Persistent spin currents in mesoscopic Heisenberg magnets

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PRL **91**, 017205 (2003); PRB (in press); cond-mat/0301351, 0308230

supported in part by DFG-Forschergruppe 412 *Spin- und Ladungsträger-korrelationen in niedrigdimensionalen metallorganischen Festkörpern*

Karlsruhe, 24-Nov-03

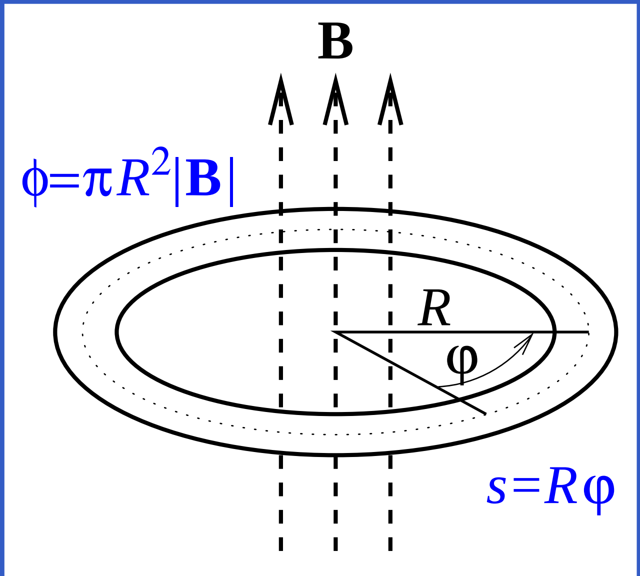
Outline

1. Introduction: persistent currents in mesoscopic normal metal rings
2. Spin transport: itinerant vs. localized spins
3. Persistent spin currents in insulators: ferromagnets vs. antiferromagnets
4. Conclusion & outlook

1. Introduction: persistent currents

metal rings in Aharonov-Bohm geometry: ground-state currents

(Pauling 1937, London 1938, Hund 1939, . . . ; Imry, *Introduction to mesoscopic physics*, 1997)



- magnetic flux $\phi = \oint \boldsymbol{A} \cdot d\boldsymbol{r}, \quad \boldsymbol{A} = \hat{\boldsymbol{e}}_\varphi Br/2$
- simplest model: free 1d electrons $(\phi_0 = hc/e)$

$$\frac{\hbar^2}{2m^*}\left(-i\frac{d}{ds} + \frac{2\pi}{L}\frac{\phi}{\phi_0}\right)^2 \psi(s) = E\psi(s)$$

- periodic boundary conditions $\psi(s+L) = \psi(s)$

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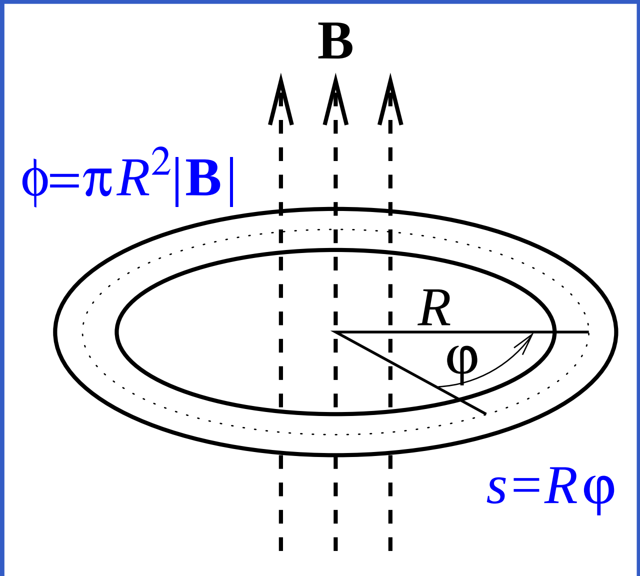

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- periodic boundary conditions $\psi(s+L) = \psi(s)$

- gauge trafo: $\tilde{\psi}(s) = \exp\left(\frac{i\phi s}{\phi_0 R}\right)\psi(s) \Rightarrow$ twisted boundary conditions

$$\frac{\hbar^2}{2m^*}\left(-i\frac{d}{ds}\right)^2 \tilde{\psi}(s) = E\tilde{\psi}(s) \quad \text{with} \quad \tilde{\psi}(s+L) = \exp\left(\frac{2\pi i\phi}{\phi_0}\right)\tilde{\psi}(s)$$

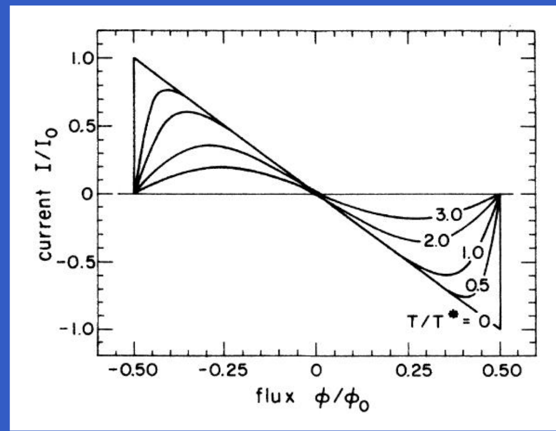
Ballistic regime

- eigenstates $\tilde{\psi}_n(s) = e^{ik_n s}/\sqrt{L}$ with eigenenergies $\epsilon_n = \dfrac{\hbar^2 k_n^2}{2m^*}$
- quantized wavevectors: $k_n = \dfrac{2\pi}{L}\left(n + \dfrac{\phi}{\phi_0}\right)$, $n = 0, \pm 1 \ldots \Rightarrow k_{-n} \neq k_n$

ballistic regime, $L \ll \ell$: (ℓ = mean free path)

- equilibrium current from thermodyn. potential

$$I(\phi) = -c\frac{\partial \Omega_{gc}(\phi)}{\partial \phi} = \frac{-e}{L}\sum_n \frac{v_n}{e^{(\epsilon_n - \mu)/T} + 1}$$



(Cheung et al. 1988)

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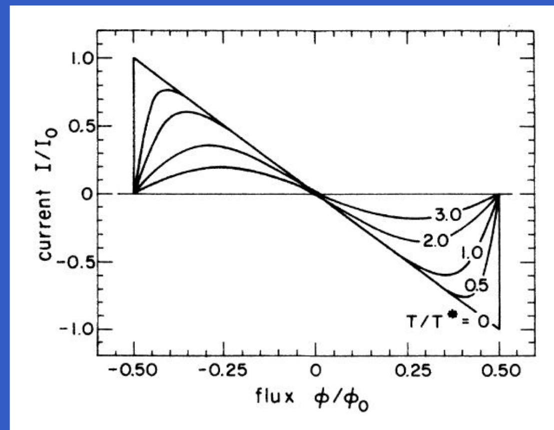

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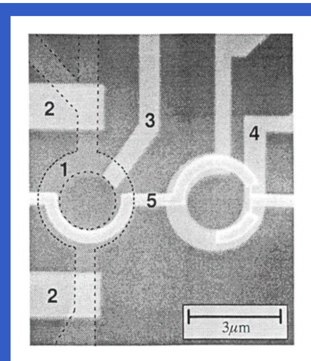

- $I_{\text{max}} = \dfrac{e v_{\text{F}}}{L} \approx 4$ nA $\Rightarrow$ observed in GaAs loop

(Mailly et al. 1993)

