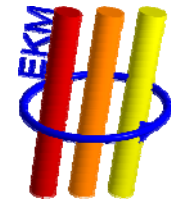




Center for Electronic Correlations and Magnetism
University of Augsburg



Construction of exact ground states for non-integrable lattice fermion models

Workshop on “*Resonating Valence Bond Physics: Spin Liquids and Beyond*”

Dedicated to the memory of Patrik Fazekas



Budapest; October 14, 2010

Dieter Vollhardt

Supported by **DFG**

TRR 80 (Augsburg-Munich)



DV 9.6.86

with compliments
Pál Fazekas

KFKI-1986-28/E

P. FAZEKAS

VARIATIONAL GROUND STATE FOR
THE PERIODIC ANDERSON MODEL

Hungarian Academy of Sciences

CENTRAL
RESEARCH
INSTITUTE FOR
PHYSICS

BUDAPEST



**First International Workshop on
Ordering Phenomena in Transition Metal Oxides
7.-10. October 2001, Kloster Irsee;**



**First International Workshop on
Ordering Phenomena in Transition Metal Oxides
7.-10. October 2001, Kloster Irsee;**

Augsburg



First International Workshop on
Ordering Phenomena in Transition Metal Oxides
7.-10. October 2001, Kloster Irsee;



Second International Workshop on
Ordering Phenomena in Transition Metal Oxides
26.-29. September 2004, Wildbad Kreuth

Construction of exact ground states for non-integrable lattice fermion models

Outline:

- Transformation of a Hamiltonian into positive semidefinite form
- Construction of exact many-electron ground states
- **Triangle** Hubbard chains
→ application to CeRh_3B_2
- **Pentagon** Hubbard chains
→ application to organic polymers

In collaboration with
Zsolt Gulacsi (Univ. Debrecen, Hungary)
Arno Kampf (Augsburg)

Highly desirable:
Exact solutions of correlation models

Construction of exact many-electron ground states from positive semidefinite Hamiltonians

Strategy

Step 1: Transform many-electron Hamiltonian into positive semidefinite form

$$\hat{H} = \hat{H}_0 + \hat{H}_U = \sum_n \hat{P}_n + E_g \equiv \hat{H}' + E_g, \quad \hat{P}_n : \text{positive semidefinite operators}$$
$$\langle \psi | \hat{P}_n | \psi \rangle \geq 0$$
$$\text{e.g., } \hat{P}_n = \Omega^\dagger \Omega, \quad \Omega \Omega^\dagger$$

→ well-defined lower bound of spectrum

Transformation feasible

- usually in several different ways
- on a subspace of the full parameter space

Construction of exact many-electron ground states from positive semidefinite Hamiltonians

Strategy

$$\hat{H} = \hat{H}_0 + \hat{H}_U = \sum_n \hat{P}_n + E_g \equiv \hat{H}' + E_g, \quad \hat{P}_n : \text{positive semidefinite}$$
$$\langle \psi | \hat{P}_n | \psi \rangle \geq 0$$

Step 2: Construct many-electron ground state

$$\hat{P}_n |\Psi_g\rangle = 0 \Rightarrow \hat{H} |\Psi_g\rangle = E_g |\Psi_g\rangle$$

ground state ground-state energy

Construction of exact many-electron ground states from positive semidefinite Hamiltonians

Strategy

$$\hat{H} = \hat{H}_0 + \hat{H}_U = \sum_n \hat{P}_n + E_g \equiv \hat{H}' + E_g, \quad \hat{P}_n : \text{positive semidefinite}$$
$$\langle \psi | \hat{P}_n | \psi \rangle \geq 0$$

Step 3: Prove uniqueness of many-electron ground state:

$$|\Psi_g\rangle \text{ spans } \ker(\hat{H}') := \{|\phi\rangle \mid \hat{H}'|\phi\rangle = 0\}$$

- Works in any dimension
- No integrability required
- Applicable to any Hamiltonian with sufficiently many microscopic parameters (so far not possible for Hamiltonians with too simple structure)

In particular: Exact ground states with flat bands

Condensed matter systems with macroscopic degeneracies

- very sensitive reaction to perturbations
- emergent behavior

Examples:

- Electrons in a magnetic field in 2D Landau (1930)
- Spins on lattices with geometric frustration Moessner, Ramirez (2006)
- Dispersionless (“flat“) bands in solids Mielke (1991)
Mielke, Tasaki (1993)
Arita, Suwa Kuroki, Aoki (2002)

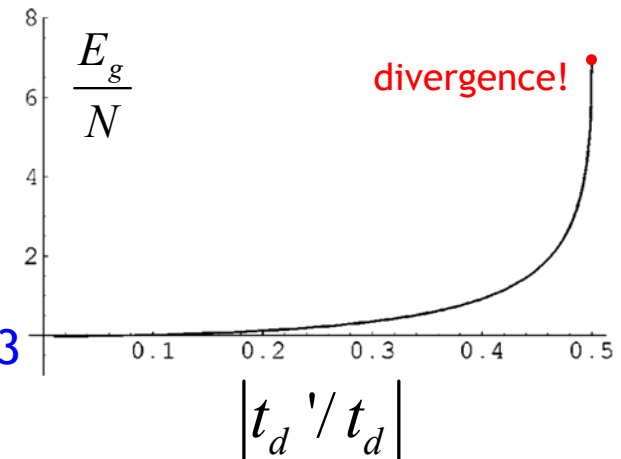
→ Find exact solutions of correlation models with flat bands

Construction of exact many-electron ground states with flat bands from positive semidefinite Hamiltonians

Hubbard model and PAM on decorated lattices in $d \geq 2$ at $U = \infty$	Brandt, Gieseke (1992)
PAM, Emery model in $d=1,2$ on regular lattices	Strack (1993)
Ferromagnetism in extended Hubbard models for arbitrary d	Strack, DV (1993, 1994, 1995)
	de Boer, Schadschneider (1995); Kollar, Strack, DV (1996)
Composite operators defined on the entire unit cell	Orlik, Gulacsi (1998, 2001)
→ PAM on regular lattices in $d=2,3$ at $U = \infty$	
Application to $d=1$	Sarasua, Continentino (2002)
PAM on regular lattices in $d=2,3$ at $U < \infty$	Gurin, Gulacsi (2001, 2002), Gulacsi (2002)
Superconductivity in the extended Hubbard model	Montorsi, Campbell (1996), Laughlin (2006)

