Tuning the Mobility of a Driven Bose-Einstein Condensate via Diabatic Floquet Bands

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We study the response of ultracold atoms to a weak force in the presence of a temporally strongly modulated optical lattice potential. It is experimentally demonstrated that the strong ac driving allows for a tailoring of the mobility of a dilute atomic Bose-Einstein condensate with the atoms moving ballistically either along or against the direction of the applied force. Our results are in agreement with a theoretical analysis of the Floquet spectrum of a model system, thus revealing the existence of diabatic Floquet bands in the atoms' band spectra and highlighting their role in the nonequilibrium transport of the atoms.

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Experiments with ultracold atomic gases in periodically modulated lattice potentials have shown that these systems represent an attractive testing ground to explore nonequilibrium quantum states [1,2]. Along these lines, the suppression of tunneling in ac-driven optical lattices was experimentally observed, both in the single-particle [3] and in the many-body [4] regimes. Other experiments have recently reported the simulation of gauge fields for neutral and spinless atoms [5].

Common knowledge in nonequilibrium statistical physics [6] indicates that not only the state of a system shifted from equilibrium but also its response to external perturbations can differ substantially from those exhibited by the system at rest. The equilibrium response of one of the simplest quantum models, a particle placed in a stationary periodic potential, when exposed to a weak constant force F is well understood: On a short-time scale, $t \ll 2\pi\hbar/FL = T_B$, where L is the period of the potential, the particle reacts by moving with the velocity determined by the local slope of the ground band. On larger time scales, $t \geq 2\pi\hbar/FL$, the response evolves into the celebrated phenomenon of Bloch oscillations [7,8]. Weak periodic modulations of the potential can change the response. It has been demonstrated that these can propel the particle over many Brillouin zones, thus rectifying Bloch oscillations into ballistic transport [9]. This observation, along with the results reported in the above-cited works [3–5], are well explained by assuming that the temporal modulations do not push the particle outside the ground band. The single-band approach is justified as long as the driving amplitude remains small, so the observed effects can be attributed to a modulation-induced renormalization of the potential [10,11]. Strong driving, however, intermingles eigenstates of the stationary system and sculpts a new spectrum of time-dependent dressed states [12,13], so that the system dynamics no longer fits the perturbative picture [14]. This idea has been exploited in recent experiments with coherent quantum ratchets [16–18] and was also used to create new topological states [19].

A distinct feature of strongly driven quantum systems is the presence in their Floquet spectra [20,21] of so-called diabatic bands [22,23]. The states belonging to these bands remain near isolated from the rest of system Hilbert space upon parameter variations. In the quasiclassical limit diabatic bands are also called regular bands because of the correspondence between the band eigenstates and regular invariant manifolds of the system in the classical limit [23]. It is possible to detect a diabatic band of a semiclassical system by populating it with an initial wave packet located in the corresponding region of the classical phase space. Evidently, this recipe no longer applies for the systems operating in the deep quantum limit.

Here we show that in this limit diabatic Floquet states reveal their presence via a strong ballisticlike response of a driven quantum particle to a weak net force. Upon changing the modulation parameters, we populated different diabatic bands and switched between regimes of positive and negative responses; i.e., the particle then moves against the applied bias. We measured the mobility [24] of a dilute atomic rubidium Bose-Einstein condensate (BEC) in an ac-driven optical potential. Good agreement between the experimental results and our theoretical model is obtained.

Model.—Consider a particle with mass M moving in a time- and space-periodic potential $U(\hat{x}, t) = V(\hat{x})A(t)$, where the periodic functions $V(x + L) = V(x)$ and $A(t + T) = A(t)$ possess the periods L and $T = 2\pi/\omega$, respectively. In addition, let the particle be exposed to a tunable net bias F. The corresponding Hamiltonian then reads

