TIME-REVERSAL SYMMETRY RELATIONS: FROM THE MULTIBAKER MAP TO OPEN QUANTUM SYSTEMS

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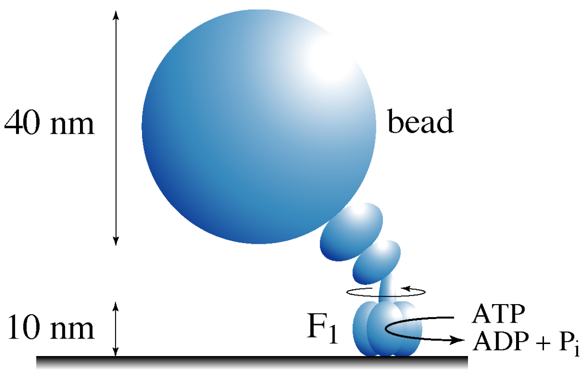
- **INTRODUCTION: TIME-REVERSAL SYMMETRY BREAKING**
- **FLUCTUATION THEOREM FOR CURRENTS & NONLINEAR RESPONSE**
- **FLUCTUATION THEOREM FOR OPEN QUANTUM SYSTEMS**
- **CONCLUSIONS**

NONEQUILIBRIUM NANOSYSTEMS MANIFEST DIRECTIONALITY & RANDOMNESS

Free-energy driving

Molecular motor: F_oF_1-ATPase

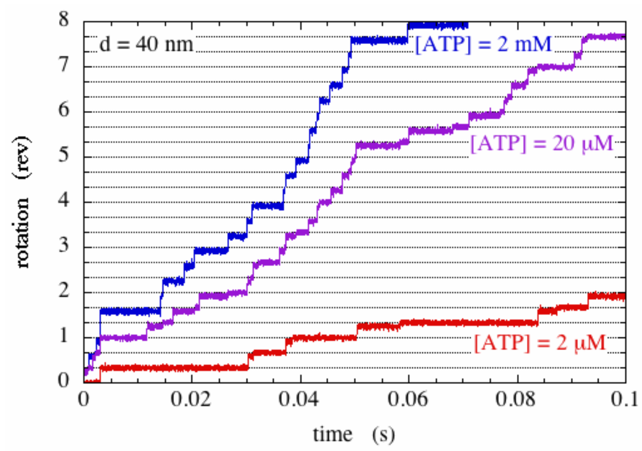
K. Kinosita and coworkers (2001): F_1-ATPase + filament/bead



$T = 300$ K

ATP hydrolysis

Power = 10^{-18} Watt



R. Yasuda, H. Noji, M. Yoshida, K. Kinosita Jr. & H. Itoh, Nature **410** (2001) 898
P. Gaspard & E. Gerritsma, J. Theor. Biol. **247** (2007) 672

Electronic nanocircuit:

quantum dot (QD) observed with a quantum point contact (QPC)

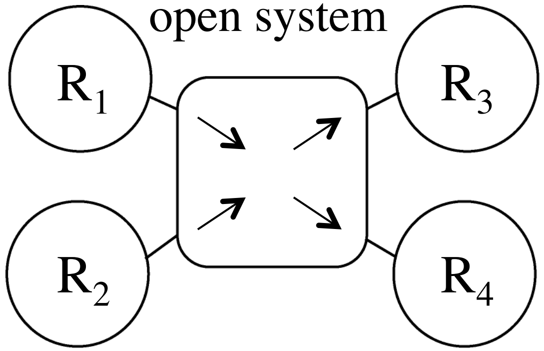
$T < 1$ K

$\text{Power}_{QD} = 10^{-19}$ Watt $\qquad$ $\text{Power}_{QPC} = 10^{-12}$ Watt

S. Gustavsson et al., Phys. Rev. Lett. **96**, 076605 (2006).

COUPLING TOGETHER DIFFERENT TRANSPORT PROCESSES

Free-energy supply from thermal or particle reservoirs:



De Donder affinities or thermodynamic forces:

thermal affinity: $$A_0 = \frac{1}{k_B T'} - \frac{1}{k_B T}$$

chemical or electrochemical affinity: $$A_\alpha = \frac{\mu_\alpha}{k_B T} - \frac{\mu_\alpha'}{k_B T'} \qquad A_\gamma = \frac{\Delta G_\gamma}{k_B T} = \frac{G_\gamma - G_\gamma^{\mathrm{eq}}}{k_B T}$$

thermodynamic entropy production: $$\frac{1}{k_B}\frac{d_i S}{dt} = \sum_\gamma A_\gamma \langle J_\gamma \rangle \geq 0$$

average currents: $$\langle J_\alpha \rangle = \boxed{\sum_\beta L_{\alpha,\beta} A_\beta} + \boxed{\frac{1}{2}\sum_{\beta,\gamma} M_{\alpha,\beta\gamma} A_\beta A_\gamma + \frac{1}{6}\sum_{\beta,\gamma,\delta} N_{\alpha,\beta\gamma\delta} A_\beta A_\gamma A_\delta + \cdots}$$

linear response: Onsager 1931 ***nonlinear response***

- Energy transduction in molecular motors: mechanochemical coupling (2 currents)
- Thermoelectric device (2 currents)
- Mass separation (3 currents)
- Electron counting statistics of quantum dots by quantum point contact (2 currents)

