Current Resonances in Graphene with Time-Dependent Potential Barriers

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A method is derived to solve the massless Dirac-Weyl equation describing electron transport in a monolayer of graphene with a scalar potential barrier \( U(x,t) \), homogeneous in the \( y \) direction, of arbitrary space and time dependence. Resonant enhancement of both electron backscattering and currents, across and along the barrier, is predicted when the modulation frequencies satisfy certain resonance conditions. These conditions resemble those for Shapiro steps of driven Josephson junctions. Surprisingly, we find a nonzero \( y \) component of the current for carriers of zero momentum along the \( y \)-axis.

\[ H_0 = \nu_F [\vec{\sigma} \cdot \vec{p} + \sigma_z \gamma_0], \]

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Growing interest in graphene [1] is stimulated by many unusual and sometimes counterintuitive properties of this two-dimensional material. Indeed, graphene supplies charge carriers exhibiting pseudorelativistic dynamics of massless Dirac fermions. As one consequence, the Klein tunneling phenomenon [2] occurs with unit probability through arbitrarily high and thick barriers at perpendicular incidence, irrespective of the particle energy, in accordance with experiment [3]. The question of how to control the electron motion in graphene arose, and hence boosted detailed studies of Dirac fermions under the influence of various forms of scalar [4–8] or vector [9] potentials.

Applying a time-dependent laser field to pristine graphene opens an alternative and efficient way [10–12] to control spectrum and transport properties. It was shown [10] that Dirac fermions across \( p-n \) junctions can acquire an effective mass when driven by a laser field. This results in an exponential suppression of chiral tunneling even for perpendicular incidence upon the junction, if the ac-electric field is directed parallel to the junction, in stark contrast to Klein tunneling occurring in the absence of the laser field. Actually, time-dependent laser fields can mimic [12] the influence of any electrostatic and/or magnetostatic graphene superlattice on the electron spectrum in graphene. The question arises whether and under which conditions time-dependent modulations of an electrostatic barrier, where energy is not conserved, would affect electron transport and generate backscattering.

In this Letter we answer this question by solving the problem for arbitrary space-time dependent scalar potentials \( U(x,t) \). Our solution is based on expanding the wave function as a power series with respect to the momentum \( k_y \), parallel to the barrier, and manifests a structure of left and right moving waves. All terms appearing in the \( k_y \) expansion can be calculated analytically, despite the fact that each term is described by a partial differential equation in \( x, t \) space. At \( k_y = 0 \) (normal incidence upon the barrier) we confirm complete Klein tunneling for any \( U(x,t) \), while for finite \( k_y \) backscattering resonances can occur at certain angles of incidence, depending on the modulation frequency of the barrier. As a counterintuitive result we find a nonzero and oscillating current \( j_y \) along the barrier, even at \( k_y = 0 \) for valley-polarized fermions. At \( k_y \neq 0 \) the current \( j_y \) arises also in valley-unpolarized situations, it can be resonantly amplified and flow in either direction. Interestingly, \( j_y \) exhibits a nonzero dc component at certain resonance frequencies, in full analogy to Shapiro steps of driven Josephson junctions.

At low energies, the honeycomb lattice of graphene engenders two copies, \( \tau_z = \pm 1 \), of Dirac-Weyl Hamiltonians [13],

\[ H_0 = \nu_F [\vec{\sigma} \cdot \vec{p} + \sigma_z \gamma_0], \]

centered about two inequivalent Dirac points ("valleys") \( K \) and \( K' \) at corners of the hexagonal first Brillouin zone where electron-hole symmetric bands touch; Pauli matrices \( \sigma_{x,y,z} \) act on two-component spinors representing sublattice amplitudes. Carriers near either of the Dirac points exhibit opposite Fermion helicities, \( \sigma \cdot \vec{p}/p = \pm 1 \). Proposals exist in literature of how to valley polarize carriers in graphene, by means of nanoribbons terminated by zigzag edges [14], by exploiting trigonal warping at elevated energies [15], or by absorbing magnetic textures [16].