Effect of disorder

- mesoscopic persistent current exists also in disordered conductor
  (Büttiker, Imry, Landauer 1983)
- $I_{\mathrm{typ}} \approx \frac{ev_{\mathrm{F}}}{L}$ in experiments on single Au rings (Chandrasekhar et al. 1991)
- $\langle I \rangle \approx 3 \cdot 10^{-3} \frac{ev_{\mathrm{F}}}{L}$ in experiments on 10^7 Cu rings (Lévy et al. 1990)
- $\langle I \rangle \approx \frac{ev_{\mathrm{F}}}{L} \frac{1}{M}$ for M-channel ring in canonical ensemble $\Rightarrow$ too small
  (Schmid 1990, v. Oppen & Riedel 1990, Altshuler et al. 1990)
- $\langle I \rangle \approx \frac{ev_{\mathrm{F}}}{L} \frac{\ell}{L} (2g_4 - g_2)$ from Coulomb interaction $\Rightarrow$ too small
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unsolved problem:

experimentally observed currents much larger than theoretical predictions

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today's topic:

spin analogue of persistent currents
for single ring in ballistic regime

2. Spin transport: itinerant vs. localized spins

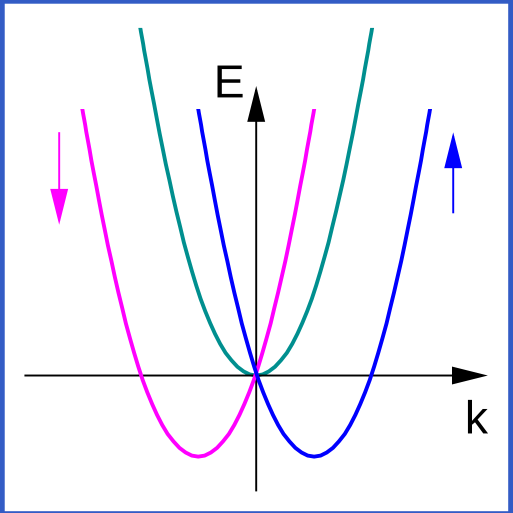
spin transport with itinerant electrons:

- conducting rings in inhomogeneous fields
  (Loss et al. 1990, Stern 1992, Balatsky & Altshuler 1993, Gao & Qian 1993)
- conducting rings with spin-orbit coupling
  (Frustaglia et al. 2001, Mal'shukov et al. 2002, Splettstoesser et al. 2003, Rashba 2003)
- thin film ferromagnets with spiral states, helimagnets
  (König et al. 2001, Heurich et al. 2003)

Rashba spin-orbit coupling in semiconductors (Rashba 1960)

$$\hat{H} = \frac{p_x^2 + p_y^2}{2m^*} + \frac{\alpha_\mathrm{R}}{\hbar}(\sigma_x \hat{p}_y - \sigma_y \hat{p}_x), \qquad \Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

eigenvalues: $E_\pm = \dfrac{\hbar^2 k^2}{2m^*} \pm \alpha_\mathrm{R}\, k, \qquad k = \sqrt{k_x^2 + k_y^2}$

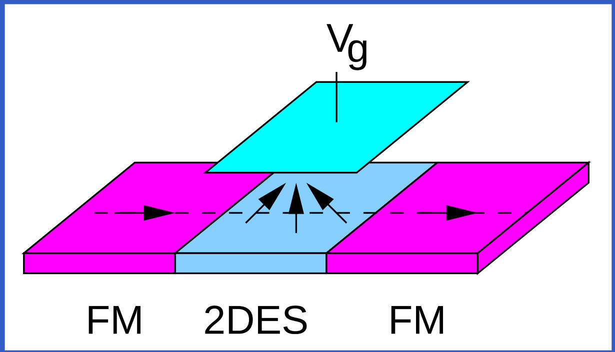


Spin-based electronic devices

spintronics: spin-based electronics for information processing

(Awschalom, Loss, Samarth, *Semiconductor spintronics and quantum computation*, 2003)

- spin transistor (Datta & Das 1990)



spin-dependent transmission
controlled by gate voltage

difficulties: spin injection, propagation, precession, collection

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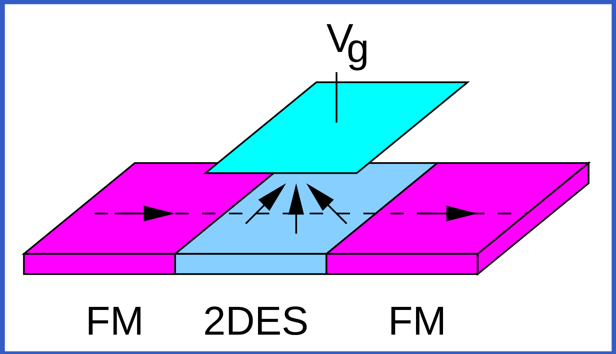

spin-dependent transmission
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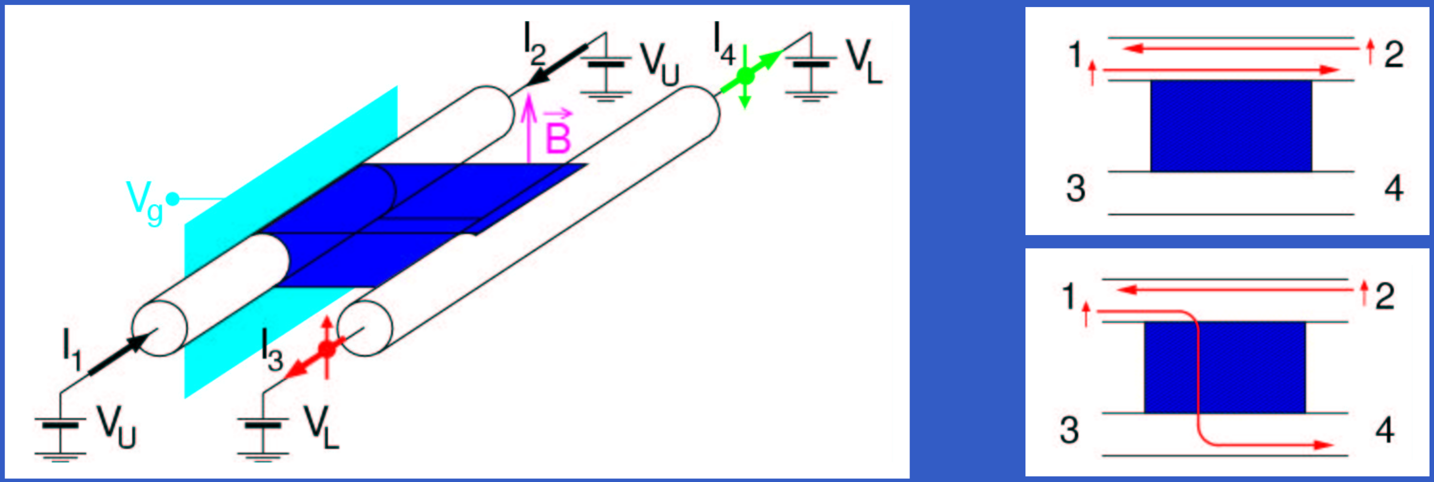
difficulties: spin injection, propagation, precession, collection

- spin filtering with quantum wires (Governale et al. 2002)



shift $E_\pm$ with V_g and B $\Rightarrow$ select tunneling events

Magnetic insulators

localized electrons: $\boldsymbol{S}_i = \frac{1}{2} \begin{bmatrix} c^+_{i\uparrow} \\ c^+_{i\downarrow} \end{bmatrix}^{\mathsf{T}} \boldsymbol{\sigma} \begin{bmatrix} c_{i\uparrow} \\ c_{i\downarrow} \end{bmatrix}$, $S^z_i = \frac{1}{2}(n_{i\uparrow} - n_{i\downarrow})$ etc.

