H_0, \rho] + \frac{\mu}{2}[x^2, \rho] - \xi(t)[x, \rho] - \frac{\hbar}{2}\nu(t)\{x, \rho\}_+$$

COMPLEX VALUED NOISE

$$\langle \xi(t)\xi(t')\rangle = \mathrm{Re}\, L(t-t') \; ; \; \langle \xi(t)\nu(t')\rangle = \frac{2i}{\hbar}\,\theta(t-t')\,\mathrm{Im}\, L(t-t')$$

$$\langle \nu(t)\nu(t')\rangle = 0$$

 PITFALLS

MARKOV MASTER EQ

$$\frac{d}{dt}\rho = -\frac{i}{\hbar} L \rho - \Gamma \rho + I(t)$$

BLOCH-REDFIELD

i.g. NO DET. BALANCE

ROTATING WAVE APPROX.

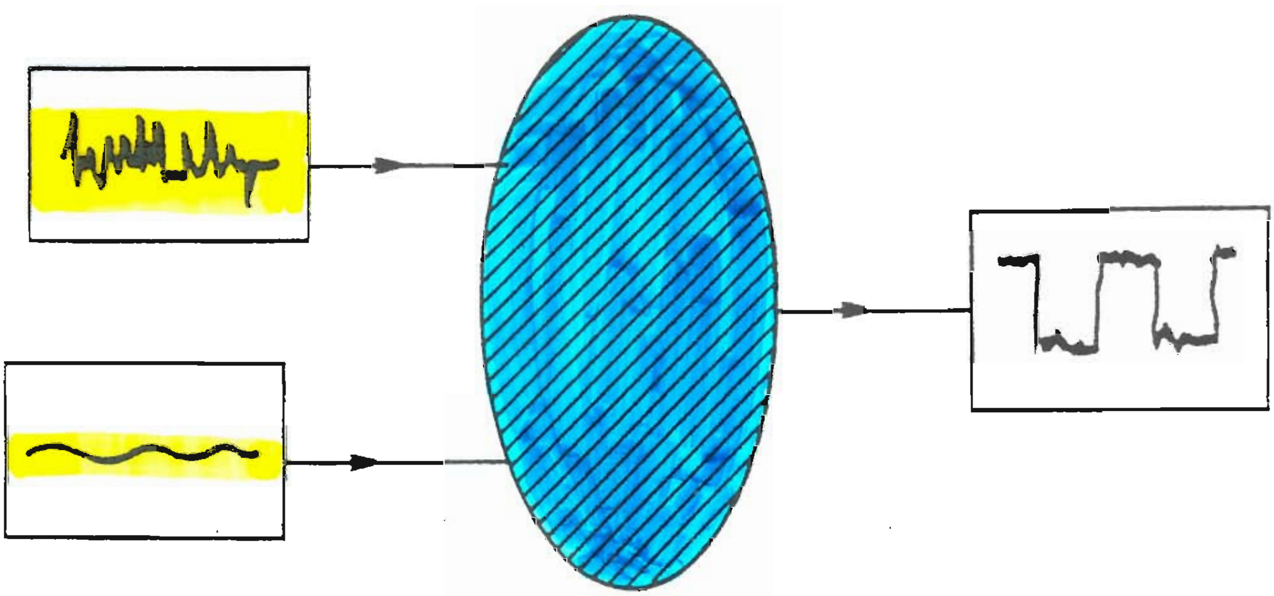
(LINDBLAD: DAVIES-APPROX.)

DET. BALANCE ✓ O.K.

BUT

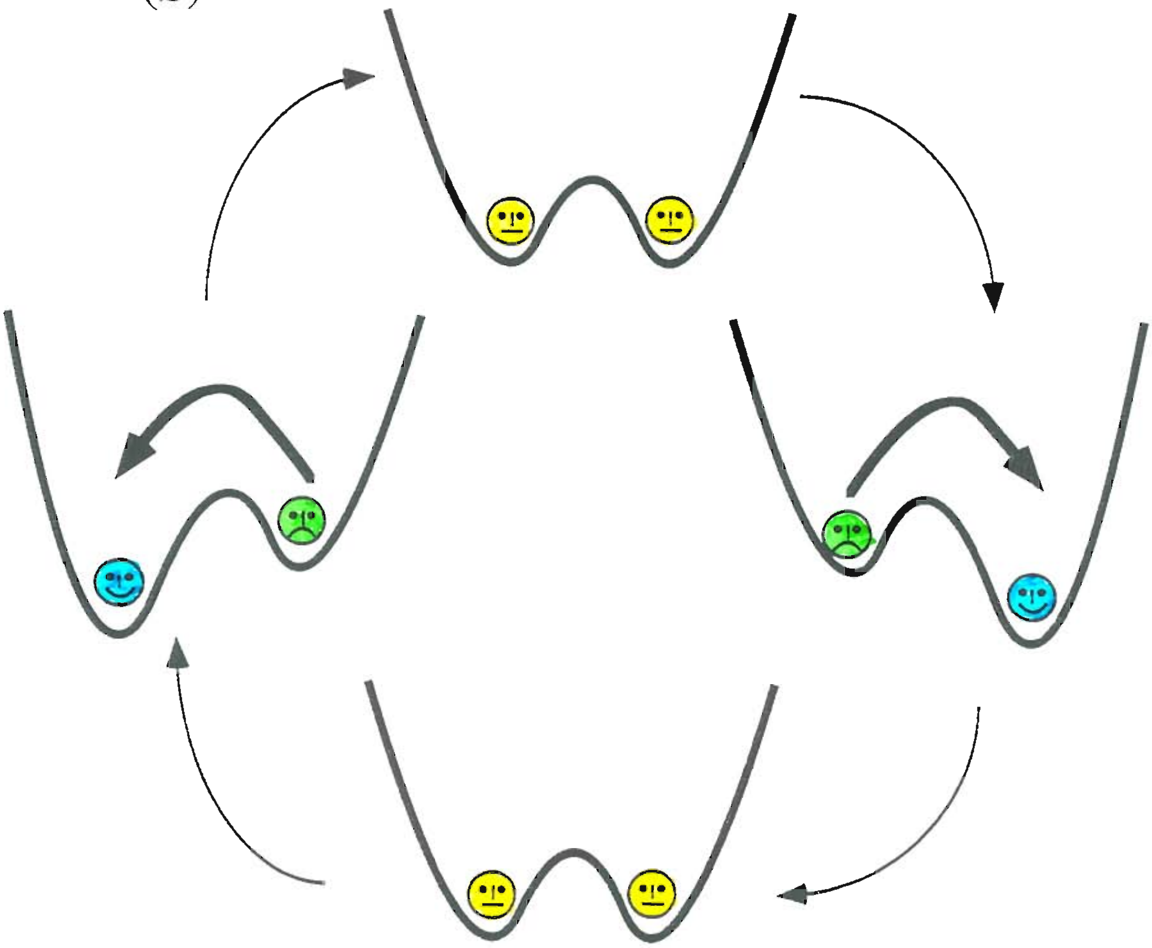
- WRONG EHRENFEST EQ.
- NO FDT
- NO KMS-COND. $\langle u(t)\, v \rangle_\beta = \langle v\, u(t + i\hbar\beta) \rangle_\beta$