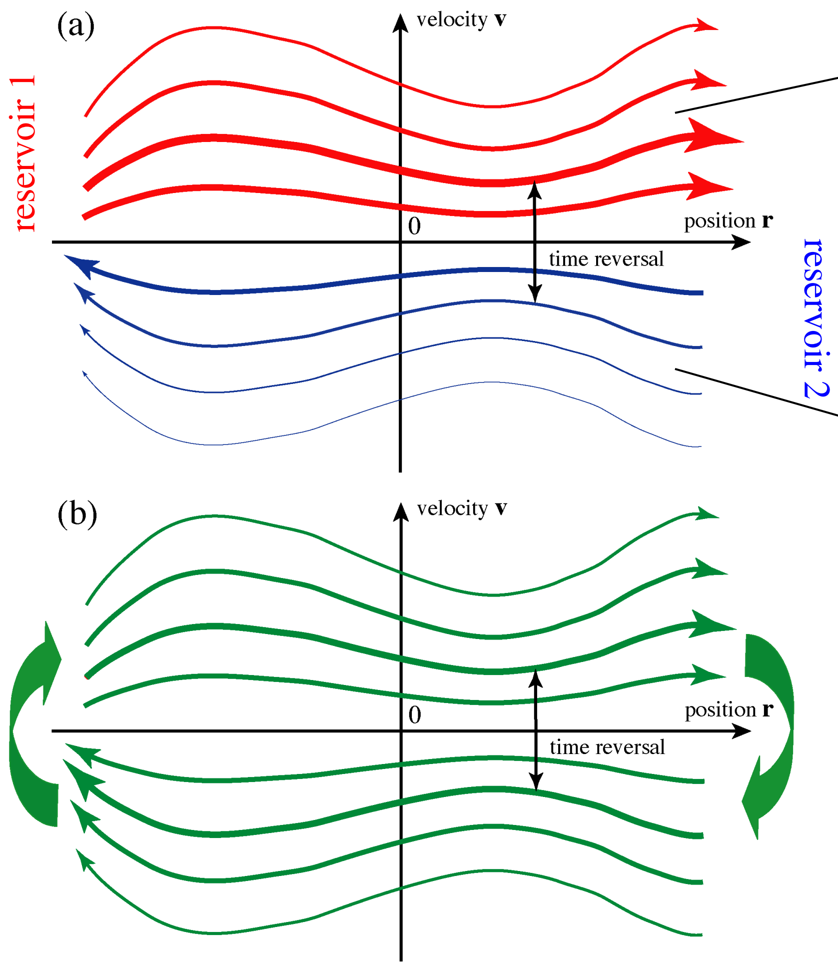
TIME ASYMMETRY IN THE STATISTICAL DESCRIPTION

nonequilibrium

entropy production
energy dissipation

equilibrium

constant entropy



more probable trajectories

Asymmetry under time reversal

less probable reversed trajectories

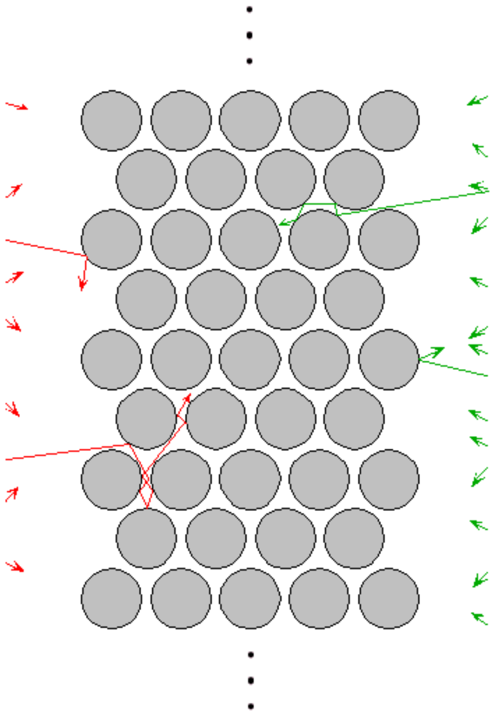
Symmetry under time reversal

Microreversibility & spontaneous symmetry breaking:
The solution of an equation may have a lower symmetry than the equation itself.

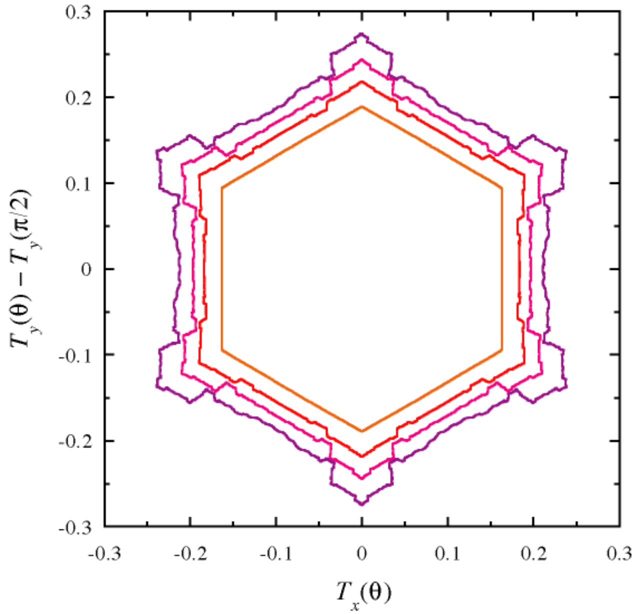
P. Gaspard, Adv. Chem. Phys. **135** (2007) 83; in: *Physics of Self-Organization Systems*, Edited by S. Ishiwata & Y. Matsunaga (World Scientific, New Jersey, 2008) p. 67.

NONEQUILIBRIUM STEADY STATES
LORENTZ GAS

no coupling between mass and energy transports



cumulative functions
of the steady state
for different interdisk distances:



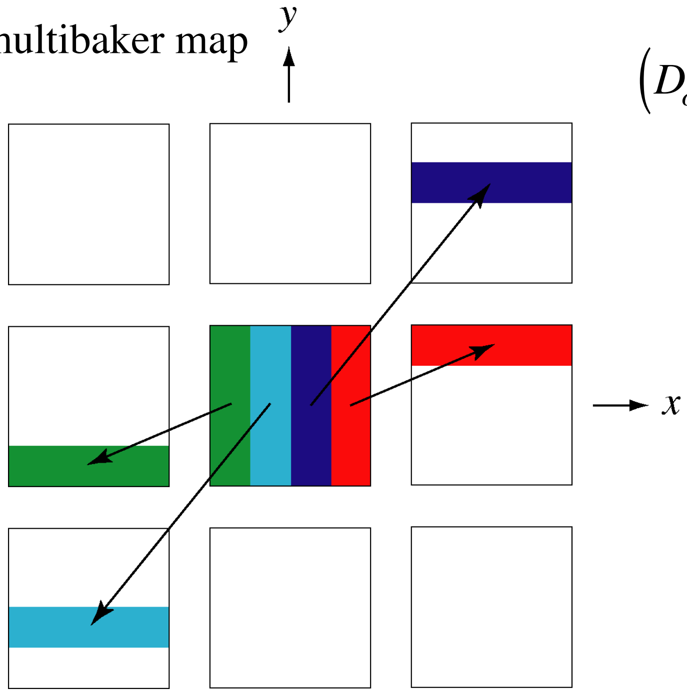
The distribution is singular in the stable direction but smooth in the unstable direction:

breaking of time-reversal symmetry since $W_u = \Theta(W_s)$ but $W_u \neq W_s$.