- motion of electrons frozen $\Rightarrow$ Heisenberg exchange interactions

$$\hat{H} = \frac{1}{2} \sum_{i,j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j - g\mu_{\mathsf{B}} \sum_i \boldsymbol{B}_i \cdot \boldsymbol{S}_i, \qquad [S^x_i, S^y_j] = i\delta_{ij} S^z_i$$

- $\boldsymbol{S}_i^2 = S(S+1)$, $S = \frac{1}{2}, 1, \ldots \Rightarrow$ Holstein-Primakoff representation

$$S_i^+ = \sqrt{2S - n_i}\, b_i$$
$$S_i^- = b_i^\dagger \sqrt{2S - n_i}$$
$$S_i^z = S - n_i$$

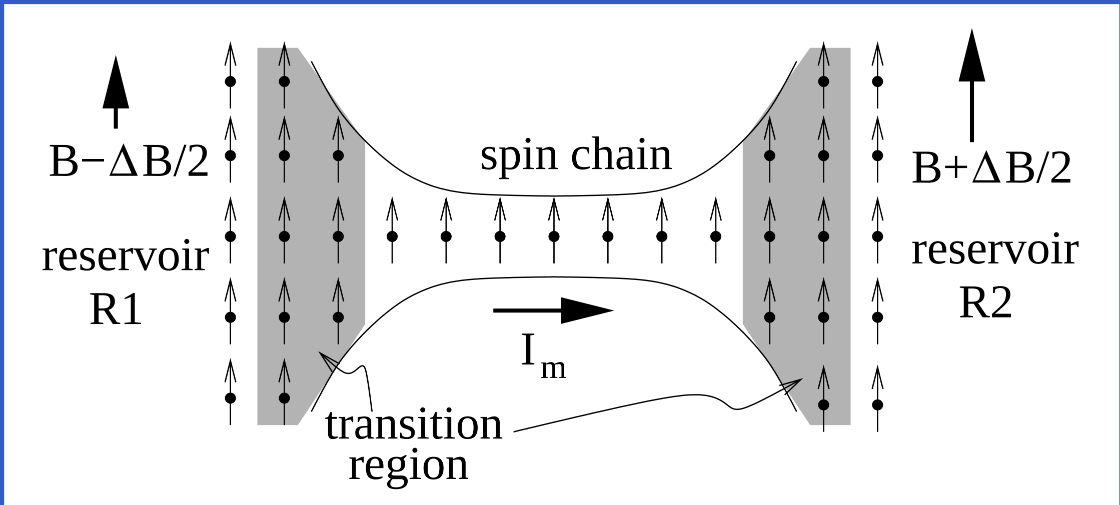
  - bosons: $[b_i, b_i^\dagger] = 1$, $n_i = b_i^\dagger b_i$
  - only $n_i \leq 2S$ is allowed
  - $S \gg \frac{1}{2} \Rightarrow$ expand in $1/S$

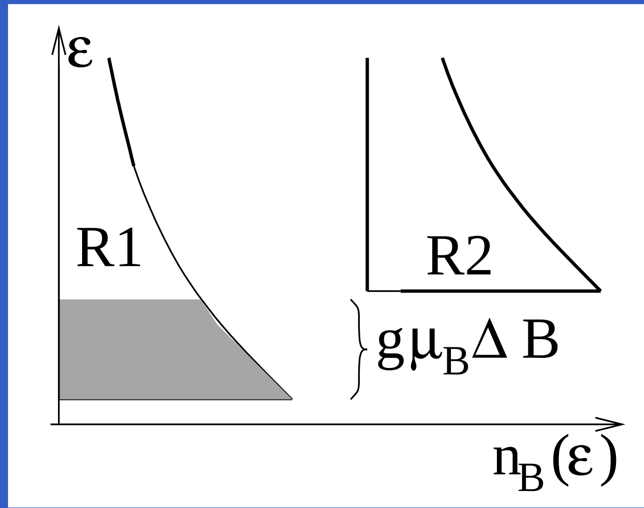
- elementary excitations: spin waves (“magnons”)

Spin transport in magnetic insulators

Heisenberg magnets in spatially varying magnetic fields:

- magnetic field difference at ends of spin system (Meier & Loss 2003)





$\Rightarrow$ magnetization transport

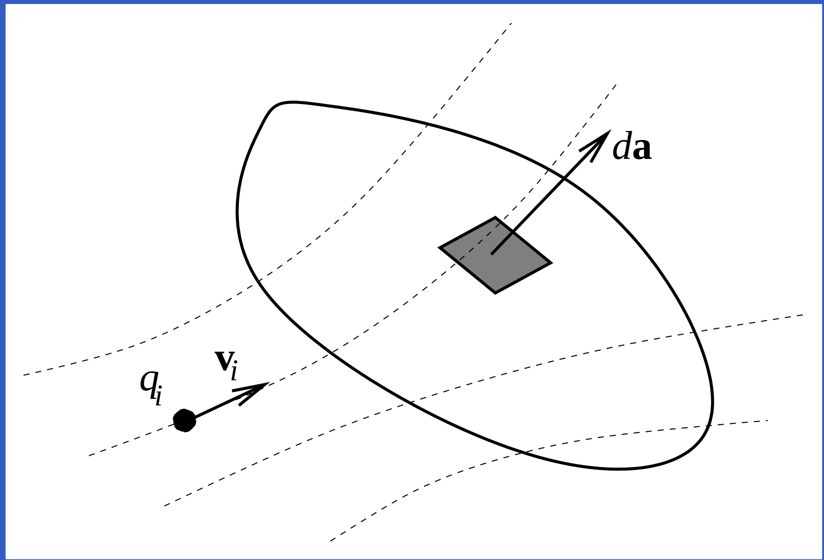
carried by magnons

semiclassical picture of spin transport:

- magnetic field $\boldsymbol{B}_i \Rightarrow$ magnetic moments $\boldsymbol{m}_i(t) = g\mu_{\mathrm{B}}\langle\psi(t)|\boldsymbol{S}_i|\psi(t)\rangle$
- magnetization $\boldsymbol{M}(\boldsymbol{r},t) = \sum_i \delta(\boldsymbol{r}-\boldsymbol{r}_i)\boldsymbol{m}_i(t)$
  time evolution of $\boldsymbol{M}(\boldsymbol{r},t) \Rightarrow$ magnetization transport

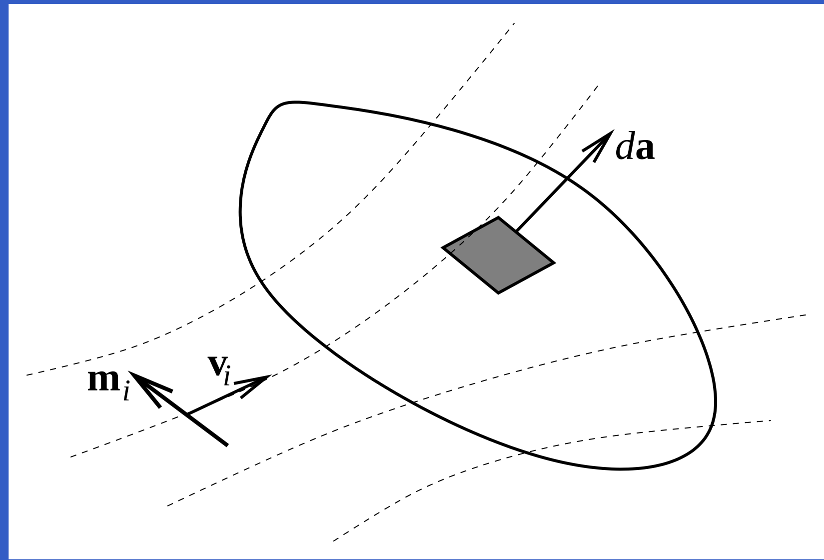
Charge current vs. magnetization current

current of electric monopoles q_i:



- current density: vector
  $j_\mu(\boldsymbol{r}) = \sum_i q_i (\boldsymbol{v}_i)_\mu \delta(\boldsymbol{r} - \boldsymbol{r}_i)$
- current through surface A:
  $$I(A) = \int_A d\boldsymbol{a} \cdot \boldsymbol{j}(\boldsymbol{r})$$

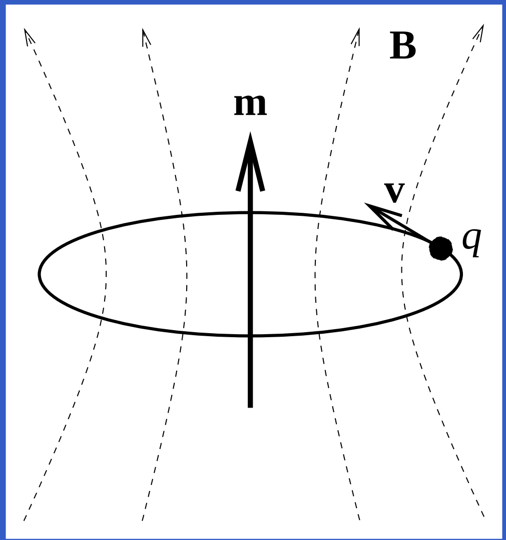
current of magnetic dipoles $\boldsymbol{m}_i$:



- magnetization current density:
  $j^\alpha_\mu(\boldsymbol{r}) = \sum_i (\boldsymbol{m}_i)^\alpha (\boldsymbol{v}_i)_\mu \delta(\boldsymbol{r} - \boldsymbol{r}_i)$
  2nd rank tensor, $\alpha = x, y, z$
- current through surface A:
  $$I^\alpha(A) = \int_A da_\mu j^\alpha_\mu(\boldsymbol{r})$$

Electrodynamics of magnetization currents

stationary charge currents
$\Rightarrow$ static magnetic fields



- Biot-Savart law:

$$\boldsymbol{B}(\boldsymbol{r}) = \int \frac{d^3r'}{c} \boldsymbol{j}(\boldsymbol{r}') \times \frac{(\boldsymbol{r}-\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|^3}$$

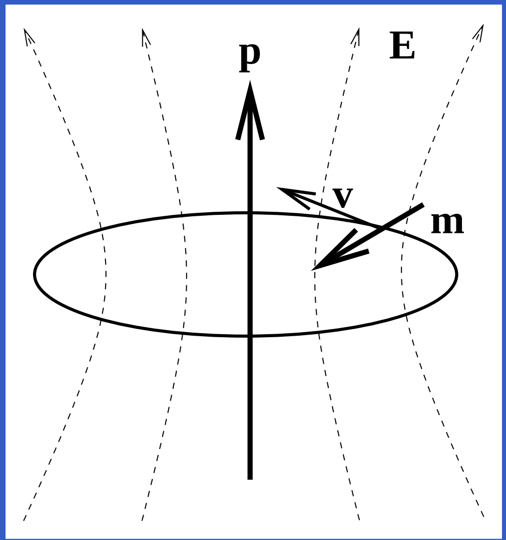
- far zone: $\boldsymbol{A} = \frac{\boldsymbol{m}\times\boldsymbol{r}}{|\boldsymbol{r}|^3}$

$$\boldsymbol{m} = \int \frac{d^3r'}{2c} \boldsymbol{r}' \times \boldsymbol{j}(\boldsymbol{r}')$$

magnetic dipole moment

stationary magnetization currents
$\Rightarrow$ static electric fields

(Hirsch 1999; Meier & Loss 2003)



- Biot-Savart-type law:

$$\phi(\boldsymbol{r}) = -\int \frac{d^3r'}{c} [\boldsymbol{M}(\boldsymbol{r}') \times \boldsymbol{v}(\boldsymbol{r}')] \cdot \frac{(\boldsymbol{r}-\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|^3}$$

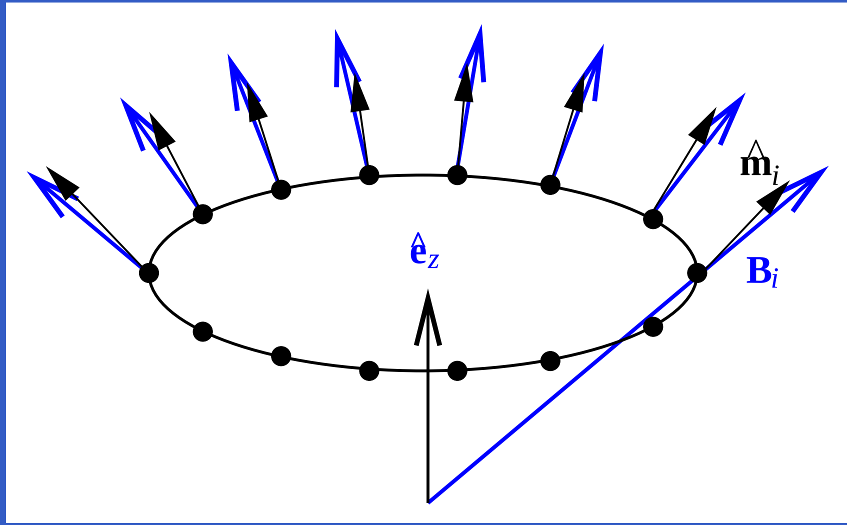
- far zone: $\phi = \frac{\boldsymbol{p}\cdot\boldsymbol{r}}{|\boldsymbol{r}|^3}$, $\boldsymbol{E} = -\nabla\phi$

$$\boldsymbol{p} = -\int \frac{d^3r'}{c} \boldsymbol{M}(\boldsymbol{r}') \times \boldsymbol{v}(\boldsymbol{r}')$$

electric dipole moment

3. Persistent spin currents in insulators

ferromagnetic Heisenberg ring in a crown-shaped magnetic field:



- Hamiltonian:

$$\hat{H} = \frac{1}{2}\sum_{ij} J_{ij}\boldsymbol{S}_i\cdot\boldsymbol{S}_j - g\mu_{\mathsf{B}}\sum_i \boldsymbol{B}_i\cdot\boldsymbol{S}_i$$

- magnetic moments

$$\boldsymbol{m}_i = g\mu_{\mathsf{B}}\langle\boldsymbol{S}_i\rangle = m_i\hat{\boldsymbol{m}}_i$$

- Heisenberg equation of motion:

$$\hbar\frac{\partial\boldsymbol{S}_i}{\partial t} + \boldsymbol{h}_i\times\boldsymbol{S}_i + \sum_j J_{ij}\boldsymbol{S}_i\times\boldsymbol{S}_j = 0 \quad \text{where} \quad \boldsymbol{h}_i \equiv g\mu_{\mathsf{B}}\boldsymbol{B}_i$$

