

Persistent spin currents in mesoscopic Heisenberg magnets

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Outline

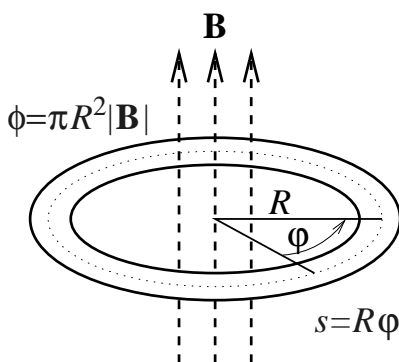
1. Introduction: persistent currents in mesoscopic normal metal rings
2. Spin transport: itinerant vs. localized spins
3. Persistent spin currents in insulators: ferromagnets vs. antiferromagnets
4. Conclusion & outlook

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1. Introduction: persistent currents

metal rings in Aharonov-Bohm geometry: ground-state currents

(Pauling 1937, London 1938, Hund 1939, ...; Imry, *Introduction to mesoscopic physics*, 1997)



- magnetic flux $\phi = \oint \mathbf{A} \cdot d\mathbf{r}$, $\mathbf{A} = \hat{e}_\varphi Br/2$
- simplest model: free 1d electrons ($\phi_0 = hc/e$)

$$\frac{\hbar^2}{2m^*} \left(-i \frac{d}{ds} + \frac{2\pi}{L} \frac{\phi}{\phi_0} \right)^2 \psi(s) = E\psi(s)$$

- periodic boundary conditions $\psi(s + L) = \psi(s)$
- gauge trafo: $\tilde{\psi}(s) = \exp\left(\frac{i\phi s}{\phi_0 R}\right) \psi(s) \Rightarrow$ twisted boundary conditions

$$\frac{\hbar^2}{2m^*} \left(-i \frac{d}{ds} \right)^2 \tilde{\psi}(s) = E\tilde{\psi}(s) \quad \text{with} \quad \tilde{\psi}(s + L) = \exp\left(\frac{2\pi i \phi}{\phi_0}\right) \tilde{\psi}(s)$$

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Ballistic regime

- eigenstates $\tilde{\psi}_n(s) = e^{ik_n s} / \sqrt{L}$ with eigenenergies $\epsilon_n = \frac{\hbar^2 k_n^2}{2m^*}$
- quantized wavevectors: $k_n = \frac{2\pi}{L} \left(n + \frac{\phi}{\phi_0} \right)$, $n = 0, \pm 1 \dots \Rightarrow k_{-n} \neq k_n$

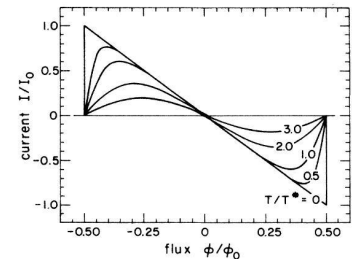
ballistic regime, $L \ll \ell$: (ℓ = mean free path)

- equilibrium current from thermodyn. potential

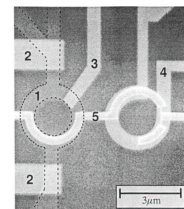
$$I(\phi) = -c \frac{\partial \Omega_{gc}(\phi)}{\partial \phi} = \frac{-e}{L} \sum_n \frac{v_n}{e^{(\epsilon_n - \mu)/T} + 1}$$

- velocities: $v_n = \frac{1}{\hbar} \frac{\partial \epsilon_n}{\partial k_n} = \frac{\hbar}{m^*} \frac{2\pi}{L} \left(n + \frac{\phi}{\phi_0} \right) \neq v_{-n}$

- $I_{\max} = \frac{ev_F}{L} \approx 4 \text{ nA} \Rightarrow$ observed in GaAs loop
(Mailly et al. 1993)



(Cheung et al. 1988)



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Effect of disorder

- mesoscopic persistent current exists also in disordered conductor
(Büttiker, Imry, Landauer 1983)
- $I_{\text{typ}} \approx \frac{ev_F}{L}$ in experiments on single Au rings
(Chandrasekhar et al. 1991)
- $\langle I \rangle \approx 3 \cdot 10^{-3} \frac{ev_F}{L}$ in experiments on 10^7 Cu rings
(Lévy et al. 1990)
- $\langle I \rangle \approx \frac{ev_F}{L} \frac{1}{M}$ for M -channel ring in canonical ensemble \Rightarrow too small
(Schmid 1990, v. Oppen & Riedel 1990, Altshuler et al. 1990)
- $\langle I \rangle \approx \frac{ev_F}{L} \frac{\ell}{L} (2g_4 - g_2)$ from Coulomb interaction \Rightarrow too small
(Ambegaokar & Eckern 1990)

unsolved problem:

experimentally observed currents much larger than theoretical predictions

today's topic:

spin analogue of persistent currents
for single ring in ballistic regime

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2. Spin transport: itinerant vs. localized spins

