Superfluid Helium-3:
From very low Temperatures to the Big Bang

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Contents:

- The quantum liquids $^3$He and $^4$He
- Superfluid phases of $^3$He
- Broken symmetries and long-range order
- Topologically stable defects
- Big Bang simulation in the low temperature lab
Helium

Two stable Helium isotopes:

\[ ^{4}\text{He}: \text{air, oil wells, ...} \quad \text{Janssen/Lockyer (1868)} \]
\[ ^{3}\text{He}: \text{Ramsay (1895)} \]

\[ ^{6}\text{Li} + ^{1}\text{n} \rightarrow ^{3}\text{H} + ^{4}\text{He} \quad \text{(1939)} \]
\[ \downarrow \]
\[ ^{3}\text{He} + ^{2}\text{He} + ^{0}\text{e}^{-} \]

\[ \frac{\text{He}}{\text{Luft}} \approx 5 \times 10^{-6}, \quad \frac{^{3}\text{He}}{^{4}\text{He}} \approx 1 \times 10^{-6} \]

Research on macroscopic samples of \(^{3}\text{He}\) since 1947
Helium

Atoms: spherical, hard core diameter $\sim 2.5$ Å

Interaction: • hard sphere repulsion
  • van der Waals dipole/multipole attraction

Boiling point: 4.2 K, $^4$He Kamerlingh Onnes (1908)
  3.2 K, $^3$He Sydoriak, et al. (1949)

Dense, simple liquid
  isotropic
  short-range interactions
  extremely pure

Nobel Prize 1913
Helium

Atoms:
- spherical shape \(\rightarrow\) weak attraction
- light mass \(\rightarrow\) strong zero-point motion

\[ T \to 0, \ P \lessapprox 30 \ \text{bar}: \ \text{Helium remains liquid} \]

\[ \lambda \propto \frac{\hbar}{\sqrt{k_B T}} \xrightarrow{T \to 0} \ \text{Macroscopic quantum phenomena} \]
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<th><strong>Helium</strong></th>
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<td><strong>2 e$^-$, $S = 0$</strong></td>
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<td><strong>Nucleus:</strong></td>
<td><strong>$S = 0$</strong></td>
<td><strong>$S = \frac{1}{2}\hbar$</strong></td>
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<td><strong>Atom(!) is a</strong></td>
<td><strong>Boson</strong></td>
<td><strong>Fermion</strong></td>
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<td><strong>Phase transition</strong></td>
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Fermi gas: **Ground state**
Fermi gas: **Excited states** \((T>0)\)

**Switch on interaction adiabatically**
Landau Fermi liquid

1-1 correspondence between k-states

(Quasi-) Particle = elementary excitation

Prototype: Helium-3

- Large effective mass
- Strongly enhanced spin susceptibility
- Strongly reduced compressibility

Instability?
Instability of Landau Fermi liquid

+ 2 non-interacting particles

Fermi sea

\[ k_x, k_y, k_z \]
Arbitrarily weak attraction $\Rightarrow$ Cooper instability

Cooper pair

Universal fermionic property
Arbitrarily weak attraction $\Rightarrow$ Cooper pair $(k, \alpha; -k, \beta)$

\[
\Psi_{L=0,2,4,\ldots} = \psi(r)|\uparrow\downarrow - \downarrow\uparrow\rangle
\]

$S=0$ (singlet)

\[
\Psi_{L=1,3,5,\ldots} = \psi_+ (r)|\uparrow\uparrow\rangle + \psi_0 (r)|\uparrow\downarrow + \downarrow\uparrow\rangle + \psi_- (r)|\downarrow\downarrow\rangle
\]

$L = 0$: isotropic wave function
$L > 0$: anisotropic wave function

Helium-3: Strongly repulsive interaction $\Rightarrow L > 0$ expected
Generalization to macroscopically many Cooper pairs

BCS theory

Bardeen, Cooper, Schrieffer (1957)

$\epsilon_c \ll E_F$

"Pair condensate"
with macroscopically coherent wave function

Transition temperature

$$T_c = 1.13 \epsilon_c \exp\left(-\frac{1}{N(0)|V_L|}\right)$$

"weak coupling theory"

$\epsilon_c, V_L$: Magnitude? Origin? $\Rightarrow T_c$?