SR



Schematic of stochastic resonance . The cross-hatched oval represents a black-box system which receives two inputs: one weak and periodic, the other strong and random. The output is relatively regular with small fluctuations.

(b)



NOISE-ASSISTED SYNCHRONIZED HOPPING

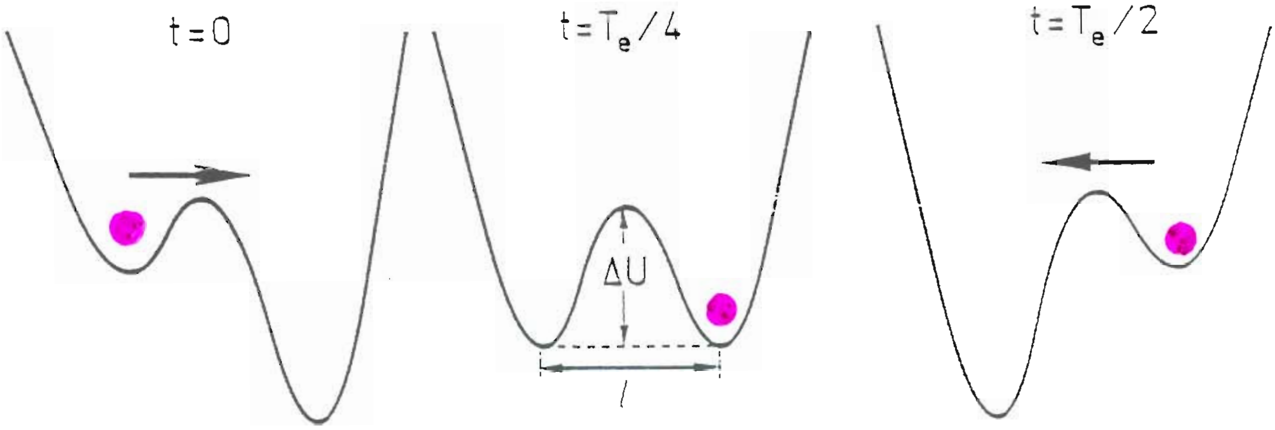
Fig 1b

Bistable Model

$$\dot{x} = x - x^3 + A\cos(\Omega t + \varphi) + \xi(t)$$

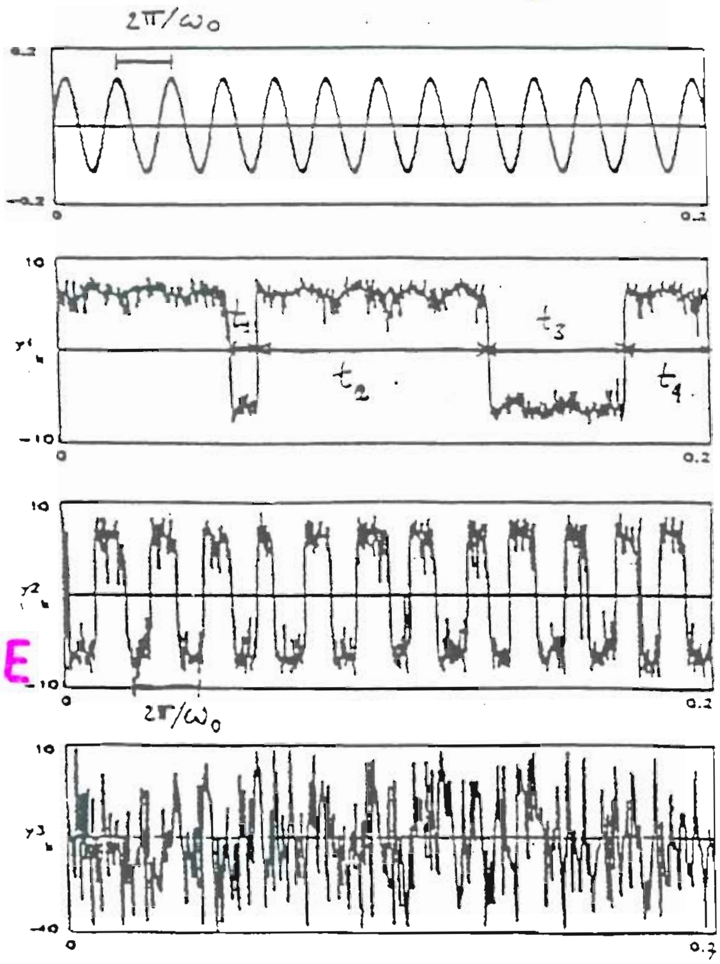
$$\langle \xi(t) \rangle = 0$$