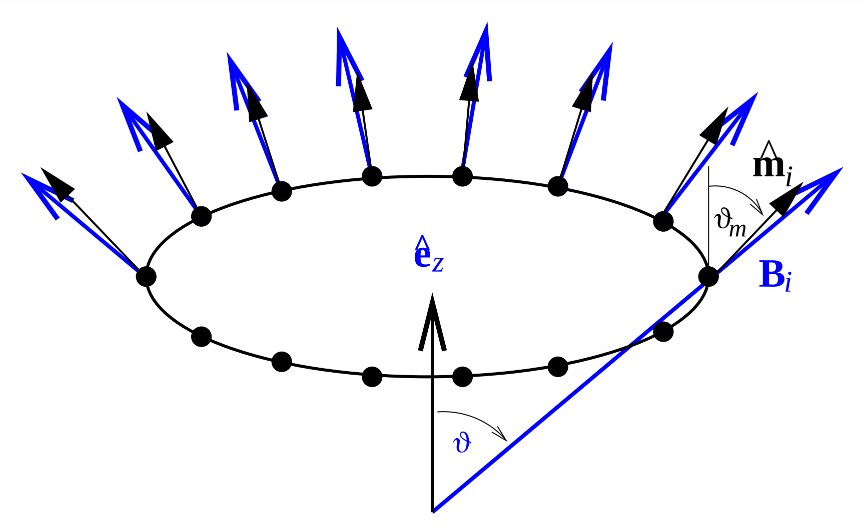
- spin current operator: $\boldsymbol{I}_{i\to j} = J_{ij}(\boldsymbol{S}_i\times\boldsymbol{S}_j)$ (Chandra, Coleman, Larkin 1990)

- longitudinal spin current: $I_s = \hat{\boldsymbol{m}}_i\cdot\langle\boldsymbol{I}_{i\to j}\rangle$

$$\frac{\partial m_i(t)}{\partial t} + \sum_j \hat{\boldsymbol{m}}_i\cdot\langle\boldsymbol{I}_{i\to j}\rangle = 0 \quad \text{continuity equation}$$

Classical ground state

- classically $\boldsymbol{S}_i = S\hat{\boldsymbol{m}}_i \quad \Rightarrow \quad$ stability: $\hat{\boldsymbol{m}}_i \times (\boldsymbol{h}_i - \sum_j J_{ij} S \hat{\boldsymbol{m}}_j) = 0$



FM ring ($J < 0$) with $\boldsymbol{B}(\boldsymbol{r}) \propto \hat{\boldsymbol{r}}$

$$\Rightarrow \sin(\vartheta - \vartheta_m) = \frac{|J|S}{h}\left[1 - \cos\frac{2\pi}{N}\right]\sin 2\vartheta$$

$\Rightarrow$ assume $h \gtrsim JS\left(\frac{2\pi}{N}\right)^2 \equiv \Delta$

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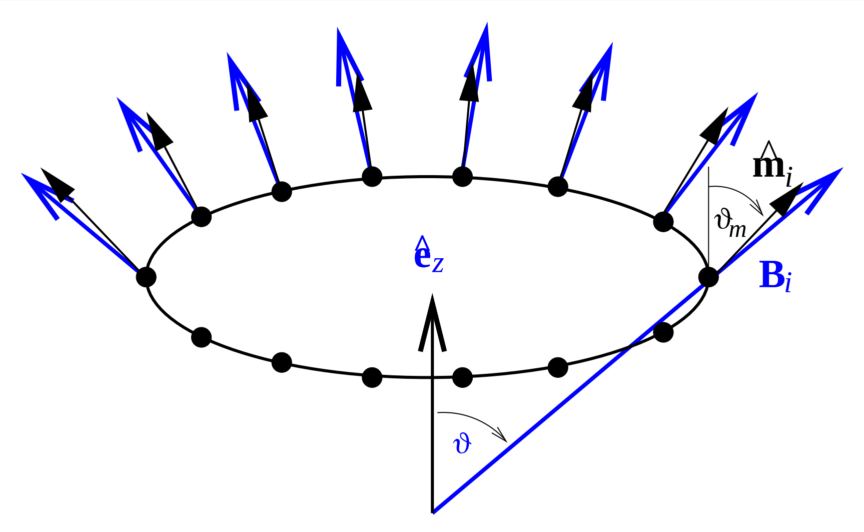

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- quantum spins: $\boldsymbol{S}_i = S_i^{\|}\hat{\boldsymbol{m}}_i + \boldsymbol{S}_i^{\perp} \quad \Rightarrow \quad$ longitudinal part $S_i^{\|} = S - b_i^{\dagger} b_i$
- decomposition of Hamiltonian: $\hat{H} = \hat{H}^{\|} + \hat{H}^{\perp} + O(S^{1/2})$
  - longitudinal: $\hat{H}^{\|} = \frac{1}{2}\sum_{i,j} J_{ij}\hat{\boldsymbol{m}}_i \cdot \hat{\boldsymbol{m}}_j S_i^{\|} S_j^{\|} - \sum_i \boldsymbol{h}_i \cdot \hat{\boldsymbol{m}}_i S_i^{\|} = O(S^2)$
  - transverse: $\hat{H}^{\perp} = \frac{1}{2}\sum_{i,j} J_{ij}\boldsymbol{S}_i^{\perp} \cdot \boldsymbol{S}_j^{\perp} = O(S)$

Transverse basis: local U(1) gauge freedom

- classical spin orientation $\hat{\boldsymbol{m}}_i \Rightarrow$ local quantization axis (fixed)
- but local triad $\{\hat{\boldsymbol{e}}_i^1, \hat{\boldsymbol{e}}_i^2, \hat{\boldsymbol{m}}_i\}$ can be arbitrarily rotated around $\hat{\boldsymbol{m}}_i$
- transverse Hamiltonian: $\quad (\boldsymbol{e}_i^{\pm} = \hat{\boldsymbol{e}}_i^1 \pm i\hat{\boldsymbol{e}}_i^2, \;\; S_i^{\pm} = \boldsymbol{e}_i^{\pm} \cdot \boldsymbol{S}_i^{\perp})$

$$\hat{H}^{\perp} = \frac{1}{8} \sum_{i,j} J_{ij} \sum_{p,p'=\pm} (\boldsymbol{e}_i^{p} \cdot \boldsymbol{e}_j^{p'})\, S_i^{-p} S_j^{-p'}$$

- local U(1) gauge transformation:

$$\boldsymbol{e}_i^{\pm} \to \tilde{\boldsymbol{e}}_i^{\pm} e^{\pm i\alpha_i} \;\; \text{and} \;\; S_i^{\pm} \to S_i^{\pm} e^{\pm i\alpha_i}, \;\; \alpha_i \text{ arbitrary}$$

- spin-wave expansion: $S_i^{+} = \sqrt{2S}\, b_i \, [1 + O(\frac{1}{S})]$

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useful reference: parallel-transported basis

- for neighbors i and j let $\{\tilde{e}_i^1, \tilde{e}_i^2, \hat{\boldsymbol{m}}_i\}$ with $\tilde{e}_i^2 = \hat{\boldsymbol{m}}_i \times \hat{\boldsymbol{m}}_j$
- express $\boldsymbol{e}_i^\pm = \tilde{\boldsymbol{e}}_i^\pm\, e^{\pm i\omega_{i\to j}}$ via angles $\omega_{i\to j}$

