What is temperature?

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In collaboration with
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Temperature sensation

- Subjective temperature sensation
- Memory effects
- Measuring the heat flow
- Heat – substance $\leftrightarrow$ Cold – substance

➡ **What is temperature?**

Overview

I. History

II. Temperature scales

III. Thermometers

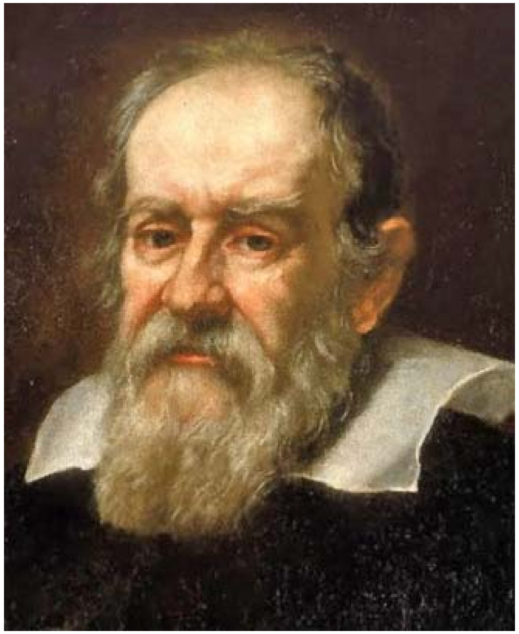
IV. Grand Laws of Thermodynamics

V. Temperature characteristics

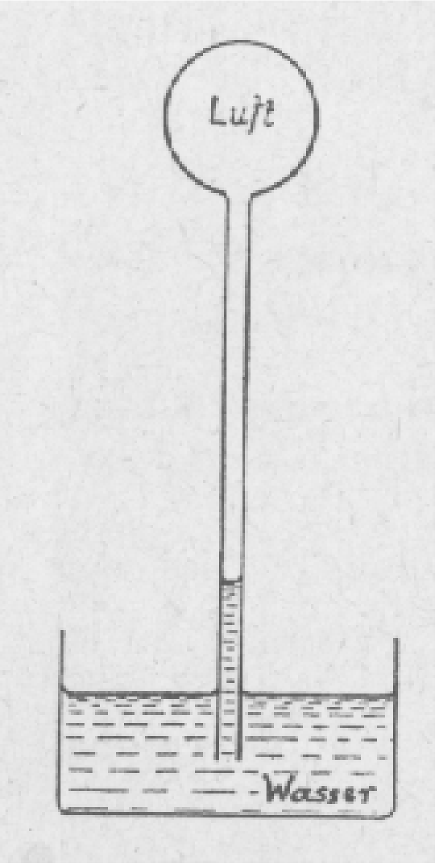
Brief history of the temperature concept

Early temperature measurements

No scale for measuring temperature between some reference points



Galileo Galilei
(1564 – 1642)



1592: first documented (gas-) thermometer invented by Galileo Galilei

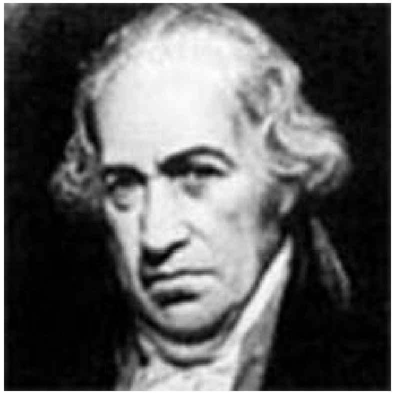
First thermometer & temperature scales

1638: Robert Fludd – air thermometer &scale
~1700: linseed oil thermometer by Newton
1701: red wine as temperature indicator by Rømer
1702: Guillaume Amontons: Absolute zero temperature?
1714: mercury and alcohol thermometer by Fahrenheit

Definition of temperature scales

Sir Isaac Newton
(1643 – 1727)

Olaf Christensen Römer
(1644 – 1710)



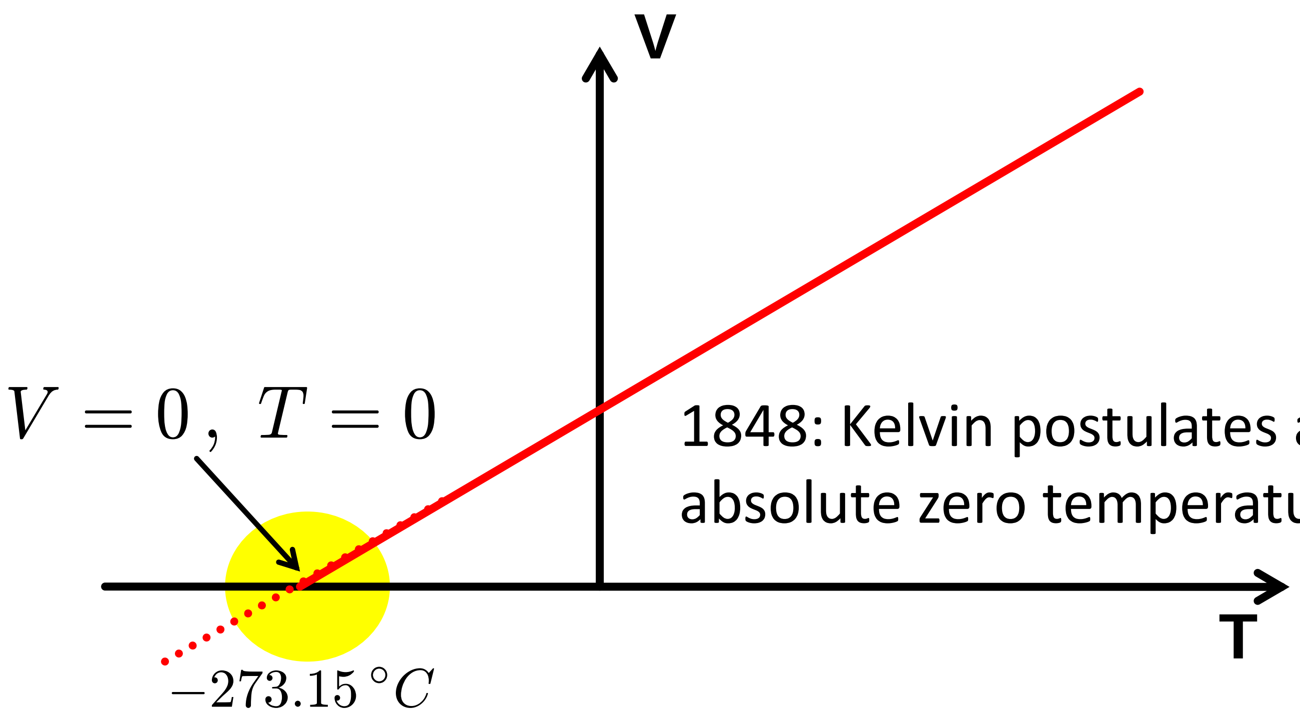
Daniel Gabriel Fahrenheit
(1686 – 1736)

René Antoine Ferchault de Réaumur
(1683 – 1757)

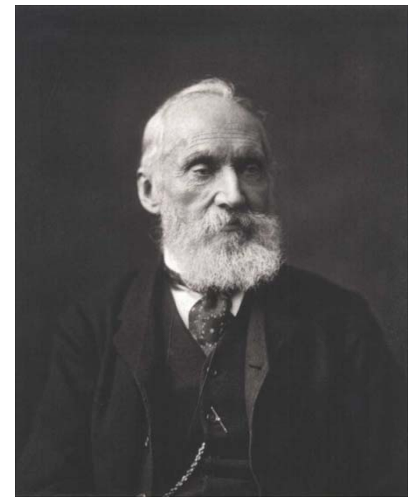
Anders Celsius
(1701 – 1744)

Absolute Zero

Ideal gas law: $pV = N\,k_{\mathrm{B}}\,T$



1848: Kelvin postulates an absolute zero temperature



William Thomson — Lord Kelvin
(1824 – 1907)

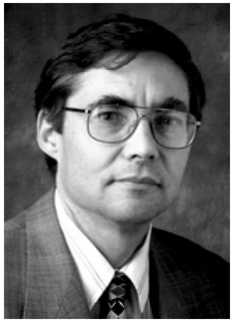
It is impossible by any procedure to reduce the temperature of a system to zero in a finite number of operations.

Low temperature milestones

1908: liquid helium; 5 K

Heike Kamerlingh Onnes
(1853 – 1926)

1995: Bose-Einstein-Condensate; 20 nK



Eric A. Cornell

Carl E. Wieman

Wolfgang Ketterle

2003: BEC; 450 pK – W. Ketterle

Temperature scales

Absolute temperature scale

Kelvin – scale (SI)

Reference: absolute zero temperature
& triple point of water (273.16K; 611.73 Pa)

Scale division: 1K is equal to the fraction $(273.16)^{-1}$ of the thermodynamic temperature of the triple point of water

$$\Delta T = 1K \equiv 1^\circ C \ (\text{degree Celsius})$$

Comparision of temperature scales

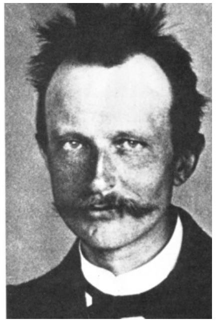
	Kelvin [K]	Celsius [°C]	Fahrenheit [°F]
Absolute zero	0.00	-273.15	-459.67
Fahrenheit's ice/salt mixture	255.37	-17.78	0.00
Ice melts (at standard pressure**)	273.15	(100) = 0.00 *	32.00
Average human body temperature	309.95	36.80	98.24
Water boils (at standard pressure)	373.13	(0) = 100 *	211.97
Titanium melts	1941.00	1668.00	3034.00

source: wikipedia.org

Further temperature scales: Newton, Delisle, Réaumur, Rømer, M. Planck (linear scales)
Rudolf Plank (logarithmic scale)

* Celsius used a reversed temperature scale with "0" indicating the boiling point of water and "100" the melting point of ice

** 1 atm = 760 mmHg = 101325 Pa



Planck units

Constant	Symbol	Value in SI units
Speed of light	c	299 792 458 m s^{-1}
Gravitational constant	G	6.67429 · 10^{-11} m^3 kg^{-1} s^{-2}
reduced Planck's constant	$\hbar$	1.054571628 ·10^{-34} J s
Coulomb force constant	$(4\pi\epsilon_0)^{-1}$	8 987 551 787.368 kg m^3 s^{-2} C^{-2}
Boltzmann constant	k_B	1.3806504·10^{-23} J K^{-1}

Name	Expression	SI equivalents
Planck temperature	$T_\mathrm{P} = \sqrt{\hbar c^5 G^{-1} k^{-2}}$	1.41168· 10^{32} K
Planck length	$l_\mathrm{P} = \sqrt{\hbar G c^{-3}}$	1.61625· 10^{-35} m
Planck mass	$m_\mathrm{P} = \sqrt{\hbar c G^{-1}}$	2.17644· 10^{-8} kg
Planck time	$t_\mathrm{P} = \sqrt{\hbar G c^{-5}}$	5.39124· 10^{-44} s

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… ihre Bedeutung für alle Zeiten und für alle, auch außerirdische und außermenschliche Kulturen notwendig behalten und welche daher als natürliche Maßeinheiten bezeichnet werden können …

… These necessarily retain their meaning for all times and for all civilizations, even extraterrestrial and non-human ones, and can therefore be designated as natural units …

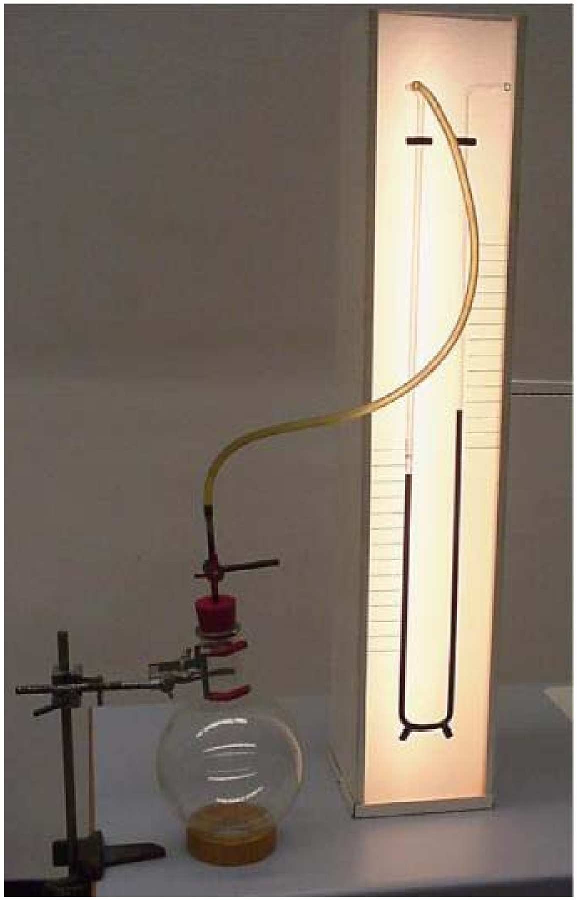
(Planck, 1899)