$$\langle \xi(t)\xi(t') \rangle = 2D\,\delta(t - t')$$



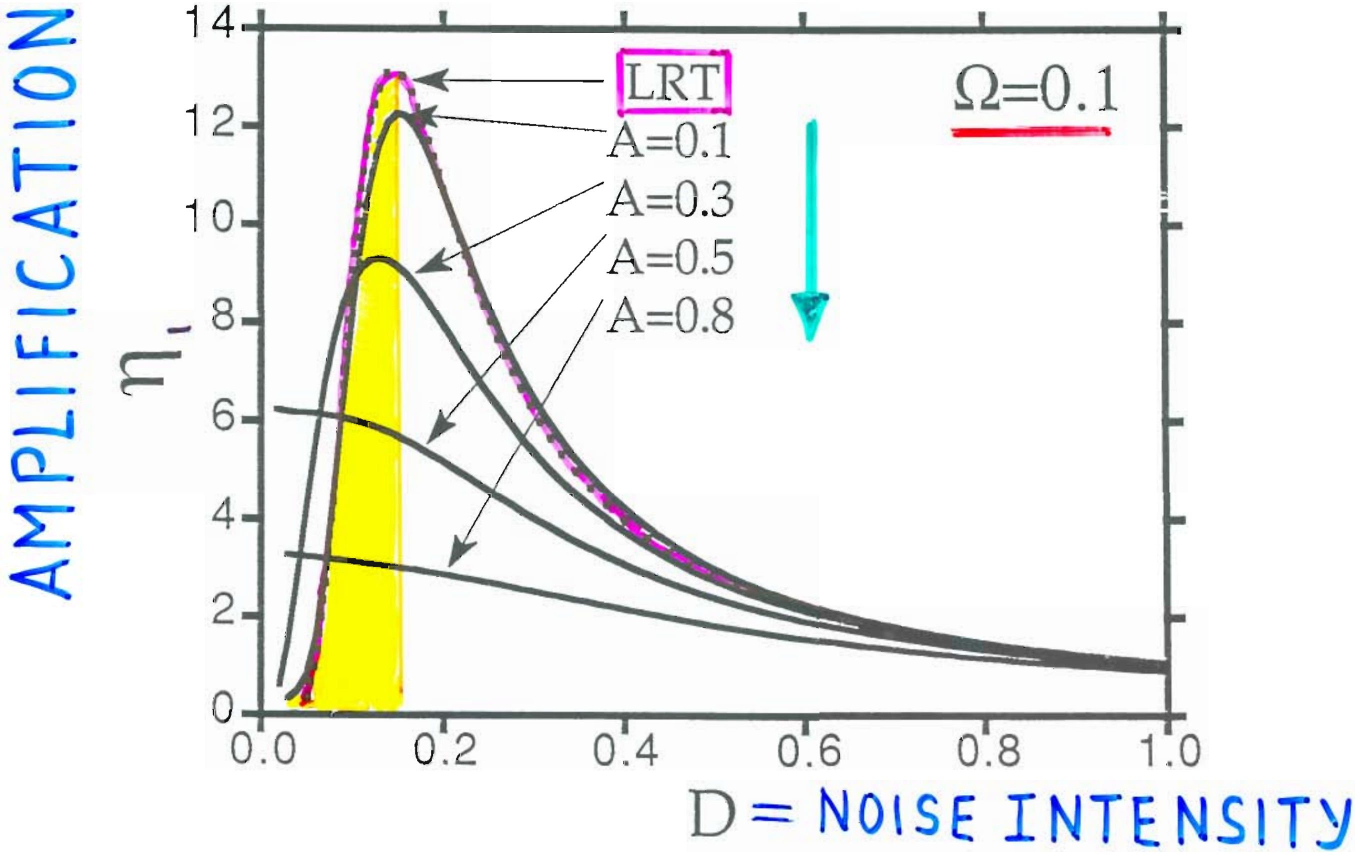
$$T_e = 2\pi/\Omega$$

SIGNAL



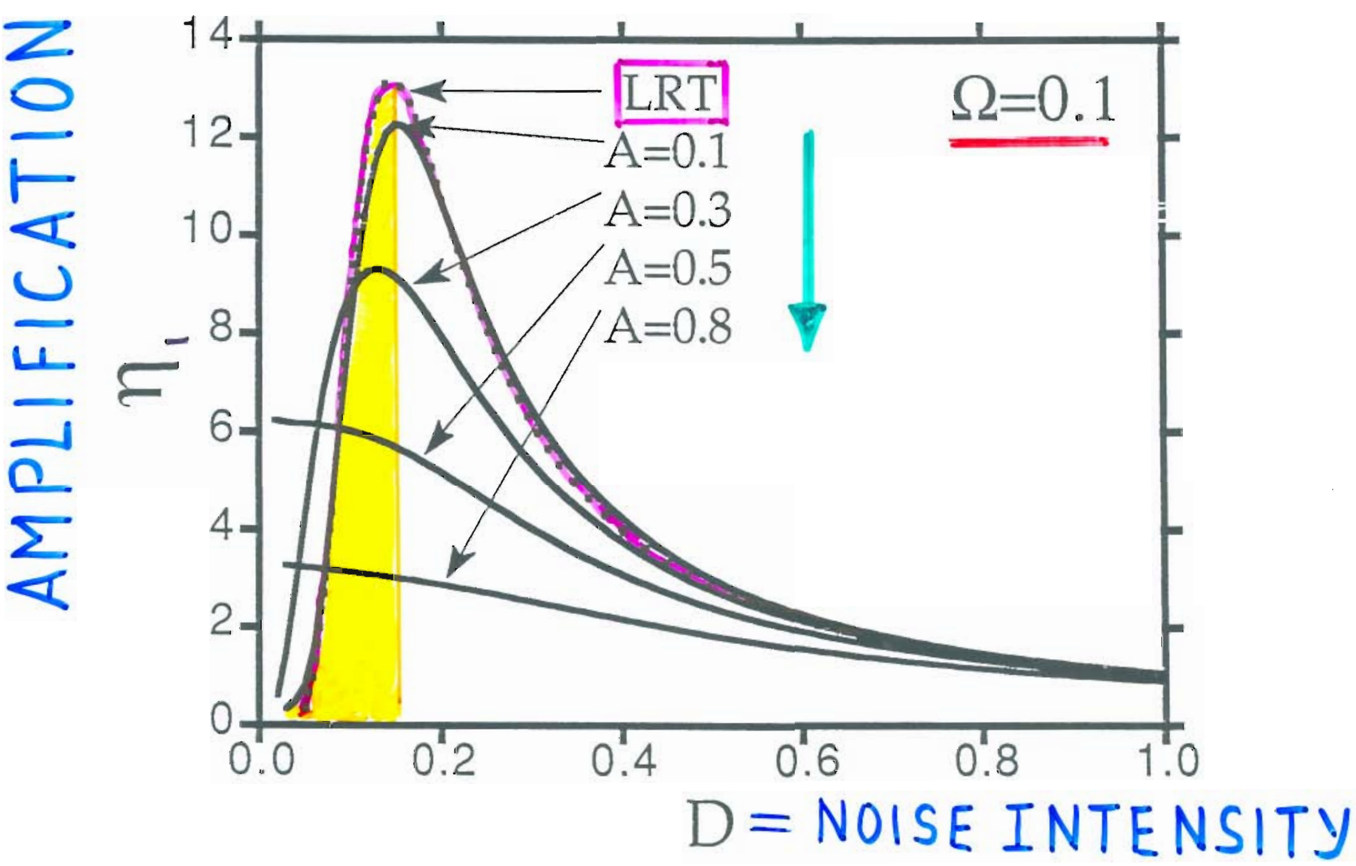
$$T_e \sim 2\Gamma^{-1}_{ESCAPE}$$

P. JUNG + P. H., PHYS. REV. A44: 8032 (91)



MORE NOISE → MORE SIGNAL

P. JUNG + P. H., PHYS. REV. A44: 8032 (91)



MORE NOISE → MORE SIGNAL

$$M_1 \sim \chi(\tau) = -\frac{1}{D}\frac{d}{d\tau}\langle \delta x(\tau)\,\delta\xi(0)\rangle$$