Smooth electromagnetic or disorder potentials [17], containing negligible Fourier components at large wave vectors of the order of \( |\vec{K}| \), will not cause scattering between valleys so that calculations can be carried out for \( \tau_z = +1 \) or \( \tau_z = -1 \) separately. Accordingly, time-dependent potentials \( U(x,t) \) should be slowly varying, without frequency components that might induce excursions to energies where the band structure of graphene starts deviating from the isotropic cone spectrum, i.e., below 0.6 eV [18]. Including \( U(x,t) \), the Dirac equation for the wave function \( \Psi_{k_y}(x,y,t) = \Psi(x,t) \exp(ik_y y) \) can be written in the form...
where from now on we assume $v_F = 1$ and $h = 1$. This equation has been solved analytically for time-independent potentials either by matching [2] of wave functions for rectangular barriers or by the WKB method [19] for smooth barriers. Time-dependent harmonic oscillations have been considered of gate voltages on either side of a graphene rectangle [20], of an electric field parallel to the barrier [10] or in resonance approximation [12], or for some class of time-dependent barriers $U(x, t)$ at $k_y = 0$ [21].

Our goal here is to construct the solution of Eq. (2) for arbitrary $U(x, t)$ acting at positive times, $U(x, t < 0) = 0$. From the ansatz

$$\Psi = \sum_{n=0}^{\infty} (ik_y)^n \left( \frac{1}{\tau_z} \right) \Psi_{n+} + \sum_{n=0}^{\infty} (ik_y)^n \left( -\frac{1}{\tau_z} \right) \Psi_{n-},$$

as a power series in $k_y$, we derive a recurrence relation for the coefficients $\Psi_{n\pm}$ which obey the inhomogeneous first order partial differential equations,

$$\left[ U(x, t) \mp i \frac{\partial}{\partial x} - i \frac{\partial}{\partial t} \right]\Psi_{n\pm} \pm \tau_z \Psi_{n-1\pm} = 0,$$

with $\Psi_{-1\pm}(x, t \geq 0) = 0$. Initial conditions can be chosen as $\Psi_{0\pm}(x, t = 0) = a_{\pm}(x) = [\Psi_A(x, t = 0) \pm \tau_z \Psi_B(x, t = 0)]/2$, $\Psi_{n>0\pm} = 0$, where $\Psi_A$, $\Psi_B$ describe electron amplitudes on either of the graphene sublattices. The two functions $a_{\pm}(x)$, providing the initial conditions, can be, e.g., a plane wave or a wave packet. We underline here the general structure of (3) as a sum of right $\Psi_+$ and left $\Psi_-$ moving waves. Using the standard d’Alembert’s ratio test, a sufficient criterion for convergence of series (3) is $|k_y| \lim_{n \to \infty} |\Psi_{n+1\pm}|/|\Psi_{n\pm}| < 1$ for all relevant $x$ and $t$.

Despite the fact that (4) are partial differential equations, we can solve them exactly using the method of characteristics [22]. The corresponding result reads

$$\Psi_{n\pm}(x, t) = a_{n\pm}(x, t) e^{-i \int_0^t dt' U(x = t' \pm t', t')},$$

with $a_{0\pm} = a_{\pm}(x \mp t)$ and

$$a_{n>0\pm} = \mp i \tau_z \int_0^t dt' \Psi_{n-1\pm}(x = t \mp t', t')$$

$$\times e^{i \int_0^t dt' U(x = t' \pm t', t')}. $$

Together with (3) the recursive solution for $\Psi_{n\pm}$ provides the exact wave function $\Psi$ to any desired accuracy. To zeroth order approximation with respect to $k_y$, we obtain

$$\psi(x, t) = a_+(x - t) \left( \frac{1}{\tau_z} \right) e^{-i \int_0^t dt' U(x = t' \pm t', t')$$

$$+ a_-(x + t) \left( -\frac{1}{\tau_z} \right) e^{-i \int_0^t dt' U(x = t' \pm t, t')} .$$

The first order corrections with respect to $k_y$ in (5) can be written as

$$a_{1\pm} = \mp i \tau_z A_{1\pm} = \mp i \tau_z \int_0^t dt' a_{\pm}(x \mp t \pm 2t')$$

$$\times e^{i \int_0^t dt' U(x = t' \pm t', t')} ,$$

so that $\Psi = \Psi_+ + \Psi_-$ as in (3), with

$$\Psi_{\pm} = [a_{0\pm} \pm k_y \tau_z A_{1\pm}] e^{-i \int_0^t dt' U(x = t' \pm t', t')} .$$

When $k_y = 0$ and when the wave packet is initially purely right moving, $a_- = 0$, Eq. (6) reveals that the electron density distribution $|a_+(x - t)|^2$ undistortedly continues to propagate to the right without reflection: $\Psi_-(x, t) = 0$ for all times $t > 0$. This proves complete Klein tunneling also in the presence of time-dependent barriers; wave functions $\Psi_{\pm}$ acquire only a phase factor by the potential at $k_y = 0$.