P. Gaspard, Physica A **369** (2006) 201.

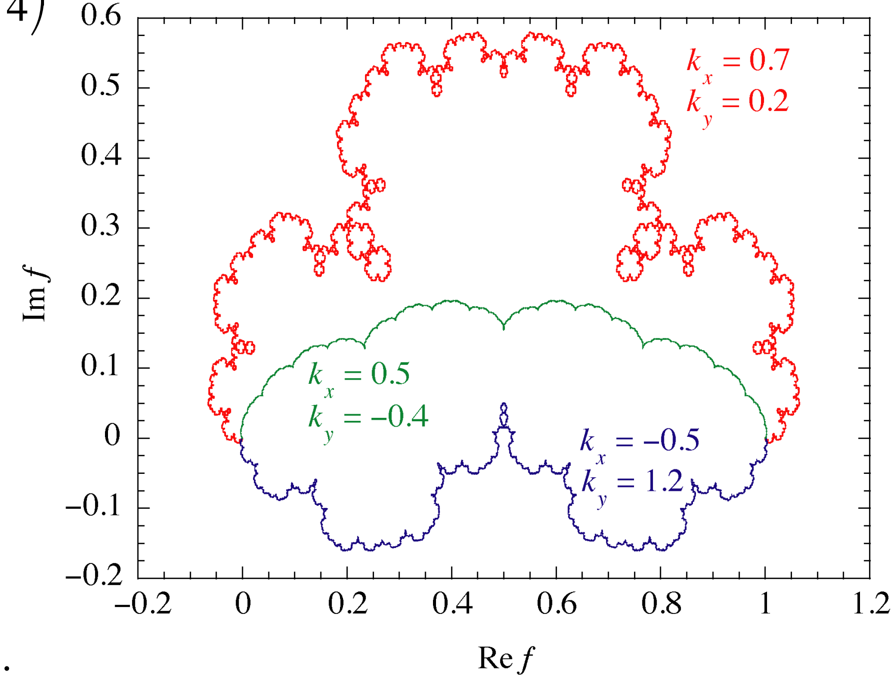
INDEPENDENT-PARTICLE SYSTEMS:
COUPLING BETWEEN MASS TRANSPORT IN DIFFERENT DIRECTIONS

multibaker map



$$\left(D_{\alpha\beta}\right) = \begin{pmatrix} 1/2 & 1/4 \\ 1/4 & 1/4 \end{pmatrix}$$

diffusive modes



Van Hove incoherent intermediate scattering function:

$$F_s(\mathbf{k},t) = \left\langle e^{i\mathbf{k}\cdot(\mathbf{r}_t - \mathbf{r}_0)} \right\rangle \propto e^{s_\mathbf{k} t}$$

dispersion relation of diffusion: diffusion coefficients, Burnett & super-Burnett coefficients,...

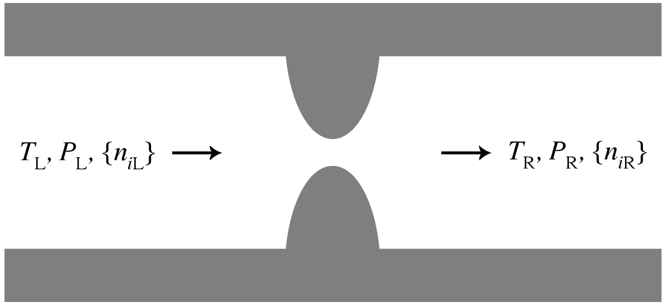
$$s_\mathbf{k} = -D_{\alpha\beta} k_\alpha k_\beta + B_{\alpha\beta\gamma\delta} k_\alpha k_\beta k_\gamma k_\delta + \cdots$$

microreversibility in a magnetic field: $s_\mathbf{k}(\mathcal{B}) = s_{-\mathbf{k}}(-\mathcal{B})$

S. Tasaki & P. Gaspard, J. Stat. Phys. **81** (1995) 935.

MANY-PARTICLE SYSTEMS: STOCHASTIC PROCESSES

Effusion of rarefied gases



ballistic motion

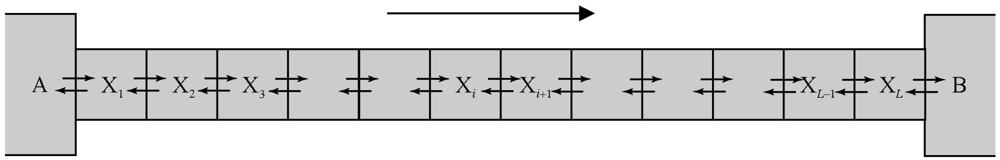
master equation:

$$\frac{d}{dt}P_t(\Delta E, \Delta N)$$
$$= \int_0^\infty d\epsilon\, W_+(\epsilon)\, P_t(\Delta E - \epsilon, \Delta N - 1)$$
$$+ \int_0^\infty d\epsilon\, W_-(\epsilon)\, P_t(\Delta E + \epsilon, \Delta N + 1)$$
$$-P_t(\Delta E, \Delta N) \int_0^\infty d\epsilon\, [W_+(\epsilon) + W_-(\epsilon)]$$

transition rates of effusion:

$$W_\pm(\epsilon) \;=\; \frac{\sigma\, n_\pm}{\sqrt{2\pi m k T_\pm}}\, \frac{\epsilon}{kT_\pm}\, \exp\left(-\frac{\epsilon}{kT_\pm}\right)$$

Diffusion between 2 reservoirs



diffusive motion

fluctuating diffusion equation:

$$\partial_t n = \vec{\nabla} \cdot (\mathcal{D}\vec{\nabla} n - \vec{\eta})$$

$$\langle \eta_k(\vec{r}, t)\rangle = 0$$
$$\langle \eta_k(\vec{r}, t)\eta_l(\vec{r}\,', t')\rangle = 2\mathcal{D}n(\vec{r})\delta(\vec{r} - \vec{r}\,')\delta(t - t')\delta_{kl}$$

Fick's law:

$$\langle J\rangle = -\mathcal{D}\nabla\langle n\rangle$$

(e.g. Lorentz gases, multibaker maps, lattice gases,…)