PAM in $d=3$

- Exact insulating and itinerant (non-Fermi liquid) ground states at $1/4$ and $3/4$ filling
- First exact proof of ferromagnetism in the PAM in $d=3$



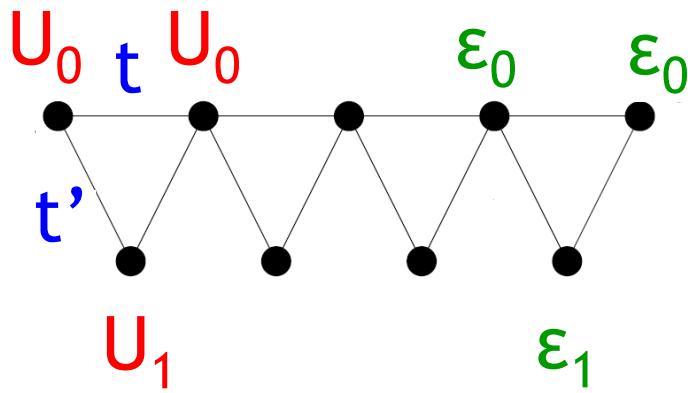
Gulacsi, DV (2003, 2005)

High sensitivity to small changes of microscopic parameters found

1. Exact many-electron ground states on triangle Hubbard chains

Flat bands in the single-electron band structure persist for $U > 0$

Z. Gulacsi, A. Kampf, DV; Prog. Theor. Phys. Suppl. 176, 1 (2008)



2 sites per cell \rightarrow 2 bands

$N_c = \#$ cells

$N = \#$ electrons

$n = \frac{N}{2N_c}$ electron density

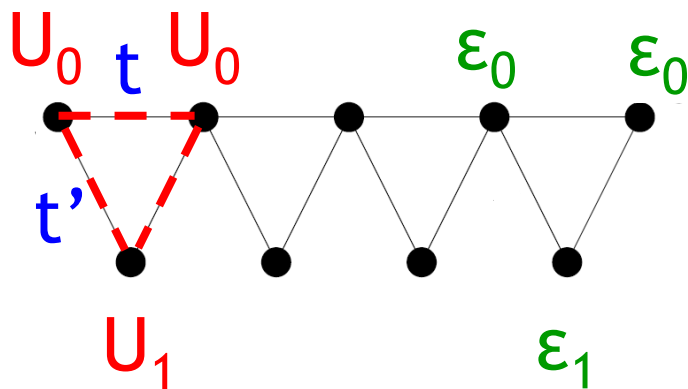
Müller-Hartmann (1995)

Penc, Shiba, Mila, Tsukagoshi (1996)

Fazekas (1997)

Derzho, Honecker, Richter (2007)

Derzho, Honecker, Richter, Maksymenko, Moessner (2010)



2 sites per cell \rightarrow 2 bands

$N_c = \#$ cells

$N = \#$ electrons

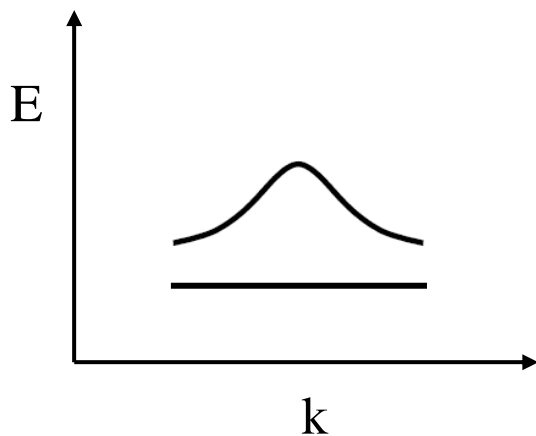
$n = \frac{N}{2N_c}$ electron density

$$\frac{(t')^2}{t} = \epsilon_1 - \epsilon_0 + 2t, \quad t > 0$$

Solution I:

$U_0, U_1 > 0$

$$\epsilon_1 - \epsilon_0 > -2t$$



Single-electron bands

Step 1 $\hat{A}_{i,\sigma} = \sqrt{t} [\hat{c}_{i,\sigma} + \hat{c}_{i+a,\sigma} + (t'/t)\hat{c}_{i+r,\sigma}]$

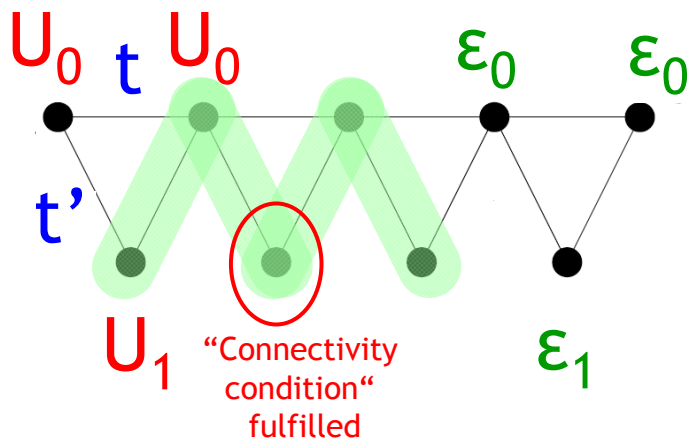
Positive semidefinite $\hat{H}^{tri} = \sum_{i,\sigma} \hat{A}_{i,\sigma}^\dagger \hat{A}_{i,\sigma} + K\hat{N} + H_U^{tri}$
 $K = \epsilon_0 - 2t$

Step 2

Construct operators B with $\{\hat{A}_{i,\sigma}, \hat{B}_{i',\sigma'}^\dagger\} = 0$

$$\hat{B}_{i,\sigma}^\dagger = [\hat{c}_{i-a+r,\sigma}^\dagger + \hat{c}_{i+r,\sigma}^\dagger - (t'/t)\hat{c}_{i,\sigma}^\dagger]$$

Ground state $|\Psi^{tri}(N)\rangle = \prod_i \hat{B}_{i,\sigma_i}^\dagger |0\rangle$