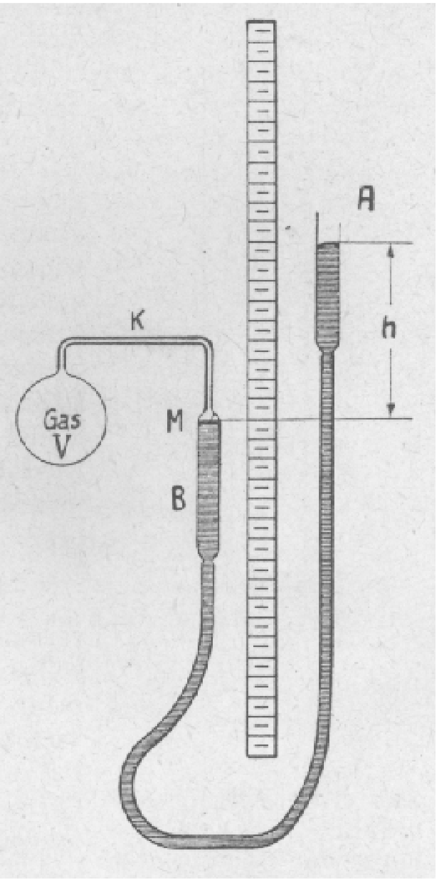
Typical temperature values [°C]

Boiling point of Nitrogen	-195.79
Lowest recorded surface temperature on Earth (Vostok, Antarctica – July 21, 1983)	-89
Highest recorded surface temperature on Earth (Al' Aziziyah, Libya – September 13, 1922)	58
Temperature in the Earth's Thermosphere (80 - 650 km above the surface)	~ 1500
Melting point of diamond	3547
Surface temperature of the sun (photosphere)	~ 5526
Temperature in the interior of the sun	~ $15 \cdot 10^6$

Thermometers

Gas Thermometers



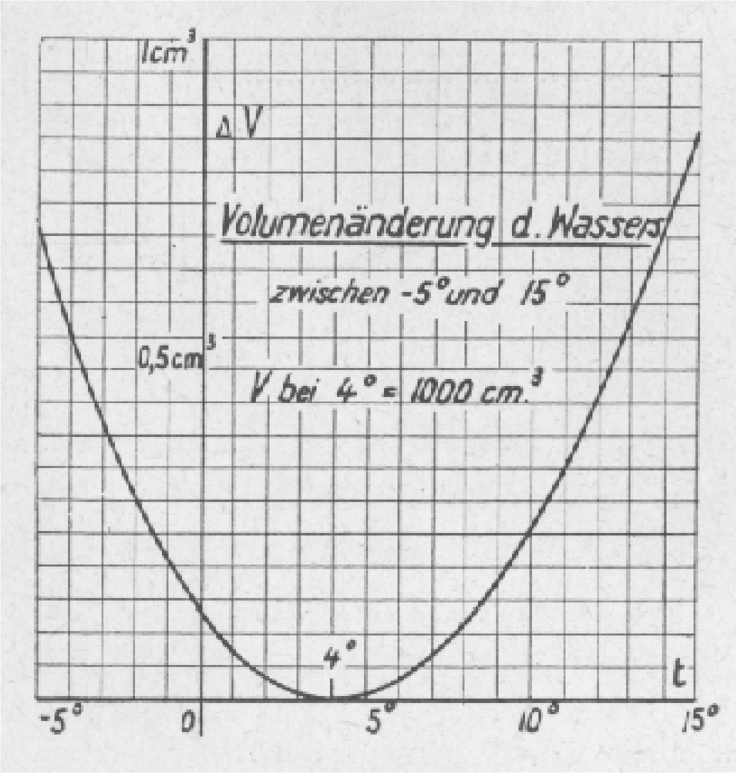


Ideal gas:

$$T = \left(\frac{V}{N\,k_{\mathrm{B}}} \right) p$$

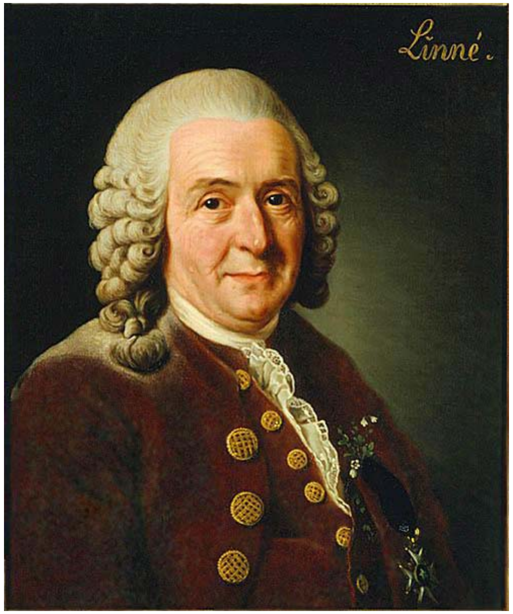
Galilei Thermometer

Volume change of water



Upon increasing T → 4°C ➡ Water volume shrinks

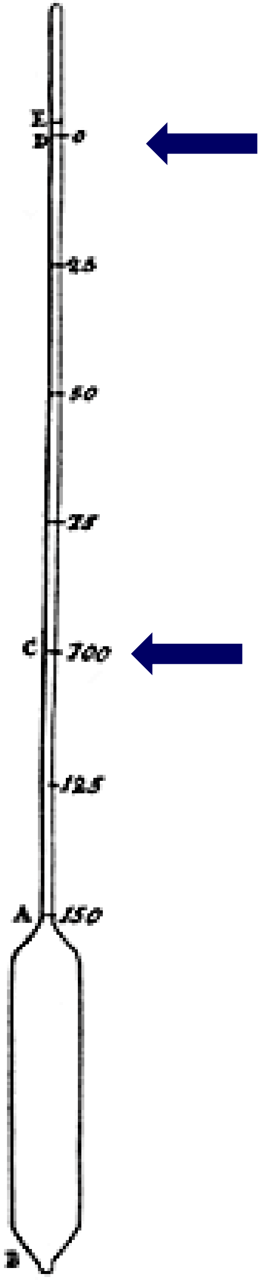
Linnaeus thermometer



Carl von Linné
(1707 – 1778)

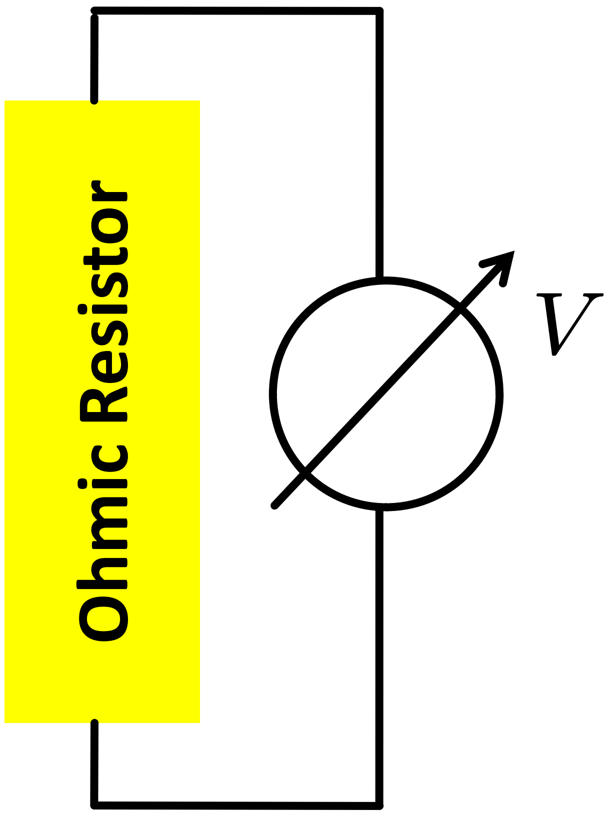
Reversed the Celsius scale

1744: broken on delivery
1745: botanical garden in Uppsala



Anders Celsius

Noise Thermometer



Johnson – Nyquist noise

$$\mathrm{PSD}_V(\omega) = 2k_\mathrm{B}TR$$

-- classical regime only --

PSD_V : power spectral density of the voltage signal

k_B : Boltzmann constant

R : resistance

Quantum fluctuation-dissipation theorem

$$PSD_I(\omega) = \hbar\omega \, \coth\left(\frac{\hbar\omega}{2k_\mathrm{B}T}\right) \, \mathrm{Re}Y(\omega)$$

$$= \left(\frac{\hbar\omega}{2} + \frac{\hbar\omega}{\exp(\beta\hbar\omega) - 1}\right) \, \mathrm{Re}Y(\omega)$$

$k_\mathrm{B}T \gg \hbar\omega$

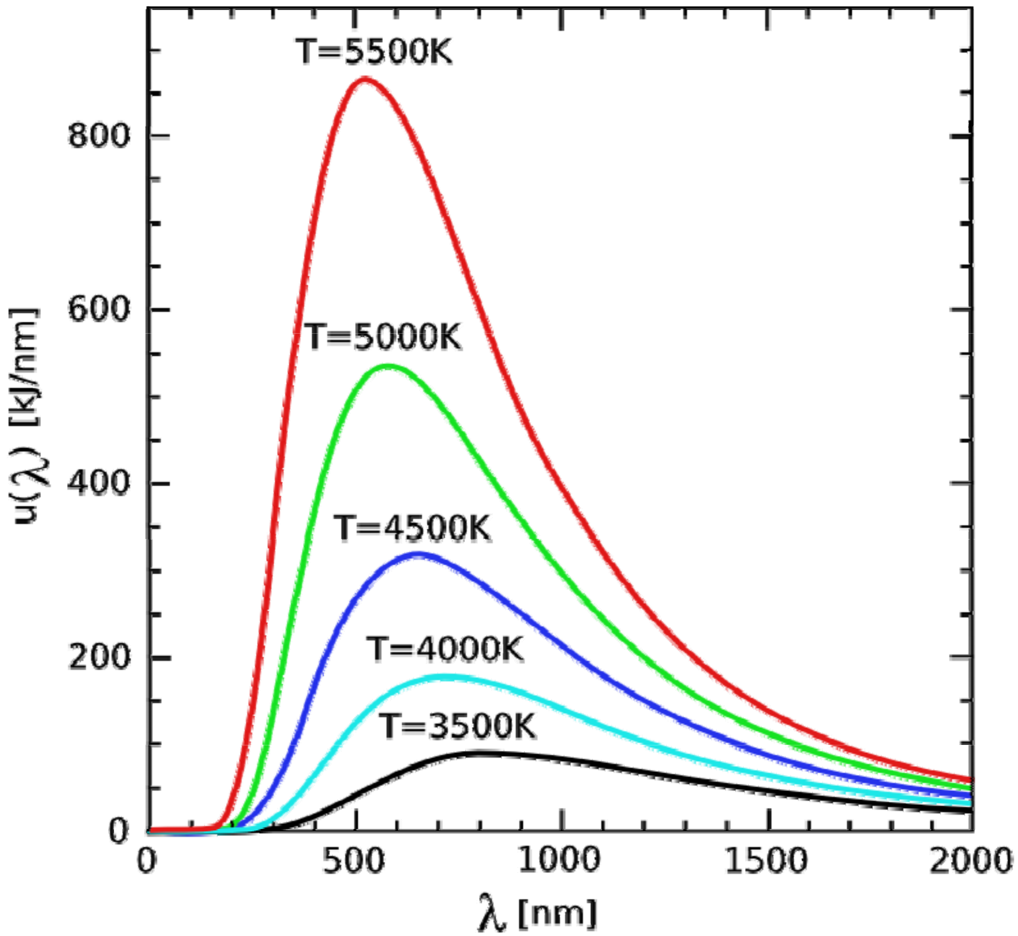
$$PSD_I(\omega) = 2k_\mathrm{B}T \, \mathrm{Re}Y(\omega)$$

$k_\mathrm{B}T \ll \hbar\omega$

$$PSD_I(\omega) = \hbar\omega \, \mathrm{Re}Y(\omega)$$

Black body radiation

Planck's law [1901]: $$u(\lambda, T) = \frac{8\pi hc}{\lambda^5} \frac{1}{\exp(\frac{hc}{\lambda k_B T}) - 1}$$