GRAPH ANALYSIS OF THE MARKOV PROCESS

J. Schnakenberg, Rev. Mod. Phys. **48** (1976) 571

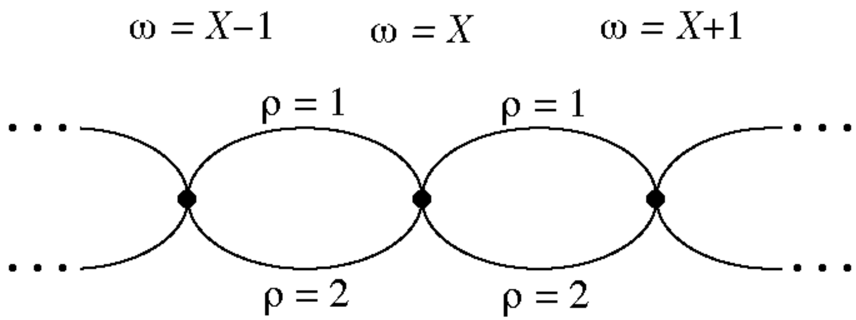
$W_\rho(\omega\,|\,\omega')$ rate of the transition $\omega \xrightarrow{\rho} \omega'$ due to the elementary process $\rho = \pm 1,\ldots,\pm r$

graph G associated with
the Markov process:

- vertices = coarse-grained states ω;
- edges = elementary processes ρ.

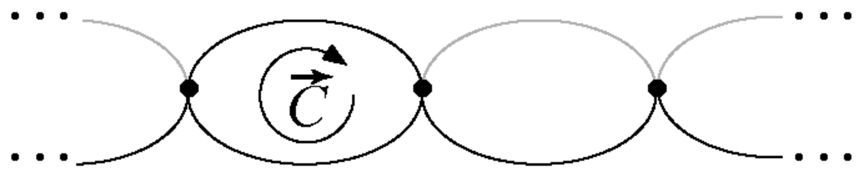
chemical network: $A \overset{\rho=1}{\leftrightarrow} X \overset{\rho=2}{\leftrightarrow} B$

G



cycle $\vec{C}$

$$\prod_{\omega\in\vec{C}} \frac{W_{+\rho}(\omega\,|\,\omega')}{W_{-\rho}(\omega'\,|\,\omega)} \equiv \exp A(\vec{C})$$



affinity A =
nonequilibrium driving potential

fluctuating current
on the edge closing the cycle

STEADY-STATE FLUCTUATION THEOREM FOR CURRENTS

fluctuating currents: $J_\gamma = \frac{1}{t}\int_0^t j_\gamma(t')\,dt'$

ex: • electric currents in a nanoscopic conductor
• rates of chemical reactions
• velocity of a molecular motor

De Donder affinities or thermodynamic forces: A_γ (free energy sources)

Stationary probability distribution P :

• No directionality at equilibrium $A_\gamma = 0$
• Directionality out of equilibrium $A_\gamma \neq 0$

$$\boxed{\frac{P\{+J_\gamma\}}{P\{-J_\gamma\}} \approx e^{t\sum_\gamma A_\gamma J_\gamma}}$$

time interval:

$t \rightarrow +\infty$

valid far from equilibrium as well as close to equilibrium

thermodynamic entropy production: $\frac{1}{k_\mathrm{B}}\frac{d_\mathrm{i}S}{dt}\bigg|_\mathrm{st} = \sum_{\gamma=1}^{c} A_\gamma \langle J_\gamma \rangle \geq 0$

D. Andrieux & P. Gaspard, J. Chem. Phys. **121** (2004) 6167.

D. Andrieux & P. Gaspard, *Fluctuation theorem for currents and Schnakenberg network theory*, J. Stat. Phys. **127** (2007) 107.

GENERATING FUNCTION OF THE CURRENTS: FULL COUNTING STATISTICS

fluctuation theorem for the currents:

with the probability distribution P

$$\boxed{\frac{P\left\{+J_\gamma\right\}}{P\left\{-J_\gamma\right\}} \approx \mathrm{e}^{t\sum_\gamma A_\gamma J_\gamma}}$$

generating function: $$Q(\left\{\lambda_\gamma, A_\gamma\right\}) = \lim_{t\to\infty} -\frac{1}{t}\ln\left\langle \exp\left[-\sum_\gamma \lambda_\gamma \int_0^t j_\gamma(t')dt'\right]\right\rangle_{\text{noneq.}}$$

counting parameters λ_γ

fluctuation theorem for the currents bis: $$\boxed{Q(\left\{\lambda_\gamma, A_\gamma\right\}) = Q(\left\{A_\gamma - \lambda_\gamma, A_\gamma\right\})}$$

average currents: $$\left\langle J_\alpha \right\rangle = \left.\frac{\partial Q}{\partial \lambda_\alpha}\right|_{\lambda_\alpha = 0} = \sum_\beta L_{\alpha,\beta} A_\beta + \frac{1}{2}\sum_{\beta,\gamma} M_{\alpha,\beta\gamma} A_\beta A_\gamma + \frac{1}{6}\sum_{\beta,\gamma,\delta} N_{\alpha,\beta\gamma\delta} A_\beta A_\gamma A_\delta + \cdots$$

D. Andrieux & P. Gaspard, J. Stat. Phys. **127** (2007) 107.

M. Esposito, U. Harbola, and S. Mukamel, Rev. Mod. Phys. **81**, 1665 (2009).

LINEAR & NONLINEAR RESPONSE THEORY

linear response coefficients

Green-Kubo formulas: 2nd cumulants

$$L_{\alpha,\beta} = -\frac{1}{2}\frac{\partial^2 Q}{\partial\lambda_\alpha \partial\lambda_\beta}\left(\{0,0\}\right) = \frac{1}{2}\int_{-\infty}^{+\infty} \left\langle \left[j_\alpha(t) - \langle j_\alpha \rangle\right]\left[j_\beta(0) - \langle j_\beta \rangle\right] \right\rangle dt$$

Onsager reciprocity relations (1931): $L_{\alpha,\beta} = L_{\beta,\alpha}$

nonlinear response coefficients at 2nd order

2nd responses of currents: $M_{\alpha,\beta\gamma} \equiv \dfrac{\partial^3 Q}{\partial\lambda_\alpha \partial A_\beta \partial A_\gamma}\left(\{0,0\}\right)$