$$|M_1|^2 \propto 1/D^2 \qquad \exp(-2\Delta U/D)$$

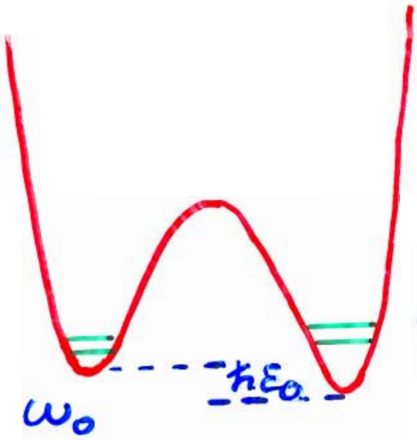
S R

IN QUANTUM MECHANICS

Q S R

$$V_0 \gg \hbar\omega_0 \gg \hbar\varepsilon_0,\, kT$$

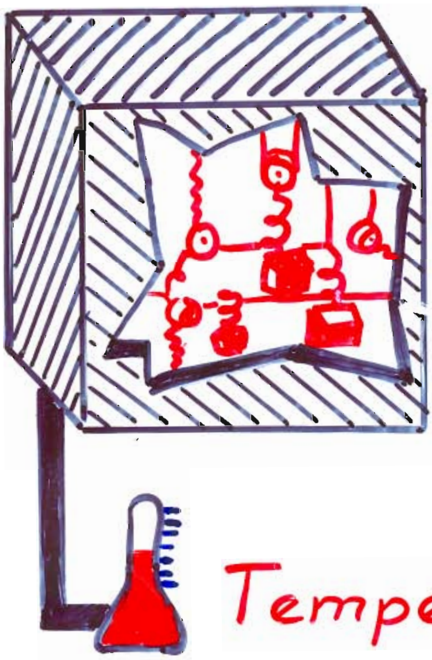
TS



$$-\frac{\hbar}{2}\left(\varepsilon_0\,\sigma_z + \Delta\,\sigma_x\right)$$

$\oplus$

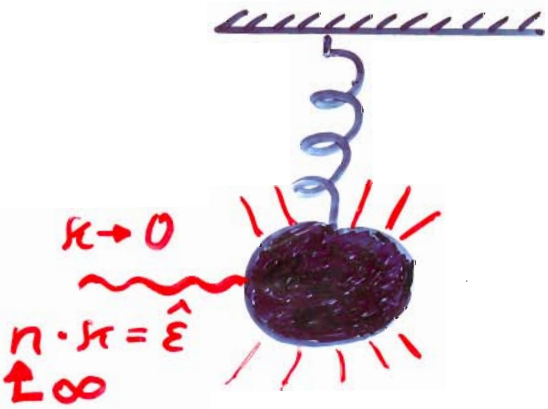
BATH
T, γ



$$\frac{1}{2}\sum_\alpha\left(\frac{p_\alpha^2}{m_\alpha} + m_\alpha\omega_\alpha^2 x_\alpha^2 - c_\alpha x_\alpha \sigma_z\right)$$

$\oplus$

DRIVE
$\Omega, \hat{\varepsilon}$



$$\frac{\hbar\hat{\varepsilon}}{2}\cos(\Omega t)\,\sigma_z$$

LINEAR RESPONSE & QSR

with $P_1 = \frac{A}{2}\chi_{qq}(\Omega) \equiv \frac{A}{2}\chi(\Omega)$

