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# ALPHA PARTICLE TUNNELLING IN THE FIELD OF A MAGNETIC MONOPOLE

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The influence of a Dirac magnetic monopole on the tunnelling probability of charged, spinless particles is studied. In a Gamow type model for the $\alpha$-decay, the change of the lifetime of an $\alpha$-unstable nucleus, like $^{147}_{62}$Sm, is treated. The change of the tunnelling probability by the magnetic monopole leads to a drastic increase in the lifetime. Possible consequences for the detection of magnetic monopoles are indicated.

The announcement of a possible candidate for the detection of a magnetic monopole [1] has spurred significant renewed interest in the physical properties of systems composed of charged particles and magnetic monopoles [‡1]. In this communication we shall elaborate only on the physical consequences which the monopole field exerts on the tunnelling probability of charged, spinless particles. Problems such as $\beta$-decay or $\gamma$-transitions give additional complications due to spin and nuclear effects, which are beyond the scope of our model study.

The archetype of this tunnelling problem is the $\alpha$-particle decay from a dyon, i.e. from a system composed of a nucleus carying a Dirac-monopole [3] of charge $g$ in the center. Dirac monopoles [3] are point-like objects with an undetermined mass. This simplification is justified because the more realistic monopole solutions of grand unified gauge theories, i.e. monopoles of the 't Hooft–Polyakov type [4], with mass $m_g \simeq 10^{16} m_p$ look for distances large compared to the Compton wave length of the very massive gauge bosons just like a Dirac monopole.

The enigma of the $\alpha$-decay problem is indeed a very old one. As is well known, the original works by Gamow [5], Condon and Gurney [6] and Laue [7] present a milestone in the theory of particle decay. Since then, this problem has been worked on extensively [‡2].

The goal of this work is to extend this type of studies to the case where the $\alpha$-particle experiences additionally the force field of a radially symmetric magnetic monopole field. In particular, we attempt the study of the influence of the monopole on the exponentially small tunnelling probability of $\alpha$-decay and contrast this with the case of tunnelling in absence of a Dirac monopole in its center. This problem is not only interesting in itself, but may also be of importance for the problem of nucleosynthesis, where the presence of many monopoles may crucially impact the lifetime

‡1 A representative sample of articles is given in ref. [2].

‡2 Ref. [8] gives a discussion of the more recent developments in both the theoretical and experimental aspects of the $\alpha$-decay problem in nuclei.

of various decay processes participating in the formation of nuclei.

As a simplified model for $\alpha$-decay in a nucleus of charge $Z$, we consider the spherically symmetric nuclear potential field $V(r)$ depicted in fig. 1. This average nuclear potential is composed of a constant attractive potential of depth $V_0$ for $r < R$ and a repulsive Coulomb barrier, $Ze^2/r$, for $r > R$. Clearly, with such a conventional one-body description of $\alpha$-decay, we are neglecting non-trivial nuclear structure effects which might not be negligible in a precise determination of the actual