2 sites per cell \rightarrow 2 bands

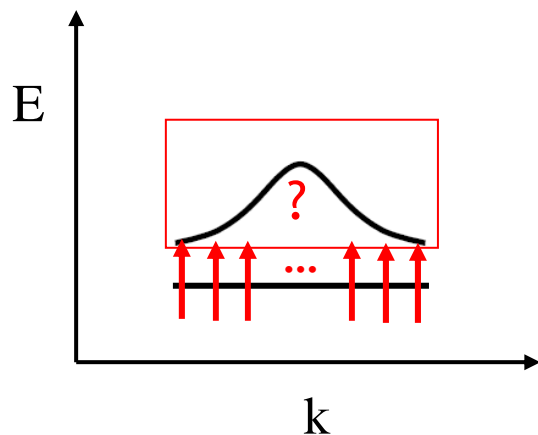
$$N_c = \# \text{ cells}$$

$$N = \# \text{ electrons}$$

$$n = \frac{N}{2N_c} \text{ electron density}$$

$$\frac{(t')^2}{t} = \epsilon_1 - \epsilon_0 + 2t, \quad t > 0$$

$$\epsilon_1 - \epsilon_0 > -2t$$



Solution I:

$$U_0, U_1 > 0$$

Physical properties

$n < 1/2$: ferromagnetic clusters

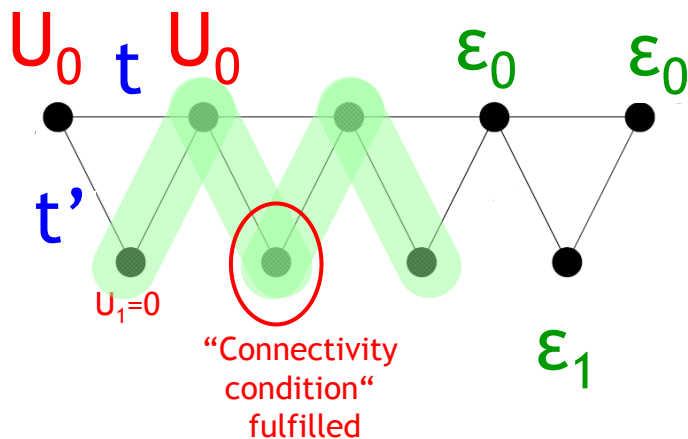
$n = 1/2$: fully saturated ferromagnet

Mielke, Tasaki (1993)

Derzho, Honecker, Richter (2007)

\rightarrow **Flat-band ferromagnetism:** Realizes ideas of Gutzwiller and Kanamori from 1963 about the origin of itinerant ferromagnetism

Not related to Stoner theory: $\chi(\omega=0, \mathbf{q}=0) = \infty \rightarrow UN(E_F)=1$



2 sites per cell \rightarrow 2 bands

$N_c = \#$ cells

$N = \#$ electrons

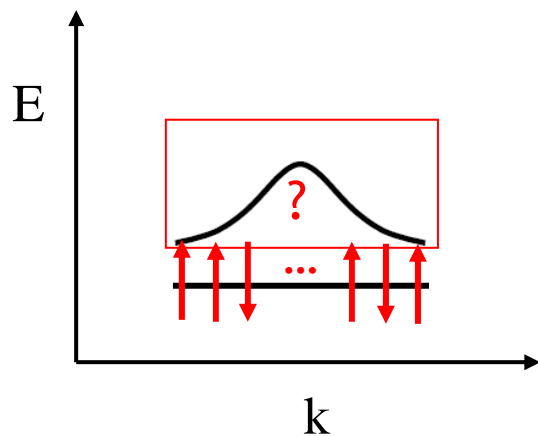
$n = \frac{N}{2N_c}$ electron density

$$\frac{(t')^2}{t} = \epsilon_1 - \epsilon_0 + 2t, \quad t > 0$$

$$\epsilon_1 - \epsilon_0 > -2t$$

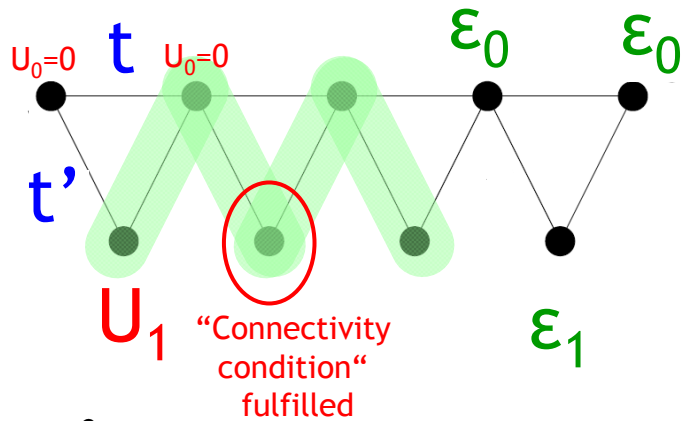
Solution II:

$$U_0 > 0, \quad U_1 = 0$$



$n=1/2$: non-magnetic

$U_1=0$: electrons uncorrelated on sites where Wannier functions connect



2 sites per cell \rightarrow 2 bands

$N_c = \#$ cells

$N = \#$ electrons

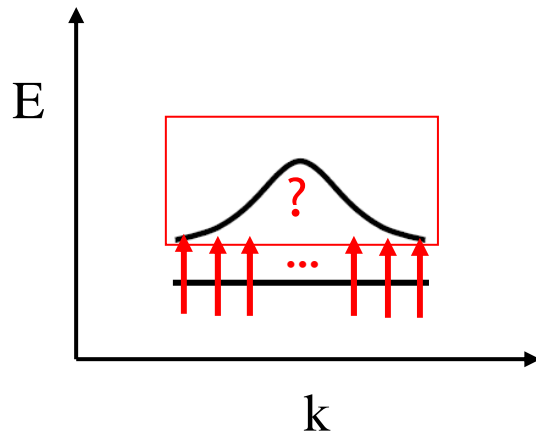
$n = \frac{N}{2N_c}$ electron density

$$\frac{(t')^2}{t} = \epsilon_1 - \epsilon_0 + 2t, \quad t > 0$$

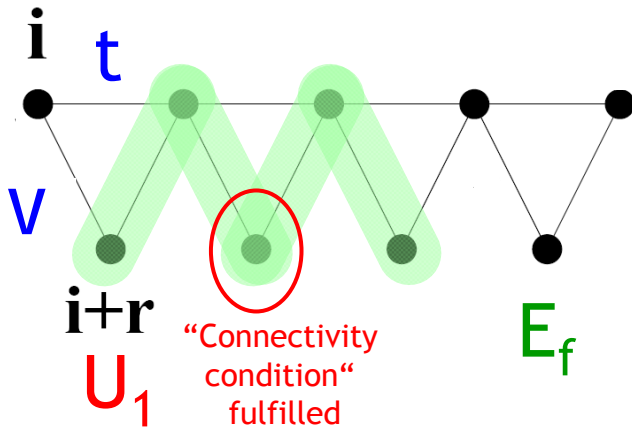
$$\epsilon_1 - \epsilon_0 > -2t$$

Solution III:

$$U_0 = 0, \quad U_1 > 0$$



$n=1/2$: fully saturated ferromagnet



2 sites per cell \rightarrow 2 bands

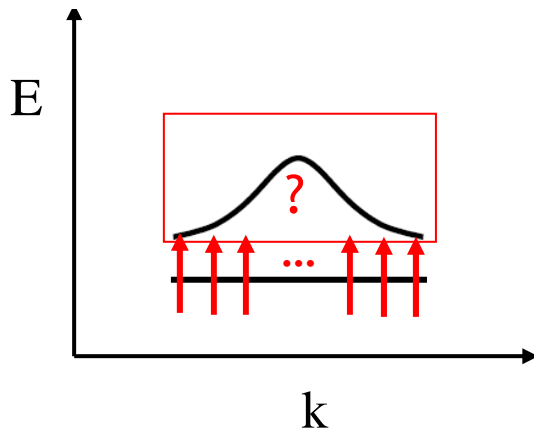
N_c = # cells

N = # electrons

$n = \frac{N}{2N_c}$ electron density

$$\frac{V^2}{t} = E_f + 2t, \quad t > 0$$

$$E_f > -2t$$



Solution III:

$$U_0 = 0, \quad U_1 > 0$$

$n=1/2$: fully saturated ferromagnet

Change of notation: $\hat{d}_{i,\sigma} \equiv \hat{c}_{i,\sigma}$,

$\hat{f}_{i,\sigma} \equiv \hat{c}_{i+r,\sigma}$, $V \equiv t'$, $E_f \equiv \epsilon_1$, $\epsilon_0 = 0$

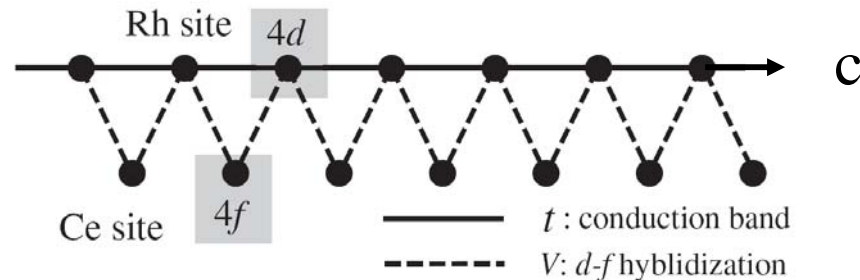
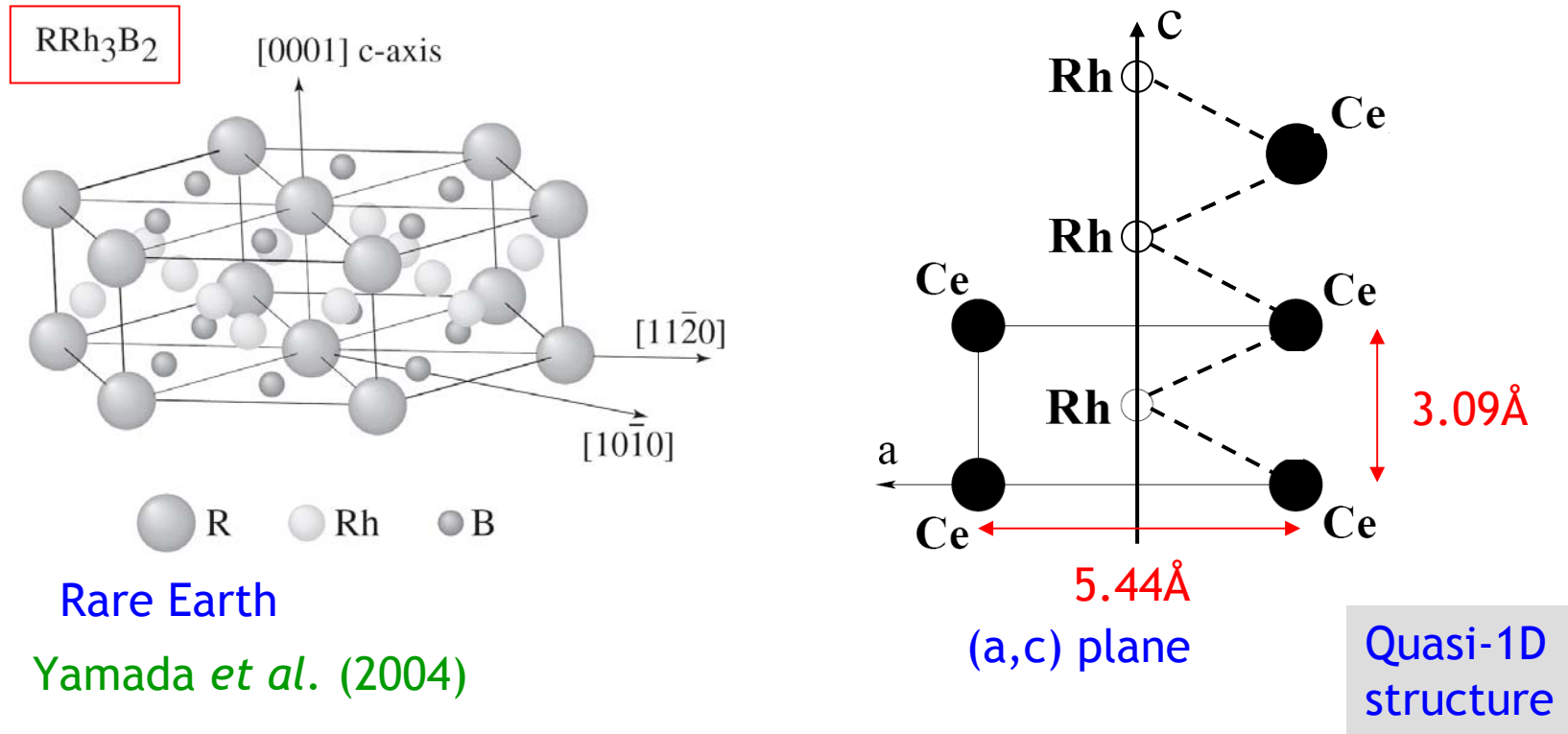
1D periodic Anderson model