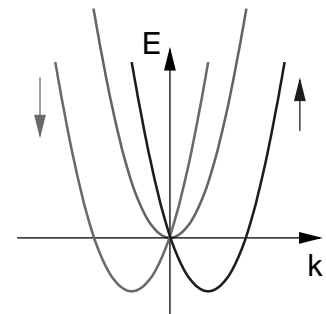
spin transport with itinerant electrons:

- conducting rings in inhomogeneous fields
(Loss et al. 1990, Stern 1992, Balatsky & Altshuler 1993, Gao & Qian 1993)
- conducting rings with spin-orbit coupling
(Frustaglia et al. 2001, Mal'shukov et al. 2002, Splettstoesser et al. 2003, Rashba 2003)
- thin film ferromagnets with spiral states, helimagnets
(König et al. 2001, Heurich et al. 2003)

Rashba spin-orbit coupling in semiconductors (Rashba 1960)

$$\hat{H} = \frac{p_x^2 + p_y^2}{2m^*} + \frac{\alpha_R}{\hbar} (\sigma_x \hat{p}_y - \sigma_y \hat{p}_x), \quad \Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

$$\text{eigenvalues: } E_{\pm} = \frac{\hbar^2 k^2}{2m^*} \pm \alpha_R k, \quad k = \sqrt{k_x^2 + k_y^2}$$



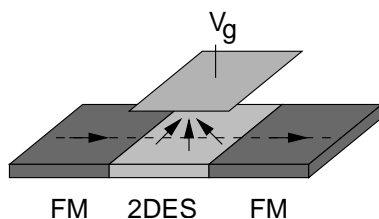
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Spin-based electronic devices

spintronics: spin-based electronics for information processing

(Awschalom, Loss, Samarth, *Semiconductor spintronics and quantum computation*, 2003)

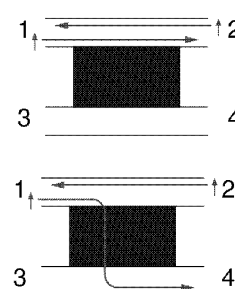
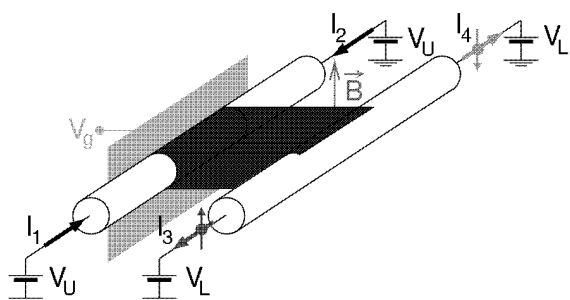
- spin transistor (Datta & Das 1990)



spin-dependent transmission
controlled by gate voltage

difficulties: spin injection, propagation, precession, collection

- spin filtering with quantum wires (Governale et al. 2002)



shift E_{\pm} with
 V_g and B
 \Rightarrow
select
tunneling
events

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Magnetic insulators

localized electrons:
$$\mathbf{S}_i = \frac{1}{2} \begin{bmatrix} c_{i\uparrow}^+ \\ c_{i\downarrow}^+ \end{bmatrix}^\top \boldsymbol{\sigma} \begin{bmatrix} c_{i\uparrow} \\ c_{i\downarrow} \end{bmatrix}, \quad S_i^z = \frac{1}{2}(n_{i\uparrow} - n_{i\downarrow}) \text{ etc.}$$

- motion of electrons frozen \Rightarrow Heisenberg exchange interactions

$$\hat{H} = \frac{1}{2} \sum_{i,j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j - g\mu_B \sum_i \mathbf{B}_i \cdot \mathbf{S}_i, \quad [S_i^x, S_j^y] = i\delta_{ij} S_i^z$$

- $S_i^2 = S(S+1)$, $S = \frac{1}{2}, 1, \dots \Rightarrow$ Holstein-Primakoff representation

$$\begin{aligned} S_i^+ &= \sqrt{2S - n_i} b_i && \blacktriangleright \text{bosons: } [b_i, b_i^\dagger] = 1, n_i = b_i^\dagger b_i \\ S_i^- &= b_i^\dagger \sqrt{2S - n_i} && \blacktriangleright \text{only } n_i \leq 2S \text{ is allowed} \\ S_i^z &= S - n_i && \blacktriangleright S \gg \frac{1}{2} \Rightarrow \text{expand in } 1/S \end{aligned}$$