1st responses of 2nd cumulants: $R_{\alpha\beta,\gamma} \equiv -\dfrac{\partial^3 Q}{\partial\lambda_\alpha \partial\lambda_\beta \partial A_\gamma}\left(\{0,0\}\right)$

= 1st responses of diffusivities: $R_{\alpha\beta,\gamma} = \dfrac{\partial}{\partial A_\gamma}\displaystyle\int_{-\infty}^{+\infty} \left\langle \left[j_\alpha(t) - \langle j_\alpha \rangle\right]\left[j_\beta(0) - \langle j_\beta \rangle\right] \right\rangle_{\text{noneq.}} dt \,\Big|_{\mathbf{A}=0}$

2nd responses of currents =
1st responses of 2nd cumulants:

$$\boxed{M_{\alpha,\beta\gamma} = \tfrac{1}{2}\left(R_{\alpha\beta,\gamma} + R_{\alpha\gamma,\beta}\right)}$$

D. Andrieux & P. Gaspard, J. Chem. Phys. **121** (2004) 6167; J. Stat. Mech. (2006) P01011.

NONLINEAR RESPONSE THEORY (cont'd)

fluctuation theorem for the currents: $$\boxed{Q(\{\lambda_\gamma, A_\gamma\}) = Q(\{A_\gamma - \lambda_\gamma, A_\gamma\})}$$

average current: $$J_\alpha = \left.\frac{\partial Q}{\partial \lambda_\alpha}\right|_{\lambda_\alpha = 0} = \sum_\beta L_{\alpha,\beta} A_\beta + \frac{1}{2}\sum_{\beta,\gamma} M_{\alpha,\beta\gamma} A_\beta A_\gamma + \frac{1}{6}\sum_{\beta,\gamma,\delta} N_{\alpha,\beta\gamma\delta} A_\beta A_\gamma A_\delta + \cdots$$

nonlinear response coefficients at 3rd order

3rd responses of currents: $$N_{\alpha,\beta\gamma\delta} \equiv \frac{\partial^4 Q}{\partial \lambda_\alpha \partial A_\beta \partial A_\gamma \partial A_\delta}(\{0,0\})$$

2nd responses of 2nd cumulants: $$T_{\alpha\beta,\gamma\delta} \equiv -\frac{\partial^4 Q}{\partial \lambda_\alpha \partial \lambda_\beta \partial A_\gamma \partial A_\delta}(\{0,0\})$$

1st responses of 3rd cumulants = 4th cumulants:

$$S_{\alpha\beta\gamma,\delta} \equiv \frac{\partial^4 Q}{\partial \lambda_\alpha \partial \lambda_\beta \partial \lambda_\gamma \partial A_\delta}(\{0,0\}) = -\frac{1}{2}\frac{\partial^4 Q}{\partial \lambda_\alpha \partial \lambda_\beta \partial \lambda_\gamma \partial \lambda_\delta}(\{0,0\})$$

relations at 3nd order: $$\boxed{N_{\alpha,\beta\gamma\delta} = \tfrac{1}{2}\left(T_{\alpha\beta,\gamma\delta} + T_{\alpha\gamma,\beta\delta} + T_{\alpha\delta,\beta\gamma} - S_{\alpha\beta\gamma,\delta}\right)}$$

D. Andrieux & P. Gaspard, J. Stat. Mech. (2007) P02006.

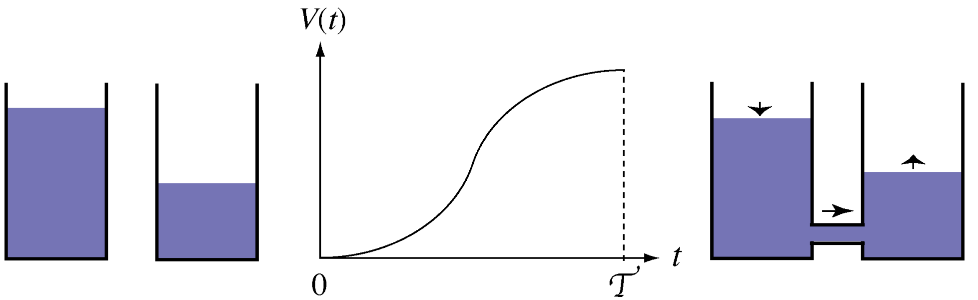
FLUCTUATION THEOREM FOR OPEN QUANTUM SYSTEMS 1

D. Andrieux, P. Gaspard, T. Monnai & S. Tasaki,
The fluctuation theorem for currents in open quantum systems,
New J. Phys. **11** (2009) 043014; *Erratum* 109802

Transient fluctuation theorem: time-dependent quantum systems

Hamiltonian:
$$H(t) = \begin{cases} H_s + \sum_j H_j & \text{for} \quad t \le 0 \\ H_s + \sum_j H_j + V(t) & \text{for} \quad 0 \le t \le \mathcal{T} \\ \tilde{H}_s + \sum_j \tilde{H}_j & \text{for} \quad \mathcal{T} \le t \end{cases}$$

Time-reversal symmetry:
$$\Theta H(t;\mathcal{B})\Theta = H(t;-\mathcal{B})$$



Quantum measurements on the reservoirs before and after the interaction $V(t)$:

$$t \le 0: \quad H_j|\Psi_k\rangle = \varepsilon_{jk}|\Psi_k\rangle$$
$$N_{j\alpha}|\Psi_k\rangle = \nu_{j\alpha k}|\Psi_k\rangle$$
$$\mathcal{T} \le t: \quad \tilde{H}_j|\tilde{\Psi}_l\rangle = \tilde{\varepsilon}_{jl}|\tilde{\Psi}_l\rangle$$
$$\tilde{N}_{j\alpha}|\tilde{\Psi}_l\rangle = \tilde{\nu}_{j\alpha l}|\tilde{\Psi}_l\rangle$$

energy transfer: $\Delta\varepsilon_j = \tilde{\varepsilon}_{jl} - \varepsilon_{jk}$