Application of the 1D periodic Anderson model to CeRh_3B_2

CeRh_3B_2 is an interesting $4f$ -system because:

- RKKY interaction cannot explain ferromagnetism
- Small f -moment $0.45 \mu_B$ (free Ce^{3+} ion: $2.14 \mu_B$)

Application of the 1D periodic Anderson model to CeRh_3B_2



Kono, Kuramoto (2006)

Mechanism for *f*-electron ferromagnetism in CeRh₃B₂?

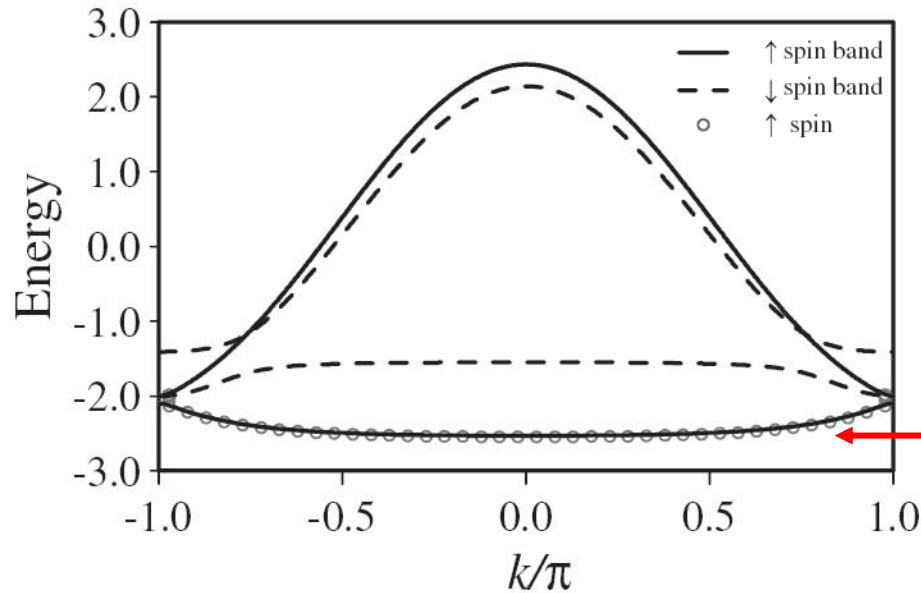
Gutzwiller projected variational wave function

$$|\Psi\rangle = P|\Phi\rangle$$

Evaluations by variational Monte Carlo (VMC)

Kono, Kuramoto (2006)

VMC

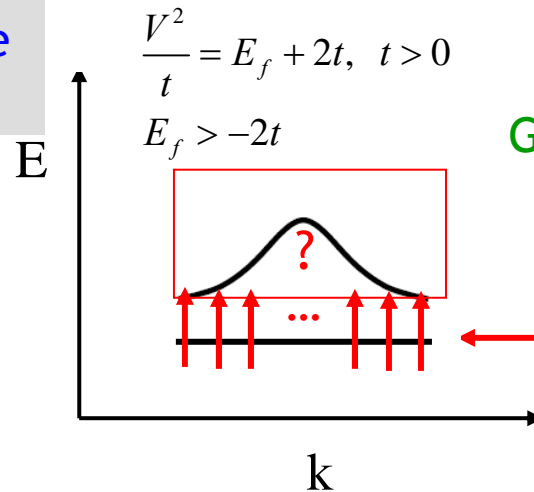


Kono, Kuramoto (2006)

almost flat band

$$t = 0.34 \text{ eV}, V = 0.24 \text{ eV}, E_f = -0.714 \text{ eV}, U = 7 \text{ eV}, n = 0.55$$

Exact ground state
(Solution III)



Gulacsi, Kampf, DV (2008)

saturated ferromagnetism,
bare flat band unchanged by U

$$\text{e.g., } t = 0.34 \text{ eV}, V = 0.23 \text{ eV}, E_f = -0.52 \text{ eV}, U > 0 \text{ arbitrary}, n = 0.5$$

Magnetic moments

Free Ce³⁺ ion

$$m_f = 2.14 \mu_B$$

Experiment:

$$m_f = 0.45 \mu_B$$

Galatanu *et al.* (2003)

VMC

$$t = 0.34 \text{ eV}, V = 0.24 \text{ eV}, E_f = -0.714 \text{ eV}, U = 7 \text{ eV}, n = 0.55$$

$$m_f = 0.94 \mu_B$$

Kono, Kuramoto (2006)

Exact ground state

$$\frac{V^2}{t} = E_f + 2t, \quad t > 0, \quad E_f > -2t$$

$$t = 0.34 \text{ eV}, V = 0.23 \text{ eV}, E_f = -0.52 \text{ eV}, U > 0 \text{ arbitrary}, n = 0.5$$

$$m_f = 0.68 \mu_B$$

Gulacsi, Kampf, DV (2008)

2. Exact many-electron ground states on pentagon chain polymers

Exact dispersive band structure at $U > 0$ can be tuned to become flat

Z. Gulacsi, A. Kampf, DV; arXiv:1007.4994

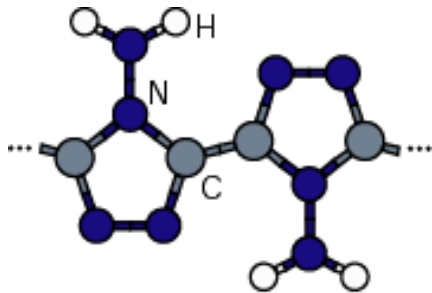
Conducting polymers: wide range of applications in

- Nanoelectronics
- Nanooptics
- Medicine

Search for plastic ferromagnets and ferromagnetism in systems with non-magnetic elements