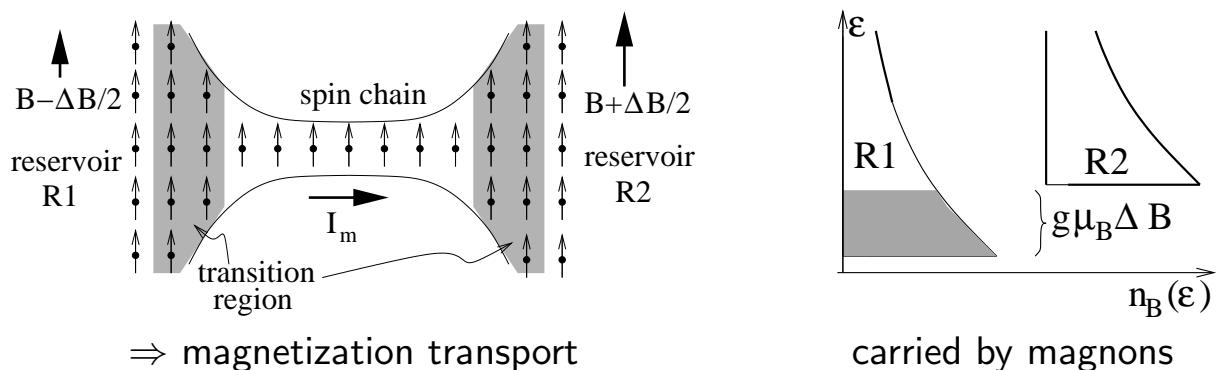
- elementary excitations: spin waves (“magnons”)

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Spin transport in magnetic insulators

Heisenberg magnets in spatially varying magnetic fields:

- magnetic field difference at ends of spin system (Meier & Loss 2003)



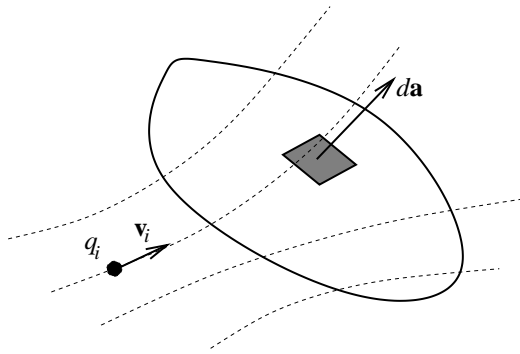
semiclassical picture of spin transport:

- magnetic field $\mathbf{B}_i \Rightarrow$ magnetic moments $\mathbf{m}_i(t) = g\mu_B \langle \psi(t) | \mathbf{S}_i | \psi(t) \rangle$
- magnetization $\mathbf{M}(\mathbf{r}, t) = \sum_i \delta(\mathbf{r} - \mathbf{r}_i) \mathbf{m}_i(t)$
time evolution of $\mathbf{M}(\mathbf{r}, t) \Rightarrow$ magnetization transport

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Charge current vs. magnetization current

current of electric monopoles q_i :

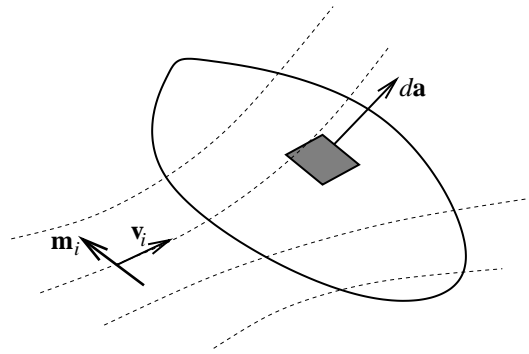


- current density: vector

$$\mathbf{j}_\mu(\mathbf{r}) = \sum_i q_i (\mathbf{v}_i)_\mu \delta(\mathbf{r} - \mathbf{r}_i)$$
- current through surface A :

$$I(A) = \int_A d\mathbf{a} \cdot \mathbf{j}(\mathbf{r})$$

current of magnetic dipoles \mathbf{m}_i :



- magnetization current density:

$$j_\mu^\alpha(\mathbf{r}) = \sum_i (\mathbf{m}_i)^\alpha (\mathbf{v}_i)_\mu \delta(\mathbf{r} - \mathbf{r}_i)$$

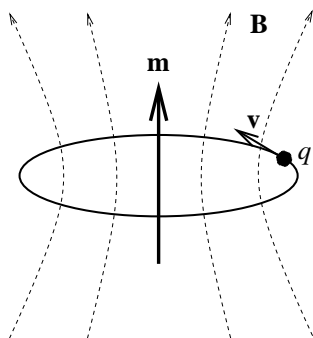
 2nd rank tensor, $\alpha = x, y, z$
- current through surface A :

$$I^\alpha(A) = \int_A da_\mu j_\mu^\alpha(\mathbf{r})$$

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Electrodynamics of magnetization currents

stationary charge currents
 \Rightarrow static magnetic fields



- Biot-Savart law:

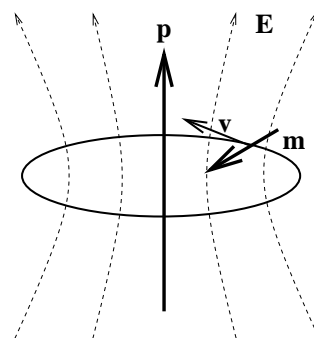
$$\mathbf{B}(\mathbf{r}) = \int \frac{d^3 r'}{c} \mathbf{j}(\mathbf{r}') \times \frac{(\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3}$$
- far zone: $\mathbf{A} = \frac{\mathbf{m} \times \mathbf{r}}{|\mathbf{r}|^3}$

$$\mathbf{m} = \int \frac{d^3 r'}{2c} \mathbf{r}' \times \mathbf{j}(\mathbf{r}')$$

 magnetic dipole moment

stationary magnetization currents
 \Rightarrow static electric fields

(Hirsch 1999; Meier & Loss 2003)



- Biot-Savart-type law:

$$\phi(\mathbf{r}) = - \int \frac{d^3 r'}{c} [\mathbf{M}(\mathbf{r}') \times \mathbf{v}(\mathbf{r}')] \cdot \frac{(\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3}$$
- far zone: $\phi = \frac{\mathbf{p} \cdot \mathbf{r}}{|\mathbf{r}|^3}$, $\mathbf{E} = -\nabla \phi$

$$\mathbf{p} = - \int \frac{d^3 r'}{c} \mathbf{M}(\mathbf{r}') \times \mathbf{v}(\mathbf{r}')$$

 electric dipole moment

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3. Persistent spin currents in insulators

ferromagnetic Heisenberg ring in a crown-shaped magnetic field:

- Hamiltonian:

$$\hat{H} = \frac{1}{2} \sum_{ij} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j - g\mu_B \sum_i \mathbf{B}_i \cdot \mathbf{S}_i$$

- magnetic moments

$$\mathbf{m}_i = g\mu_B \langle \mathbf{S}_i \rangle = m_i \hat{\mathbf{m}}_i$$

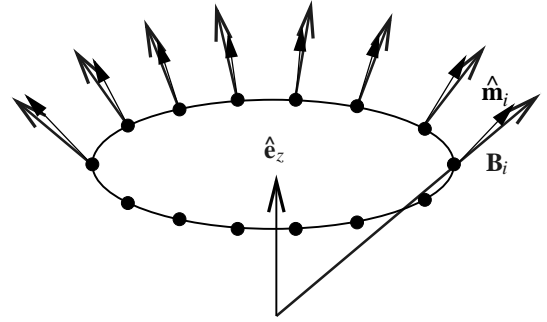
- Heisenberg equation of motion:

$$\hbar \frac{\partial \mathbf{S}_i}{\partial t} + \mathbf{h}_i \times \mathbf{S}_i + \sum_j J_{ij} \mathbf{S}_i \times \mathbf{S}_j = 0 \quad \text{where} \quad \mathbf{h}_i \equiv g\mu_B \mathbf{B}_i$$

- spin current operator: $\mathbf{I}_{i \rightarrow j} = J_{ij} (\mathbf{S}_i \times \mathbf{S}_j)$ (Chandra, Coleman, Larkin 1990)

- longitudinal spin current: $I_s = \hat{\mathbf{m}}_i \cdot \langle \mathbf{I}_{i \rightarrow j} \rangle$

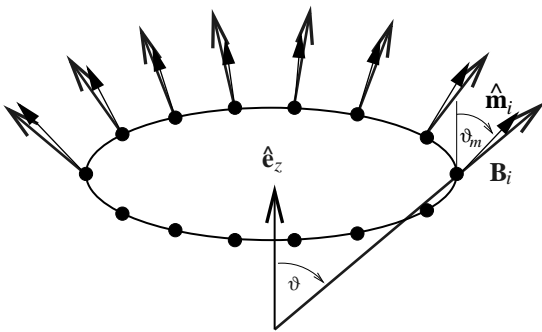
$$\frac{\partial m_i(t)}{\partial t} + \sum_j \hat{\mathbf{m}}_i \cdot \langle \mathbf{I}_{i \rightarrow j} \rangle = 0 \quad \text{continuity equation}$$



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Classical ground state

- classically $\mathbf{S}_i = S \hat{\mathbf{m}}_i \Rightarrow$ stability: $\hat{\mathbf{m}}_i \times (\mathbf{h}_i - \sum_j J_{ij} S \hat{\mathbf{m}}_j) = 0$