As a measurable quantity, we now evaluate the current density in Cartesian components, $j_x = \Psi^\dagger \tau_z \sigma_x \Psi = 2\Psi_+^\dagger \tau_z \Psi_+ - 2\Psi_-^\dagger \Psi_- = j_{0x} + j_{1x}$ and $j_y = \Psi^\dagger \tau_z \sigma_y \Psi = 2i\tau_z (\Psi_+^\dagger \Psi_- - \Psi_+ \Psi_-^\dagger) = j_{0y} + j_{1y}$. Here, the last equals signs refer to zeroth and first order contributions with respect to $k_y$, respectively, yielding

$$j_{0x} = 2(|a_{0+}|^2 - |a_{0-}|^2),$$

$$j_{0y} = 4\tau_z |a_{0+} a_{0-}| \sin(\varphi + \phi_0),$$

$$j_{1x} = 4k_y \tau_z |\text{Re}(a_{0+} A^*_1 - a_{0-} A^*_1)|,$n

$$j_{1y} = 4k_y |[A_{1+} a_{0-}] \sin(\varphi - \phi_-) - [A^*_1 a_{0+}] \sin(\varphi + \phi_+)|,$n

with $\varphi = \int_0^t [U(x = t - t', t') - U(x + t - t', t')] dt'$, $\phi_0 = \text{arg}(a_{0+} a_{0-}^*)$, and $\phi_\pm = \text{arg}(a_{0\pm} A^*_{1\pm})$. We distinguish two cases: (i) $\tau_z$-independent contributions $j_{0x}$ and $j_{1x}$ which can be observed for valley-unpolarized carriers and (ii) $\tau_z$-dependent contributions $j_{1x}$ and $j_{0y}$, where detection calls for valley polarization.

Equations (9) and (10) describe the current density at normal incidence, $k_y = 0$. Then $j_x$ stays unaffected by the barrier, irrespective of $\tau_z$, which rephrases the above result of complete Klein tunneling. Surprisingly, a current $j_y$ flows perpendicular to the momentum in graphene, provided the sample is valley polarized [the total current $j = j^+ + j^-$, where $j^+$ originates from states near valley
$K (\tau_z = +1)$ or $K' (\tau_z = -1)$, respectively. This current (10) results from interfering left and right moving waves, which both need to have nonzero amplitudes, $a_0, a_0 \neq 0$.

Equations (11) and (12) describe corrections to the current density at small but finite angles of incidence, $k_y \neq 0$. Thereby, $j_{1y}$ exhibits qualitatively similar properties to $j_{0y}$; in particular, it stays nonvanishing at finite valley polarization only. By contrast, the current density $j_{1y}$ now exhibits striking current oscillations and current reversals already in valley-unpolarized situations, as we show in more detail below.

Next we turn to the question of how carriers are reflected by $U(x, t)$. Let us consider an initially right moving plane wave, $\Psi_{k_y} = e^{i(k_y x + \gamma_y)} (1)$ at $t = 0$ which produces a current density $j_{0y} = +2$ pointing to the right. Using Eqs. (3) and (8), and assuming small $k_y$, the leading contribution to the reflected current density $j_{2y} = -k_y^2 |A_{1-}|^2$ arises in $O(k_y^2)$ under the action of the barrier at $t > 0$ and is proportional to $|j_{0y}|$, cf. Eqs. (7) and (9). This suggests to employ the ratio

$$R(x, t) := \frac{j_{2y} / j_{0y}}{k_y^2 |A_{1-}(x, t)|^2 / 2}$$

as a measure for the reflectivity at small $k_y$. While the quantity $R(x, t)$ evolves in time, together with $U(x, t)$, it is independent of $\tau_z$ and, thus, measurable without valley polarization. Moreover, we also analyze the time averaged reflectivity $R(x) := \lim_{t \to \infty} \frac{1}{T} \int_0^T R(x, t) dt / T$, which can be measured just by means of dc equipment.