Thanksgiving 1971: Transition in $^3$He at $T_c = 0.0026$ K

Osheroff, Richardson, Lee (1972)
The Nobel Prize in Physics 1996
"for their discovery of superfluidity in helium-3"

David M. Lee
Cornell (USA)

Douglas D. Osheroff
Stanford (USA)

Robert C. Richardson
Cornell (USA)
Phase diagram of Helium-3

P-T phase diagram
Phase diagram of Helium-3

P-T phase diagram

Dense, simple liquid

- isotropic short-range interactions
- extremely pure
- nuclear spin $S=1/2$

Ordered spins
Disordered spins
Solid (bcc)

Superfluid A phase
Superfluid B phase

Normal Fermi liquid

Gas
Phase diagram of Helium-3

P-T-H phase diagram

“Very low temperatures”: $T \ll T_{\text{boiling}} \sim 3-4 \text{ K}$
$\ll T_{\text{backgr. rad.}} \sim 3 \text{ K}$
Superfluid phases of $^3$He

Theory + experiment: $L=1$, $S=1$ in all phases
Leggett
Wölfle
Mermin, ...

Attraction due to spin fluctuations
Anderson, Brinkman (1973)

→ anisotropy directions in a $^3$He Cooper pair
orbital part
spin part
... and a mystery!

NMR experiment on nuclear spins $I = \frac{1}{2} \hbar$  

$\omega^2 - \omega_L^2 \propto \Delta^2(T)$

$\omega_L$  

$Larmor$ frequency: $\omega_L = \gamma H$

Shift of $\omega_L \iff$ spin-nonconserving interactions  

$\rightarrow$ nuclear dipole interaction $g_D \sim 10^{-7} K \ll T_C$

Origin of frequency shift ?!  

Leggett (1973)
The superfluid phases of $^3$He
B-phase

\[ \Psi = |↑↑⟩ + |↑↓⟩ + |↓↑⟩ + |↓↓⟩ \]

\[ \Delta(\mathbf{k}) = \Delta_0 \]

Balian, Werthamer (1963)  
Vdovin (1963)

(pseudo-) isotropic state ↔ s-wave superconductor

Weak-coupling theory: stable for all \( T < T_c \)
A-phase

\[ \Psi = |\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle \rightarrow \text{strong anisotropy} \]

\[ \Delta(\hat{k}) = \Delta_0 \sin(\hat{k},\hat{l}) \]  
Anderson, Morel (1961)

Cooper pair
orbital angular momentum

“Axial state” has point nodes

→ “unconventional” pairing in
  • heavy fermion/high-\(T_c\) superconductors
  • \(\text{Sr}_2\text{RuO}_4\)

Strong-coupling effect
$E_k^2 = v_F^2 (k - k_F)^2 + \Delta_0^2 \sin^2 (\hat{k}, \hat{i}) = g_{ij} p_i p_j$

e = \begin{cases} 
+1 & \hat{k} \parallel + \hat{i} \\
-1 & \hat{k} \parallel - \hat{i} 
\end{cases} \quad 2 \text{ chiralities}

A = k_F \hat{i}

p = k - eA

Lorentz invariance: Symmetry enhancement at low energies
$E_k^2 = v_F^2 (k - k_F)^2 + \Delta_0^2 \sin^2 (\hat{k}, \hat{l}) = g^{ij} p_i p_j$

\[
e = \begin{cases} 
+1 & \hat{k} \parallel \hat{i} \\
-1 & \hat{k} \parallel -\hat{i} 
\end{cases} 
\]