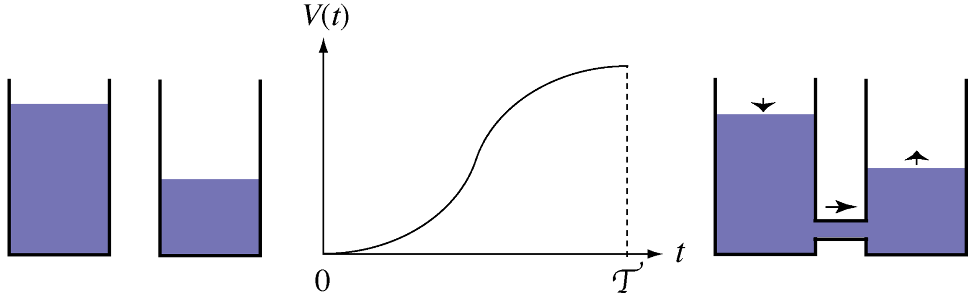
particle transfer: $\Delta\nu_{j\alpha} = \tilde{\nu}_{j\alpha l} - \nu_{j\alpha k}$

Kurchan (2000)

FLUCTUATION THEOREM FOR OPEN QUANTUM SYSTEMS 2

Transient fluctuation theorem: time-dependent quantum systems

Jarzynski (1997)



Forward protocol: $i\hbar \frac{\partial}{\partial t} U_{\mathrm{F}}(t;\mathcal{B}) = H(t;\mathcal{B}) U_{\mathrm{F}}(t;\mathcal{B})$ $\quad \rho(0;\mathcal{B}) = \prod_j \mathrm{e}^{\beta_j \left[\Phi_j(\mathcal{B}) - H_j + \sum_\alpha \mu_{j\alpha} N_{j\alpha}\right]}$

Reversed protocol: $i\hbar \frac{\partial}{\partial t} U_{\mathrm{R}}(t;\mathcal{B}) = H(\mathcal{T} - t;\mathcal{B}) U_{\mathrm{R}}(t;\mathcal{B})$ $\quad \rho(\mathcal{T};-\mathcal{B}) = \prod_j \mathrm{e}^{\beta_j \left[\tilde{\Phi}_j(-\mathcal{B}) - \tilde{H}_j + \sum_\alpha \mu_{j\alpha} N_{j\alpha}\right]}$

Time-reversal symmetry: $U_{\mathrm{R}}(\mathcal{T};-\mathcal{B}) = \Theta U_{\mathrm{F}}^{-1}(\mathcal{T};\mathcal{B})\Theta$

Probabilities of energy and particle transfers during the forward and reversed protocols:

$$\frac{P_{\mathrm{F}}\left(\Delta\varepsilon_j, \Delta\nu_{j\alpha};\mathcal{B}\right)}{P_{\mathrm{R}}\left(-\Delta\varepsilon_j, -\Delta\nu_{j\alpha};-\mathcal{B}\right)} = \mathrm{e}^{\sum_j \beta_j \left(\Delta\varepsilon_j - \sum_\alpha \mu_{j\alpha}\Delta\nu_{j\alpha} - \Delta\Phi_j\right)} \qquad \Delta\Phi_j \equiv \tilde{\Phi}_j(-\mathcal{B}) - \Phi_j(\mathcal{B})$$

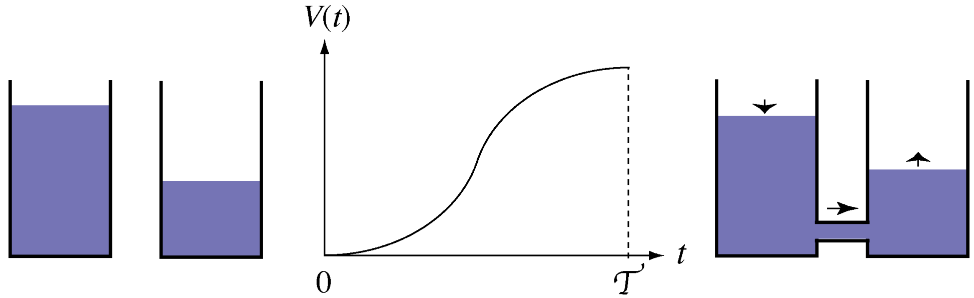
Crooks fluctuation theorem:

$$\frac{P_{\mathrm{F}}(W;\mathcal{B})}{P_{\mathrm{R}}(-W;-\mathcal{B})} = \mathrm{e}^{\beta(W-\Delta F)} \qquad \Delta F \equiv \tilde{F}(-\mathcal{B}) - F(\mathcal{B})$$

P. Talkner, M. Campisi, and P. Hänggi, J. Stat. Mech. (2009) P02025.

FLUCTUATION THEOREM FOR OPEN QUANTUM SYSTEMS 3

Transient fluctuation theorem: time-dependent quantum systems



$$\frac{P_{\mathrm{F}}\left(\Delta\varepsilon_j,\Delta\nu_{j\alpha};\mathcal{B}\right)}{P_{\mathrm{R}}\left(-\Delta\varepsilon_j,-\Delta\nu_{j\alpha};-\mathcal{B}\right)} = \mathrm{e}^{\sum_j \beta_j\left(\Delta\varepsilon_j - \sum_\alpha \mu_{j\alpha}\Delta\nu_{j\alpha} - \Delta\Phi_j\right)} \qquad \Delta\Phi_j \equiv \tilde{\Phi}_j(-\mathcal{B}) - \Phi_j(\mathcal{B})$$

Generating function of the statistical moments:

$$G_{\mathrm{F,R}}\left(\xi_j,\eta_{j\alpha};\mathcal{B}\right) \equiv \int \prod_{j\alpha} \Delta\varepsilon_j \Delta\nu_{j\alpha}\, \mathrm{e}^{-\sum_j \xi_j\Delta\varepsilon_j - \sum_{j\alpha}\eta_{j\alpha}\Delta\nu_{j\alpha}}\, P_{\mathrm{F,R}}\left(\Delta\varepsilon_j,\Delta\nu_{j\alpha};\mathcal{B}\right)$$

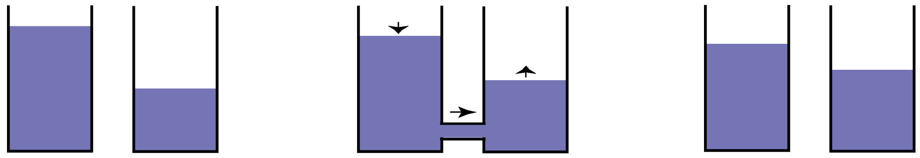
Time-reversal symmetry relation:

$$G_{\mathrm{F}}\left(\xi_j,\eta_{j\alpha};\mathcal{B}\right) = \mathrm{e}^{-\sum_j \beta_j\Delta\Phi_j}\, G_{\mathrm{R}}\left(\boxed{\beta_j} - \xi_j, \boxed{-\beta_j\mu_{j\alpha}} - \eta_{j\alpha};-\mathcal{B}\right)$$

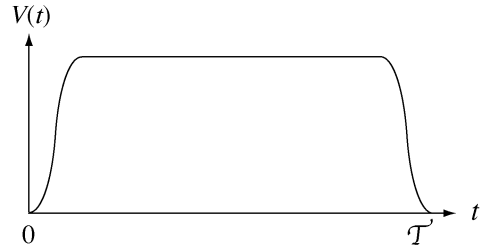
DIFFICULTY: The symmetry is NOT with respect to the thermodynamic forces or affinities.