In the following, two specific examples $U(x, t) = U(1, 2)(x, t)$ are considered. As initial conditions we take into account two cases: (i) a superposition of equal amplitudes of right and left propagating plane waves, $a_\pm = \exp(\pm ik_y x)$ and $\tau_z = +1$ when calculating $j_{0y}$, and (ii) an incidently right moving wave $a_+ = \exp(ik_y x), a_- = 0$ when calculating $R$ and $j_{1y}$ for valley-unpolarized systems. Our first example is $U(1, 2)(x, t) = U_0 \cos(x / \lambda) \cos(\omega t)$, calculated by numerical integration of Eq. (17), solid blue line. The leading contribution to the current density $j_{2y}$ occurs in $O(k_y^2)$.

Using $j_{0y}(x, t) = \sin 2 \left[\frac{\lambda}{\omega} - \frac{\sin \omega t}{\omega^2}\right]$ (14), which can be rewritten as a sum

$$j_{0y}(x, t) = \sum_{n=-\infty}^{\infty} J_n \left(\frac{2U_0}{\omega^2}\right) \sin(2k_y x + \frac{2U_0 t}{\omega} - n \omega t)$$

(15)

using Bessel functions $J_n$. This form (15) reveals a peculiarity at $\omega = \omega_n$ with

$$\omega_n = \sqrt{2U_0 / n}, \quad n \in \mathbb{N},$$

(16)

similar to Shapiro steps [23] of a driven Josephson junction. As depicted in Fig. 1(a), frequencies $\omega = \omega_n$ generate periodic oscillations, which again, as in the case of

FIG. 1 (color online). (a) Current $j_{0y}$ (10) perpendicular to $k$ versus time for $U(1, 2)(x, t) = U_0 \cos(x / \lambda) \cos(\omega t), k_y = \pi / 2$, and $U_0/\omega^2 = 0.49$. (b) Same as (a) but for potential $U(1, 2)(x, t) = U_0 \cos(x / \lambda) \cos(\omega t)$, and frequencies $\omega = \pi / 2 (1 / \lambda)$ [light gray (red) line] and $\omega = 1 / \lambda$ [dark gray (blue) line]. Both currents are periodic. For the matching condition $\omega = 1 / \lambda$ [dark gray (blue)] a considerable enhancement followed by a saturation of the amplitude of the current oscillations occurs, while away from this resonance no enhancement is seen versus time. (c) Current $j_{1y}$ (12) for $U(1, 2)(x, t)$ at $x = 0, k_y = 0$. At Shapiro resonance $|U_0|/\omega^2 = 1 / 2, k_y = 0, U_0 \lambda = 1, 0.1$ and frequencies $\omega = (\pi / 2) / \lambda$ [light gray (red) line] and $\omega = 1 / \lambda$ [dark gray (blue) line]. Both currents are aperiodic. For the matching condition $\omega = 1 / \lambda$ [dark gray (blue)] a considerable enhancement followed by a saturation of the amplitude of the current oscillations occurs, while away from this resonance no enhancement is seen versus time. (d) Current $j_{1y}$ (12) for $U(1, 2)(x, t)$ at $x = 0, k_y = 0$. At Shapiro resonance $|U_0|/\omega^2 = 1 / 2, k_y = 0, U_0 \lambda = 1, 0.1$ and frequencies $\omega = (\pi / 2) / \lambda$ [light gray (red) line] and $\omega = 1 / \lambda$ [dark gray (blue) line]. Both currents are aperiodic. For the matching condition $\omega = 1 / \lambda$ [dark gray (blue)] a considerable enhancement followed by a saturation of the amplitude of the current oscillations occurs, while away from this resonance no enhancement is seen versus time. (e) Current $j_{1y}$ (12) for $U(1, 2)(x, t)$ at $x = 0, k_y = 0$. At Shapiro resonance $|U_0|/\omega^2 = 1 / 2, k_y = 0, U_0 \lambda = 1, 0.1$ and frequencies $\omega = (\pi / 2) / \lambda$ [light gray (red) line] and $\omega = 1 / \lambda$ [dark gray (blue) line]. Both currents are aperiodic. For the matching condition $\omega = 1 / \lambda$ [dark gray (blue)] a considerable enhancement followed by a saturation of the amplitude of the current oscillations occurs, while away from this resonance no enhancement is seen versus time. (f) Current $j_{1y}$ (12) for $U(1, 2)(x, t)$ at $x = 0, k_y = 0$. At Shapiro resonance $|U_0|/\omega^2 = 1 / 2, k_y = 0, U_0 \lambda = 1, 0.1$ and frequencies $\omega = (\pi / 2) / \lambda$ [light gray (red) line] and $\omega = 1 / \lambda$ [dark gray (blue) line]. Both currents are aperiodic. For the matching condition $\omega = 1 / \lambda$ [dark gray (blue)] a considerable enhancement followed by a saturation of the amplitude of the current oscillations occurs, while away from this resonance no enhancement is seen versus time. (g) Current $j_{1y}$ (12) for $U(1, 2)(x, t)$ at $x = 0, k_y = 0$. At Shapiro resonance $|U_0|/\omega^2 = 1 / 2, k_y = 0, U_0 \lambda = 1, 0.1$ and frequencies $\omega = (\pi / 2) / \lambda$ [light gray (red) line] and $\omega = 1 / \lambda$ [dark gray (blue) line]. Both currents are aperiodic. For the matching condition $\omega = 1 / \lambda$ [dark gray (blue)] a considerable enhancement followed by a saturation of the amplitude of the current oscillations occurs, while away from this resonance no enhancement is seen versus time.
Shapiro steps, induce a nonzero dc component in the current at given $x$. Modulating the potential with $\omega \neq \omega_n$ results in aperiodic oscillations and zero dc component.