$$\hat{H} = \hat{p}^2/2M + V(\hat{x})A(t) - F\hat{x}. \tag{1}$$

The Hamiltonian of the nondriven system, i.e., when $A(t) \equiv 1$ and $F = 0$, is a spatially periodic operator and its reciprocal space is spanned by the Bloch bands, $E_n(\kappa)$, $\kappa \in [-\pi/L, \pi/L]$, $n = 1, 2, \ldots$. By assuming that the initial state is localized at the point with quasimomentum $\kappa = 0$, the action of a bias F can be considered as a linear ramp of the quasimomentum, $\kappa(t) = Ft/\hbar$. On a time scale $t \ll T_B$, the response of the system to a weak bias is determined by the band curvature $\hbar^{-2}\partial^2 E_n(k)/\partial\kappa^2$, i.e., by the effective mass [25]. The curvature of the ground-state band is small and positive near the center of the Brillouin zone. One could, in principle, obtain a variety of mobility responses by placing the particle into different excited bands; however, the exponentially small splitting between neighboring bands makes this idea less practical. It is possible to prepare an initial atomic sample outside the center of the Brillouin zone, where the slope of the ground band is nonzero [26]; this would mean that the atoms were already set into a slow motion in the absence of any bias.

The dynamics of such a time-modulated system (1), $A(t) \neq$ const, is more diverse. The Hamiltonian is periodic in time and the solution of the corresponding Schrödinger equation for a given value of the quasimomentum κ, i.e.,

$$[\hat{H}(\kappa, t) - i\hbar\partial_t]\Psi_\kappa(t) = 0, \qquad \hat{H}(\kappa, t+T) = \hat{H}(\kappa, t), \tag{2}$$

can be obtained by solving the eigenvalue problem for the operator that propagates the system over one period of driving [21],

$$|\psi_{n,\kappa}(T)\rangle = \hat{U}(T)|\psi_{n,\kappa}(0)\rangle = \exp[-i\epsilon_n(\kappa)t/\hbar]|\phi_{n,\kappa}(0)\rangle. \tag{3}$$

The eigenfunctions obey the Floquet theorem, $|\psi_{n,\kappa}(t)\rangle = \exp[-i\epsilon_n(\kappa)t/\hbar]|\phi_{n,\kappa}(t)\rangle$, where Floquet states are time periodic, $|\phi_{n,\kappa}(t+T)\rangle = |\phi_{n,\kappa}(t+T)\rangle$. The quasienergies $\epsilon_n(\kappa)$ are conventionally restricted to the interval $[-\hbar\pi/T, \hbar\pi/T]$. Figure 1 depicts Floquet band spectra of the temporally periodically driven optical lattice system, for typical experimental parameters and two different drive frequencies. The spectra exhibit a complex, weblike structure, which can be tailored by varying the modulation $A(t)$. The finiteness of the quasienergy range creates a certain problem with the ordering of Floquet bands. The concept of adiabatic following of a band loses its mathematical rigor here because the set of avoided crossing points is dense everywhere [13,27]. However, there exists no problem with the concept of *diabatic* following Ref. [22]. A diabatic band can be obtained by moving through the parameter space and connecting segments of different bands by ignoring all avoided crossings met on the way when the gap width of the upcoming avoided crossing is below certain threshold ε; see Refs. [22,23] for details. The threshold is related to the velocity of the excursion through the parameter space, which here is given by the strength of the bias F [28]. The corresponding diabatic bands assume straight lines, running across the Brillouin zone [23,29]; cf. Fig. 1(b).

Transport properties of the nth Floquet state are characterized by the average velocity [29], $\bar{v}_{n,\kappa} = \langle v_{n,\kappa}(t)\rangle_T$, where $v_{n,\kappa}(t)$ is the instantaneous expectation value of the velocity operator, $\hat{v} = (-i\hbar/M)\partial_x$, and $\langle\ldots\rangle_T$ denotes a time average over one period of the driving [30].