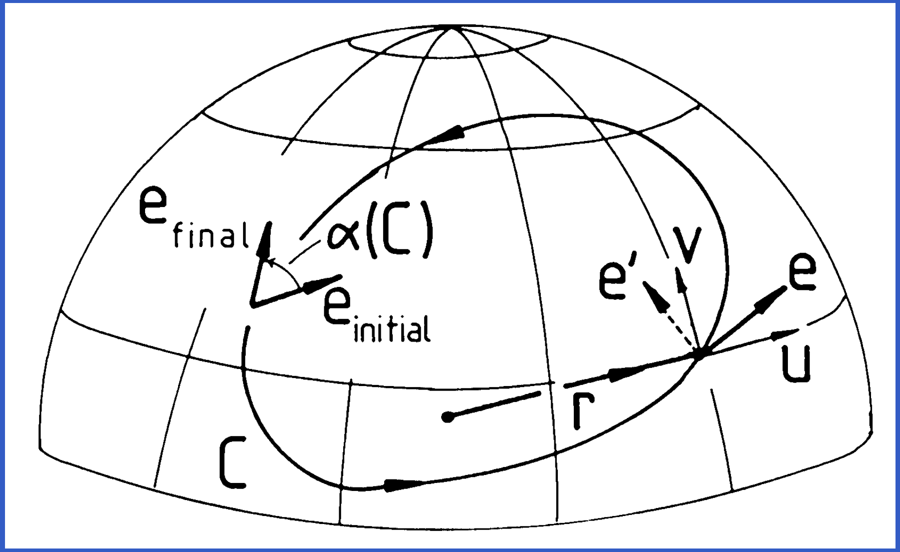
Parallel transport on surface of unit sphere

given:

- closed curve $\mathcal{C}$ on unit sphere, parametrized by $\hat{\boldsymbol{m}}(s)$

- transverse vector defined by angular velocity $\boldsymbol{\omega}(s)$:

$$\frac{d\boldsymbol{e}(s)}{ds} = \boldsymbol{\omega}(s) \times \boldsymbol{e}(s)$$



(M.V.Berry, in: Shapere & Wilczek, *Geometric Phases in Physics*, 1989)

parallel transport:

- $\boldsymbol{\omega}(\boldsymbol{s}) = \hat{\boldsymbol{m}}(s) \times \dfrac{d\hat{\boldsymbol{m}}(s)}{ds}$
- $\hat{\boldsymbol{m}}(s) \cdot \boldsymbol{\omega}(s) = 0 \;\Rightarrow$ transverse basis does not twist around $\hat{\boldsymbol{m}}(s)$
- $\boldsymbol{e}_{\text{initial}} \neq \boldsymbol{e}_{\text{final}}$ (“anholonomy”)

  $\Rightarrow$ defect angle $\alpha(\mathcal{C}) = \angle(\boldsymbol{e}_{\text{initial}}, \boldsymbol{e}_{\text{final}}) =$ solid angle subtended by $\mathcal{C}$

Gauge invariance and current conservation

transverse Hamiltonian:

$$\hat{H}^{\perp} = \frac{1}{8}\sum_{i,j} J_{ij}\Big[(1+\hat{\boldsymbol{m}}_i\cdot\hat{\boldsymbol{m}}_j)\,e^{i(\omega_{i\to j}-\omega_{j\to i})}\,S_i^- S_j^+ - (1-\hat{\boldsymbol{m}}_i\cdot\hat{\boldsymbol{m}}_j)\,e^{i(\omega_{i\to j}+\omega_{j\to i})}\,S_i^- S_j^- + \text{h.c.}\Big]$$

- ferromagnet: $\hat{\boldsymbol{m}}_i\cdot\hat{\boldsymbol{m}}_j = 1 + O(\frac{1}{N})$ $\Rightarrow$ FM magnons
- antiferromagnet: $\hat{\boldsymbol{m}}_i\cdot\hat{\boldsymbol{m}}_j = -1 + O(\frac{1}{N})$ $\Rightarrow$ AF magnons

gauge invariance: $\omega_{i\to j} \to \omega_{i\to j} + \alpha_i, \quad S_i^p \to S_i^p e^{ip\alpha_i}$

- conserved spin current: (Noether's theorem)

$$0 = \langle\frac{\partial\hat{H}^{\perp}}{\partial\alpha_i}\rangle = \langle\sum_j \frac{\partial\hat{H}^{\perp}}{\partial\omega_{i\to j}}\rangle = -\sum_j \underbrace{\langle\hat{\boldsymbol{m}}_i\cdot J_{ij}(\boldsymbol{S}_i^{\perp}\times\boldsymbol{S}_j^{\perp})\rangle}_{=\hat{\boldsymbol{m}}_i\cdot\langle\boldsymbol{I}_{i\to j}\rangle}$$

- agrees with Heisenberg equation of motion

Gauge invariant persistent current

- ferromagnetic next-neighbor coupling $J < 0 \;\Rightarrow\; \hat{\boldsymbol{m}}_i \cdot \hat{\boldsymbol{m}}_{i+1} \approx 1$
- gauge phases away $\Rightarrow$ twisted boundary conditions $S^{\pm}_{i+N} = \exp(\pm i\Omega)\, S^{\pm}_i$

where

$$\Omega = \sum_{i=1}^{N} (\omega_{i\to i+1} - \omega_{i\to i-1}) = \text{gauge invariant geometric flux}$$

$$= \text{total defect angle of } \hat{\boldsymbol{m}}_i \text{ on the unit sphere}$$

$$= 2\pi(1 - \cos\vartheta_m) \text{ for crown-shaped configuration}$$

Gauge invariant persistent current

- ferromagnetic next-neighbor coupling $J < 0 \;\Rightarrow\; \hat{\boldsymbol{m}}_i \cdot \hat{\boldsymbol{m}}_{i+1} \approx 1$
- gauge phases away $\Rightarrow$ twisted boundary conditions $S^{\pm}_{i+N} = \exp(\pm i\Omega)\, S^{\pm}_i$

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gauge invariant persistent spin current:

$$I_s(\Omega) = -\frac{\partial F(\Omega)}{\partial \Omega} = J \langle\, \hat{\boldsymbol{m}}_i \cdot (\boldsymbol{S}^{\perp}_i \times \boldsymbol{S}^{\perp}_{i+1})\,\rangle$$

compare with persistent charge current:

$$I(\phi) = -c\,\frac{\partial \Omega_{gc}(\phi)}{\partial \phi}, \qquad \phi = \text{ magnetic flux}$$

Persistent spin current carried by magnons

- FM spin waves:

$$\hat{H}_{\mathsf{sw}} = \sum_k (\epsilon_k + h)\, b_k^\dagger b_k \quad \text{with} \quad \epsilon_k = 2JS(1-\cos(ka)) \propto k^2$$

- quantized wavevectors: $k = \frac{2\pi}{L}\left(n + \frac{\Omega}{2\pi}\right), \quad n = 0, \pm 1, \ldots, \pm\frac{N}{2}$

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$$I_m(\Omega) = -\frac{g\mu_{\text{B}}}{L}\sum_k \frac{v_k}{e^{(\epsilon_k + h)/T} - 1} \qquad \text{where } v_k = \frac{1}{\hbar}\frac{\partial \epsilon_k}{\partial k} \propto k$$

$$\approx \frac{g\mu_{\text{B}} T}{\hbar} \frac{\sin\Omega}{\cos\Omega - \cosh(2\pi\sqrt{h/\Delta})} \qquad \text{for} \quad \Delta \equiv JS\left(\tfrac{2\pi}{N}\right)^2 \ll T \ll J$$