$u(\lambda, T)$: spectral energy density

λ : wavelength

h : Planck constant

c : speed of light

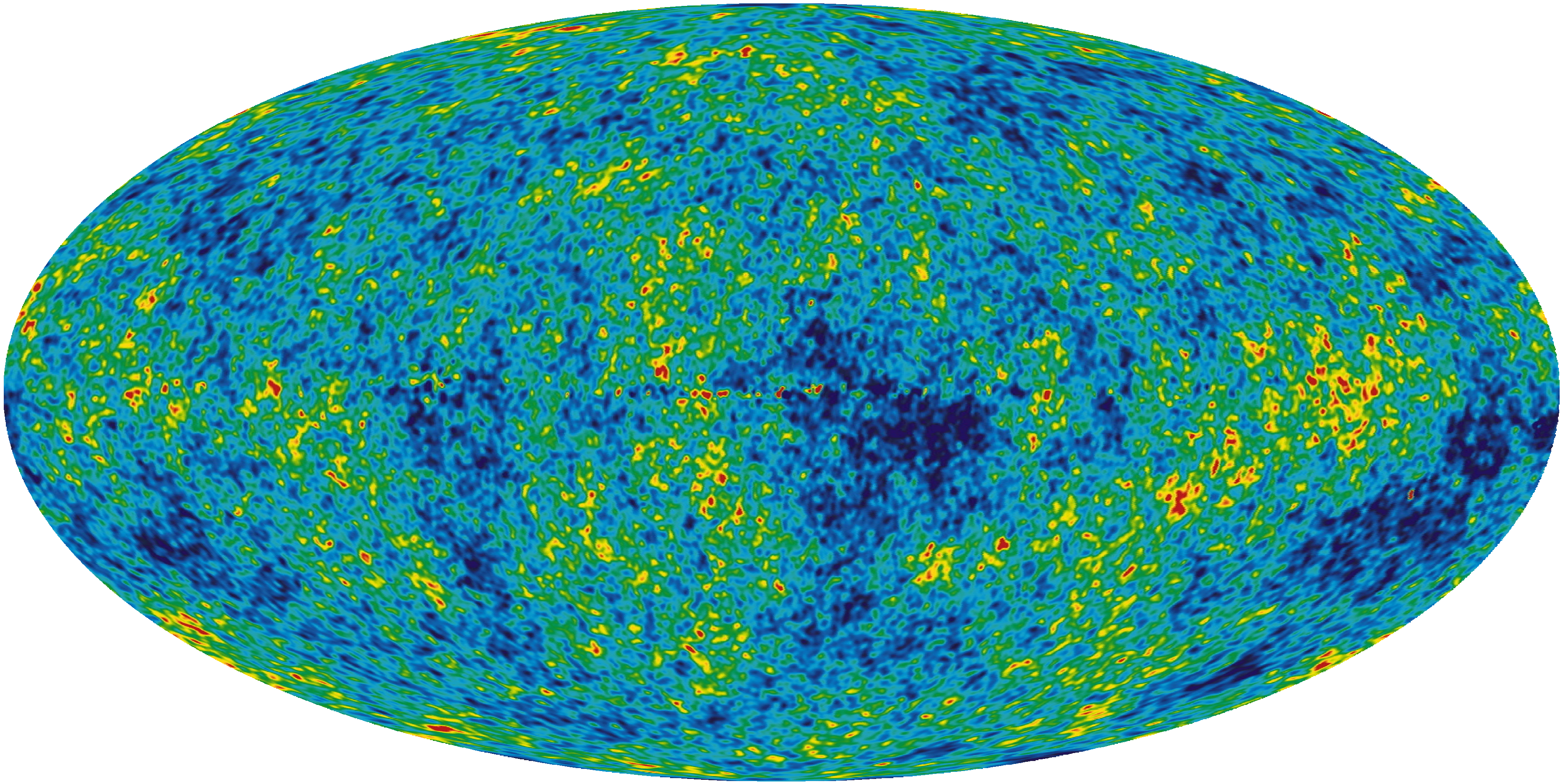
k_B : Boltzmann constant

Stefan – Boltzmann law:

$$E \propto T^4$$

Thermometer !

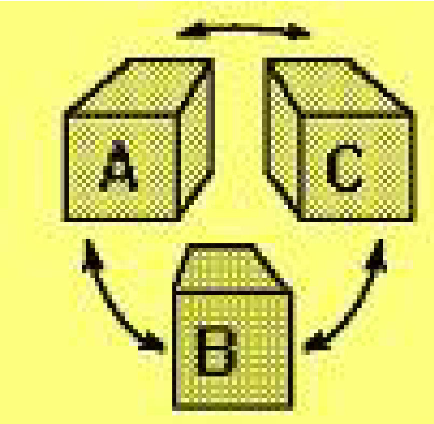
Cosmic background temperature



T = 2.725 ± …. K

Four Grand Laws of Thermodynamics

Zeroth Law



Transitivity !

$$\text{A in equilibrium with B:} \quad f_{\mathrm{AB}}(p_{\mathrm{A}}, V_{\mathrm{A}}; p_{\mathrm{B}}, V_{\mathrm{B}}, ...) = 0$$

$$\text{B in equilibrium with C:} \quad f_{\mathrm{BC}}(p_{\mathrm{B}}, V_{\mathrm{B}}; p_{\mathrm{C}}, V_{\mathrm{C}}, ...) = 0$$

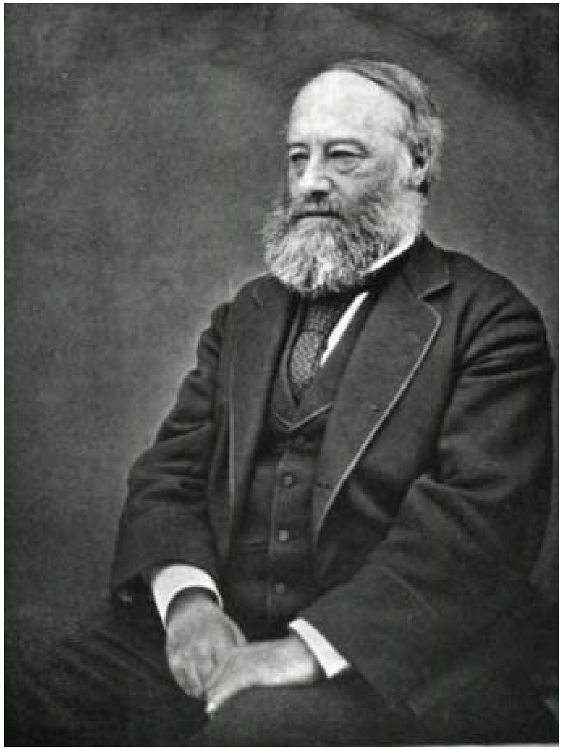
$$\Rightarrow \text{A in equilibrium with C} \Leftrightarrow f_{\mathrm{AC}}(p_{\mathrm{A}}, V_{\mathrm{A}}; p_{\mathrm{C}}, V_{\mathrm{C}}, ...) = 0$$

Allows the formal introduction of a temperature:

$$T = T_{\mathrm{A}}(p_{\mathrm{A}}, V_{\mathrm{A}}; ...) = T_{\mathrm{B}}(p_{\mathrm{B}}, V_{\mathrm{B}}; ...) = T_{\mathrm{C}}(p_{\mathrm{C}}, V_{\mathrm{C}}; ...)$$

First Law

Julius Robert von Mayer
(1814 – 1878)



James Prescott Joule
(1818 – 1889)

Hermann von Helmholtz
(1821 – 1894)

First Law – Energy Conservation

$$\Delta E = \Delta Q + \Delta W$$

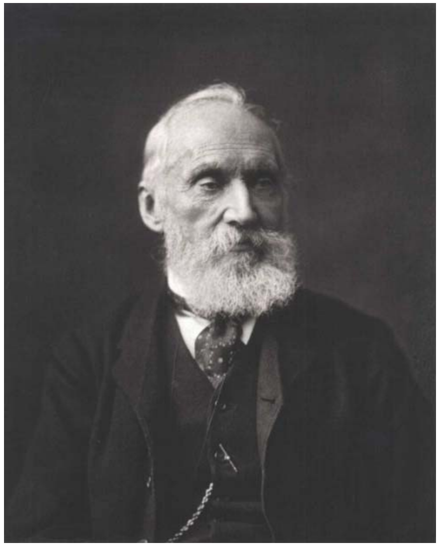
ΔE	change in internal energy
ΔQ	heat added on the system
ΔW	work done on the system

H. von Helmholtz: “Über die Erhaltung der Kraft” (1847)

$$\Delta E = (T\Delta S)_{\text{quasi-static}} - (p\Delta V)_{\text{quasi-static}}$$

Second Law

Rudolf Julius Emanuel Clausius
(1822 – 1888)



William Thomson alias Lord Kelvin
(1824 – 1907)

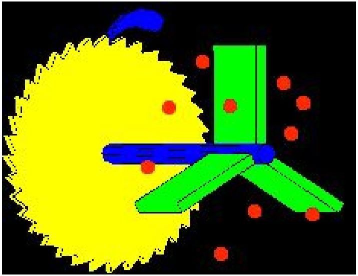
Heat generally cannot spontaneously flow from a material at lower temperature to a material at higher temperature.

No cyclic process exists whose sole effect is to extract heat from a single heat bath at temperature T and convert it entirely to work.

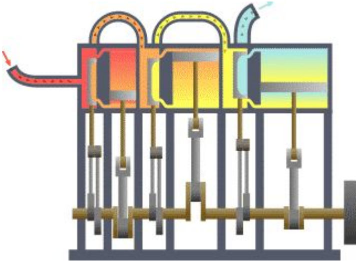
$$\delta Q = T\mathrm{d}S \quad \text{(Zürich, 1865)}$$

Perpetuum mobile of the second kind ?

One heat reservoir



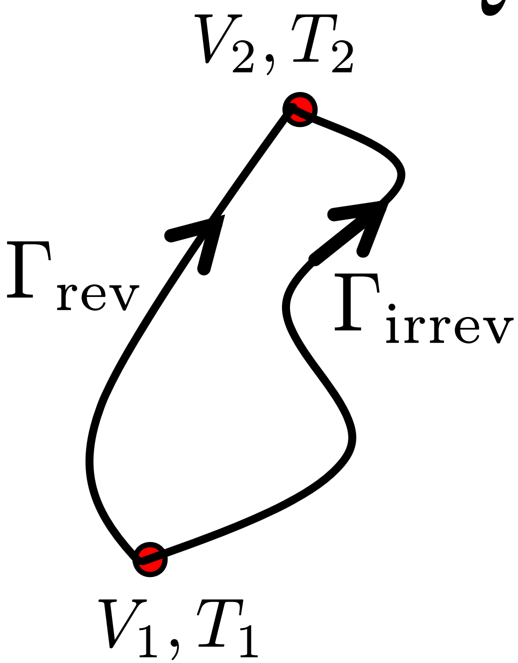
Two heat reservoirs



NO !

Entropy S – content of *transformation* „Verwandlungswert“

$$d\,S = \delta Q^{\mathrm{rev}} / T\,; \quad \delta Q^{\mathrm{irrev}} < \delta Q^{\mathrm{rev}}$$



$$\oint_C \frac{\delta Q}{T} \leq 0$$

$$C = \Gamma_{\mathrm{rev}}^{-1} + \Gamma_{\mathrm{irrev}}$$

$$S(V_2,T_2) - S(V_1,T_1) \geq \int_{\Gamma_{\mathrm{irrev}}} \frac{\delta Q}{T}$$

$$S(V_2,T_2) - S(V_1,T_1) = \int_{\Gamma_{\mathrm{rev}}} \frac{\delta Q}{T}$$

$$\Bigg\} \quad \frac{\partial S}{\partial t} \geq 0$$

NO !