$$\boxed{\eta_1 = 4\pi |P_1|^2 = \pi A^2 |\chi(\Omega)|^2}$$

$$\boxed{SNR = \frac{\pi A^2 |\chi(\Omega)|^2}{S_{qq}(\Omega, A=0)} = \frac{\pi A^2 |\chi(\Omega)|^2}{Im\,\chi(\Omega)\,\hbar\coth(\hbar\Omega\beta/2)}}$$

¡valid at all temperatures!

PROBLEM: QUANTUM $\chi_{qq}(\Omega) \overset{FDT}{\rightsquigarrow} S_{qq}(\Omega)$

$$S_{qq}(t) = \frac{1}{2}\langle \delta\hat{q}(t)\delta\hat{q}(0) + \delta\hat{q}(0)\delta\hat{q}(t)\rangle_\beta$$

¡DIFFICULT!

LINEAR RESPONSE & QSR

with $P_1 = \frac{A}{2}\chi_{qq}(\Omega) \equiv \frac{A}{2}\chi(\Omega)$

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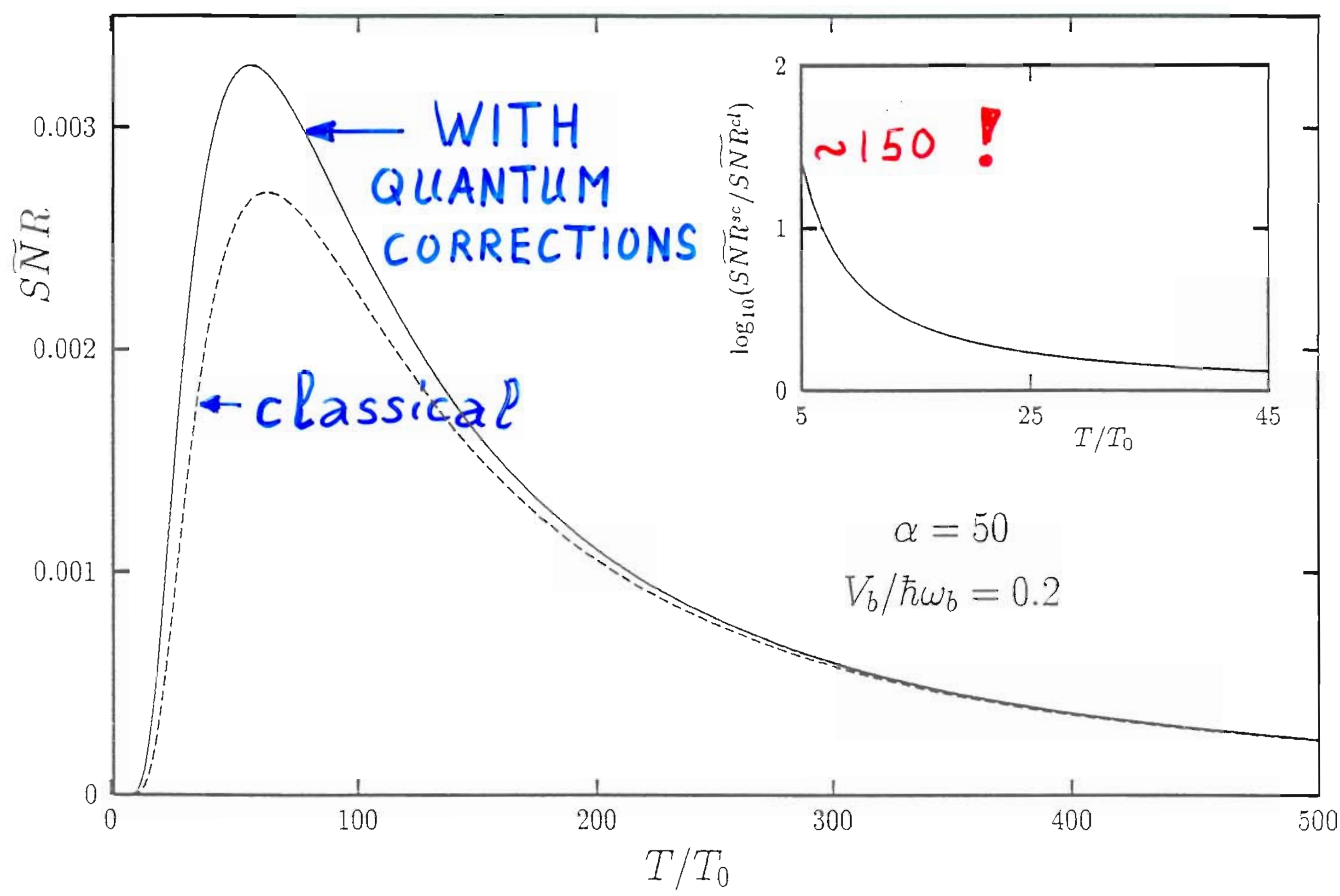
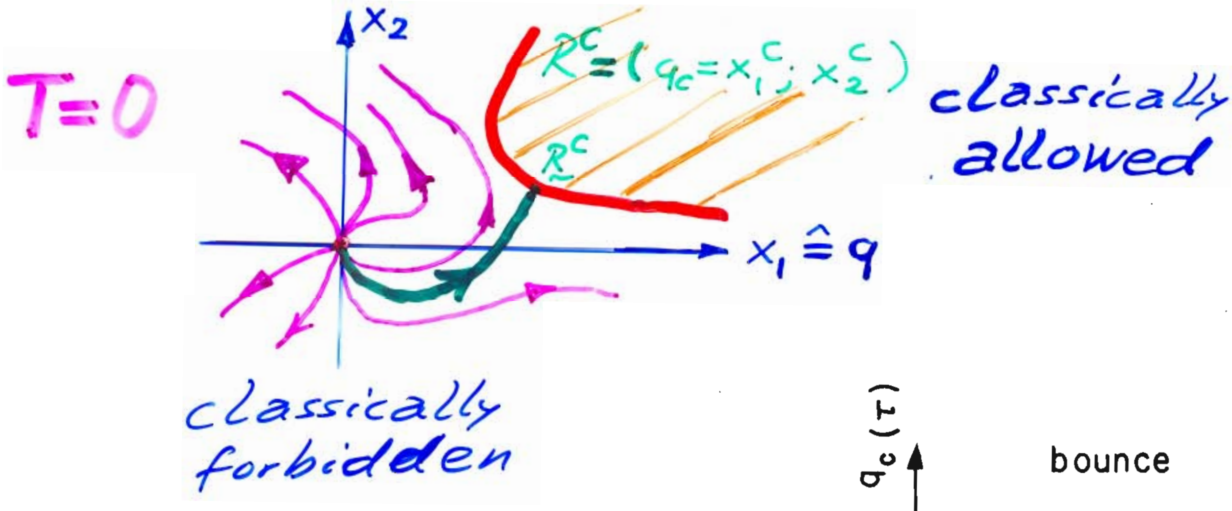
¡DIFFICULT!