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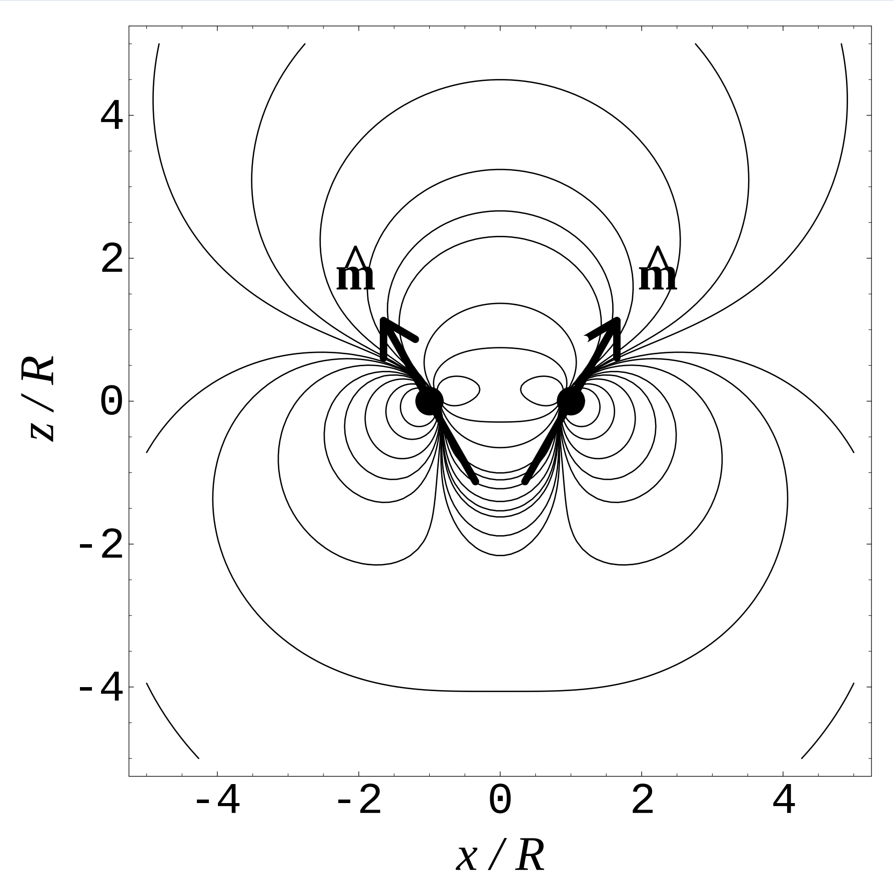
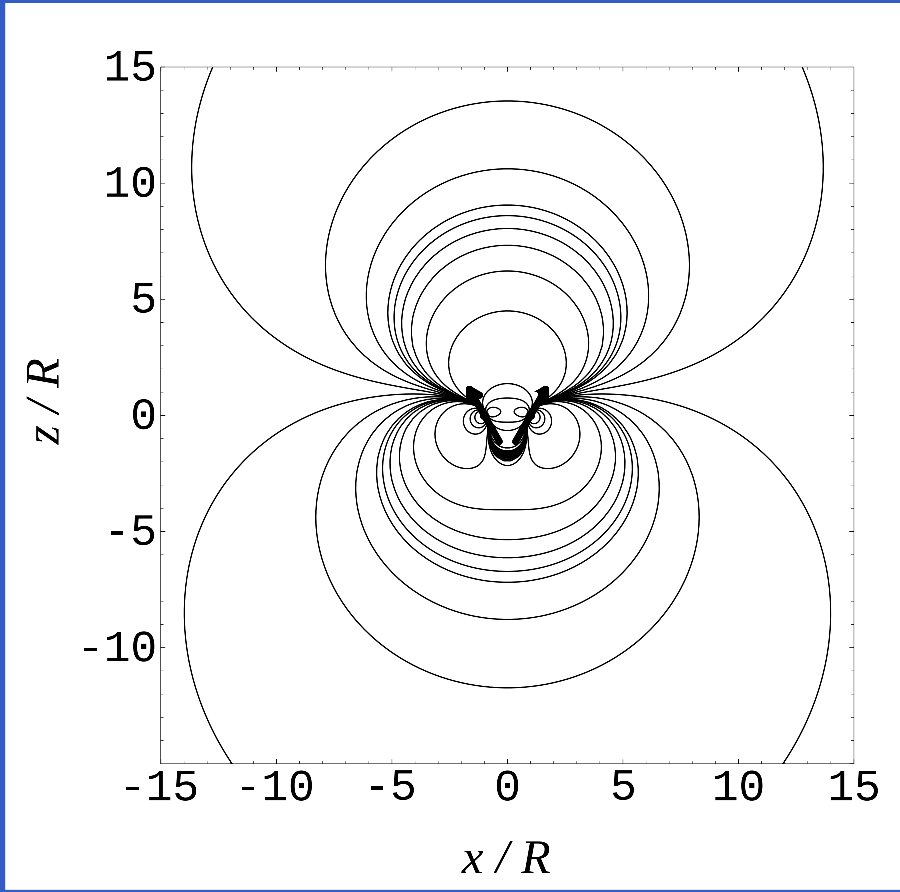
- bosonic analogue of charge current: $I(\phi) = \frac{-e}{L} \sum_k \frac{v_k}{e^{(\epsilon_k - \mu)/T} + 1}$
- thermal fluctuations: $I_m \propto T$
- mesoscopic quantum effect: I_m vanishes in bulk limit or classical limit

Electric field generated by spin current

lines of constant electric potential for $\vartheta_m = 30^o$





generalized Biot-Savart law:

$$\phi(\boldsymbol{r}) = \frac{I_m}{c} \oint [d\boldsymbol{r}' \times \hat{\boldsymbol{m}}(\boldsymbol{r}')] \cdot \frac{(\boldsymbol{r} - \boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|^3}$$

far zone: $\phi(\boldsymbol{r}) = \boldsymbol{p} \cdot \boldsymbol{r}/|\boldsymbol{r}|^3$
with electric dipole moment

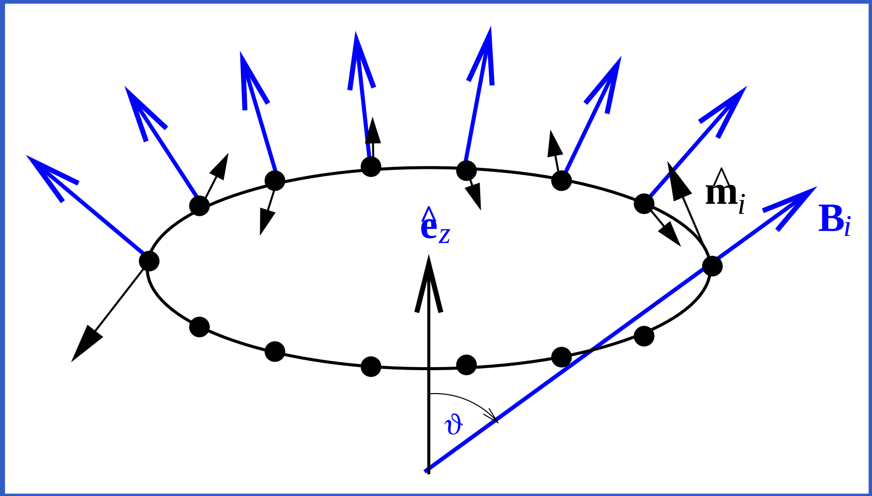
$$\boldsymbol{p} = -\hat{\boldsymbol{e}}_z \frac{I_m}{c} \sin \vartheta_m$$

Experimental parameters

- desired:
  measurement of voltage difference ΔU
  at distance $\approx L$ above and below Heisenberg ring
- optimal parameters:
  - solid angle $\Omega = 2\pi(1 - \cos\vartheta_m) = \pi/2 \Rightarrow \vartheta_m \approx 41^o$
  - magnetic field: $g\mu_{\mathrm{B}} B \approx \Delta = JS(2\pi/N)^2$
  - temperature: $\Delta \ll T \ll J$
- $T = 50\,\mathrm{K}$, $L = 100\,\mathrm{nm}$, $N = 100$, $J = 100\,\mathrm{K} \Rightarrow$ $\boxed{\Delta U \approx 0.2\mathrm{nV},\ B \approx 0.1\mathrm{T}}$
- requirements:
  - well-characterized rings with large Heisenberg coupling J
  - submicron B inhomogeneity
  - nanovolt sensitivity
- detection may be difficult due to screening
- spin rings in inhomogeneous electric fields: Aharonov-Casher effect