Thermodynamic potentials

Internal energy: $E(S,V,\{N_i\}) = T\,S - p\,V + \sum \mu_i\,N_i$

Helmholtz free energy: $F(T,V,\{N_i\}) = E - T\,S$

Enthalpy: $H(S,p,\{N_i\}) = E + p\,V$

Gibbs free energy: $G(T,p,\{N_i\}) = E + p\,V - T\,S = \sum \mu_i\,N_i$

Planck potential: $\Phi(T,p,\{N_i\}) = -G/T$

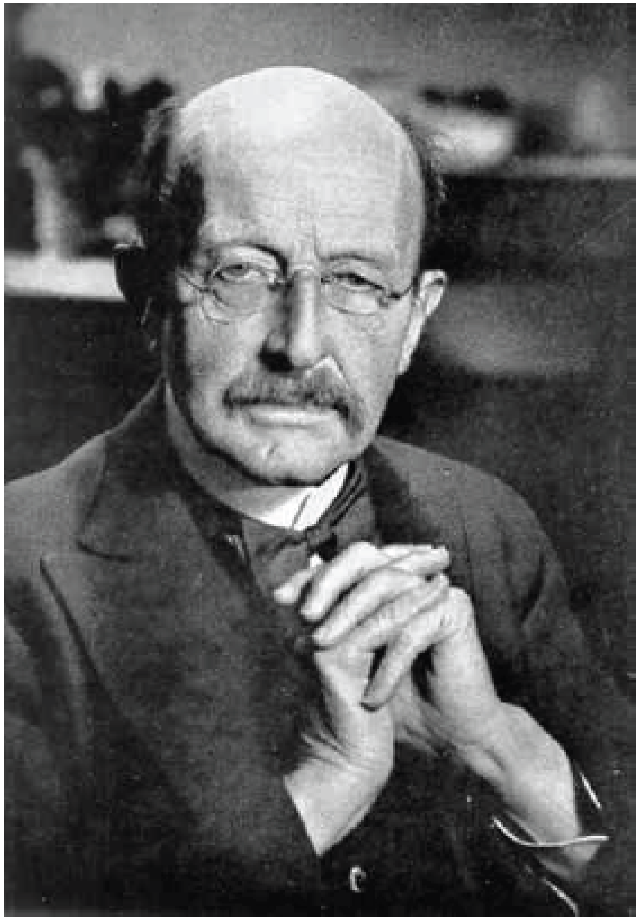
Third Law

Walter Hermann NERNST
(1864 - 1941)

„mein Wärmesatz“
(during his lecture August 15, 1905)

$$\frac{\Delta H - \Delta G}{T} = \Delta S \longrightarrow 0 \quad \text{as} \quad T \longrightarrow 0$$

… and Planck's version

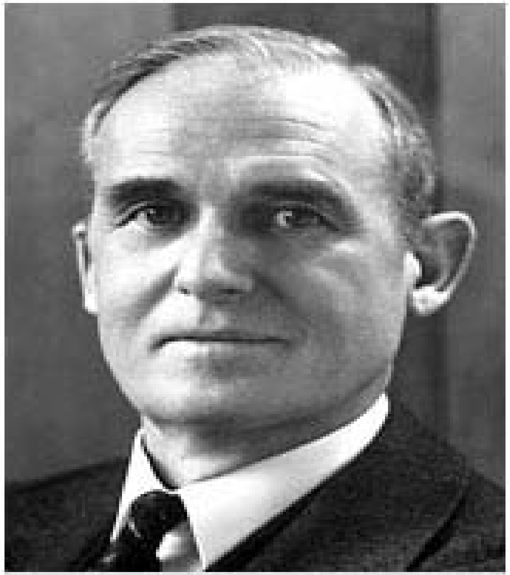


Max PLANCK
(1858 - 1947)

The entropy $s = S/N$ per particle approaches at $T = 0$ a constant ($s_0 = k_B \ln g(N)/N$) value that possibly depends on the chemical composition of the system. This limiting value can generally be set to zero.

The Nobel Prize in Chemistry 1949

"for his contributions in the field of chemical thermodynamics, particularly concerning the behaviour of substances at extremely low temperatures"



William Francis Giauque

USA

University of California
Berkeley, CA, USA

b. 1895
d. 1982

Approaching zero temperature

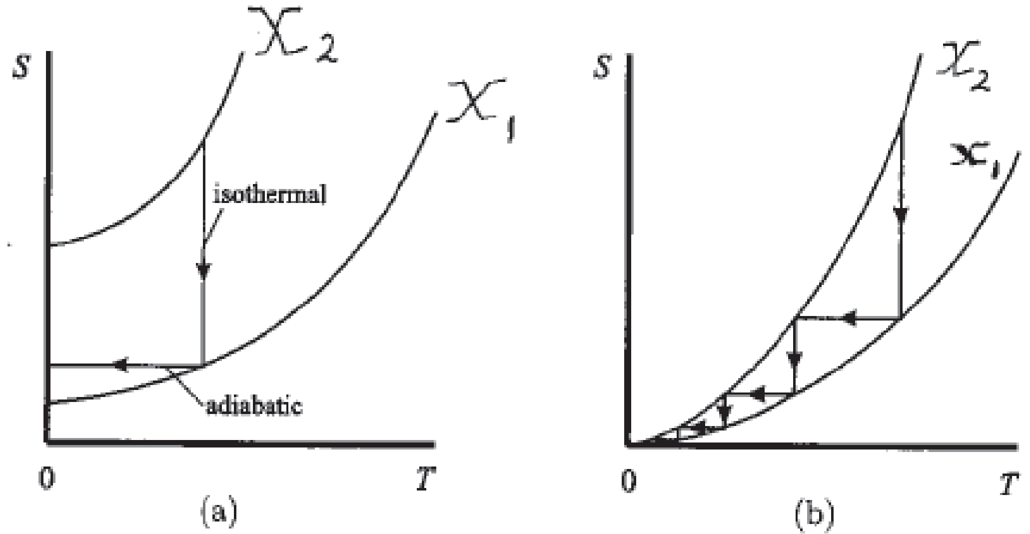


Fig. 1.13. Approaching absolute zero. System (a) not obeying the Third Law can get to $T = 0$ in two steps. System (b) obeying the Third Law cannot get to $T = 0$ in a finite number of steps.

Nernst 1912 (after being criticized by Einstein in Brussels):
One cannot reach $T = 0$ with a finite number of (reversible) isothermal and adiabatic steps.

corollary: The $T = 0$ isotherm is also an isentrope. (H.B. Callen)

Famous exception of the 3rd law

classical ideal gas

$$S = N\left[c_V \ln(T) + k_B \ln(V/N) + \sigma\right]$$

Moreover:

classical statistical mechanics: n-vector model with n-dimensional vectors > 1 violates third law. (e.g. planar Heisenberg ($n = 2$) or the $n = 3$ Heisenberg model)

Temperature definitions

Thermodynamic Temperature

$$\delta Q^{\mathrm{rev}} = T\,dS \leftarrow \text{thermodynamic entropy}$$

$$S = S(E, V, N_1, N_2, ...; M, P, ...)$$

$S(E, ...)$: (continuous) & differentiable and monotonic function of the internal energy E

$$\left(\frac{\partial S}{\partial E}\right)_{...} = \frac{1}{T}$$

How to determine the absolute thermodynamic temperature

VORLESUNGEN

ÜBER

THERMODYNAMIK

VON

DR. MAX PLANCK,

PROFESSOR DER THEORETISCHEN PHYSIK
AN DER UNIVERSITÄT BERLIN.

VIERTE AUFLAGE.

MIT FÜNF FIGUREN IM TEXT.

LEIPZIG,
VERLAG VON VEIT & COMP.
1913

$$\log \frac{T}{T_0} = \int_{t_0}^{t} \frac{\left(\frac{\mathrm{d}v}{\mathrm{d}t}\right)_p \mathrm{d}t}{v + c_p' \left(\frac{\mathrm{d}t}{\mathrm{d}p}\right)}$$

t : arbitrary temperature scale

v : volume

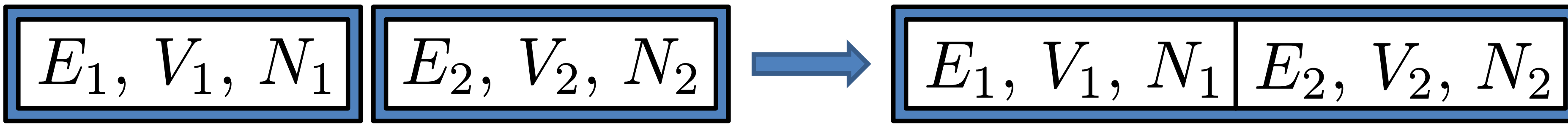
c_p' : specific heat at constant pressure

" ... wo nun wieder unter dem Integralzeichen lauter direkt und verhältnismäßig bequem meßbare Größen stehen. ..."

M. Planck

A statistical definition of Temperature

$\Omega(E,V,N)$: number of microstates for a given macrostate (E,V,N)



$\Omega_1(E_1,V_1,N_1) \quad \Omega_2(E_2,V_2,N_2) \Rightarrow \Omega = \Omega_1 \cdot \Omega_2$

$$\text{in equilibrium: } \delta\Omega = \frac{\partial \Omega_1}{\partial E_1}\delta E_1\,\Omega_2 + \Omega_1\,\frac{\partial \Omega_2}{\partial E_2}\delta E_2 = 0$$

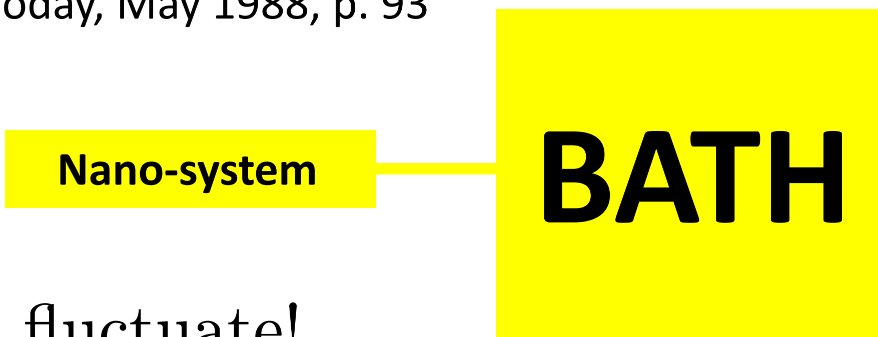
$$\text{energy conservation: } \delta E_1 = -\delta E_2 \Rightarrow \frac{1}{\Omega_1}\frac{\partial \Omega_1}{\partial E_1} = \frac{1}{\Omega_2}\frac{\partial \Omega_2}{\partial E_2}$$

Boltzmann (Planck): $\mathbf{S = k_B \ln\Omega}$

$$\Rightarrow \frac{1}{T_1} = \frac{1}{T_2} \Rightarrow \mathbf{T_1 = T_2 = T}$$

Temperature Fluctuations – An Oxymoron

Ch. Kittel, Physics Today, May 1988, p. 93



$$\beta = \frac{1}{k_\mathrm{B} T}$$

T does NOT fluctuate!

$$|\Delta U|\,|\Delta\beta| = 1 \leftrightarrow |\Delta U|\,|\Delta T| = k_\mathrm{B} T^2 \quad \textbf{NO!}$$

"…Temperature is precisely defined only for a system in thermal equilibrium with a heat bath: The temperature of a system A, however small, is defined as equal to the temperature of a very large heat reservoir B with which the system is in equilibrium and in thermal contact. Thermal contact means that A and B can exchange energy, although insulated from the outer World. …"

Molecular Dynamics

- Every quadratic term in the Hamiltonian contributes $\frac{1}{2}k_{\mathrm{B}}T$ to the internal energy
- Classical systems: **VIRIAL THEOREM**

$$[H_i \propto q_i^n \text{ or } p_i^n \Rightarrow \langle E_i \rangle = k_{\mathrm{B}}T/n]$$

- Classical harmonic Oscillator

$$\frac{\langle p_i^2 \rangle}{2m} = \frac{1}{2}m\langle v_i^2 \rangle = \frac{1}{2}k_{\mathrm{B}}T$$

$$\frac{1}{2}m\omega^2\langle x_i^2 \rangle = \frac{1}{2}k_{\mathrm{B}}T$$

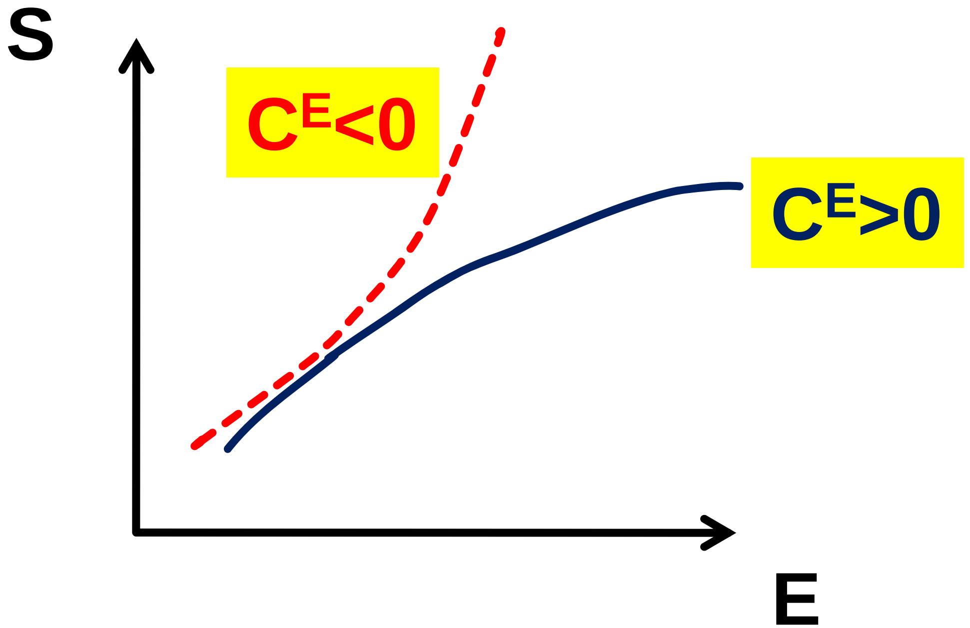
NOT SO FOR A QUANTUM OSCILLATOR

Specific Heat

$$E = \sum_i E_i \exp(-\beta E_i) / \sum_i \exp(-\beta E_i)$$

$$C^{\mathrm{E}} = \frac{\mathrm{d}E}{\mathrm{d}T} = (k_{\mathrm{B}} T^2)^{-1} \langle (E_i - \langle E_i \rangle)^2 \rangle > \mathbf{0}\,!$$

Note also: $\frac{\partial S}{\partial E} = 1/T \,, \quad \frac{\partial^2 S}{\partial E^2} = - \frac{1}{C^{\mathrm{E}} T^2}$



Specific heat paradox (microcanonical)

R. Emden (1907); A.S. Eddington (1926);
D. Lynden-Bell & R. Wood (1968); W. Thirring (1970);
W. Thirring, H. Narnhofer & H. A. Posch (2003)

Coulomb: $\langle \Phi_i \rangle = -k_\mathrm{B} T$ ¡ CLASSICAL !