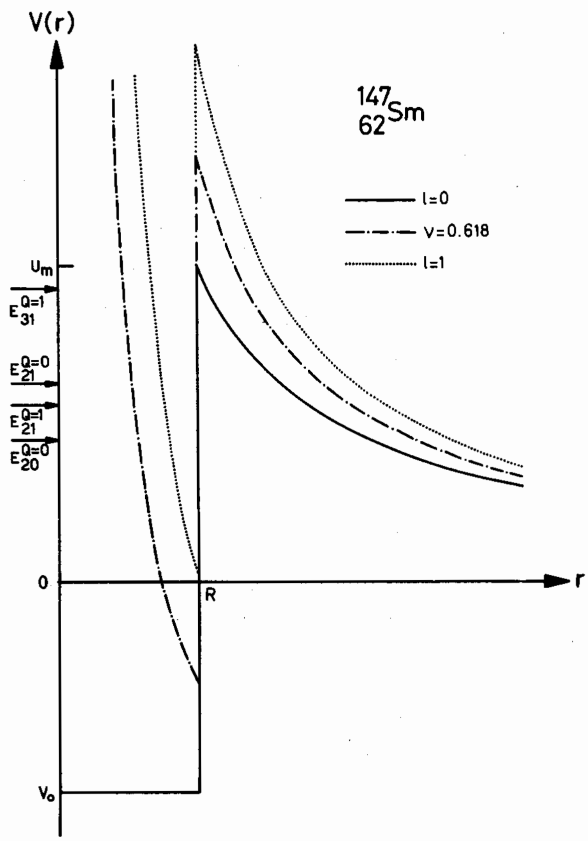


Fig. 1. A sketch of the nuclear potential field used in text. Dash-dotted line: effective potential field including the centrifugal angular momentum barrier induced by the charge–monopole system ($\nu = \frac{1}{2}(\sqrt{5}-1) = 0.618$). Dotted line: corresponding effective potential field for a system without monopole with angular momentum $l = 1$. The arrows indicate the locations of corresponding resonance energies.

tunnelling probability. However, our focus will not be on the precise values of the corresponding resonance energies of the tunnelling $\alpha$-particle, but rather in the characteristic shifts of these resonance energies, being induced by the typical lowering of the (one-body) centrifugal barrier of the $\alpha$-particle. Thus, we will neglect the interaction of the nuclear core with the monopole field. This can be justified since the electromagnetic interaction effect of the core with the monopole is negligibly small compared with the strong nuclear binding potential $V(r)$. Moreover, an important advantageous feature of our one-body description is the fact that it is still possible to separate the angular and radial parts of the underlying Schrödinger dynamics in presence of the singular monopole field [9,10]. This will no longer hold within a many-body approach to $\alpha$-decay with a monopole at the center of the nuclear core. Therefore, by use of the scheme of wave sections, as pioneered by Wu and Yang [9], we separate the angular and radial parts of the wave function $\psi(r, \theta, \phi)$:

$$\psi(r, \theta, \phi) = R(r) Y_{Q,l_Q,m}(\theta, \phi). \tag{1}$$

Hereby, $Y_{Q,l_Q,m}(\theta, \phi)$ are the monopole harmonics [9,10] and $Q = Z_\alpha eg/\hbar c = Z_\alpha N/2 = 1$ is the magnitude of angular momentum generated by the monopole–$\alpha$-particle system for $N = 1$ and $l_Q = Q, Q+1, \ldots$ is the quantum number of the total angular momentum in the presence of a singular vector potential $\boldsymbol{A}_{\mathrm{m}}$ for a magnetic monopole:

$$\boldsymbol{L} = \boldsymbol{r} \times [\boldsymbol{p} - 2(e/c)\boldsymbol{A}_{\mathrm{m}}] - Q\boldsymbol{r}/r. \tag{2}$$

By use of the usual substitution $R(r) \cdot r = u(r)$, the radial part of the Schrödinger equation for the $\alpha$-particle–monopole–nucleus system reads explicitly

$$\mathrm{d}^2u/\mathrm{d}r^2 + \{(2\mu/\hbar^2)[E - V(r)] - [l_Q(l_Q+1) - Q^2]/r^2\} u = 0, \tag{3}$$

where $\mu$ denotes the reduced mass of the $\alpha$-particle.

By use of the substitution

$$\nu(\nu+1) = l_Q(l_Q+1) - Q^2, \tag{4}$$

i.e.

$$\nu\left[\left(l_Q+\tfrac{1}{2}\right)^2-Q^2\right]^{1/2}-\tfrac{1}{2},$$

we observe that the problem of $\alpha$-decay in presence of a trapped magnetic monopole in the center of the nucleus can be mapped onto the ordinary problem of $\alpha$-decay in a nucleus if we only substitute the integer value of total angular momentum by the irrational, effectively reduced value

$$l_Q-\tfrac{1}{2}<\nu<l_Q. \tag{5}$$

The first few effective angular momentum values are – for $Q=1$ – then explicitly given by $l_Q=1 \to \nu=\frac{1}{2}(\sqrt{5}-1)=0.618\ldots$; $l_Q=2\to\nu=\frac{1}{2}(\sqrt{21}-1)=1.791\ldots$; $l_Q=3\to\nu=\frac{1}{2}(\sqrt{45}-1)=2.854\ldots$. Thus, due to this characteristic reduction of the corresponding effective angular momentum value in (3), which in turn yields a characteristic reduction for the centrifugal barrier in (3), we can already expect a characteristic reduction of the resonance energy values towards lower values. This, of course, then also implies that the tunnelling probability of $\alpha$-decay with a monopole is exponentially suppressed over the case of usual $\alpha$-decay in absence of a magnetic monopole.