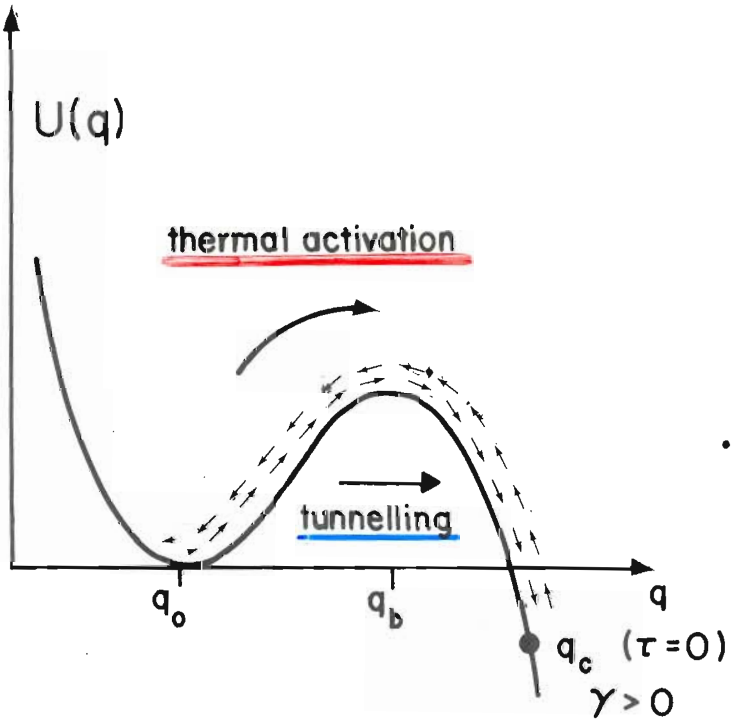
2 LIMITS

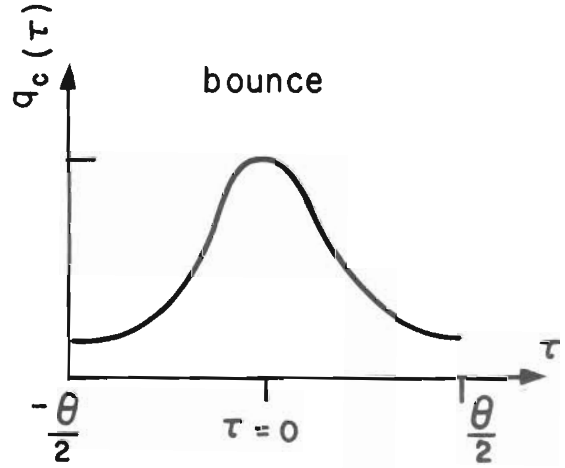
above ~ near crossover to thermal hopping