2 chiralities

$g^{ij} = v_F^2 l i l_j + \left( \frac{\Delta}{k_F} \right)^2 (\delta_{ij} - l_i l_j)$

$\Leftrightarrow$ Massless, chiral leptons, e.g., neutrino $E(p) = cp$

$\Rightarrow$ Chiral anomaly of standard model

$A_1$-phase

finite magnetic field

$\Psi = |\uparrow\uparrow\rangle$

Long-range ordered magnetic liquid
Broken Symmetries, Long Range Order
Broken Symmetries, Long Range Order

Normal $^3\text{He} \leftrightarrow ^3\text{He-A}, ^3\text{He-B}$: 2. order phase transition

$T<T_c$: higher order, lower symmetry of ground state

I. Ferromagnet

$T>T_c$:
- Average magnetization: $\langle M \rangle = 0$
- Symmetry group: $\text{SO}(3)$

$T<T_c$:
- Average magnetization: $\langle M \rangle \neq 0$
- Order parameter
- Symmetry group: $\text{U}(1) \subseteq \text{SO}(3)$

$T<T_c$: $\text{SO}(3)$ rotation symmetry in spin space spontaneously broken
2. order phase transition

\(T < T_c\): higher order, lower symmetry of ground state

II. Liquid crystal

Symmetry group:
- \(T > T_c\): \(SO(3)\)
- \(T < T_c\): \(U(1) \subset SO(3)\)

\(T < T_c\): \(SO(3)\) rotation symmetry in real space spontaneously broken
Broken Symmetries, Long Range Order

2. order phase transition

$T<T_c$: higher order, lower symmetry of ground state

III. Conventional superconductor

$T>T_c$

Pair amplitude $\left\langle c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger \right\rangle = 0$

Gauge transf. $c_{k\sigma}^\dagger \rightarrow c_{k\sigma}^\dagger e^{i\varphi}$: gauge invariant

Symmetry group $U(1)$

$T<T_c$

$\Delta e^{i\varphi}$ “Order parameter“

not gauge invariant

---

Gauge transformation:

$|\psi\rangle \rightarrow e^{i\varphi} |\psi\rangle$
Broken Symmetries, Long Range Order

2. order phase transition

$T < T_c$: higher order, lower symmetry of ground state

III. Conventional superconductor

$T > T_c$  
$T < T_c$: $U(1)$ “gauge symmetry“ spontaneously broken
Cooper pair: $\hat{d}$ orbital part

$L=1$, $S=1$ in all phases

$\hat{l}$ spin part

$SO(3)_S \times SO(3)_L \times U(1)_\phi$ symmetry spontaneously broken

Characterized by $2 \times (2L + 1) \times (2S + 1) = 18$ real numbers

3x3 order parameter matrix $A_{i\mu}$

Quantum coherence in

- phase
- anisotropy direction for spin
- anisotropy direction in real space

Superfluid, magnetic liquid crystal

Leggett (1975)
Broken symmetries in superfluid $^3$He

$3$He-$A$  $\text{SO}(3)_S \times \text{SO}(3)_L \times U(1)_\varphi$ symmetry broken

$U(1)_{S_z} \times U(1)_{L_z-\varphi}$

"Unconventional" pairing

Cooper pairs

Fixed absolute orientation

Mineev (1980)
Bruder, DV (1986)
Resolution of the NMR puzzle

\[ \omega^2 - \omega_L^2 \propto \Delta^2(T) \]
Superfluid $^3$He - a quantum amplifier

Cooper pairs in $^3$He-A

What determines the actual relative orientation of $\hat{d}, \hat{i}$?