FM ring ($J < 0$) with $\mathbf{B}(\mathbf{r}) \propto \hat{\mathbf{r}}$

$$\Rightarrow \sin(\vartheta - \vartheta_m) = \frac{|J|S}{h} \left[1 - \cos \frac{2\pi}{N} \right] \sin 2\vartheta$$

$$\Rightarrow \text{assume } h \gtrsim JS \left(\frac{2\pi}{N} \right)^2 \equiv \Delta$$

- quantum spins: $\mathbf{S}_i = S_i^{\parallel} \hat{\mathbf{m}}_i + \mathbf{S}_i^{\perp} \Rightarrow$ longitudinal part $S_i^{\parallel} = S - b_i^{\dagger} b_i$

- decomposition of Hamiltonian: $\hat{H} = \hat{H}^{\parallel} + \hat{H}^{\perp} + O(S^{1/2})$

▶ longitudinal: $\hat{H}^{\parallel} = \frac{1}{2} \sum_{i,j} J_{ij} \hat{\mathbf{m}}_i \cdot \hat{\mathbf{m}}_j S_i^{\parallel} S_j^{\parallel} - \sum_i \mathbf{h}_i \cdot \hat{\mathbf{m}}_i S_i^{\parallel} = O(S^2)$

▶ transverse: $\hat{H}^{\perp} = \frac{1}{2} \sum_{i,j} J_{ij} \mathbf{S}_i^{\perp} \cdot \mathbf{S}_j^{\perp} = O(S)$

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Transverse basis: local U(1) gauge freedom

- classical spin orientation $\hat{\mathbf{m}}_i \Rightarrow$ local quantization axis (fixed)
- but local triad $\{\hat{\mathbf{e}}_i^1, \hat{\mathbf{e}}_i^2, \hat{\mathbf{m}}_i\}$ can be arbitrarily rotated around $\hat{\mathbf{m}}_i$
- transverse Hamiltonian: $(\mathbf{e}_i^\pm = \hat{\mathbf{e}}_i^1 \pm i\hat{\mathbf{e}}_i^2, S_i^\pm = \mathbf{e}_i^\pm \cdot \mathbf{S}_i^\perp)$

$$\hat{H}^\perp = \frac{1}{8} \sum_{i,j} J_{ij} \sum_{p,p'=\pm} (\mathbf{e}_i^p \cdot \mathbf{e}_j^{p'}) S_i^{-p} S_j^{-p'}$$

- local U(1) gauge transformation:

$$\mathbf{e}_i^\pm \rightarrow \tilde{\mathbf{e}}_i^\pm e^{\pm i\alpha_i} \quad \text{and} \quad S_i^\pm \rightarrow S_i^\pm e^{\pm i\alpha_i}, \quad \alpha_i \text{ arbitrary}$$

- spin-wave expansion: $S_i^+ = \sqrt{2S} b_i [1 + O(\frac{1}{S})]$

useful reference: parallel-transported basis

- for neighbors i and j let $\{\tilde{\mathbf{e}}_i^1, \tilde{\mathbf{e}}_i^2, \hat{\mathbf{m}}_i\}$ with $\tilde{\mathbf{e}}_i^2 = \hat{\mathbf{m}}_i \times \hat{\mathbf{m}}_j$
- express $\mathbf{e}_i^\pm = \tilde{\mathbf{e}}_i^\pm e^{\pm i\omega_{i \rightarrow j}}$ via angles $\omega_{i \rightarrow j}$

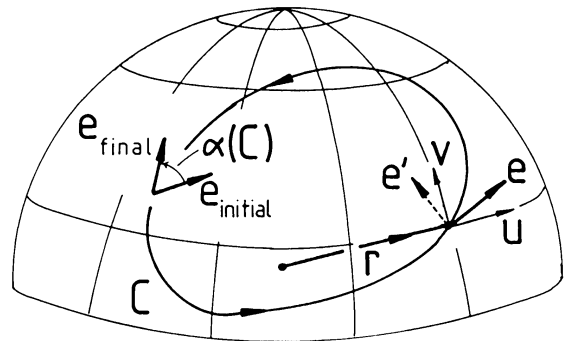
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Parallel transport on surface of unit sphere

given:

- closed curve \mathcal{C} on unit sphere, parametrized by $\hat{\mathbf{m}}(s)$
- transverse vector defined by angular velocity $\boldsymbol{\omega}(s)$:

$$\frac{d\mathbf{e}(s)}{ds} = \boldsymbol{\omega}(s) \times \mathbf{e}(s)$$



(M.V.Berry, in: Shapere & Wilczek, *Geometric Phases in Physics*, 1989)

parallel transport:

- $\boldsymbol{\omega}(s) = \hat{\mathbf{m}}(s) \times \frac{d\hat{\mathbf{m}}(s)}{ds}$
- $\hat{\mathbf{m}}(s) \cdot \boldsymbol{\omega}(s) = 0 \Rightarrow$ transverse basis does not twist around $\hat{\mathbf{m}}(s)$
- $\mathbf{e}_{\text{initial}} \neq \mathbf{e}_{\text{final}}$ ("anholonomy")
 \Rightarrow defect angle $\alpha(\mathcal{C}) = \angle(\mathbf{e}_{\text{initial}}, \mathbf{e}_{\text{final}}) =$ solid angle subtended by \mathcal{C}

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Gauge invariance and current conservation

transverse Hamiltonian:

$$\hat{H}^\perp = \frac{1}{8} \sum_{i,j} J_{ij} \left[(1 + \hat{\mathbf{m}}_i \cdot \hat{\mathbf{m}}_j) e^{i(\omega_{i \rightarrow j} - \omega_{j \rightarrow i})} S_i^- S_j^+ - (1 - \hat{\mathbf{m}}_i \cdot \hat{\mathbf{m}}_j) e^{i(\omega_{i \rightarrow j} + \omega_{j \rightarrow i})} S_i^- S_j^- + \text{h.c.} \right]$$

- ferromagnet: $\hat{\mathbf{m}}_i \cdot \hat{\mathbf{m}}_j = 1 + O(\frac{1}{N}) \Rightarrow$ FM magnons
- antiferromagnet: $\hat{\mathbf{m}}_i \cdot \hat{\mathbf{m}}_j = -1 + O(\frac{1}{N}) \Rightarrow$ AF magnons

gauge invariance: $\omega_{i \rightarrow j} \rightarrow \omega_{i \rightarrow j} + \alpha_i, S_i^p \rightarrow S_i^p e^{ip\alpha_i}$

- conserved spin current: (Noether's theorem)

$$0 = \left\langle \frac{\partial \hat{H}^\perp}{\partial \alpha_i} \right\rangle = \left\langle \sum_j \frac{\partial \hat{H}^\perp}{\partial \omega_{i \rightarrow j}} \right\rangle = - \sum_j \underbrace{\langle \hat{\mathbf{m}}_i \cdot J_{ij} (\mathbf{S}_i^\perp \times \mathbf{S}_j^\perp) \rangle}_{= \hat{\mathbf{m}}_i \cdot \langle \mathbf{I}_{i \rightarrow j} \rangle}$$

- agrees with Heisenberg equation of motion

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Gauge invariant persistent current

- ferromagnetic next-neighbor coupling $J < 0 \Rightarrow \hat{\mathbf{m}}_i \cdot \hat{\mathbf{m}}_{i+1} \approx 1$
- gauge phases away \Rightarrow twisted boundary conditions $S_{i+N}^\pm = \exp(\pm i\Omega) S_i^\pm$

where

$$\begin{aligned} \Omega &= \sum_{i=1}^N (\omega_{i \rightarrow i+1} - \omega_{i \rightarrow i-1}) = \text{gauge invariant geometric flux} \\ &= \text{total defect angle of } \hat{\mathbf{m}}_i \text{ on the unit sphere} \\ &= 2\pi(1 - \cos \vartheta_m) \text{ for crown-shaped configuration} \end{aligned}$$

gauge invariant persistent spin current:

$$I_s(\Omega) = - \frac{\partial F(\Omega)}{\partial \Omega} = J \langle \hat{\mathbf{m}}_i \cdot (\mathbf{S}_i^\perp \times \mathbf{S}_{i+1}^\perp) \rangle$$

compare with persistent charge current:

$$I(\phi) = -c \frac{\partial \Omega_{gc}(\phi)}{\partial \phi}, \quad \phi = \text{magnetic flux}$$

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Persistent spin current carried by magnons

- FM spin waves:

$$\hat{H}_{\text{sw}} = \sum_k (\epsilon_k + h) b_k^\dagger b_k \quad \text{with} \quad \epsilon_k = 2JS(1 - \cos(ka)) \propto k^2$$

- quantized wavevectors: $k = \frac{2\pi}{L} \left(n + \frac{\Omega}{2\pi} \right)$, $n = 0, \pm 1, \dots, \pm \frac{N}{2}$

magnetization current: $I_m = \frac{g\mu_B}{\hbar} I_s = -\frac{g\mu_B}{\hbar} \frac{\partial F_{\text{sw}}}{\partial \Omega}$

$$I_m(\Omega) = -\frac{g\mu_B}{L} \sum_k \frac{v_k}{e^{(\epsilon_k + h)/T} - 1} \quad \text{where} \quad v_k = \frac{1}{\hbar} \frac{\partial \epsilon_k}{\partial k} \propto k$$