Candidate: **Flat-band ferromagnetism in organic polymers**

Polymethylaminotriazole



Spin-DFT

Suwa, Arita, Kuroki, Aoki (2003, arXiv:0907.2477)

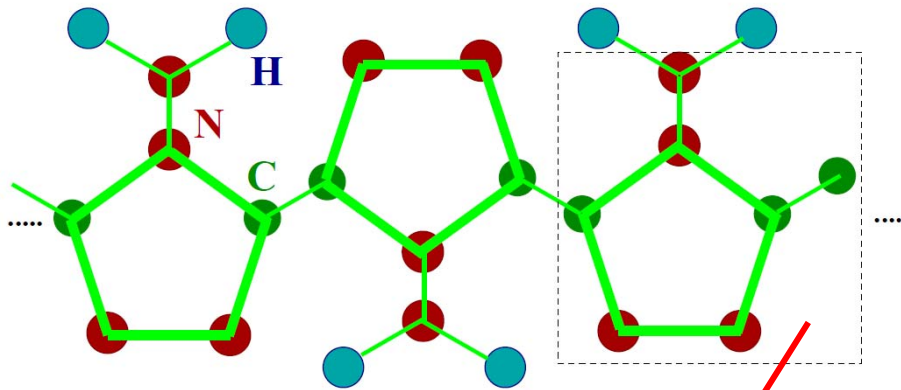
Arita, Suwa, Kuroki, Aoki (2002, 2003)

- Strong correlations in acene and thiophene organic molecular crystals
- stabilization of magnetic phases at high electron densities

Brocks, van den Brink, Morpurgo (2004)

Polymethylaminotriazole

Gulacsi, Kampf, DV (arXiv:1007.4994)

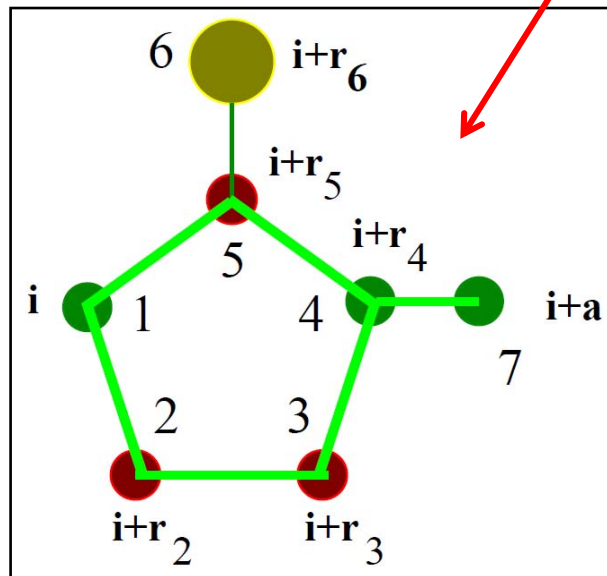


6 sites per cell \rightarrow 6 bands

$N_c = \# \text{ cells}$

$N = \# \text{ electrons}$

$n = \frac{N}{6N_c}$ electron density



Arbitrary

- local interactions $U_n > 0$
- on-site potentials ϵ_n
- hopping amplitudes $t_{n,n'}$

Single-particle bands

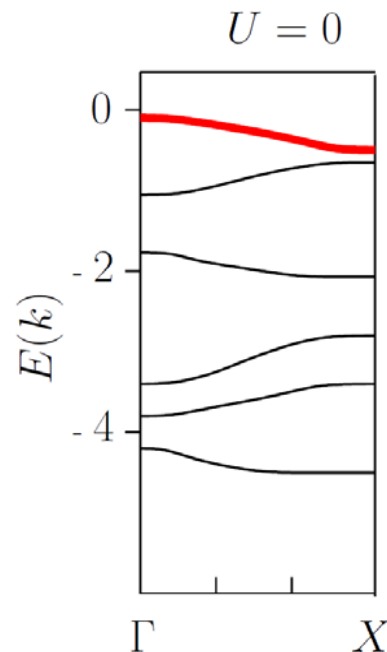
Gulacsi, Kampf, DV (arXiv:1007.4994)

Band dispersion

$$\epsilon = E_\nu(k), \nu \leq 6 \quad k = \mathbf{k} \cdot \mathbf{a}$$

$$2t_c t^2 [(\epsilon_6 - \epsilon)T_h - t_h T_f] \cos k + T_f \{t_h^2 t_c^2 - (\epsilon_2 - \epsilon)^2 t_c^2 + [(\epsilon_1 - \epsilon)(\epsilon_2 - \epsilon) - t^2]^2 - (\epsilon_1 - \epsilon)^2 t_h^2\} + 2(\epsilon_6 - \epsilon)t^2 \{t^2(\epsilon_2 + t_h - \epsilon) - (\epsilon_1 - \epsilon)T_h\} = 0,$$

with $T_h = (\epsilon_2 - \epsilon)^2 - t_h^2$, $T_f = (\epsilon_6 - \epsilon)(\epsilon_5 - \epsilon) - t_f^2$



$$t_c \equiv t_{4,7} = 0.5, \quad t_h \equiv t_{3,2} = -1.1, \quad t_f \equiv t_{5,6} = 1.2$$

$$\epsilon_1 = \epsilon_4 = -2.5, \quad \epsilon_2 = \epsilon_3 = -2.0, \quad \epsilon_5 = -2.1, \quad \epsilon_6 = -2.1$$

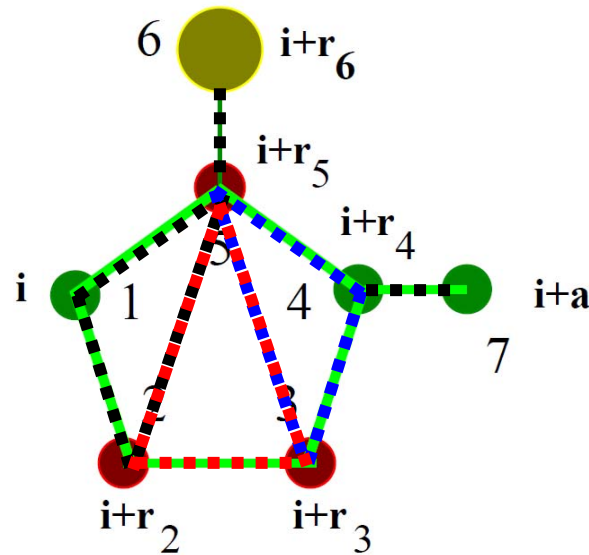
Transform Hamiltonian in positive semidefinite form

