

Campisi *et al.* Reply: The logarithmic oscillator possesses a spectacular property: its heat capacity is infinite, hence it can lead a second system to Gibbs equilibrium by means of weak interactions [1]. The criticism of Meléndez *et al.* is that this can neither be implemented in simulations nor in experiments due to the length and time scales involved [2]. That our method can be employed in computer simulations is an incontrovertible fact that both we (see Figs. 2 and 3 in Ref. [1]) and, as well, the authors of the Comment (see Figs. 1 and 3 of Ref. [3]) have convincingly demonstrated with the number of particles ranging from $N = 1$ to $N = 18$. In Table I below we show that this is also experimentally feasible.

The table reports data referring to a one-dimensional (1D) implementation in which N Rb atoms ($m = 85.4678$ amu) move in a 1D box, and collide with themselves and with a particle subject to the potential $\varphi_b(x) = T \ln \sqrt{1 + x^2/b^2}$. This setup can be implemented with current cold-atom physics technology [4]. In the first column we have the number f of degrees of freedom of the system. In the second column we give the accuracy with which the actual distribution $p(v)$ of the absolute value v of any of the f velocities approximates the target Maxwell distribution $p_\beta(v) = (\pi/2\beta m)^{-1/2} e^{-\beta m v^2/2}$. In Fig. 3 of our Letter [1], the red solid line is $p(v)$ and the black dashed line is $p_\beta(v)$. The accuracy is here calculated as the Kolmogorov-Smirnov distance $H_{KS}[p|p_\beta] = \max_u |\int_0^u dv [p(v) - p_\beta(v)]|$ [5]. In the third and fourth columns we have, respectively, the corresponding ratio E_{tot}/T , and trap lengths calculated as $L = 2b\sqrt{e^{2E_{\text{tot}}/T} - 1} \simeq 2be^{E_{\text{tot}}/T}$, with a cutoff length of $b = 10^{-8}$ m [6]. In the fifth column we report the corresponding collision times $\tau = L/N\bar{v}$ where $\bar{v} = \sqrt{k_B T/m}$ is the average velocity. Following Meléndez *et al.* we use $T = 1$ K. The upper part of the table is for fixed f and varying accuracy H_{KS} . It shows how the length and time scales vary accordingly. The lower part is for fixed accuracy H_{KS} and varying f . In agreement with our estimate [1], the ratio E_{tot}/T scales approximately as $E_{\text{tot}}/T \sim f/2$. Note that, accordingly, the box size scales exponentially with f , i.e., $L \sim 2be^{f/2}$, which, as stressed in our Letter, limits the applicability to small systems [1]. As the table clearly shows, for f sufficiently small, good accuracies can be achieved with experimentally accessible length and time scales [7].

We also respond to the second criticism raised in the Comment. Since in our method the temperature appears as a parameter in the Hamiltonian, one can use it to study the response to a temporally varying temperature, using the theory of fluctuations in time-dependent Hamiltonians [1,8]. The authors of the comment instead studied the issue

TABLE I. Length (L) and time (τ) scales of possible implementations of logarithmic oscillators as thermostats for cold Rb atoms confined into a 1D trap, depending on the number $f = N$ of atoms and the required accuracy H_{KS} .

f	H_{KS}	E_{tot}/T	L [m]	τ [sec]
20	0.005	16.45	2.78724×10^{-1}	1.41295×10^{-3}
20	0.01	14.8	5.35289×10^{-2}	2.71358×10^{-4}
20	0.02	13.1	9.77885×10^{-3}	4.95726×10^{-5}
20	0.03	11.9	2.94533×10^{-3}	1.4931×10^{-5}
20	0.04	11.05	1.25888×10^{-3}	6.38172×10^{-6}
10	0.02	7.75	4.64314×10^{-5}	4.70756×10^{-7}
20	0.02	13.1	9.77885×10^{-3}	4.95726×10^{-5}
30	0.02	18.1	1.45131×10^0	4.90482×10^{-3}
40	0.02	23.1	2.15393×10^2	5.45955×10^{-1}
50	0.02	28	2.89251×10^4	5.86529×10^1

of a spatially varying temperature. Not only is this second criticism not pertinent to our Letter, but it is also neither sufficiently documented nor conclusive [9].

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- [6] The choice $b = \sigma = 10^{-10}$ m of Meléndez *et al.* is presently too small to be experimentally achievable.
- [7] In clear contrast, the choice of 26 particles in three dimensions, i.e., $f = 3 \times 26 = 78$, used in the Comment, yields, due to the exponential growth, astronomical length and time scales. Such a choice evidently violates our criterion of “smallness” [1].
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