Single atom energy-conversion device with a quantum load
SUPPLEMENTARY INFORMATION

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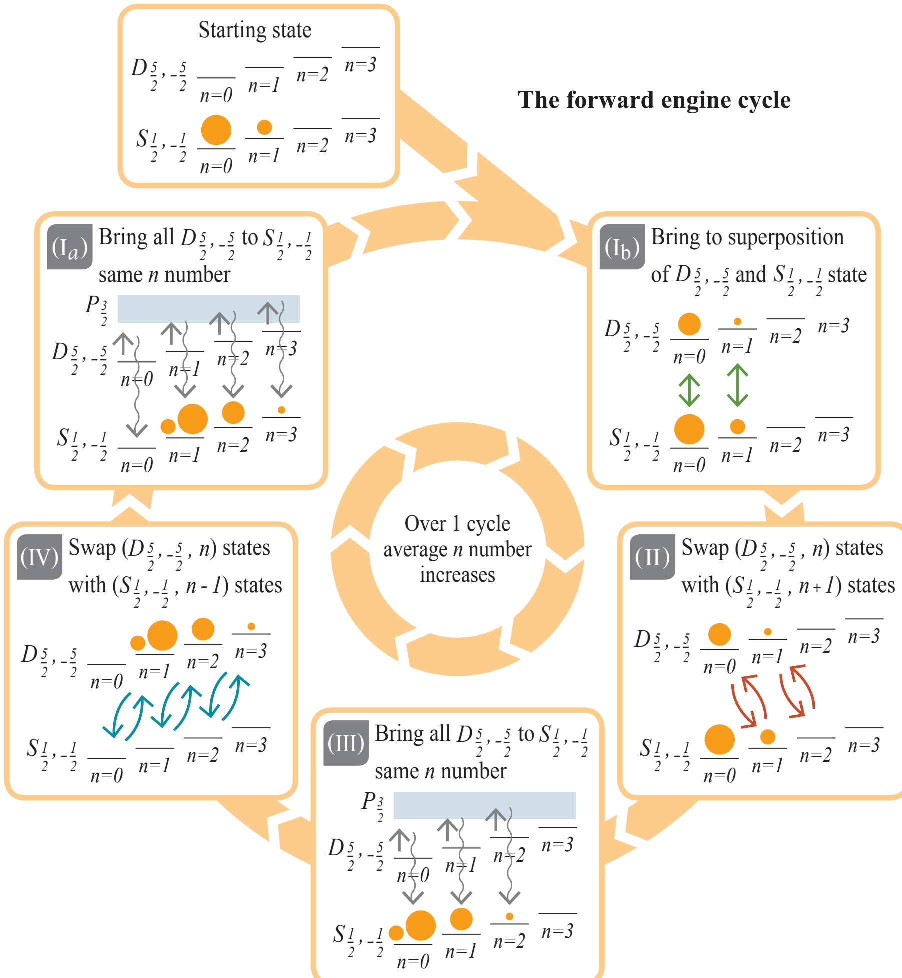

Supplementary Figure 1: **Pictorial description of the forward engine cycle.** The initialized starting state is shown, followed by strokes ($\mathrm{I_b}$) through (IV), and finally back to (I_a). The diameter of the orange circles represents the probability of the ion being in the corresponding state.

A DETAILED GRAPHICAL REPRESENTATION OF THE CYCLE

Supplementary Figure 1 provides an intuitive understanding of the forward engine cycle. The system we study is composed of a two-level system and a harmonic oscillator, which can be represented in the basis $|\sigma, n\rangle$. The notation $\sigma =\uparrow,\downarrow$ is used to indicate, respectively, the excited and the ground state of the two-level system, while n represents the level of the harmonic oscillator. The energy difference between the two levels $\sigma = \uparrow,\downarrow$ is much larger than between two levels of the harmonic oscillator. For clarity, the allowed energy states of the system can be spread out horizontally and represented as two "staircases" on top of each other. The bottom staircase corresponds to the lower energy state of the two-level system, ($\mathrm{S}_{\frac{1}{2},-\frac{1}{2}}$), while the top staircase corresponds to the upper energy state, ($\mathrm{D}_{\frac{5}{2},-\frac{5}{2}}$). In each staircase, different steps correspond to different levels n of the harmonic oscillator, increasing from left to right. The diameter of the orange circles schematically represents the probability of the ion being in the corresponding energy state, and is equivalent to p_n values in Fig.4 (a,b) (see main article).