Moreover, due to its reduced centrifugal potential barrier energy, a new, higher-lying resonance state may be pulled down from the set of continuum of energies (see fig. 1): with $l>\nu$ its corresponding energy in absence of the monopole would not fit below the effective potential barrier. In fig. 2 we depict the resonance energies for the model potential in fig. 1 for the element $^{147}_{62}$Sm with a typical nuclear radius $R=8.677$ fm and a typical attractive nuclear potential well of $V_0=-13.27$ MeV, i.e. the height of the potential barrier at $R$ is then $U_m=33.18$ MeV. With this choice of the potential parameters, the experimental values for the energy and the decay width of the ground state of $^{147}_{62}$Sm are reproduced with a deviation of less than 0.01%. Fig. 2 compares the resonance energies $E^Q_{nl}$ ($n=1,2\ldots$, order of resonance energy) for the case with a monopole, $Q=1$, with the corresponding ones without a monopole, $Q=0$. Indeed, there occurs always a characteristic decrease in the resonance energies. In addition, the resonance energy $E^{Q=1}_{3,1}$, absent for $Q=0$ is a

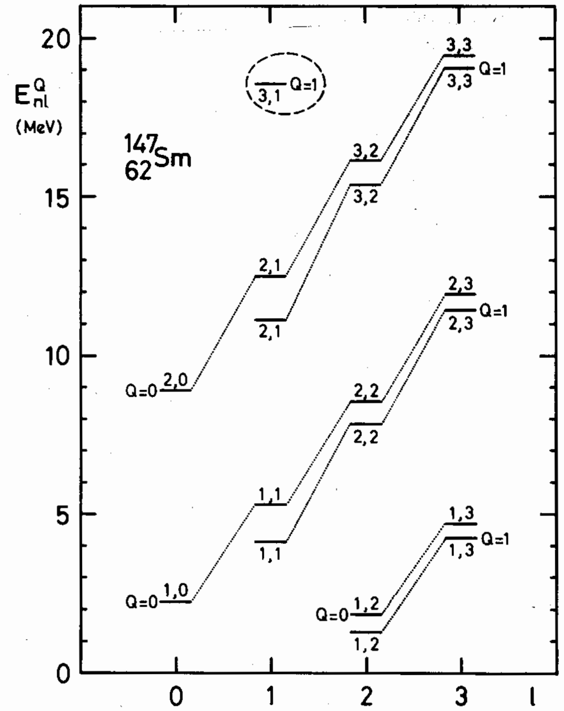


Fig. 2. Schematic alpha-particle resonance levels in $^{147}_{62}$Sm for various angular momentum values $l$, without monopole, $Q=0$, and, correspondingly, with monopole, $Q=1$. With a monopole, there occurs a new resonance level, indicated by the enclosed, dashed ellipse.

new resonance state which is pulled in from the continuum of states.

In order to determine the decay rate $\lambda$, connected with the resonance width $\Gamma$ by $\Gamma=\hbar\lambda$, we make use of scattering theory. Following Bohr and Mottelson [11], the scattering phase shift can be written in the following form:

$$\exp(2i\delta_\nu)=\left[(L_\nu-\Delta_\nu+is_\nu)/(L_\nu-\Delta_\nu-is_\nu)\right]\times\exp(2i\xi_\nu), \tag{6a}$$

where

$$\xi_\nu=\arctan\left(F_\nu(\eta,\rho)/G_\nu(\eta,\rho)\right)|_{r=R}$$
$$\left(\text{with } \rho=kr,\ k=\left(2\mu E/\hbar^2\right)^{1/2},\ \eta=2Ze^2\mu/\hbar^2k\right) \tag{6b}$$