⟹ $$T_\mathrm{kin} = -\frac{1}{2}\langle \Phi_\mathrm{tot} \rangle = 3Nk_\mathrm{B}T/2$$

$$2T_\mathrm{kin} + \langle \Phi_\mathrm{tot} \rangle = 0 = E + T_\mathrm{kin}$$

⟹ $$C^E = \frac{\mathrm{d}E}{\mathrm{d}T} = -\frac{\mathrm{d}T_\mathrm{kin}}{\mathrm{d}T} = -3Nk_\mathrm{B}/2 < 0$$!

Quantum Regime

$$h = 6.62606896 \cdot 10^{-34}\,\mathrm{J\ s} = \frac{4}{K_{\mathrm{J}}^2 R_{\mathrm{K}}}$$

$$\text{with } K_{\mathrm{J}} = 2e/h\,,\ R_{\mathrm{K}} = h/e^2$$

$$h\nu = 1\,\mathrm{kg}\,c^2$$

$$\rightarrow \nu = 135639273 \cdot 10^{42}\mathrm{Hz}$$

Quantum Harmonic Oscillator

$$H_{\mathrm{S}} = \frac{p^2}{2M} + \frac{M}{2}\omega_0^2 x^2 \,, \quad E_n = (n + \frac{1}{2})\hbar\omega_0 \,, \quad \beta = \frac{1}{k_{\mathrm{B}}T}$$

partition function

$$Z = \sum_{n=0}^{\infty} \mathrm{e}^{-\beta E_n} = \frac{1}{2\sinh[\hbar\beta\omega_0/2]}$$

entropy

$$S = k_{\mathrm{B}}\left[\ln(Z) - \beta\frac{\partial}{\partial\beta}\ln(Z)\right]$$

$$= k_{\mathrm{B}}\left[\frac{\hbar\beta\omega_0}{\exp(\hbar\beta\omega_0) - 1} - \ln\left(1 - \exp(-\hbar\beta\omega_0)\right)\right]$$

Specific heat: $C^{\mathrm{E}} = \frac{\partial E}{\partial T}\,, \quad E = \frac{\hbar\omega_0}{2} + \frac{\hbar\omega_0}{\exp\left(\hbar\omega_0\beta\right) - 1}$

Quantum harmonic Oscillator: $C^E = k_B \left(\frac{\hbar\beta\omega_0}{2\sinh(\hbar\beta\omega_0/2)} \right)^2$

low-temperature behavior

$$C^E = k_B \left(\frac{\hbar\omega_0}{k_B T} \right)^2 \exp\left(-\frac{\hbar\omega_0}{k_B T} \right)$$

high-temperature behavior

$$C^E = k_B \left[1 - \frac{1}{12} \left(\frac{\hbar\omega_0}{k_B T} \right)^2 + O(T^{-4}) \right]$$

the limit $\omega_0 \to 0$ does NOT yield the specific heat $C^{\mathrm{E}} = k_{\mathrm{B}}/2$ of the free particle

$$\Rightarrow \quad E \neq \frac{1}{2} k_{\mathrm{B}} T$$

No virial theorem !

No equipartition th.!

? Negative Temperature ?

Spin system: $|\vec{S}| = 1/2\,; \quad \vec{\mu} = \gamma \vec{S}\,; \quad H = -\sum \vec{\mu}_i \cdot \vec{B}$

$$\vec{S} \parallel \vec{B} \Rightarrow \text{ Two-State-System: } \quad \epsilon_g = -\frac{1}{2}\gamma B < \epsilon_e = +\frac{1}{2}\gamma B = \mu B$$

$$N = n_g + n_e \quad \& \; E = \mu B (n_e - n_g)\,, \text{ typically } \; E < 0$$

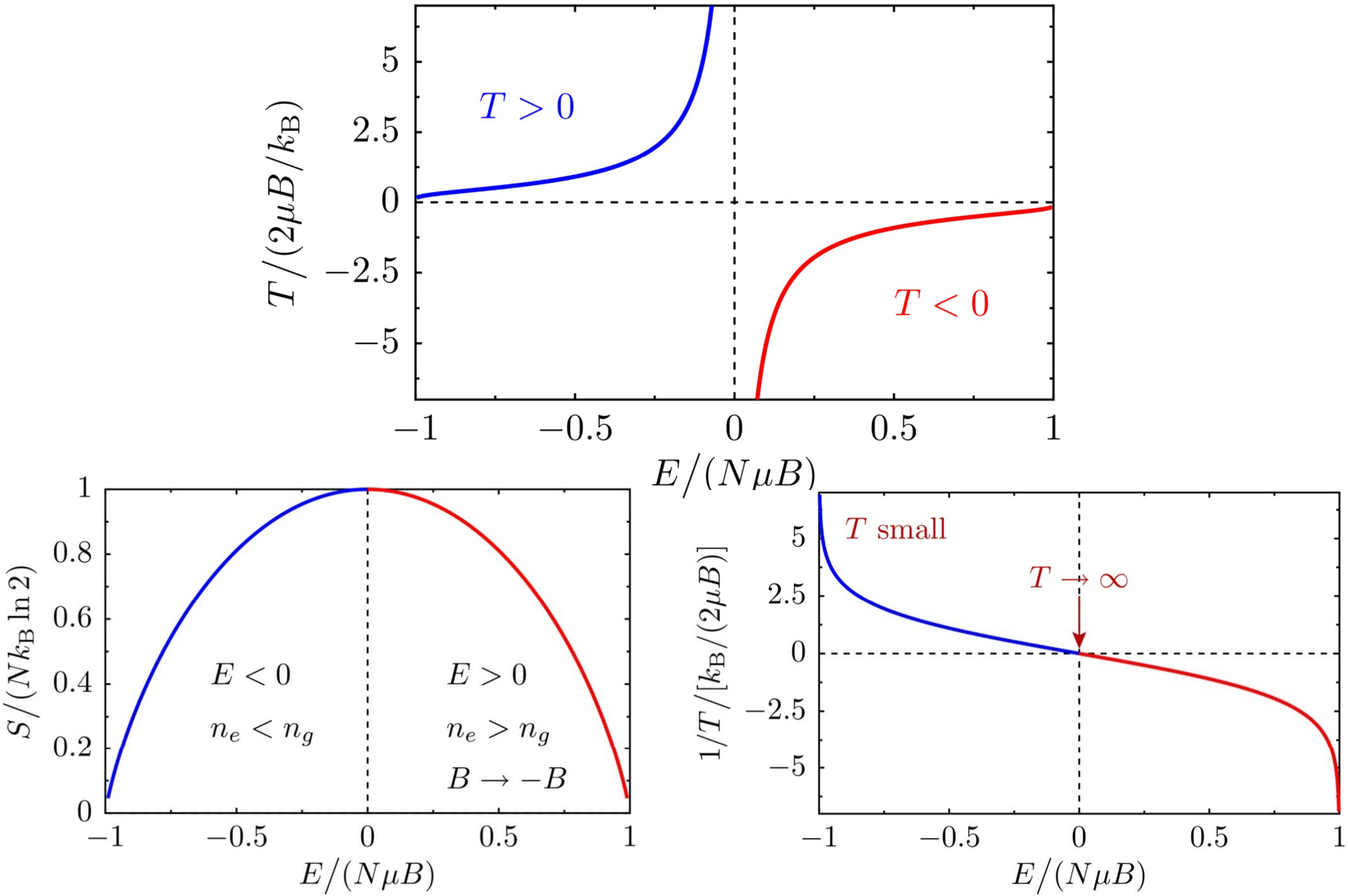
$$\Omega = \frac{N!}{n_g!\, n_e!} \Rightarrow S = k_{\mathrm{B}} \ln \Omega$$

$$\Rightarrow n_g = \frac{1}{2}\left(N - \frac{E}{\mu B}\right)$$

$$\Rightarrow n_e = \frac{1}{2}\left(N + \frac{E}{\mu B}\right)$$

$$\Rightarrow \frac{1}{T} = \frac{\partial S}{\partial E}$$

Negative (Spin)-Temperature!



Relativistic Regime

$$\Sigma(u = 0) \longrightarrow \Sigma'(u \neq 0)$$

$$x' = \gamma(u)\,(x - u \cdot t)$$

$$t' = \gamma(u)\left(t - ux/c^2\right)$$

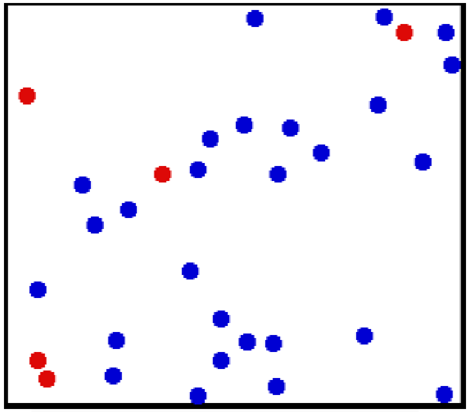
$$\text{with } \gamma(u) = \left(1 - \frac{u^2}{c^2}\right)^{-1/2}$$

$$x^2 - c^2 t^2 = (x')^2 - c^2 (t')^2$$

Note: thermodynamic observables are **NONLOCAL**

J. Dunkel & P.H., *Relativistic Brownian motion*, Phys. Rep. **471**, 1 - 73 (2009)

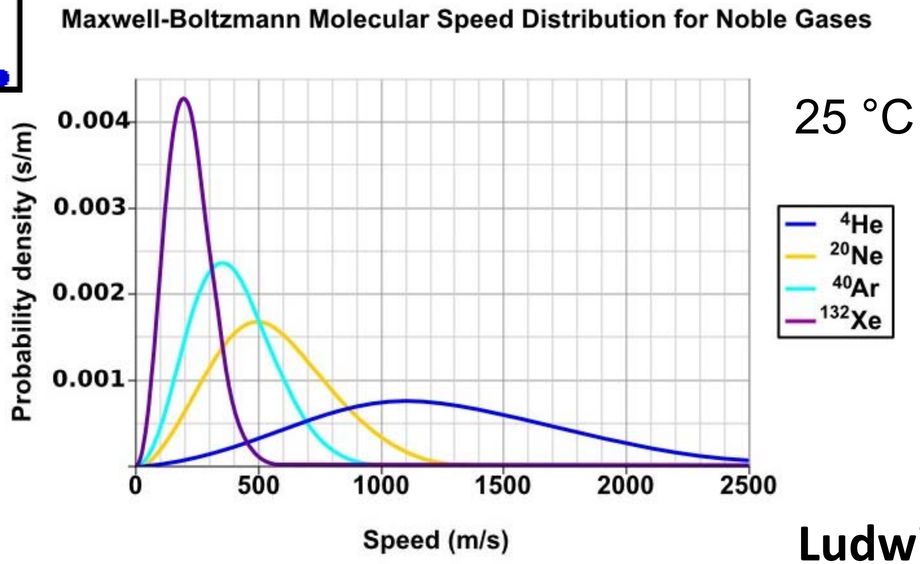
Maxwell-Boltzmann distribution

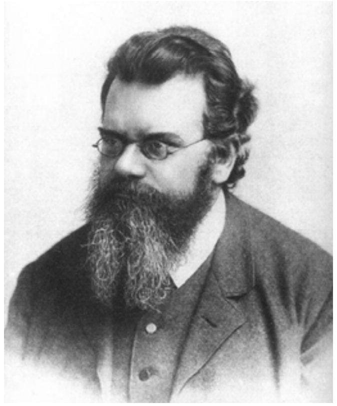


velocity distribution:

$$f_{\vec{v}}(v_x, v_y, v_z) = \left(\frac{m}{2\pi\, k_\mathrm{B} T}\right)^{3/2} \exp\left(-\frac{v_x^2 + v_y^2 + v_z^2}{2\, k_\mathrm{B} T/m}\right)$$

James Clerk Maxwell
(1831 – 1879)