FLUCTUATION THEOREM FOR OPEN QUANTUM SYSTEMS 4

From the *transient* to the *steady-state* fluctuation theorem



$$H(t) = H_1 + H_2 + V(t)$$



$$V(t) = V(\mathcal{T} - t)$$

$$G_{\mathrm{F}} = G_{\mathrm{R}} \equiv G$$

Generating function of the cumulants:

$$Q\left(\xi_j, \eta_{j\alpha}; \mathcal{B}\right) = -\lim_{\mathcal{T}\to\infty} \frac{1}{\mathcal{T}} \ln G\left(\xi_j, \eta_{j\alpha}; \mathcal{B}\right)$$

Time-reversal symmetry relation:

$$Q\left(\xi_j, \eta_{j\alpha}; \mathcal{B}\right) = Q\left(\beta_j - \xi_j, -\beta_j \mu_{j\alpha} - \eta_{j\alpha}; -\mathcal{B}\right)$$

Proposition:

$$\boxed{Q\left(\xi_1, \xi_2, \eta_{1\alpha}, \eta_{2\alpha}; \mathcal{B}\right) = \tilde{Q}\left(\xi_1 - \xi_2, \eta_{1\alpha} - \eta_{2\alpha}; \mathcal{B}\right)}$$

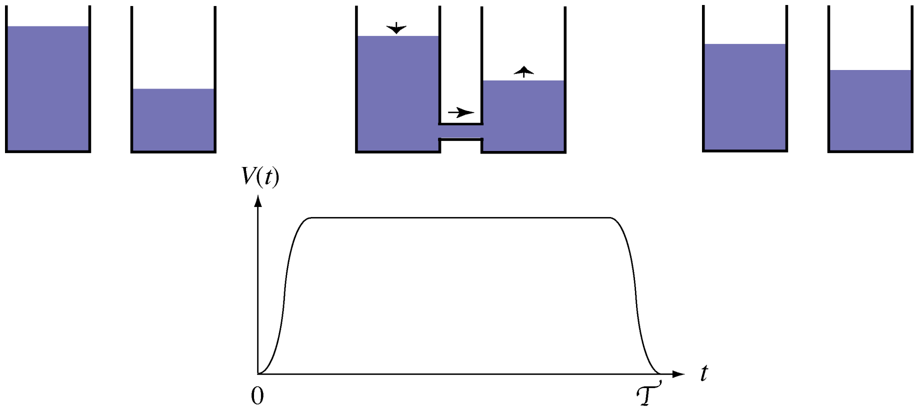
Principle of the proof: $\quad L\tilde{G} \leq G \leq K\tilde{G}$

with L and K independent of $\mathcal{T}$ and $\tilde{G} = \tilde{G}\left(\xi_1 - \xi_2, \eta_{1\alpha} - \eta_{2\alpha}; \mathcal{B}\right)$

D. Andrieux, P. Gaspard, T. Monnai & S. Tasaki, New J. Phys. **11** (2009) 043014; *Erratum* 109802.

FLUCTUATION THEOREM FOR OPEN QUANTUM SYSTEMS 5

From the *transient* to the *steady-state* fluctuation theorem



Proposition:

$$\tilde{Q}(\xi,\eta_\alpha;\mathcal{B}) = -\lim_{\mathcal{T}\to\infty}\frac{1}{\mathcal{T}}\ln\tilde{G}(\xi,\eta_\alpha;\mathcal{B})$$

$$\xi \equiv \xi_1 - \xi_2$$

$$\eta_\alpha \equiv \eta_{1\alpha} - \eta_{2\alpha}$$

$$\tilde{G}(\xi,\eta_\alpha;\mathcal{B}) = \mathrm{tr}\rho_0 \mathrm{e}^{iHt}\mathrm{e}^{-\frac{\xi}{2}(H_1-H_2)-\sum_\alpha \frac{\eta_\alpha}{2}(N_{1\alpha}-N_{2\alpha})}\mathrm{e}^{-iHt}\mathrm{e}^{\frac{\xi}{2}(H_1-H_2)+\sum_\alpha \frac{\eta_\alpha}{2}(N_{1\alpha}-N_{2\alpha})}$$

equivalent to Levitov-Lesovik formula for full counting statistics

affinities or thermodynamic forces:

$$A_0 \equiv \beta_1 - \beta_2$$

$$A_\alpha \equiv -\beta_1\mu_{1\alpha} + \beta_2\mu_{2\alpha}$$

steady-state fluctuation theorem:

$$\boxed{\tilde{Q}(\xi,\eta_\alpha;\mathcal{B}) = \tilde{Q}(A_0-\xi, A_\alpha-\eta_\alpha;-\mathcal{B})}$$