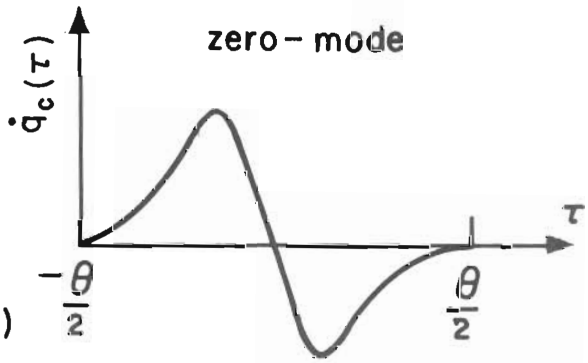
AT LOW T


WITH QUANTUM CORRECTIONS
classical
~150 !
$\alpha = 50$
$V_b/\hbar\omega_b = 0.2$
$\widetilde{SNR}$
T/T_0
$\log_{10}(\widetilde{SNR}^{sc}/\widetilde{SNR}^{cl})$
0
0.001
0.002
0.003
100
200
300
400
500
1
2
5
25
45





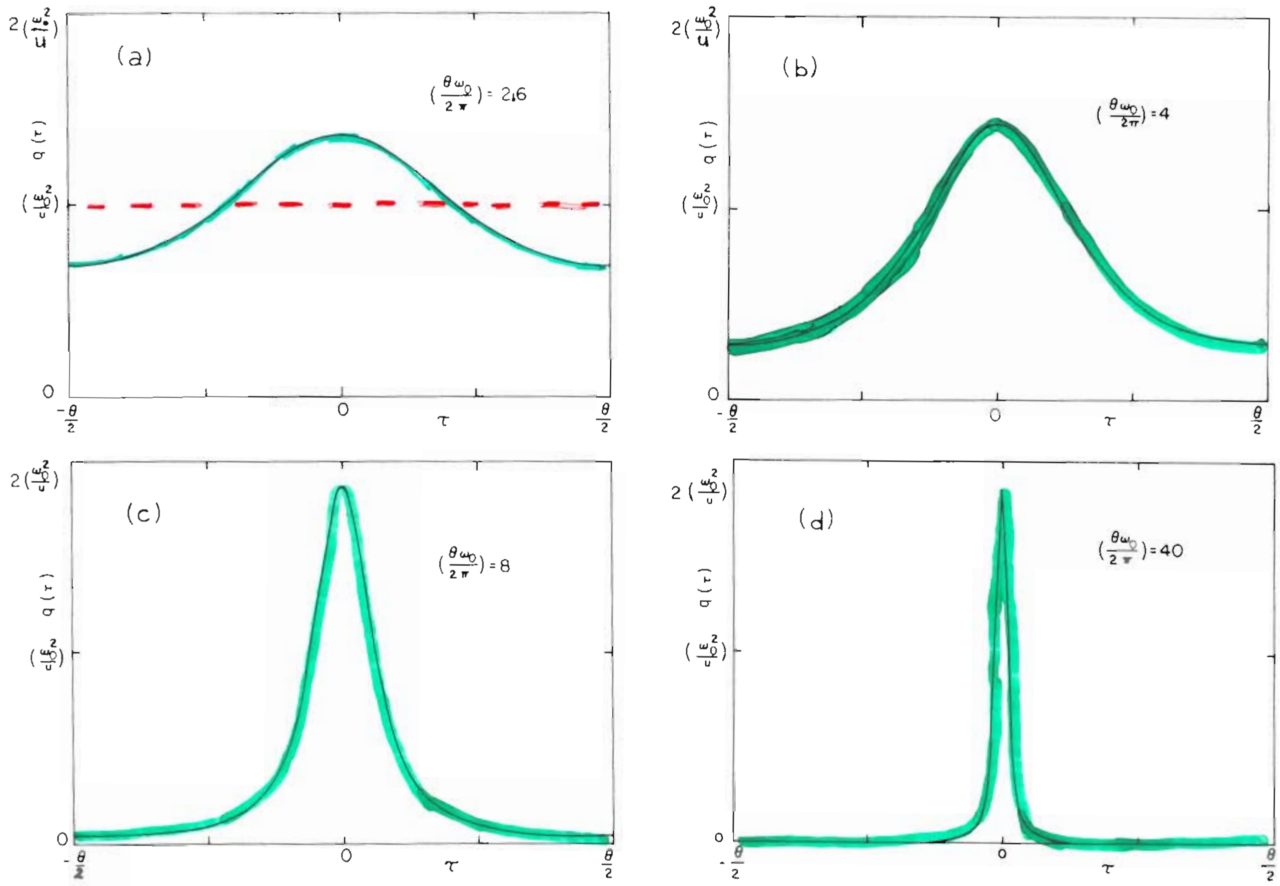




$\gamma = 0$:

$$M \frac{d^2}{d\tau^2} q_B(\tau) = \frac{\partial U}{\partial q_B}$$

$$- M \frac{d^2}{d\tau^2} \dot{q}_B(\tau) + \left(\frac{\partial^2 U}{\partial q_B^2} \right) \dot{q}_B(\tau) = 0$$