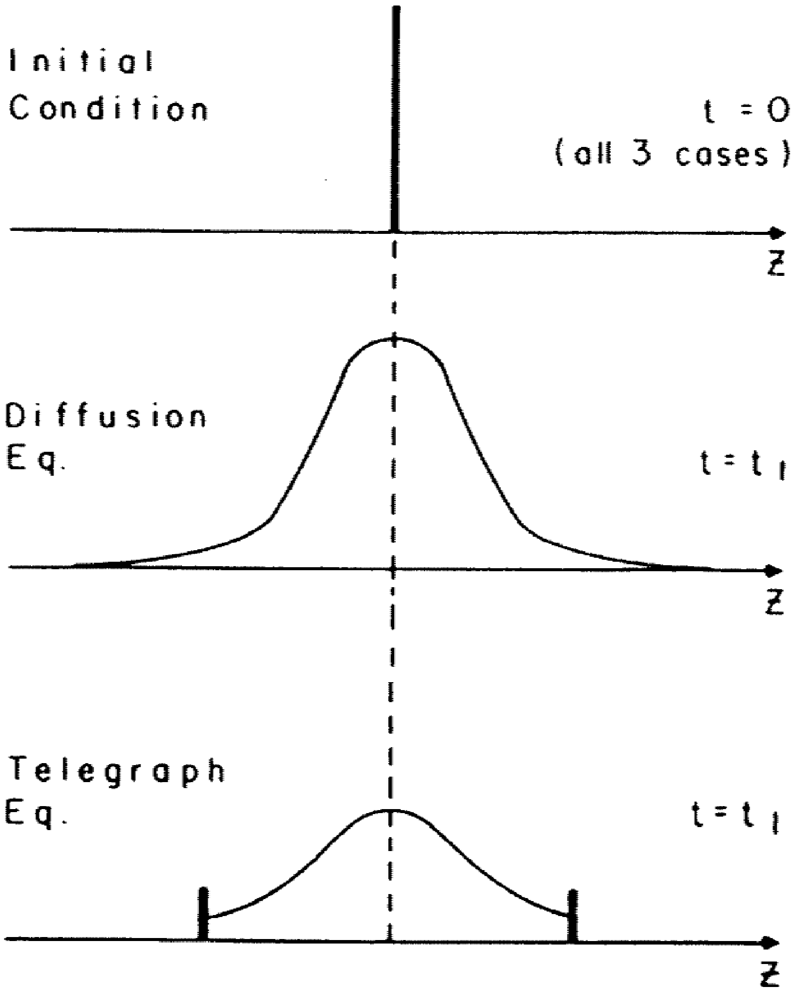




Ludwig Eduard Boltzmann
(1844 – 1906)

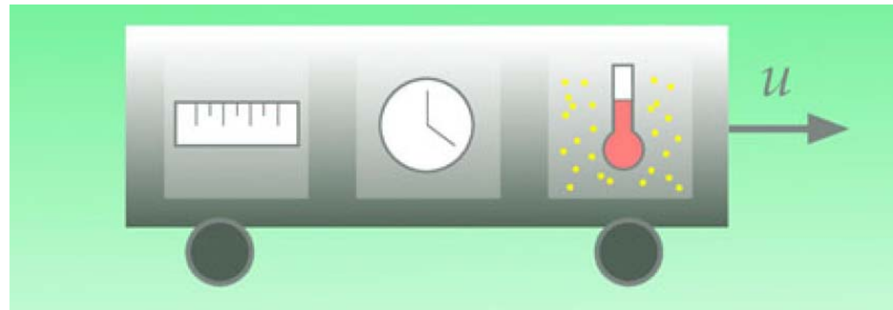
! v < c violated !

Relativistic Brownian motion



Masoliver & Weiss,
Eur. J. Phys. **17**: 190 (1996)

J. Dunkel, P.H., Phys. Rep. **471**, 1 (2009)

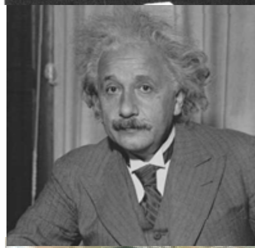


$$\gamma = \left[1 - (u/c)^2\right]^{-1/2}$$

$$l'(u) = l_0 \gamma^{-1} \qquad t'(u) = t_0 \gamma^{+1}$$

$$T'(u) = T_0 \gamma^{-\alpha}$$
$$\alpha = ?$$

Relativistic thermodynamics



$$T'(w) = T\;(1 - w^2)^{\alpha/2}$$

$$\alpha = \begin{cases} +1 & \text{Planck, Einstein} \\ \;\;0 & \text{Landsberg, van Kampen} \\ -1 & \text{Ott} \end{cases}$$

Why this confusion?

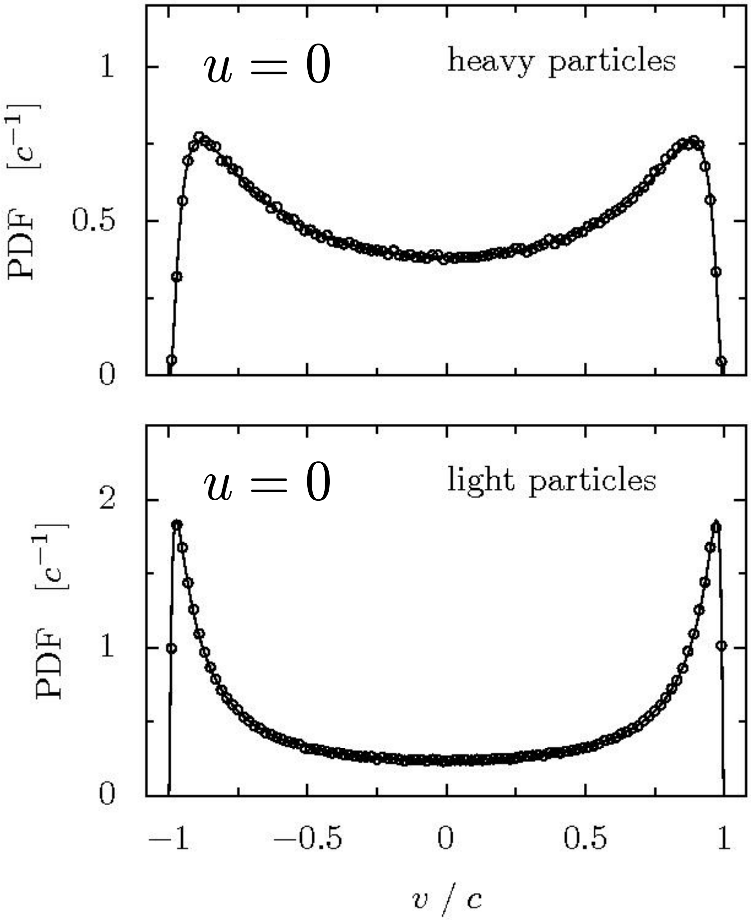
(i) R-TD non-local theory

(ii) different definitions of work and heat

Jüttner Gas

$$f_{\text{Maxwell}}(\vec{p}) = [\beta/(2\pi m)]^{d/2} \exp\left(-\beta p^2/2m\right)$$

$$f_{\text{Jüttner}}(\vec{p}) = Z_d^{-1} \exp\left[-\beta_{\text{J}}(m^2c^4 + p^2c^2)^{1/2}\right]$$



$$\langle \vec{p} \cdot \vec{v} \rangle = d k_{\text{B}} \mathcal{T} = d/\beta_{\text{J}}$$

statistical relativistic temperature

$$T = \mathcal{T} = (k_{\text{B}}\beta_{\text{J}})^{-1}$$

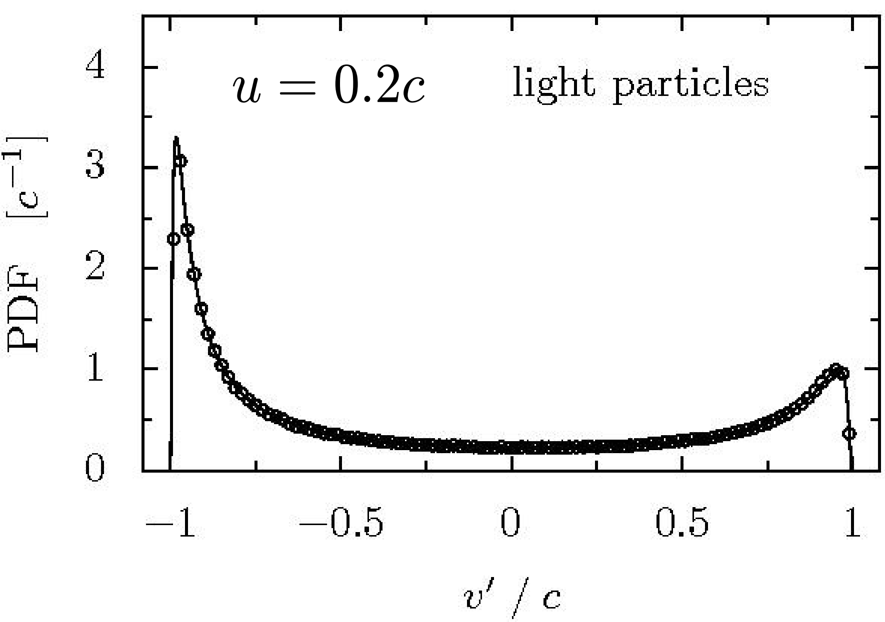
J. Dunkel & P.H., Phys. Rep. **471**, 1-73 (2009)

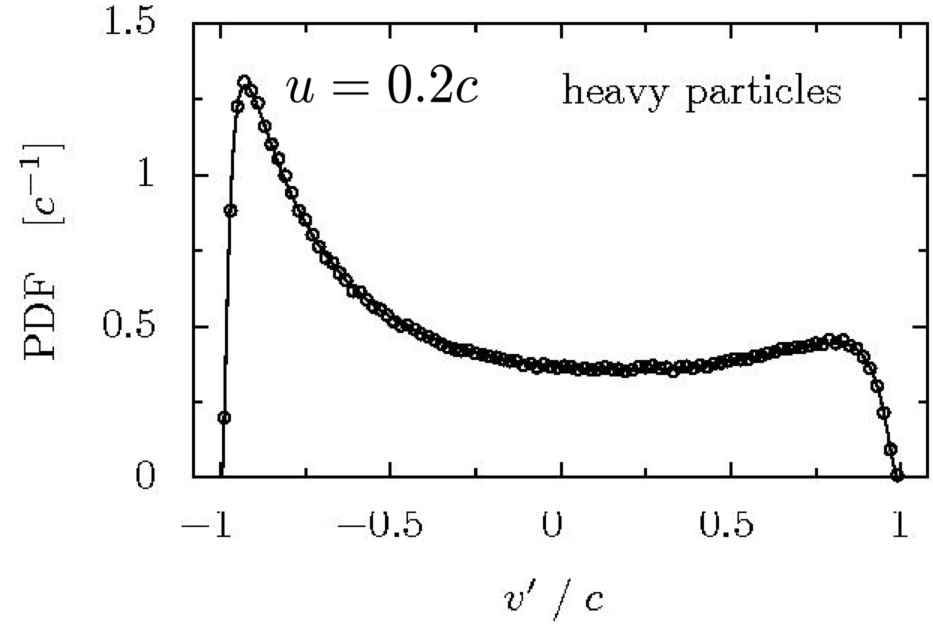
Moving Observers

$$\Sigma(u=0) \longrightarrow \Sigma'(u \neq 0)$$

$$f'_{\text{Jüttner}}(v',u) = Z_1^{-1} \frac{m\gamma(v')^3}{\gamma(u)} \exp\left[-\beta_{\text{J}}\gamma(u)m\gamma(v')(c^2+uv')\right]$$

v' : velocity in moving frame





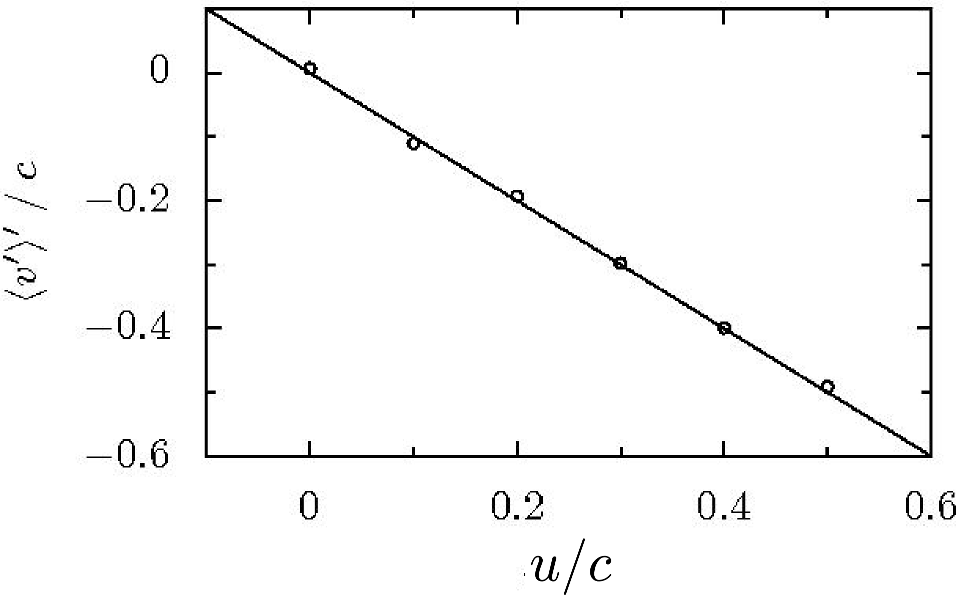
J. Dunkel & P.H., Phys. Rep. **471**, 1-73 (2009)