The engine cycle is composed of 4 strokes, shown clockwise from (I) to (IV) in Supplementary Figure 1. The top left box (without a number), shows the initial state. In the initial state, the two-level system is in the ground state, and the harmonic oscillator has the largest probability of the ion being in the ground state ($n = 0$ of the harmonic oscillator), and a decreasing probability for the higher levels. The cycle then proceeds to the second part of the first stroke, (I_b). Applying the $1,762$ nm laser for a predetermined amount of time brings the engine to a superposition of its two levels. In stroke (II), a Jaynes-Cummings coupling moves the population from state $|\downarrow, n+1\rangle$ to $|\uparrow, n\rangle$, and the population from state $|\uparrow, n\rangle$ to $|\downarrow, n+1\rangle$, as represented by the red arrows. For the values of $p_{\mathrm{D}}^{\mathcal{A}}$ studied, this stroke results in a decrease in the average n number, $\langle n\rangle_{\mathrm{L}}$, of the harmonic oscillator. In stroke (III) the two-level system is no longer coupled to the harmonic oscillator, and it is reset to its ground state by a dissipative process. Stroke (IV) is similar to stroke (II), but here an anti-Jaynes-Cummings interaction is used between the two-level system and the harmonic oscillator. For this case, $|\downarrow, n\rangle$ is swapped with $|\uparrow, n+1\rangle$, and vice versa (as represented by the blue arrows). This step produces the most significant increase in the average phonon number, $\langle n\rangle_{\mathrm{L}}$, of the harmonic oscillator. Finally, stroke (I_a) resets the two-level system to its ground state, with the two-level system decoupled from the harmonic oscillator. We note that the physics leading to the redistribution of the harmonic oscillator population can be captured in steps (I) and (II).

IDEAL EVOLUTION OF THE ENGINE AND LOAD

To develop a clearer understanding of how the energy-conversion device works, it is useful to consider its evolution under ideal conditions. In this section, we consider (i) that the lasers do not induce any dephasing, (ii) that there are no spontaneous emissions outside of the optical pumping during strokes (I_a) and (III), and (iii) that transfers in strokes (II) and (IV) have an efficiency of 100% which is independent of the n level. We begin our analysis with both the engine and the load in their ground state and, to simplify the explanation, we start our analysis of the cycle after stroke (III). The initial condition is $\rho_{\mathrm{E+L}}^{\mathcal{C}_0} = |\downarrow, 0\rangle\langle\downarrow, 0|$, where $\rho_{\mathrm{E+L}}^{\mathcal{C}_0}$ is the density matrix before the blue sideband operation (which takes the ion from $\mathcal{C}$ to $\mathcal{D}$ in Fig. 2 (bottom) of the main article. The subscript "0" refers to the number of times that the operation ($\mathcal{C}$ to $\mathcal{D}$) has been implemented *prior* to the given density-matrix expression.

We now proceed by implementing stroke (IV) of the cycle. The effect of the anti-Jaynes-Cummings coupling is to shift the engine to the $D_{\frac{5}{2},-\frac{5}{2}}$ state, while simultaneously transferring n to $n+1$. Therefore, stroke (IV) gives

$$\rho_{\mathrm{E+L}}^{\mathcal{D}_1} = |\uparrow, 1\rangle\langle\uparrow, 1|. \tag{1}$$

We now apply stroke (I). For the sake of giving a generic example, we consider the case in which after step (I_b), $p_{\mathrm{D}}^{\mathcal{A}} = 1/4$. Stroke (I) produces a pure state

$$\rho_{\mathrm{E+L}}^{\mathcal{A}_1} = \frac{1}{4}\left(|\uparrow\rangle + \sqrt{3}e^{\mathrm{i}\phi}|\downarrow\rangle\right)\left(\langle\uparrow| + \sqrt{3}e^{-\mathrm{i}\phi}\langle\downarrow|\right) \otimes |1\rangle\langle 1|, \tag{2}$$

where $\rho_{\mathrm{E+L}}^{\mathcal{A}_1}$ is the density matrix at point $\mathcal{A}_1$. Here, ϕ is a phase which can be determined experimentally, and that can be fixed such that it returns to the same value at the end of any integer number of cycles. Now we proceed to stroke (II) of the cycle, an ideal Jaynes-Cummings coupling which transfers $|\downarrow, n\rangle$ to $|\uparrow, n-1\rangle$ and viceversa. Stroke (II) thus results in an entangled state

$$\rho_{\mathrm{E+L}}^{\mathcal{B}_1} = \frac{1}{4}\left(|\downarrow, 2\rangle + \sqrt{3}e^{\mathrm{i}\phi}|\uparrow, 0\rangle\right)\left(\langle\downarrow, 2| + \sqrt{3}e^{-\mathrm{i}\phi}\langle\uparrow, 0|\right). \tag{3}$$

