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DISSIPATIVE QUANTUM DECAY AT FINITE TEMPERATURES

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In various experiments performed over the last four years a typical quantum behavior of macroscopic variables of Josephson systems has been observed. Those experiments spurred a rapid development of the theory which accounts for the effects of damping upon the quantum behavior including tunneling, coherence and fluctuations. In the following, the discussion is confined to the topic of dissipative quantum decay.

We consider a quantum system, interacting with a thermal environment at temperature T , which can decay out of a metastable state. The metastable state is rendered unstable via both, thermal activation over the barrier and quantum tunneling. Kramers' classical theory of the activation rate becomes modified at lower temperatures when quantum fluctuations and tunneling prevail. An efficient technique to calculate the decay rate Γ in presence of dissipative quantum effects (damping) is given by Feynman's path-integral approach. To determine the decay rate Γ , which is proportional to the imaginary part of the free energy, the Euclidian time version of Feynman's path-integral is used advantageously, and the so-called "bounce" technique originally introduced by Langer and beautifully popularized

by Coleman, is employed. Within the WKB approximation, there occurs a crossover temperature T_0

$$T_0 = \frac{\hbar}{2\pi k_B} \alpha \quad (1)$$

where a transition between classical and quantum behavior takes place. T_0 is characterized by the fact that the "bounce" solution at low temperatures ceases to exist. The quantity α occurring in the expression for the crossover temperature T_0 denotes the memory-renormalized reactive frequency which enters the classical prefactor of the activation rate $\Gamma_{c\ell}$ in presence of generally frequency dependent damping. At $T = T_0$, the classical Arrhenius factor, $\exp - U_b/k_B T$ (with U_b denoting the barrier height) matches smoothly with the one-bounce contribution, $\exp - S_B(T)/\hbar$, where S_B is the effective Euclidian action evaluated along the "bounce"-trajectory with period $\hbar/k_B T$.

For temperatures $T > T_0$, the classical prefactor of the rate becomes modified due to quantum fluctuations in order \hbar^2 . The behavior very close to $T = T_0$ is complicated by the fact that the fluctuation modes about the bounce trajectory include, besides an unstable (negative eigenvalue) mode, two dangerous modes; an exact zero-mode, which describes phase fluctuations of the "bounce" and a quasi zero-mode describing amplitude fluctuations. Below T_0 , the fluctuation modes about the "bounce"-trajectory include the zero-mode of the "bounce" phase and an unstable mode. In this temperature range, the thermal enhancement of the rate is different from the classical enhancement inherent in the Arrhenius factor. For a dissipative quantum system the effective action $S_B(T)$ near $T = 0$ shrinks algebraically with increasing temperature, i.e.

$$S_B(T) = S_B(T=0) - cT^n \quad (2)$$

The exponent n is independent of the particular form of the metastable potential and is a distinctive feature of the dissipative mechanism at low frequencies.

For the important case of "Ohmic-like" dissipation, i.e. a damping mechanism $\gamma(\omega)$ that becomes independent of frequency, at low frequency, $\gamma(\omega) \rightarrow \gamma_0$ ($\omega \rightarrow 0$), the enhancement near $T \geq 0$ takes on an universal dependence on temperature as

$$\Gamma(T) = \Gamma(T=0) \exp cT^2/k \quad (3)$$

with a coefficient c depending on the damping coefficient γ_0 and the form of the metastable potential. For an undamped system, $\gamma(\omega) = 0$, the thermal enhancement is exponentially small. Obviously, a precise measurement of the temperature dependence near $T = 0$ would render a crucial test of currently discussed theories modeling the thermal heat bath interaction.

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