is the "hard-sphere" phase. In terms of the regular and irregular Coulomb functions [12] $F_\nu(\eta, \rho)$ and $G_\nu(\eta, \rho)$, respectively, (evaluated at real $\nu$) (eq. (5)), the logarithmic derivative $L_\nu$ is given by

$$L_\nu = [R/u_\nu(R)]\, du_\nu(r)/dr\,|_{r=R} = (\cos\delta_\nu F_\nu' - \sin\delta_\nu G_\nu')/(\cos\delta_\nu F_\nu - \sin\delta_\nu G_\nu)\,|_{r=R}, \quad (6c)$$

with $X' \equiv R\, dX/dr$. By use of the well-known wronskian relation for the Coulomb functions we get for $s_\nu$ and $\Delta_\nu$ [13]

$$s_\nu = (F_\nu^2 + G_\nu^2)^{-1}, \quad \Delta_\nu = s_\nu(F_\nu F_\nu' + G_\nu G_\nu'). \quad (6d)$$

The resonance energies are then determined by the resonance condition

$$L_\nu(E_r) = \Delta_\nu(E_r). \quad (7)$$

The decay width $\Gamma$ follows from the Breit–Wigner form [11] as

$$\Gamma = 2s_\nu\gamma_\nu \quad \text{with} \quad \gamma_\nu^{-1} = (-\partial L_\nu/\partial E + \partial\Delta_\nu/\partial E)\,|_{E=E_r}. \quad (8)$$

The quantities $L_\nu$ and $\partial L_\nu/\partial E$ are readily evaluated as

$$L_\nu = 1 + \kappa R\, j_\nu'(\kappa R)/j_\nu(\kappa R), \quad \kappa^2 = (2\mu/\hbar^2)(E + |V_0|), \quad (9a)$$

and

$$\partial L_\nu/\partial E\,|_{E=E_r} = (\mu R^2/\hbar^2)\{[\nu(\nu+1) - \Delta_\nu(\Delta_\nu - 1)]/\kappa^2 R^2 - 1\}, \quad (9b)$$

where $j_\nu$ denotes a spherical Bessel function at irrational index $\nu$. In order to arrive at a closed expression for the decay rate, respectively for $\Delta_\nu$ and $\partial\Delta_\nu/\partial E$, we use in the regime $r \geqslant R$ for the regular and irregular Coulomb functions [14] the WKB-approximation. A somewhat cumbersome evaluation then yields

$$\Delta_\nu \simeq -R\{[1/4f(R)]\, df(r)/dr\,|_{r=R} + [f(R)]^{1/2}\}, \quad (10a)$$

with

$$f(r) = (2\mu/\hbar^2)[V(r) - E] + (\nu + \tfrac{1}{2})^2/r^2, \quad (10b)$$

and for its derivative we get

$$\partial\Delta_\nu/\partial E\,|_{E=E_r} \simeq (\mu R^2/\hbar^2)\{1/R[f(R)]^{1/2} - [1/2Rf^2(R)]\, df(r)/dr\,|_{r=R}\}. \quad (10c)$$

Upon combining (8), (6), (9) and (10) we then obtain a closed form expression for the decay width:

$$\Gamma_{nl}^{Q=1} = (2\hbar^2/\mu R^2)\{\rho/[F_\nu^2(\eta,\rho) + G_\nu^2(\eta,\rho)]\} \times \{1 - [\nu(\nu+1) - \Delta_\nu(\Delta_\nu - 1)]/\kappa^2 R^2 - [1/2Rf^2(R)]\, df(r)/dr\,|_{r=R} + 1/R[f(R)]^{1/2}\}^{-1}\,|_{E=E_r}. \quad (11)$$