D. Andrieux, P. Gaspard, T. Monnai & S. Tasaki, New J. Phys. **11** (2009) 043014; *Erratum* 109802.

FLUCTUATION THEOREM FOR OPEN QUANTUM SYSTEMS 6

affinities or thermodynamic forces:

$$A_0 \equiv \beta_1 - \beta_2$$

$$A_\alpha \equiv -\beta_1 \mu_{1\alpha} + \beta_2 \mu_{2\alpha} \qquad \alpha = 1,2,\ldots,c$$

counting parameters:

$$\lambda_0 \equiv \xi_1 - \xi_2$$

$$\lambda_\alpha \equiv \eta_{1\alpha} - \eta_{2\alpha}$$

quantum steady-state fluctuation theorem for currents:

$$\boxed{\tilde{Q}\left(\left\{\lambda_\gamma, A_\gamma\right\}; \mathcal{B}\right) = \tilde{Q}\left(\left\{A_\gamma - \lambda_\gamma, A_\gamma\right\}; -\mathcal{B}\right)}$$

linear response coefficients

Casimir-Onsager reciprocity relations: $\quad L_{\alpha,\beta}(\mathcal{B}) = L_{\beta,\alpha}(-\mathcal{B})$

nonlinear response coefficients: magnetic-field asymmetry

$$\frac{\partial^3 Q}{\partial\lambda_\alpha \partial A_\beta \partial A_\gamma}(\mathbf{0},\mathbf{0};\mathcal{B}) = -\frac{\partial^3 Q}{\partial\lambda_\alpha \partial A_\beta \partial A_\gamma}(\mathbf{0},\mathbf{0};-\mathcal{B})$$

$$R_{\alpha\beta,\gamma}(\mathcal{B}) = R_{\alpha\beta,\gamma}(-\mathcal{B}) + \frac{\partial^3 Q}{\partial\lambda_\alpha \partial A_\beta \partial A_\gamma}(\mathbf{0},\mathbf{0};\mathcal{B})$$

$$M_{\alpha,\beta\gamma}(\mathcal{B}) + M_{\alpha,\beta\gamma}(-\mathcal{B}) = R_{\alpha\beta,\gamma}(-\mathcal{B}) + R_{\alpha\gamma,\beta}(-\mathcal{B}) + \frac{\partial^3 Q}{\partial\lambda_\alpha \partial A_\beta \partial A_\gamma}(\mathbf{0},\mathbf{0};\mathcal{B})$$

D. Andrieux, P. Gaspard, T. Monnai & S. Tasaki, New J. Phys. **11** (2009) 043014; *Erratum* 109802

APPLICATION TO QUANTUM ELECTRON TRANSPORT

K. Saito and Y. Utsumi, Phys. Rev. B **78**, 115429 (2008).
Y. Utsumi and K. Saito, Phys. Rev. B **79**, 235311 (2009).
S. Nakamura et al., Phys. Rev. Lett. **104**, 080602 (2010).

Aharonov-Bohm ring on a GaAs/AlGaAs heterostructure 2DEG

measurement of current and fluctuations

current: $$I = G_1 V + \frac{1}{2!} G_2 V^2 + \frac{1}{3!} G_3 V^3 + \ldots$$

noise power: $$S = S_0 + S_1 V + \frac{1}{2!} S_2 V^2 + \ldots$$

	Theory:	Experiment:
Johnson-Nyquist relation:	$S_0 = 4 k_B T G_1$	$S_0 / 4 k_B T G_1 \approx 1$
Casimir-Onsager relation:	$G_1(\mathcal{B}) = G_1(-\mathcal{B})$	
Förster-Büttiker relation:	$S_1^S = 2 k_B T G_2^S$	$S_1^S / 2 k_B T G_2^S \approx 5$
fluctuation theorem:	$S_1^A = 6 k_B T G_2^A$	$S_1^A / 6 k_B T G_2^A \approx 1.5$

$$X^S(\mathcal{B}) \equiv X(\mathcal{B}) + X(-\mathcal{B})$$

$$X^A(\mathcal{B}) \equiv X(\mathcal{B}) - X(-\mathcal{B})$$

FULL COUNTING STATISTICS OF ELECTRONS 1

S. Gustavsson et al., *Counting Statistics of Single Electron Transport in a Quantum Dot*,
Phys. Rev. Lett. **96**, 076605 (2006).

current fluctuations in a GaAs-GaAlAs quantum dot (QD):
real-time detection with a quantum point contact (QPC)

$$\frac{I_{\mathrm{QPC}}}{I_{\mathrm{QD}}} \approx 3.5 \times 10^{7}$$

350 mK

quantum dot (QD): $V_{\mathrm{QD}} = 1.2$ mV $\quad I_{\mathrm{QD}} = 127$ aA $\quad P_{\mathrm{QD}} = 1.5 \times 10^{-19}$ W

quantum point contact (QPC): $V_{\mathrm{QPC}} = 0.5$ mV $\quad I_{\mathrm{QPC}} = 4.5$ nA $\quad P_{\mathrm{QPC}} = 2.3 \times 10^{-12}$ W

Counting statistics: number of electrons entering the QD during a given time

limit of a large bias voltage: $\left| \pm eV/2 - \varepsilon \right| >> k_{\mathrm{B}} T$ *bidirectionality not observed*

The coupling between the QPC and QD currents performs the quantum measurement of the QD state. The dissipation in the QPC is much larger than in the QD in agreement with the fact that quantum measurement generates entropy production.

FULL COUNTING STATISTICS OF ELECTRONS 2

T. Fujisawa et al., *Bidirectional Counting of Single Electrons*, Science **312**, 1634 (2006).
Y. Utsumi et al., *Bidirectional single-electron counting and the fluctuation theorem*, Phys. Rev. B **81**, 125331 (2010).

current fluctuations in a AlGaAs/GaAs quantum dot:
real-time detection with a quantum point contact

There are two QDs in series, which allows bidirectional counting.

$$\frac{I_{\text{QPC}}}{I_{\text{QD}}} \approx 2 \times 10^{8}$$

quantum dot (QD):

$$A_{\text{QD}} = \frac{eV_{\text{QD}}}{k_{\text{B}}T} \approx 27$$

quantum point contact (QPC):

$$A_{\text{QPC}} = \frac{eV_{\text{QPC}}}{k_{\text{B}}T} \approx 71$$

$$A_{\text{eff}} \approx 3 \qquad \frac{P(N,t)}{P(-N,t)} \approx \exp\left(A_{\text{eff}} N\right) \qquad t \to \infty$$

The fluctuation theorem apparently holds but with a low effective affinity because of a very important **back-action** of the QPC on the QD.

CONCLUSIONS

Breaking of time-reversal symmetry

in the statistical description of nonequilibrium systems

Stochastic processes:

Steady-state fluctuation theorem for the currents:

Generalization of Onsager reciprocity relations to nonlinear response coefficients

Open quantum systems:

Transient fluctuation theorem for the currents *in a magnetic field*

$$\Downarrow$$

Steady-state fluctuation theorem for the currents *in a magnetic field*:

Generalization of Casimir-Onsager reciprocity relations to nonlinear response coefficients