Finally, stroke (III) brings the engine and the load to the state

$$\rho_{\mathrm{E+L}}^{\mathcal{C}_1} = \frac{1}{4}|\downarrow\rangle\langle\downarrow| \otimes (3|0\rangle\langle 0| + |2\rangle\langle 2|). \tag{4}$$

Note that after the four strokes, the engine is back to its initial state, $|\downarrow\rangle\langle\downarrow|$, but the load has a higher average occupation, $\langle n\rangle_{\rm L}$. We also see the importance of preparing a superposition state in step (I_b), which allows correlations to be created between engine and load. Furthermore, the superposition state in step (I_b) leads to the load going from a pure state in $\rho_{\rm E+L}^{\mathcal{A}_1}$, to a mixed state by the end of $\rho_{\rm E+L}^{\mathcal{C}_1}$, resulting in an increase in its information entropy. Continuing from $\rho_{\rm E+L}^{\mathcal{C}_1}$ and progressing through an additional cycle, the reduced density matrix of the load evolves as

$$\rho_{\rm E+L}^{{\rm L},\mathcal{D}_2} = \rho_{\rm E+L}^{{\rm L},\mathcal{A}_2} = \frac{1}{4}(3|1\rangle\langle 1| + |3\rangle\langle 3|) \tag{5}$$

$$\rho_{\rm E+L}^{{\rm L},\mathcal{B}_2} = \rho_{\rm E+L}^{{\rm L},\mathcal{C}_2} = \frac{1}{16}(9|0\rangle\langle 0| + 6|2\rangle\langle 2| + |4\rangle\langle 4|). \tag{6}$$

For the given parameter value, $p_{\rm D}^{\mathcal{A}} = 1/4$, it can be shown that the change in the average phonon number of the load with each cycle is $\Delta\langle n\rangle_{\rm L} = 1/2$.

To generalize, for the specified initial condition we can write the probability of occupation, $p_n^{\mathcal{C}_{N_{\rm c}}}$, of the $n-$th level of the harmonic oscillator. The superscript $\mathcal{C}_{N_{\rm c}}$ denotes that this is the probability of occupation after stroke (III) (which brings the ion to point $\mathcal{C}$ in Fig. 1(c)), of the $N_{\rm c}-$th cycle. We get

$$p_n^{\mathcal{C}_{N_{\rm c}}} = \sum_{k=0}^{N_{\rm c}} {\rm C}_k^{N_{\rm c}}\ (p_{\rm D}^{\mathcal{A}})^k (1-p_{\rm D}^{\mathcal{A}})^{N_{\rm c}-k} \delta_{n,2k+{\rm mod}(N_{\rm c},2)} \tag{7}$$

where ${\rm C}_a^b = b!/[a!(b-a)!]$, once again $p_{\rm D}^{\mathcal{A}}$ is the value to which the engine population is reset once in each cycle, at the end of stroke (I), and $\delta_{a,b}$ is the Kronecker delta function. The sum over integers k, together with the Kronecker delta, ensures that only even (odd) levels are occupied for even (odd) values of $N_{\rm c}$. Eq. (7) describes the evolution of a probability distribution of a biased normal diffusive process, and this implies three things: (i) that the change in average phonon occupation of the load is $\Delta\langle n\rangle_{\rm L} = 1/2$ with each additional cycle, (ii) that the information entropy of the load grows asymptotically as the logarithm of the number of cycles, i.e. $\mathcal{S}_{\rm L} \propto ln N_{\rm c}$, and (iii) that the distribution will tend towards a Gaussian. This last point is particularly important because it implies that the load is not in a passive state, and that its ergotropy increases with $N_{\rm c}$. From Eq. (7) we can compute that the ergotropy grows as $\mathcal{E}_{\rm L} \propto N_{\rm c} - \alpha\sqrt{N_{\rm c}}$, where α is a constant. This expression comes from the fact that the energy of the load grows linearly with $N_{\rm c}$, while the energy of the "passified" load only grows as $\sqrt{N_{\rm c}}$.