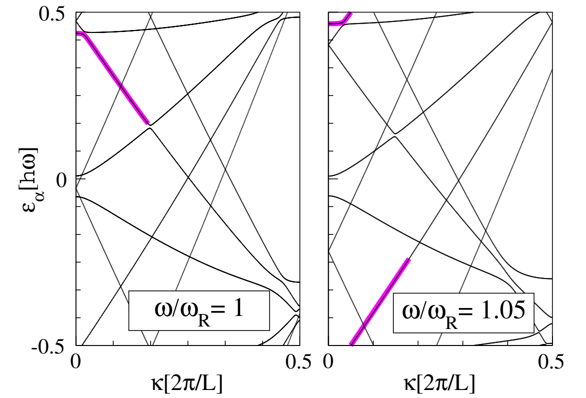

FIG. 1 (color online). Calculated Floquet band spectra of the system, Eqs. (1), (5), and (6), for typical experimental parameters. Only the first eight Floquet bands, maximally overlapping with the Bloch band of the undriven potential, are depicted. The heavy (magenta) lines mark the Floquet states with the largest overlap to the initial atomic state (see text), which here transform themselves into ballistic diabatic bands after passing a first, relatively broad, avoided crossing. A small change in the driving frequency allows switching from a ballistic band with negative velocity (left panel) to the symmetry-related band of positive velocity (right panel). The time-reversal symmetry of the system dynamics implies that the negative part of the Brillouin zone is a mirror image of the presented one. The used parameters are $V_1 = 0.352 \times 16E_r$, $V_2 = 0.11 \times 16E_r$, $A_1 = 0.66$, $A_2 = 0.4$. The driving frequency is measured in units of $\omega_R = 8\omega_r$ (see text). The parameters ω_r and $E_r = \hbar\omega_r$ denote the recoil frequency and the recoil energy, respectively.

By virtue of the Hellmann-Feynman theorem [20,21], the average velocity of the state is equal to the local slope of the corresponding Floquet band [29],

$$\bar{v}_{n,\kappa} = \hbar^{-1}\partial\epsilon_n(\kappa)/\partial\kappa. \tag{4}$$

Because diabatic bands have near constant velocities, they are entitled to be termed ballistic bands. The issue of symmetry is significant here: When the system Hamiltonian equation (1) is invariant under time or space reversal, all Floquet bands are flat at the center of the Brillouin zone, thus exhibiting zero average velocities [16,17]. Formally, the presence of a symmetry implies that the potential function $U(x,t) = A(t)V(x)$ remains invariant either under the transformation $\hat{S}_t$: $t \to -t + \tau$, $x \to x + \chi$ or $\hat{S}_x$: $t \to t + \tau$, $x \to -x + \chi$, where τ and χ are appropriate shift constants; see in Ref. [17]. In contrast to genuine Floquet bands, a diabatic band does need to be periodic in quasimomentum κ space [16] and sometimes can wrap the Brillouin zone several times before meeting a broad avoided crossing and losing the diabaticity property. This fact does not contradict the above symmetry statement because the presence of a symmetry only means that two ballistic bands with opposite velocities cross each other at the point $\kappa = 0$. Note, however, that such a crossing mimics a narrow avoided crossing between the corresponding pair of genuine Floquet bands.

Consider next the situation when at time $t = 0$ a dilute and delocalized cloud of ultracold atoms is loaded into the temporally driven lattice potential. The initial atomic wave function $|\varphi_0\rangle$ has the form of a wave packet, well localized in quasimomentum at the point $\kappa = 0$. For our calculations, we assume that the initial atomic wave function is of the form of the Bloch ground state of the nondriven lattice potential, which is experimentally reasonable. We can order the Floquet states, $FS[n]$, according to their overlaps with the initial state, i.e., $c_n = |\langle \phi_{n,\kappa=0}(0)|\varphi_0\rangle|^2$, $c_1 > c_2 > \ldots$. We find that, for our experimental parameters, the overlap with the first Floquet band is $c_1 \simeq 0.85$, and it is natural to expect that the system mobility is determined mainly by the properties of this band. We are interested in the situation where this state transforms itself into a ballistic band outside the center of the Brillouin zone, for example, due to a broad avoided crossing with another Floquet state, as is the case for the parameters used in Fig. 1. A motion through the κ space, induced by a weak bias force, is slow enough for the system to remain on the band when passing through this ac. We expect that the nonvanishing slope of the band beyond this point will determine the mean particle velocity. In general, the described scenario assumes that (i) a significant population of the relevant band can be achieved and (ii) the band group velocity is high. There is no general recipe how to manage such a situation. There are, however, several prerequisites serving as a guide. First, the system should operate in the deep quantum regime, ideally with only few Floquet states within the potential range. Otherwise, the initial wave function will be distributed among several Floquet states of different group velocities. Their joint contributions then yield a inconclusive asymptotic response, while interference effects will blur the finite-time response even more. Second, the modulation frequency should be chosen close to the frequency of oscillations at the bottom of the potential well, $\omega_R = 4\pi^2\hbar/L^2M$. In numerical studies we have observed that the resonant modulation typically produces at the vicinity of the initially populated $FS[1]$ state at $\kappa = 0$ two well-separated Floquet states that are ballistic, with opposite velocities just outside the center of the zone. By slightly adjusting the frequency of the driving, one can then map the initially populated Floquet state (which has a vanishing group velocity) into a ballistic state by means of a broad avoided crossing between the $FS[1]$ state and one of the states of the ballistic pair.