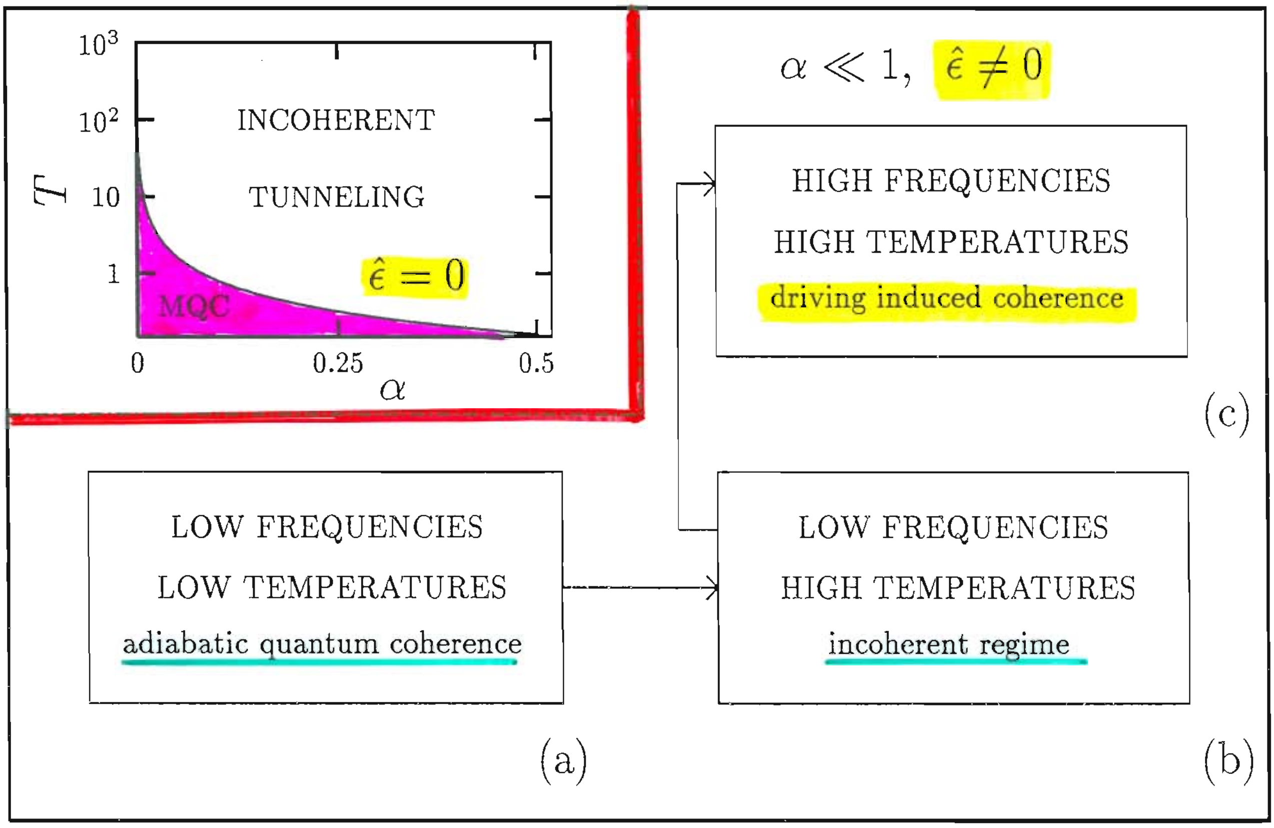


$$-M\ddot{q}_B + \frac{\partial U(q_B)}{\partial q_B} + \int_{-\Theta/2}^{\Theta/2} k(\tau-\tau')\, q_B(\tau')\, d\tau' = 0$$

$$\Theta = \hbar/kT$$

$$q_B(\tau+\Theta) = q_B(\tau)$$

QUANTUM SR



10^3
10^2
10
1
T
INCOHERENT
TUNNELING
$\hat{\epsilon} = 0$
MQC
0
0.25
0.5
α
$\alpha \ll 1, \ \hat{\epsilon} \neq 0$
HIGH FREQUENCIES
HIGH TEMPERATURES
driving induced coherence
(c)
LOW FREQUENCIES
LOW TEMPERATURES
adiabatic quantum coherence
(a)
LOW FREQUENCIES
HIGH TEMPERATURES
incoherent regime
(b)

DRIVEN QUANTUM TUNNELING

M. GRIFONI, P.H. PHYS. REP. 304: 229-358 (98)

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http://www.physik.uni-augsburg.de/theo1/hanggi/

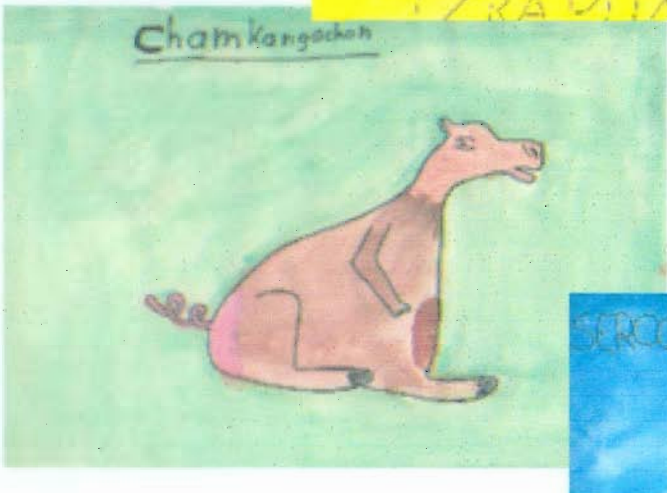
DRIVEN - TUNNELING - ZOO

SUPPR. vs. ENH.

CDT

CHAOS-ASSISTED

EHG



QSR

COHERENT TUNNELING CONTROL

- DRIVING (Ω, A, ..)
- BATH SPECTRUM
- NOISE INPUT

HOMEPAGE
„HANGGI“

GO TO : FEATURE ARTICLES

- **Quantum Dissipation**
  and
  Quantum Transport

http://www.physik.uni-augsburg.de/theo1/hanggi/Quantum.html