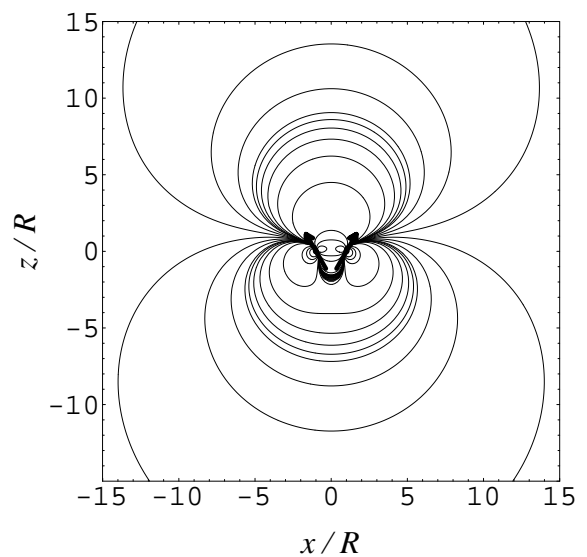
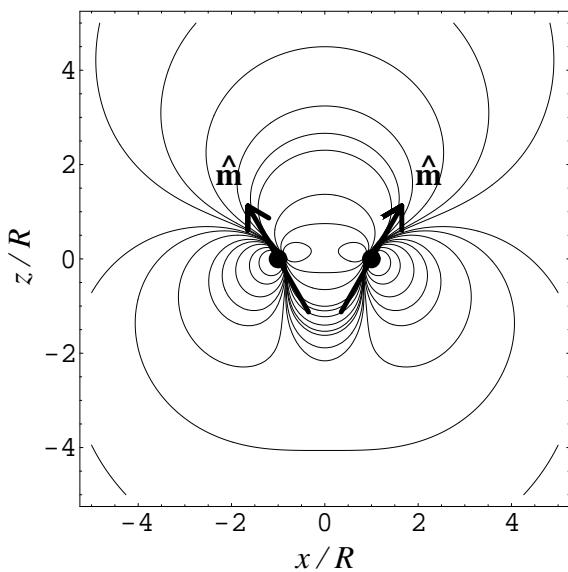
$$\approx \frac{g\mu_B T}{\hbar} \frac{\sin \Omega}{\cos \Omega - \cosh(2\pi\sqrt{h/\Delta})} \quad \text{for} \quad \Delta \equiv JS\left(\frac{2\pi}{N}\right)^2 \ll T \ll J$$

- bosonic analogue of charge current: $I(\phi) = \frac{-e}{L} \sum_k \frac{v_k}{e^{(\epsilon_k - \mu)/T} + 1}$
- thermal fluctuations: $I_m \propto T$
- mesoscopic quantum effect: I_m vanishes in bulk limit or classical limit

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Electric field generated by spin current

lines of constant electric potential for $\vartheta_m = 30^\circ$



generalized Biot-Savart law:

$$\phi(\mathbf{r}) = \frac{I_m}{c} \oint [d\mathbf{r}' \times \hat{\mathbf{m}}(\mathbf{r}')] \cdot \frac{(\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3}$$

far zone: $\phi(\mathbf{r}) = \mathbf{p} \cdot \mathbf{r} / |\mathbf{r}|^3$
with electric dipole moment

$$\mathbf{p} = -\hat{\mathbf{e}}_z \frac{I_m}{c} \sin \vartheta_m$$

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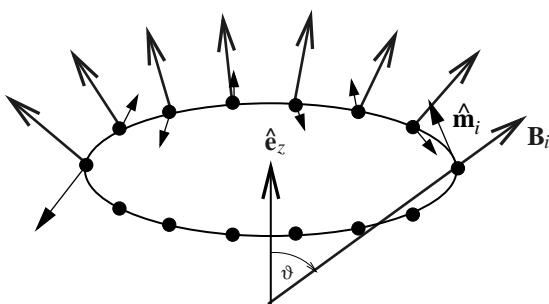
Experimental parameters

- desired:
 - measurement of voltage difference ΔU
 - at distance $\approx L$ above and below Heisenberg ring
- optimal parameters:
 - ▶ solid angle $\Omega = 2\pi(1 - \cos \vartheta_m) = \pi/2 \Rightarrow \vartheta_m \approx 41^\circ$
 - ▶ magnetic field: $g\mu_B B \approx \Delta = JS(2\pi/N)^2$
 - ▶ temperature: $\Delta \ll T \ll J$
- $T = 50 \text{ K}, L = 100 \text{ nm}, N = 100, J = 100 \text{ K} \Rightarrow \boxed{\Delta U \approx 0.2 \text{ nV}, B \approx 0.1 \text{ T}}$
- requirements:
 - ▶ well-characterized rings with large Heisenberg coupling J
 - ▶ submicron B inhomogeneity
 - ▶ nanovolt sensitivity
- detection may be difficult due to screening
- spin rings in inhomogeneous electric fields: Aharonov-Casher effect