Antiferromagnetic spin ring

- antiferromagnetic next-neighbor coupling $J > 0$



crown-shaped field $\boldsymbol{B}(\boldsymbol{r}) \propto \hat{\boldsymbol{r}}$
$\Rightarrow$ spins flip for $h > JS \equiv h_c$

stability of Néel-like state:

$\Rightarrow \sin 2(\bar{\vartheta} - \vartheta) = (\frac{JS}{h})^2 \sin^2(\frac{2\pi}{N}) \sin(2\bar{\vartheta})$

$\Rightarrow$ assume $h \gtrsim JS/N$ ($\ll h_c$)

- two sublattices with $\hat{\boldsymbol{m}}_i \approx (-1)^i \hat{\boldsymbol{n}}_i$ and $\hat{\boldsymbol{n}}_i \cdot \boldsymbol{h}_i \approx 0$

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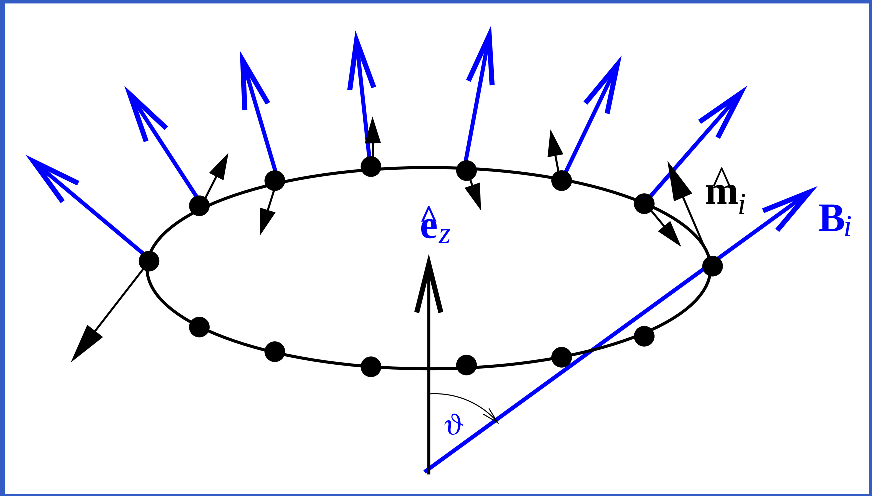

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- twisted boundary conditions $b_{i+N} = e^{\pm i\Omega}\, b_i$
  with $\Omega = \sum_{i=1}^{N} (-1)^i (\omega_{i\to i+1} + \omega_{i\to i-1})$ = total defect angle of $\hat{\boldsymbol{n}}_i$

AF spin waves: quantum fluctations $\Rightarrow$ new magnon vacuum

$$\hat{H}_{\text{sw}} = \sum_{k\geq 0} \epsilon_k \,(\alpha_k^\dagger \alpha_k + \beta_k^\dagger \beta_k - 1), \qquad \epsilon_k = 2JS\sin(ka) \propto k$$

Persistent spin current carried by AF magnons

- spin current: $I_s(\Omega) = J\langle \hat{\boldsymbol{n}}_i \cdot (\boldsymbol{S}_i^{\perp} \times \boldsymbol{S}_{i+1}^{\perp})\rangle = -\partial F_{\mathsf{sw}}/\partial\Omega$
- magnetization current:

$$I_m(\Omega) = -\frac{2g\mu_{\mathrm{B}}}{L}\sum_k c_k\left(n_k + \frac{1}{2}\right) \qquad n_k = \frac{1}{e^{\epsilon_k/T} - 1}, \quad c_k = \frac{1}{\hbar}\frac{\partial \epsilon_k}{\partial k}$$

$$\xrightarrow{T=0} \boxed{I_m = I_m^0\left(1 - \frac{\Omega}{\pi}\right)} \qquad I_m^0 = \frac{g\mu_{\mathrm{B}} c_0}{L} = \frac{g\mu_{\mathrm{B}}}{\hbar}\frac{JS}{N}$$

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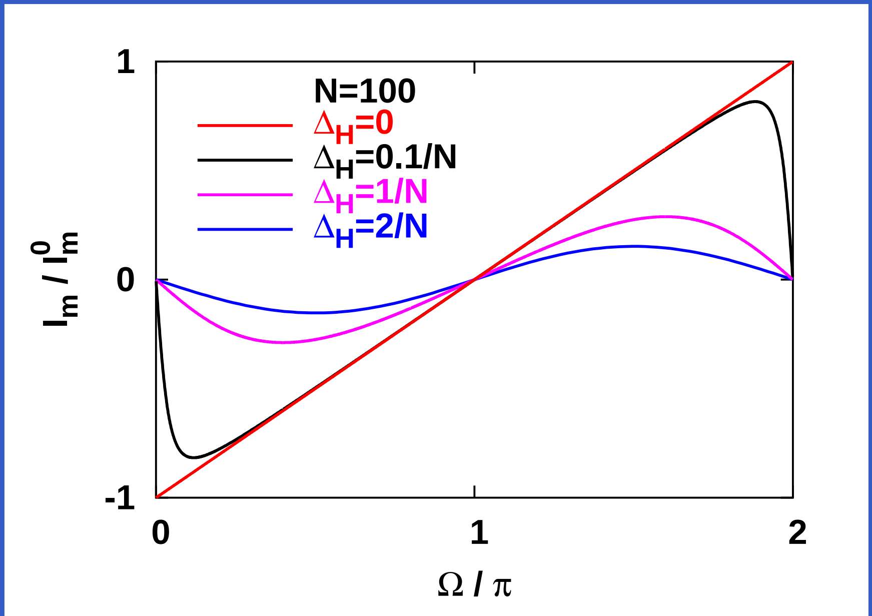
integer spin: Haldane gap $= 2JS\Delta_\text{H} \propto e^{-\pi S} \propto a/\xi$, $\quad \Delta_\text{H}(S{=}1) \approx 0.2$



"modified" spin-wave theory: suppress Neél order on average $\sum_i\langle \boldsymbol{S}_i \cdot \boldsymbol{m}_i\rangle = 0$ via staggered field

(Takahashi 1989, Hirsch & Tang 1989)

$$\Delta_\text{H} \gg \frac{1}{N} \Rightarrow \frac{I_m}{I_m^0} \propto \sqrt{\frac{L}{\xi}}\, e^{-L/\xi}\, \sin\Omega$$

4. Conclusion & outlook

persistent spin currents in Heisenberg magnets:

- mesoscopic quantum interference effect
- bosonic analogue of charge currents in metal rings: $\frac{\Omega}{2\pi} \leftrightarrow \frac{\phi}{\phi_0}$
- magnetization transported by magnons due to
  - thermal fluctuations (ferromagnets)
  - quantum fluctuations (antiferromagnets)
- spin current induces dipole-like electric field
- experimental requirements:
  - well-defined Heisenberg rings
  - submicron field inhomogeneities
  - detection of nanovoltages

open questions:

- diffusive regime of disordered magnets

Metallasiloxanolates

current activities in DFG-Forschergruppe FOR 412: (Molodtsova et al. 2003)

$C_{68}H_{172}Cu_{10}N_{16}O_{56}Si_{20}$