Experiment.—In our experiments we used an optical potential, $U(x, t) = V(x)A(t)$, where $V(x)$ and $A(t)$ are of the form [17,18,23]

$$V(x) = (V_1/2)\cos(2kx) + (V_2/2)\sin(4kx), \qquad (5)$$

$$A(t) = A_1\sin^2(\omega t/2) + A_2\cos^2(\omega t). \qquad (6)$$

Here, λ is the wavelength of the driving laser field and $k = 2\pi/\lambda$. Using the spatial lattice period, $L = \lambda/2$, we arrive at a resonance frequency of $\omega_R = 8\omega_r$, where $\omega_r = \hbar k^2/2M_{\mathrm{Rb}}$ is the recoil frequency, and $E_r = \hbar\omega_r$ denotes the recoil energy. Note that this setup, although possessing a ratchetlike spatial profile [31], is perfectly *time symmetric* since $A(-t) = A(t)$ [32]. Therefore, in the absence of an external bias force, $F = 0$, no transport occurs [17,34]; i.e., the average current vanishes for any initial atomic wave packet with zero average velocity. We performed the experiments by loading a Bose-Einstein condensate of rubidium atoms into a periodically time-modulated optical potential, of the form given by Eq. (4), to an external dc bias force.

We start the experimental sequence by preparing a Bose-Einstein condensate of rubidium (^{87}Rb) atoms in a far detuned optical dipole trap. During the next step the atoms are allowed to freely expand ballistically for 2.5 ms, which reduces the atomic density. This leaves the atomic cloud with essentially kinetic energy only, while the interaction energy is strongly diminished. After then loading the atoms into an optical lattice potential, a dc bias is realized by moving the lattice potential with a constant acceleration. This acceleration emulates a constant force in the comoving frame (see the Supplemental Material [33]). The measured mean velocity of the atomic cloud versus the bias strength F is depicted in Fig. 2(b). For small values of $F > 0$, a negative response is clearly detectable; i.e., the average atomic velocity is opposite to the applied bias force. We attribute this to the population of the Floquet state marked by the heavy line in the left panel of Fig. 1, which for positive quasimomentum values has a negative group velocity. For larger absolute values of the dc force, above roughly $|F| = 0.02E_r/\lambda$, the mobility again becomes positive. This is attributed to the population of other Floquet states when moving beyond the quasimomentum region indicated by the thick (magenta) lines in Fig. 1. We have also investigated the dependence of the atomic transport on the drive frequency; see Fig. 3(a). By changing the frequency to slightly higher values, the atomic mobility can be tuned to a normal, i.e., positive, response. This response is attributed to the overlap of the $FS[1]$ state with the symmetry-related counterpart of the previously involved ballistic band, as shown in the right panel of Fig. 1.

An important issue is the stability of the mobility response. Let us first discuss this issue in two different limits, classical and quantum ones. In the classical dissipationless limit, a stationary motion against a constant bias is possible [35] due to the existence of invariant manifolds, i.e. transporting regular islands, periodic orbits, and cantori [36], in the phase space of an ac-driven Hamiltonian system [29]. Diabatic bands can be viewed as the quantum counterpart of classical ballistic manifolds. However, this analogy is not exact. In contrast to classical manifolds, diabatic bands are not completely