Similar resonance effects can also be seen in both the reflectivity $R$ (13) and the current $j_{1y}$ (12). By inserting $U^{(1)}(x, t)$ into Eq. (7), we derive

$$A_{1-} = e^{ik_x(x+it)} \int_0^t dt' e^{-2ik_xt'} e^{2iU[t'[\omega t'-\sin(\omega t')]/\omega^2}$$

$$= e^{ik_x(x+it)} \sum_{n=-\infty}^\infty J_n \left( \frac{2U_0}{\omega^2} \right) e^{2iU[t'-(2k_x-n\omega)t]}/(2U_0/\omega - 2k_x - n\omega)}, \quad \text{Eq. (17)},$$

from which we read off a Shapiro step resonance condition

$$k_n = k_x = -\frac{n}{2} \frac{U_0}{\omega}, \quad n \in \mathbb{N},$$

specifying now a directional dependence of the momentum $k$. From (17) together with (12) we conclude that in valley-unpolarized samples the current $j_y \propto k_y$ parallel to the barrier oscillates as a function of time and may take either sign [despite the fixed $k_y$, see Fig. 1(c)]. In addition, the amplitude of these oscillations increases with time as the Shapiro step resonance condition (18) is met [compare light gray (red) and dark gray (blue) curves in Fig. 1(c)].

Analogous resonances also show up in both reflectivities, $R$ and $\tilde{R}$. The latter allows experimental observation of the here predicted behavior without time-domain measurements. Indeed, near the Shapiro step resonance (18) we can keep only one summand in the expansion (17), yielding

$$R(t) = \frac{1}{2} k_x^2 J^2 \left( \frac{2U_0}{\omega^2} \right) \sin^2 \left[ \frac{(U_0 - k_x - n\omega/2)t}{2} \right], \quad \text{Eq. (19)},$$

This equation is in a good agreement with numerical integration of (17); see Figs. 1(d) and 1(e). Averaging (19) with respect to time results in

$$\bar{R} = \frac{1}{4} k_x^2 J^2 \left( \frac{2U_0}{\omega^2} \right) \frac{1}{(U_0/\omega - k_x - n\omega/2)^2}, \quad \text{Eq. (20)},$$

so that the barrier will become transparent near momenta $k_x = k_n$ [see Fig. 1(f)], already for small $U_0$. This produces strong anomalies in transport properties at angles $\arctan(k_n/k_x)$ of the incidence. Instead of sweeping the directions of $k$ one may alternatively sweep $\omega$ at fixed $k$, cf. (18); ensuing resonance peaks are clearly observed in Fig. 1(g). The constraint $R < 1$ determines the maximum value

$$|k_y| \leq \frac{|U_0 - k_x - n\omega/2|}{J_n \left( \frac{2U_0}{\omega^2} \right)}, \quad \text{Eq. (21)}.$$
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