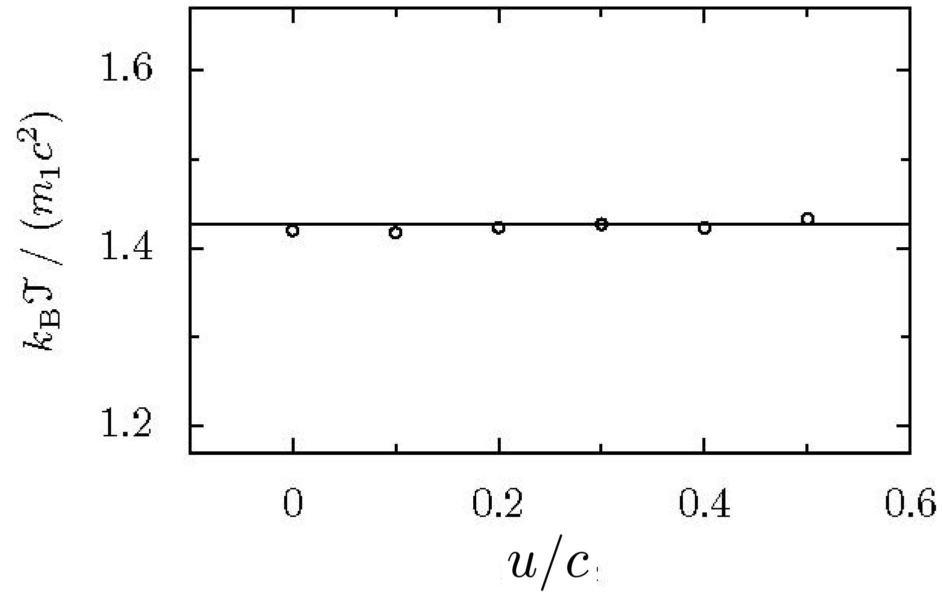
Measuring temperature in Lorentz invariant way

Lorentz invariant equipartition theorem

$$k_B \mathcal{T} = m\gamma(u)^3 \langle \gamma(v')(v'+u)^2 \rangle_{t'}$$

with $u = -\langle v' \rangle_{t'}$





Starting points in RTD?

"BAD" macroscopic variables $\mathcal{N}, \mathcal{U}^0, \boldsymbol{\mathcal{U}}$

- ambiguous definition
- hide non-local character

"GOOD" mesoscopic tensor densities $j^\mu(t,x),\ \theta^{\mu\nu}(t,x),\ \ldots$

- uniquely defined, e.g., can be derived from Lagrangians
- encode symmetries ⇔ conserved Noether currents

Thermodynamic variables

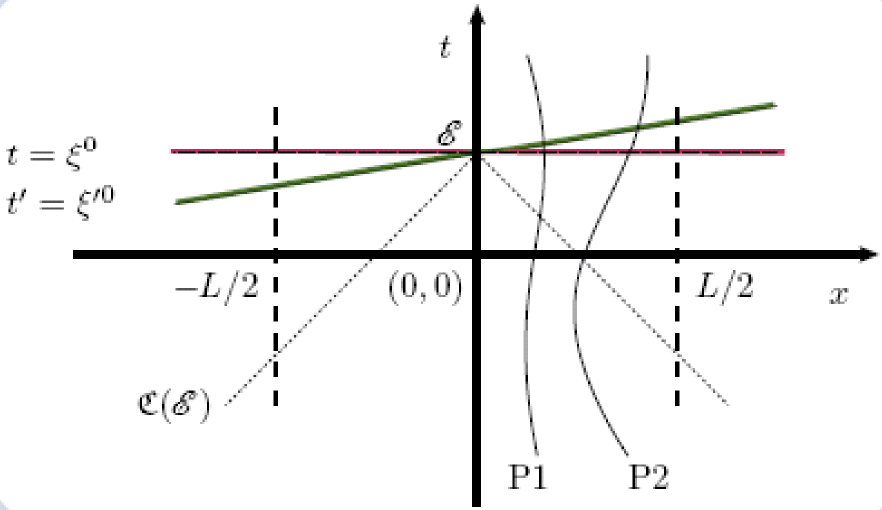
$$\partial_\mu j^\mu \equiv 0 \qquad \partial_\nu s^\nu \equiv 0 \qquad \partial_\mu \theta^{\mu i} \not\equiv 0$$

non-local TD variables ⇔ surface integrals in space-time

$$\mathcal{N}[\mathfrak{H}] := \int_{\mathfrak{H}} d\sigma_\nu \, j^\nu = \mathcal{N}[\mathfrak{H}'] \quad \text{scalar}$$

$$\mathcal{S}[\mathfrak{H}] := \int_{\mathfrak{H}} d\sigma_\nu \, s^\nu = \mathcal{S}[\mathfrak{H}'] \quad \text{scalar}$$

$$\mathcal{U}^\nu[\mathfrak{H}] := \int_{\mathfrak{H}} d\sigma_\mu \, \theta^{\mu\nu} \neq \mathcal{U}^\nu[\mathfrak{H}'] \quad \text{4-vector}$$



$$\mathcal{N}[\mathfrak{H}] = \mathcal{N}'[\mathfrak{H}] = \mathcal{N}'[\mathfrak{H}'] = \mathcal{N}[\mathfrak{H}'] \qquad \mathcal{S}[\mathfrak{H}] = \mathcal{S}'[\mathfrak{H}] = \mathcal{S}'[\mathfrak{H}'] = \mathcal{S}[\mathfrak{H}'] \qquad \Lambda^\nu{}_\mu \mathcal{U}^\mu[\mathfrak{H}] = \mathcal{U}'^\nu[\mathfrak{H}] \neq \mathcal{U}'^\nu[\mathfrak{H}'] = \Lambda^\nu{}_\mu \mathcal{U}^\mu[\mathfrak{H}']$$

Heat, second law & temperature

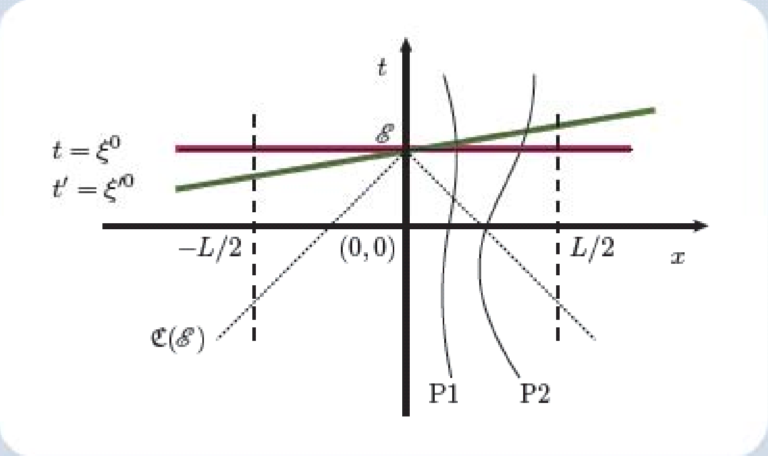
Planck-Einstein

$$\text{đ}\mathcal{Q}'[\mathcal{J}'] := \mathcal{T}'\mathrm{d}\mathcal{S}' := \mathrm{d}\mathcal{U}'^0[\mathcal{J}'] - w'\mathrm{d}\mathcal{U}'^1[\mathcal{J}'] + \mathcal{P}'\mathrm{d}V'$$

$$\gamma = (1 - w'^2)^{-1/2}$$

$$\mathcal{T}'\,\mathrm{d}\mathcal{S}' := \text{đ}\mathcal{Q}'^0[\mathcal{J}'] = \gamma^{-1}\text{đ}\mathcal{Q}^0[\mathcal{J}] = \gamma^{-1}\,\mathcal{T}\,\mathrm{d}\mathcal{S}$$

cooler



Ott

$$\text{đ}\mathcal{Q}^\mu[\mathcal{J}] := \mathrm{d}\mathcal{U}^\mu[\mathcal{J}] - \text{đ}\mathcal{A}^\mu[\mathcal{J}],$$

$$(\text{đ}\mathcal{A}^\mu[\mathcal{J}]) := (-\mathcal{P}\mathrm{d}V, \mathbf{0}).$$

$$\mathcal{T}'\,\mathrm{d}\mathcal{S}' := \text{đ}\mathcal{Q}'^0 = \gamma\text{đ}\mathcal{Q}^0 = \gamma\,\mathcal{T}\,\mathrm{d}\mathcal{S}.$$

hotter

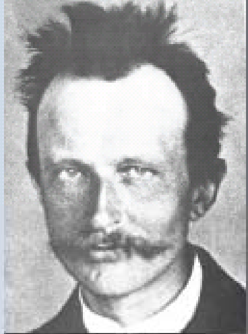
van Kampen / Landsberg

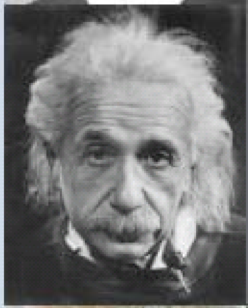
$$\text{đ}\mathcal{Q}^\mu[\mathcal{J}] := \mathrm{d}\mathcal{U}^\mu[\mathcal{J}] - \text{đ}\mathcal{A}^\mu[\mathcal{J}],$$

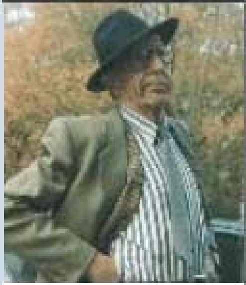
$$\text{đ}\mathcal{Q}' := -w'_\mu\text{đ}\mathcal{Q}'^\mu = -w_\mu\text{đ}\mathcal{Q}^\mu = \text{đ}\mathcal{Q} = \text{đ}\mathcal{Q}^0.$$

$$\mathcal{T}'\mathrm{d}\mathcal{S}' := \text{đ}\mathcal{Q}' = \text{đ}\mathcal{Q} = \mathcal{T}\mathrm{d}\mathcal{S},$$

T'=T







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“Isochronous” RTD useful at all?

shortcomings

(i) not feasible experimentally

(ii) extension to General Relativity problematic

“Isochronous” RTD useful at all?

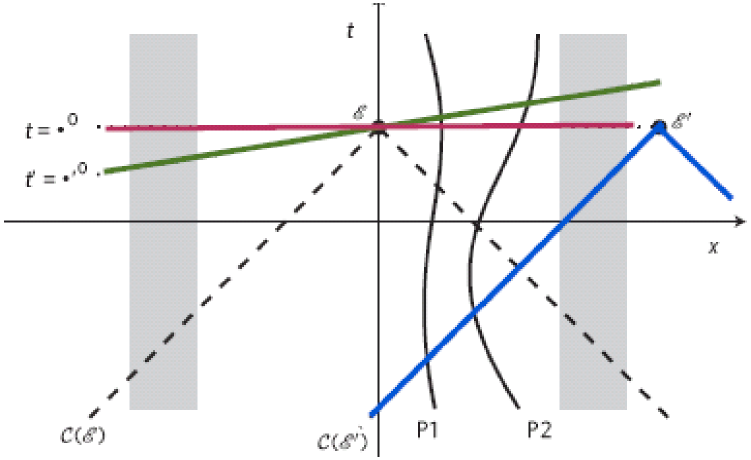
shortcomings

(i) not feasible experimentally
(ii) extension to General Relativity problematic

SOLUTION: replace isochronous hyperplanes by lightcones

Why?