$\rightarrow$ Anisotropic spin-orbit interaction of nuclear dipoles:

$\uparrow \uparrow \neq \rightarrow \rightarrow$

Dipole-dipole coupling of $^3$He nuclei: $g_D \sim 10^{-7} K \ll T_C$

Unimportant ?!
Superfluid $^3$He - a quantum amplifier

Cooper pairs in $^3$He-A

Fixed absolute orientation

• Long-range order in $\hat{d}, \hat{l}$
• $g_D \sim 10^{-7} K$: tiny, but lifts degeneracy of relative orientation

Quantum coherence

$\hat{d}, \hat{l}$ locked in all Cooper pairs

NMR frequency increases:

$$\omega^2 = (\gamma H)^2 + g_D \Delta^2(T)$$

Leggett (1973)

→ Nuclear dipole interaction macroscopically measurable
The Nobel Prize in Physics 2003
"for pioneering contributions to the theory of superconductors and superfluids"

Alexei A. Abrikosov
USA and Russia

Vitaly L. Ginzburg
Russia

Anthony J. Leggett
UK and USA
Order parameter textures and topological defects
Order parameter textures

Orientation of anisotropy directions $\hat{d}, \hat{l}$ in $^3$He-A?

Magnetic field $\rightarrow \hat{d}$

Walls $\rightarrow \hat{l}$

$\rightarrow$ “Textures“ in $\hat{d}, \hat{l}$ $\leftrightarrow$ liquid crystals

$\rightarrow$ Topologically stable defects: Classification by homotopy theory
Order parameter textures and topological defects

D=2: domain walls in $\hat{d}$ or $\hat{l}$

Single domain wall

Domain wall lattice
Order parameter textures and topological defects

D=1: Vortices

Vortex formation (rotation experiments)
e.g., Mermin-Ho vortex (non-singular)
Order parameter textures and topological defects

D=0: Monopoles

“Boojum” in $\hat{\ell}$-texture of $^3$He-A (geometric constraint)

Defect formation by, e.g.,

- rotation
- geometric constraints
- rapid crossing through phase transition
Big bang simulation in the low temperature lab
Universality in continuous phase transitions

- **High symmetry, short-range order**
  - $T > T_c$
  - Spins: paramagnetic
  - Defects: domain walls

- **Broken symmetry, long-range order**
  - $T = T_c$
  - Spins: ferromagnetic
  - Defects: vortices, etc.

- **$T < T_c$**
  - Helium: normal liquid
  - Cosmic strings, etc. Kibble (1976)
  - Nucleation of galaxies?

**Universe:**
- Unified forces and fields

**Helium:**
- Superfluid

**Spins:**
- Paramagnetic
- Ferromagnetic

**Defects:**
- Domain walls
- Vortices, etc.

**Kibble (1976):** BANG!
Rapid thermal quench through 2. order phase transition  

Kibble (1976)

1. Local temperature $T \gg T_C$

   Expansion + rapid cooling

2. Nucleation of independently ordered regions

   Clustering of ordered regions

   $\rightarrow$ Defects

3. 

   Defects overlap

4. 

   $T < T_C$ : Vortex tangle

Estimate of density of defects  

Zurek (1985)

"Kibble-Zurek mechanism": How to test?
Big bang simulation in the low temperature laboratory

Grenoble: Bäuerle et al. (1996), Helsinki: Ruutu et al. (1996)

Measured vortex tangle density:
Quantitative support for Kibble-Zurek mechanism
Present research on superfluid $^3$He: Quantum Turbulence

**Classical Turbulence**

Leonardo da Vinci (1452-1519)

Flow through grid

**Quantum Turbulence** = Turbulence in the absence of viscous dissipation (superfluid at $T \to 0$)

- Why are quantum and classical turbulence so similar?
- What provides dissipation in the absence of friction?

Test system: $^3$He-B

Vinen, Donnelly: *Physics Today* (April, 2007)
Superfluid Helium-3:

- **Anisotropic superfluid**
  - 3 different bulk phases
  - Cooper pairs with internal structure

- **Large symmetry group broken**
  - Close connections to particle theory
  - Zoo of topological defects
  - Kibble-Zurek mechanism quantitatively verified