Define operators acting on blocks \mathcal{B}_i

$$\hat{G}_{\alpha, i, \sigma}^\dagger = \sum_{l \in \mathcal{B}_{i, \alpha}} a_{\alpha, l} \hat{c}_{i+r_l, \sigma}^\dagger \quad \alpha=1, \dots, 5$$

3 three-site blocks

2 two-site blocks



Original form

$$\begin{aligned}\hat{H}_0 &= \sum_{\sigma, \mathbf{i}} \sum_{n, n' (n > n')} (t_{n, n'} \hat{c}_{\mathbf{i}+\mathbf{r}_n, \sigma}^\dagger \hat{c}_{\mathbf{i}+\mathbf{r}_{n'}, \sigma} + H.c.) + \\ &\quad + \sum_{\sigma, \mathbf{i}} \sum_{n=1}^m \epsilon_n \hat{n}_{\mathbf{i}+\mathbf{r}_n, \sigma}, \\ \hat{H}_U &= \sum_{\mathbf{i}} \sum_{n=1}^m U_n \hat{n}_{\mathbf{i}+\mathbf{r}_n, \uparrow} \hat{n}_{\mathbf{i}+\mathbf{r}_n, \downarrow}.\end{aligned}$$

Positive semidefinite form

$$\hat{H} - C_g = \hat{H}_G + \hat{H}_P$$

with $\hat{H}_G = \sum_{\mathbf{i}, \sigma} \sum_{\alpha=1}^{m-1} \hat{G}_{\alpha, \mathbf{i}, \sigma} \hat{G}_{\alpha, \mathbf{i}, \sigma}^\dagger$, $\hat{H}_P = \sum_{n=1}^m U_n \sum_{\mathbf{i}} \hat{P}_{\mathbf{i}+\mathbf{r}_n}$, $\hat{P}_{\mathbf{j}} = \hat{n}_{\mathbf{j}, \uparrow} \hat{n}_{\mathbf{j}, \downarrow} - (\hat{n}_{\mathbf{j}, \uparrow} + \hat{n}_{\mathbf{j}, \downarrow}) + 1$

$$C_g = q_U N - N_c [\sum_{n=1}^m U_n + 2 \sum_{\alpha=1}^{m-1} z_\alpha], \quad z_\alpha = \sum_{\ell} |a_{\alpha, \ell}|^2$$

$$q_U = (1/2)[(U_1 + \epsilon_1 + |t_c|) + (U_2 + \epsilon_2 + |t_h|) + \{[(U_2 + \epsilon_2 + |t_h|) - (U_1 + \epsilon_1 + |t_c|)]^2 + 4t^2\}^{1/2}]$$

Original form

$$\begin{aligned}\hat{H}_0 &= \sum_{\sigma, \mathbf{i}} \sum_{n, n' (n > n')} (t_{n, n'} \hat{c}_{\mathbf{i}+\mathbf{r}_n, \sigma}^\dagger \hat{c}_{\mathbf{i}+\mathbf{r}_{n'}, \sigma} + H.c.) + \\ &\quad + \sum_{\sigma, \mathbf{i}} \sum_{n=1}^m \epsilon_n \hat{n}_{\mathbf{i}+\mathbf{r}_n, \sigma}, \\ \hat{H}_U &= \sum_{\mathbf{i}} \sum_{n=1}^m U_n \hat{n}_{\mathbf{i}+\mathbf{r}_n, \uparrow} \hat{n}_{\mathbf{i}+\mathbf{r}_n, \downarrow}.\end{aligned}$$

Positive semidefinite form

$$\hat{H} - C_g = \hat{H}_G + \hat{H}_P$$

with $\hat{H}_G = \sum_{\mathbf{i}, \sigma} \sum_{\alpha=1}^{m-1} \hat{G}_{\alpha, \mathbf{i}, \sigma} \hat{G}_{\alpha, \mathbf{i}, \sigma}^\dagger$, $\hat{H}_P = \sum_{n=1}^m U_n \sum_{\mathbf{i}} \hat{P}_{\mathbf{i}+\mathbf{r}_n}$

Matching conditions:

$$\begin{aligned}\sum_{\alpha=1}^{m-1} z_\alpha &= 4Q_1 + Q_1^2/|t_h| + 2t^2/Q_1 + 2(|t_h| + |t_c|) + Q_3^2 + t_f^2/Q_3^2 \\ Q_1 &= q_U - U_2 - \epsilon_2 - |t_h|, \quad Q_2 = q_U - U_1 - \epsilon_1 - |t_c|, \\ Q_3 &= |t_f| \sqrt{|t_h|} / [|t_h| (q_U - U_5 - \epsilon_5) - (q_U - U_2 - \epsilon_2)^2 + t_h^2]^{1/2}\end{aligned}$$

free parameters in block operators < # matching conditions

Parameter space for which the transformation holds

$$t_h < 0, \quad Z = (q_U - Q_3^2) > \epsilon_6,$$

$$W = q_U - [(q_U - U_2 - \epsilon_2)^2 - t_h^2]/|t_h| > \epsilon_5,$$

$$W - \epsilon_5 > U_5 > 0, \quad U_6 = Z - \epsilon_6.$$

No strong restrictions!

Explicit form of block operators:

$$\hat{G}_{1,i,\sigma} = q_S \hat{b}_{i,5,2} - t/q_S \hat{c}_{i+r_1,\sigma},$$

$$\hat{G}_{2,i,\sigma} = -t_S \hat{b}_{i,2,3} + Q_1 \hat{c}_{i+r_5,\sigma}/t_S,$$

$$\hat{G}_{3,i,\sigma} = q_S \hat{b}_{i,5,3} - t/q_S \hat{c}_{i+r_4,\sigma},$$

$$\hat{G}_{4,i,\sigma} = Q_3 \hat{c}_{i+r_6,\sigma} - t_f/Q_3 \hat{c}_{i+r_5,\sigma},$$

$$\hat{G}_{5,i,\sigma} = |t_c|^{1/2} [\hat{c}_{i+r_4,\sigma} - t_c \hat{c}_{i+a,\sigma}/|t_c|],$$

with $q_S = Q_1^{1/2}$, $t_S = |t_h|^{1/2}$, $\hat{b}_{i,n,m} = \sum_{p=n,m} \hat{c}_{i+r_p,\sigma}$