This expression presents the main result of this communication. Alternatively, we may recast the result in (11) in the Gamow form

$$\Gamma_{nl}^{Q} = A_{nl}^{Q} \exp(-G), \quad (12)$$

where for the model potential in (3) the Gamow factor is given by

$$G = 2\int_R^{r_0} \{(2\mu/\hbar^2)[V(r) - E] + \nu(\nu+1)/r^2\}^{1/2}\, dr = r_0 k\Big\{\arcsin\big([2\sqrt{c}(1 - 2R/r_0) + 2a]/(4c+1)\big) + 2\sqrt{c} - 2a + 2\sqrt{c}\log\Big[\Big\{1 + 2cr_0/R + 2\big[cr_0/R + c^2(r_0/R)^2 - c\big]^{1/2}\Big\}/(4c+1)\Big]\Big\}, \quad (13a)$$

with

$$c = (\hbar^2/2\mu)[\nu(\nu+1)/4Z^2e^4]\,E, \quad r_0 = 2Ze^2/E, \quad a = [c + R/r_0 - (R/r_0)^2]^{1/2}. \quad (13b)$$

The results of (11) are depicted in fig. 3 for the

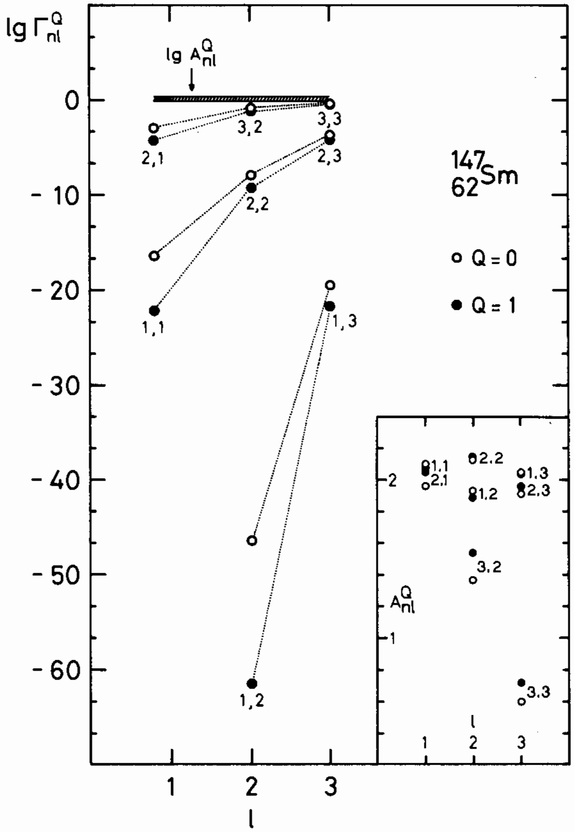


Fig. 3. Resonance widths (in MeV) for $^{147}_{62}$Sm without monopole, (○), and with monopole, (●). The prefactor varies weakly within the hatched region. The inset exhibits the more detailed behaviour of this prefactor.

typical case of $^{147}_{62}$Sm. The widths for the nucleus–monopole system are exponentially suppressed over the case without a monopole. The prefactor $A^Q_{nl}$, defined in (12), varies rather weakly as function of angular momentum and resonance energy $E_r$ (see fig. 3 and inset), i.e. the suppression of the tunnelling probability is thus controlled by a typical increase of the Gamow factor, given in (13). The range of relative magnitude of this exponential suppression for the decay widths, or the corresponding exponential enhancement of the lifetime, respectively, may become of the order of $10^{15}$! Moreover, the calculated prefactor in (12) coincides typically up to a factor of 2 with the simple WKB estimate

$$A^Q_{nl}(\mathrm{WKB}) = \left[\left(\hbar^2/2\mu R^2\right)\left(E + |V_0|\right)\right]^{1/2}. \qquad (14)$$

In conclusion, the characteristic result of exponential suppression of the tunnelling probability might be of importance for aspects of the nucleosynthesis. For example, the element $^{147}_{62}$Sm has an $\alpha$-decay lifetime of $1.06 \times 10^{11}$ yr which compares with the age of the universe $T_u \simeq 2 \times 10^{10}$ yr [15]. An alternative experimental scheme for possible monopole detection, which makes use of this characteristic suppression, might be as follows: Considering $\gamma$-induced resonance activation of $\alpha$-decay tuned for the case of nucleus plus monopole might result in a measurable response, thereby indicating the existence of a monopole.

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