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Antiferromagnetic spin ring

- antiferromagnetic next-neighbor coupling $J > 0$



crown-shaped field $\mathbf{B}(\mathbf{r}) \propto \hat{\mathbf{r}}$
 \Rightarrow spins flip for $h > JS \equiv h_c$

stability of Néel-like state:

$$\Rightarrow \sin 2(\bar{\vartheta} - \vartheta) = \left(\frac{JS}{h}\right)^2 \sin^2\left(\frac{2\pi}{N}\right) \sin(2\bar{\vartheta})$$

$$\Rightarrow \text{assume } h \gtrsim JS/N (\ll h_c)$$

- two sublattices with $\hat{\mathbf{m}}_i \approx (-1)^i \hat{\mathbf{n}}_i$ and $\hat{\mathbf{n}}_i \cdot \mathbf{h}_i \approx 0$
- twisted boundary conditions $b_{i+N} = e^{\pm i\Omega} b_i$
 with $\Omega = \sum_{i=1}^N (-1)^i (\omega_{i \rightarrow i+1} + \omega_{i \rightarrow i-1}) = \text{total defect angle of } \hat{\mathbf{n}}_i$

AF spin waves: quantum fluctuations \Rightarrow new magnon vacuum

$$\hat{H}_{\text{sw}} = \sum_{k \geq 0} \epsilon_k (\alpha_k^\dagger \alpha_k + \beta_k^\dagger \beta_k - 1), \quad \epsilon_k = 2JS \sin(ka) \propto k$$

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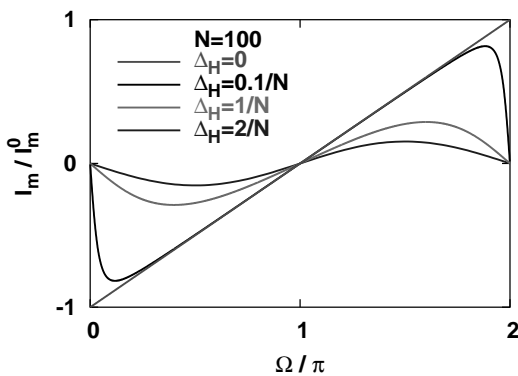
Persistent spin current carried by AF magnons

- spin current: $I_s(\Omega) = J \langle \hat{n}_i \cdot (\mathbf{S}_i^\perp \times \mathbf{S}_{i+1}^\perp) \rangle = -\partial F_{\text{sw}} / \partial \Omega$
- magnetization current:

$$I_m(\Omega) = -\frac{2g\mu_B}{L} \sum_k c_k \left(n_k + \frac{1}{2} \right) \quad n_k = \frac{1}{e^{\epsilon_k/T} - 1}, \quad c_k = \frac{1}{\hbar} \frac{\partial \epsilon_k}{\partial k}$$

$$T \xrightarrow{=} 0 \quad \boxed{I_m = I_m^0 \left(1 - \frac{\Omega}{\pi} \right)} \quad I_m^0 = \frac{g\mu_B c_0}{L} = \frac{g\mu_B}{\hbar} \frac{JS}{N}$$

integer spin: Haldane gap $= 2JS\Delta_H \propto e^{-\pi S} \propto a/\xi$, $\Delta_H(S=1) \approx 0.2$



“modified” spin-wave theory: suppress Neél order on average $\sum_i \langle \mathbf{S}_i \cdot \mathbf{m}_i \rangle = 0$ via staggered field

(Takahashi 1989, Hirsch & Tang 1989)

$$\Delta_H \gg \frac{1}{N} \Rightarrow \frac{I_m}{I_m^0} \propto \sqrt{\frac{L}{\xi}} e^{-L/\xi} \sin \Omega$$

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4. Conclusion & outlook

persistent spin currents in Heisenberg magnets:

- mesoscopic quantum interference effect
- bosonic analogue of charge currents in metal rings: $\frac{\Omega}{2\pi} \leftrightarrow \frac{\phi}{\phi_0}$
- magnetization transported by magnons due to
 - ▶ thermal fluctuations (ferromagnets)
 - ▶ quantum fluctuations (antiferromagnets)
- spin current induces dipole-like electric field
- experimental requirements:
 - ▶ well-defined Heisenberg rings
 - ▶ submicron field inhomogeneities
 - ▶ detection of nanovoltages

open questions:

- diffusive regime of disordered magnets

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Metallasiloxanolates

current activities in DFG-Forschergruppe FOR 412:

(Molodtsova et al. 2003)