Effective band structure

$$\hat{H}_G = \hat{H}_{kin} + \text{const}$$

$$\hat{H}_{kin} = - \sum_{\mathbf{i}, \sigma} \sum_{\alpha=1}^{m-1} \hat{G}_{\alpha, \mathbf{i}, \sigma}^\dagger \hat{G}_{\alpha, \mathbf{i}, \sigma}$$

↑
quadratic in the original c-operators

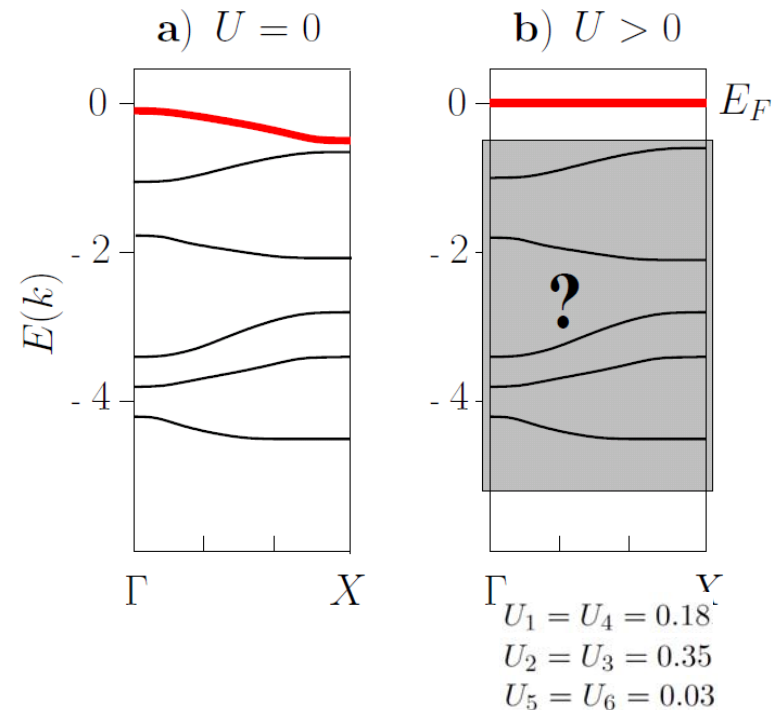
→ Effective, interaction dependent band structure:
same as for H_0 but with renormalization

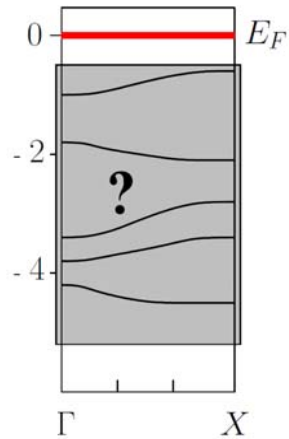
$$\epsilon_n \rightarrow \epsilon_n^R = \epsilon_n + U_n - qU$$

free parameters in block operators
< # matching conditions

→ exact effective band structure is
in general **dispersive**

Upper band can be **tuned** to
become flat due to interactions





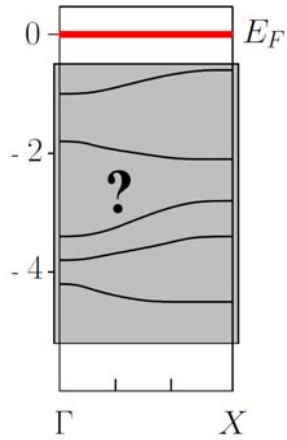
Upper flat band half filled for $N=N^*=11N_c \rightarrow$ density $n=11/6$

Ground state for $n=11/6$: $|\Psi_g(N^*)\rangle = \left[\prod_{\sigma} \hat{G}_{\sigma}^{\dagger} \right] \hat{F}^{\dagger} |0\rangle$

$\hat{G}_{\sigma}^{\dagger} = \prod_{\mathbf{i}} \prod_{\alpha=1}^{m-1} \hat{G}_{m,\mathbf{i},\sigma}^{\dagger}$ creates $(m-1)N_c$ σ -electrons

$\hat{F}^{\dagger} = \prod_{\mathbf{i}} \hat{c}_{\mathbf{i}+\mathbf{r}_{n_i},\sigma}^{\dagger}$ creates one σ -electrons in each unit cell

one σ -electron per site \rightarrow localized
 $-\sigma$ -electron are mobile (= Slater determinant)



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Physical properties

$$\Gamma_{\mathbf{i}}(\mathbf{r}) = \langle \Psi_g(N^*) | (\hat{c}_{\mathbf{i}+\mathbf{r}_n,-\sigma}^{\dagger} \hat{c}_{\mathbf{i}+\mathbf{r}_n+\mathbf{r},-\sigma} + H.c.) | \Psi_g(N^*) \rangle \xrightarrow{N_c \rightarrow \infty} e^{-r/\xi}$$

Ground state for $n = 11/6$: localized ferromagnet

Ground state for $n > 11/6$ ($S_{z,\max}$ sector)

charge gap $\Delta\mu = E_g(N) - 2E_g(N-1) + E_g(N-2) = 0$, $\xi = \infty \rightarrow$ itinerant ferromagnet

Matching conditions allow for parameters

$$n \geq 11/6, \quad W/t = 0.15, \quad \text{with } 0.2 \leq U_n/W \leq 2.3$$

Fulfills experimental conditions:

- Strong correlations in acene and thiophene organic molecular crystals
- stabilization of magnetic phases at high electron densities

Brocks, van den Brink, Morpurgo (2004)

Dispersive band may be tuned by interactions to become flat

→ new route for the design of ferromagnetic pentagon-chain polymers

Conclusion

Strategy for the construction of exact many-electron ground states:

Step 1: Transform many-electron Hamiltonian into positive semidefinite form

Step 2: Construct many-electron ground state

Step 3: Prove uniqueness of many-electron ground state:

- Works in any dimension
- No integrability required
- Applicable to any Hamiltonian with sufficiently many microscopic parameters

E.g.: Pentagon chain polymers

Tune dispersive band structure by the interaction

→ design flat band structure with ferromagnetic or half metallic ground state