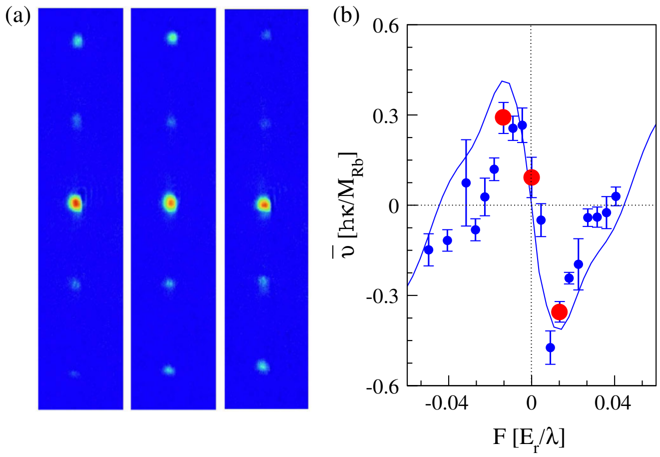

FIG. 2 (color online). (a) Time-of-flight (TOF) images of the atomic cloud after $t_{\rm exp} = 26T$ periods of modulation for $F = -0.0181 E_r/\lambda$ (left), 0 (center), and $0.0181 E_r/\lambda$ (right). The visible atomic diffraction orders are $s = -2, \ldots, 2$ (from bottom to top). (b) Measured mean velocity of the atomic cloud in the laboratory frame versus applied bias F. Note that the overall antisymmetric response behavior; i.e., $\bar{v}(F) = -\bar{v}(-F)$. The mean velocity of the atomic cloud was calculated as $\bar{v} = \bar{p}/M_{\rm Rb}$ with $\bar{p} = 2\hbar k \sum_s s|c_s|^2$, where $|c_s|^2$ denotes the fraction of atoms in the sth order momentum state, $|2s\hbar k\rangle$, with $s = \pm 1, \pm 2$. The error bars show the standard deviation of the mean value. The enlarged (red) data points correspond to the TOF images. The solid lines depict the results of numerical simulations of the theory. The initial state was chosen in the form of a narrow Gaussian packet in the quasimomentum space, with the center at $\kappa = 0$ and dispersion $\sigma_\kappa = 0.04\hbar k$ (see the Supplemental Material [33]). The other parameters are the same as in Fig. 1.

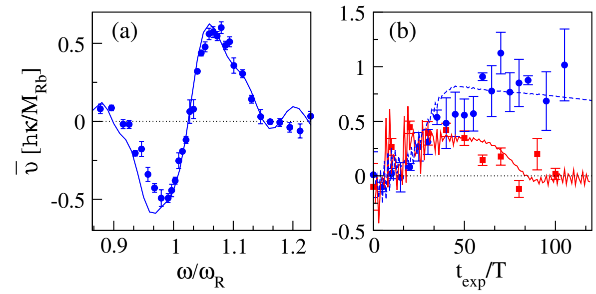

FIG. 3 (color online). (a) Average velocity of the atomic cloud in the laboratory frame as a function of the modulation frequency ω (blue dots) for a bias strength $F = -0.0181 E_r/\lambda$ and exposition time $26T$; (b) Time evolution of the average cloud velocity for driving frequencies $\omega/\omega_R = 1.05$ (red squares) and $\omega/\omega_R = 1.076$ (blue dots). The solid lines depict the results of numerical simulations (see the Supplemental Material [33]). The error bars show the standard deviation of the mean. The remaining parameters are the same as in Fig. 1.

isolated because any finite bias sets these bands into a contact with other states [37]. In addition, the band can lose its diabatic property by encountering a broad avoided crossing; see left panel of Fig. 1. However, the band could run over a substantial region of the κ space—or even perform several revolutions around the Brillouin zone—before this happens, thus transforming the ballistic response into a long-lasting metastable phenomenon. In order to experimentally investigate this issue, we have measured the velocity of the atomic cloud as a function of the exposition time for two different values of the driving frequency, both of which produced negative responses. Figure 3(b) presents the results of the measurements. For a modulation frequency $\omega/\omega_R = 1.05$ (red squares), the mean momentum of the atomic cloud initially increases, reaching a broad maximum value of $0.4\hbar k$ after around 30 modulation periods, and then starts to decrease again, approaching zero at $t \approx 80T$. When slightly changing the modulation frequency to $\omega/\omega_R = 1.076$ (blue dots), the observed atomic momentum increases to roughly $0.9\hbar k$. This value is maintained to much longer modulation times. This finding is in agreement with the fact that the structure of Floquet bands is very sensitive to variations of the system parameters and, correspondingly, the system response can be controlled over a wide range.

Conclusions.—The experimental detection of diabatic Floquet states in a time-dependent driven Bose-Einstein condensate opens several directions that are worth exploring further. For example, there is the issue relating to the constructive role that decoherence may play for the stabilization of the asymptotic response, similar to what has been observed with stochastic models operating in the classical regime [38]. Another promising research avenue is the implementation of the tunable dispersion relation, stemming from the Floquet spectrum of temporally modulated optical lattices, for simulations of relativistic physics effects with ultracold atoms [39,40]. Finally, the inclusion of interaction between atoms of a dense condensate [41,42] may open a way to the detection of many-body diabatic Floquet states.

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