✓ photographic measurements
✓ equivalent for moving observers
✓ GR extension
✓ nonrelativistic limit



“Photographic” thermodynamics

light-cone averaging

$$\mathfrak{C}(\mathscr{E}) \;:=\; \{\; (t, \boldsymbol{x}) \mid t = \xi^0 - |\boldsymbol{x} - \boldsymbol{\xi}| \;\}$$

photographic state variables

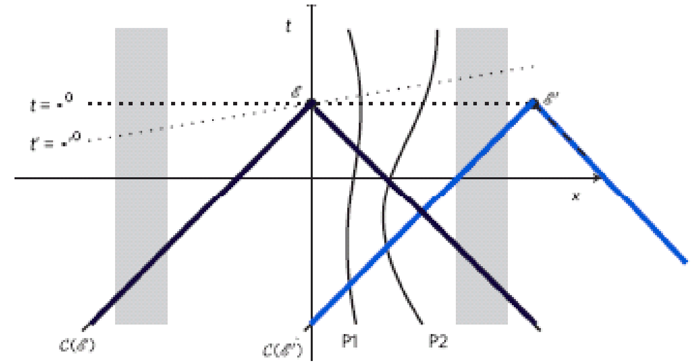
$$\mathcal{U}^0[\mathfrak{C}] \;=\; N \langle p^0 \rangle$$

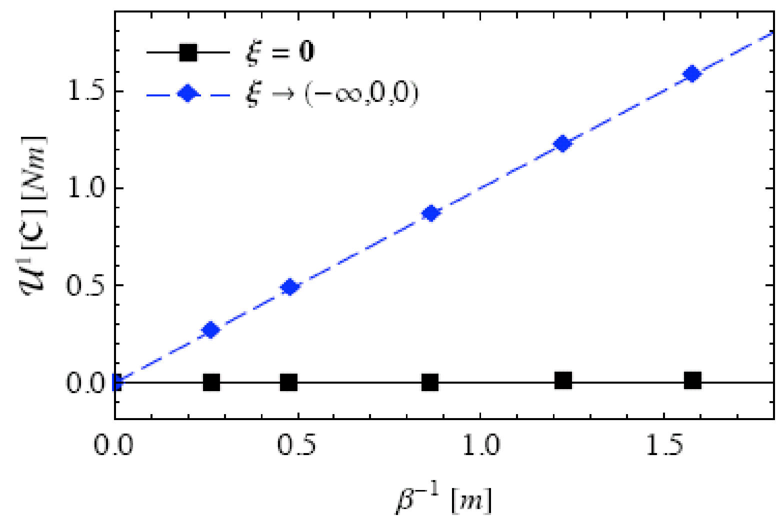
$$\mathcal{U}^i[\mathfrak{C}] \;=\; \frac{N}{\beta} \int \mathrm{d}^3 x \, \frac{x^i - \xi^i}{|\boldsymbol{x} - \boldsymbol{\xi}|} \, \varrho(\boldsymbol{x})$$

distant observer (at rest !)

$$\mathcal{U}^i[\mathfrak{C}] = -\frac{\xi^i}{|\boldsymbol{\xi}|} N k_{\mathrm{B}} \mathcal{T}$$

apparent drift !





JD, Peter Hanggi & Stefan Hilbert, Nature Physics 5:741, 2009

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"Photographic" thermodynamics

light-cone averaging

$$\mathfrak{C}(\mathscr{E}) \;:=\; \{\, (t, \boldsymbol{x}) \mid t = \xi^0 - |\boldsymbol{x} - \boldsymbol{\xi}| \,\}$$

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$$\mathfrak{U}^0[\mathfrak{C}] \;=\; N\,\langle p^0 \rangle$$

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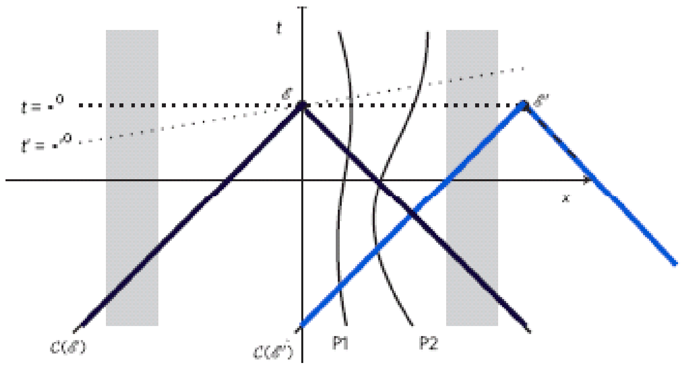
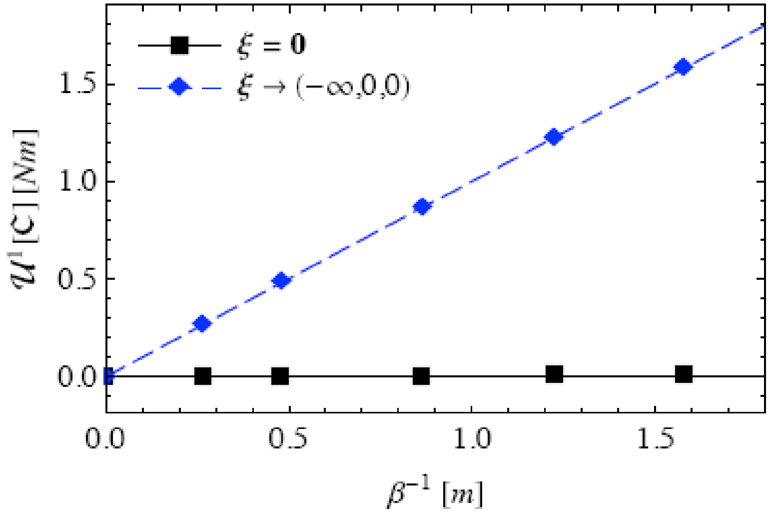
distant observer (at rest !)

$$\mathfrak{U}^i[\mathfrak{C}] = -\frac{\xi^i}{|\boldsymbol{\xi}|} N k_{\mathrm{B}} \mathfrak{T}$$

apparent drift !





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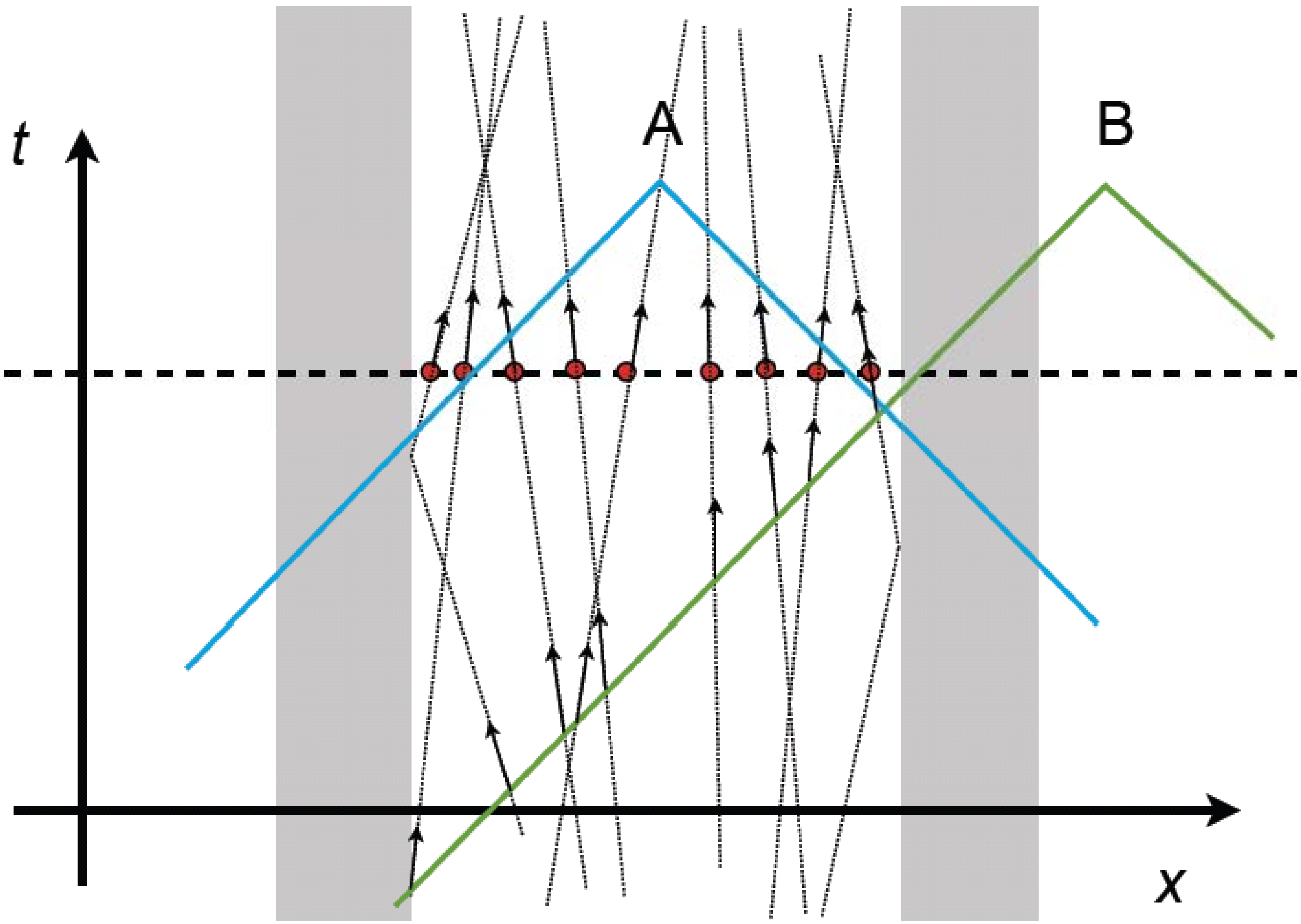

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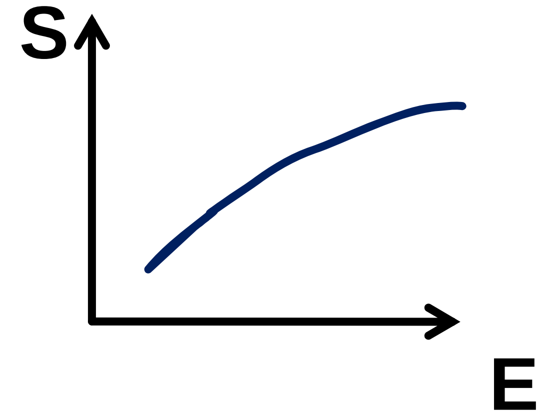
Take Home Messages

- $\exists$ absolute temperature $T \geq 0$



- $T = \frac{\partial E}{\partial S} \dots$

  with $S(E)$ a concave function of E !

- $S(E)$: microcanonical NOT always $=$ $S(E)$: canonical (long range, non-ergodic parts)

- T does **NOT** fluctuate

- quantum mechanics: no virial theorem; no equipartion

- relativistic statistical thermometer

$$\mathcal{T}_\Sigma = \mathcal{T}_{\Sigma'}$$

ENDE

FIN

THAT'S IT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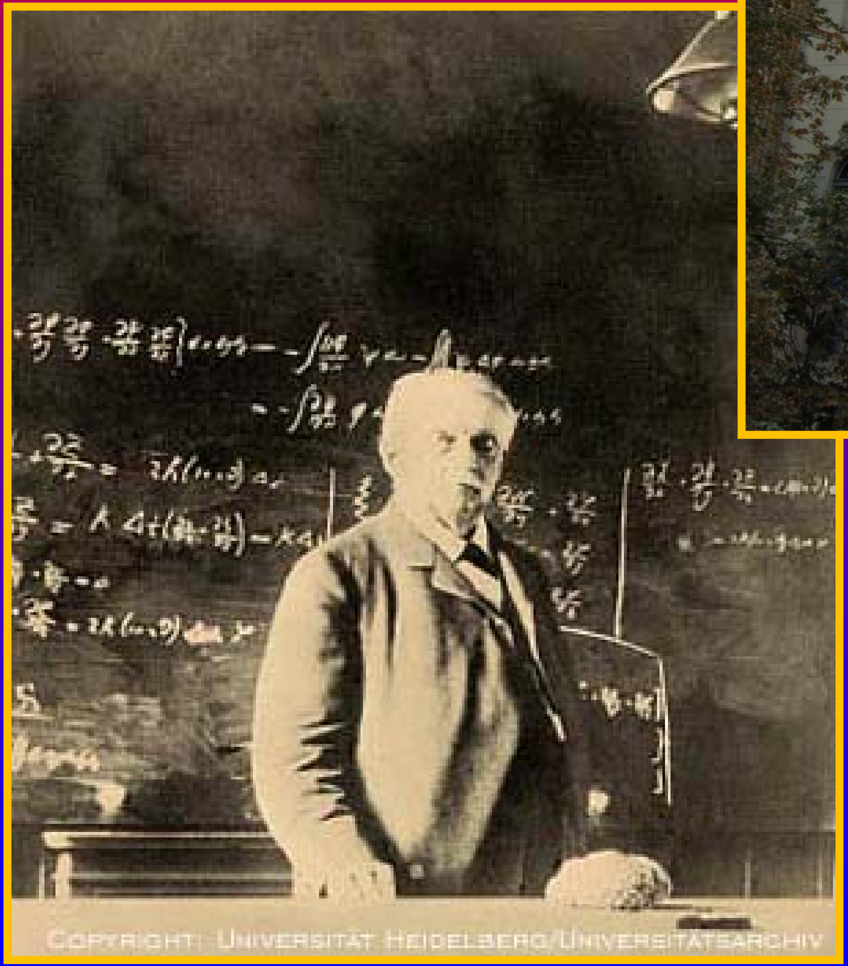
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