Optical instabilities

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OPTICAL INSTABILITIES DRIVEN BY COLORED NOISE

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Over the last years, there has been a steadily growing interest in nonlinear systems in which the nonlinear coupling can induce phenomena such as a Hopf bifurcation, bistable behavior or chaos[1]. Here our focus will be on nonlinear, noisy optical systems which either undergo a continuous Hopf bifurcation or which exhibit bistability. The control paramerters may also be subject to fluctuations. Using the standard assumption that those fluctuations evolve on an entirely different time scale, one usually approximates the noise by white (Gaussian) fluctuations. Typical examples are the treatments of the single mode laser[2] or a dye laser at threshold [3] . Above threshold, the whole matter can complicate considerably. In particular, it has been realized recently that finite noise correlation effects, e.g. in pump laser fluctuations [4] or in noisy external driving fields can play a crucial role. Because optical systems which do exhibit bistability [5,6] are particularly sensitive to the details of the noise properties , as is manifested by the exponential suppression of probability of the locally unstable state , they are ideal to put to a critical test the various different theories and approximation schemes.

First, let us briefly summarize the present theoretical status for Gaussian white noise (Fokker-Planck Eqs.) and white shot noise (Master Eqs.). Some interesting recent results are:

(i) For Master equations with nearest neighbor transitions (one-photon transitions) and Master equations with both one-step and two-step transitions (i.e. two-photon transitions) it is possible to derive exact analytical results for the stationary probability and mean first passage times[7].

(ii) There exists a novel Fokker-Planck approximation to the <u>long-time</u> dynamics of Master equations yielding identical stationary probabilities and escape rates [8].

(iii) A truncated Kramers-Moyal approximation (at 2-nd order) to a given Master equation <u>overestimates</u> exponentially the escape rates [8,9].

In presence of colored noise, the system is generally no longer readily tractable [10]. As the archetype of a bistable system which is driven by correlated Gaussian noise, we consider with $a > 0$, $b > 0$, the flow :

$$\dot{x} = ax - bx^3 + \xi(t) \quad , \quad <\xi(t)\xi(s)> = D/\tau \exp-|t-s|/\tau \qquad (1)$$

Recently, we have investigated (1) more closely [10]. We have found that the usual approximation schemes which expand around the Markovian theory (for the details see in Ref. 10a) converge non-uniformly and give in this case exponentially wrong results for the stationary probability $\overline{p}(x;\tau)$ at small but finite noise correlation time τ (Ref. 10a). A novel nonlinear approximation scheme has been proposed in Ref. 10b, which gives results which are in close agreement with measured values of sojourn times or escape rates (for figures see in Ref. 10a and 10b). With weak noise, the colored noise activation rates over the barrier of eq. (1) are estimated as :

$$\Gamma(\tau) = \frac{a}{\pi\sqrt{2}} \exp - \Delta\phi(\tau,D)/D , \qquad (2a)$$

where ($<x^2>$: D-dependent and τ-dependent stationary 2-nd mean)

$$\Delta\phi(\tau,D) = \frac{a^2}{4b} \{ 1 + \tau(3b<x^2> - a) \} > \Delta\phi(\tau=0,D) \qquad (2b)$$

Thus, the rate $\Gamma(\tau)$ is predicted to undergo an exponential decrease with increasing noise correlation time τ. Recent experiments for the sojourn time (inverse of the rate) confirm this feature (see Fig. 1).

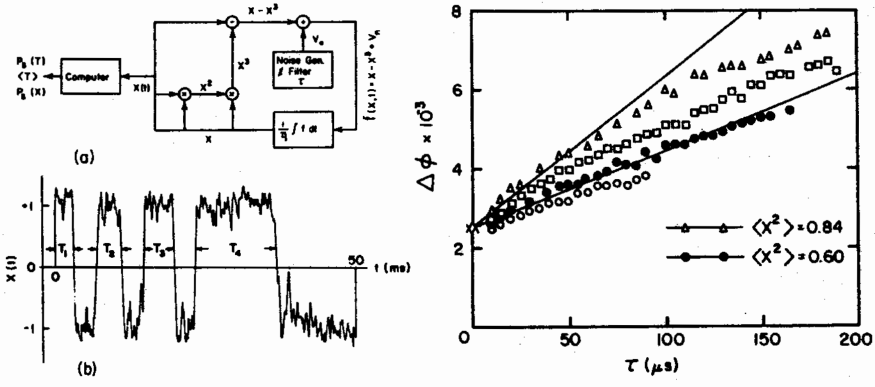

Fig. 1: Experiment (a), typical realization (b) and Arrhenius factor $\Delta\phi$ for the system in eq. (1) ($a= 10^4 sec^{-1}$, $b = 10^4 sec^{-1}V^{-2}$), taken from Ref. 10b. triangles: D=0.212;squares: D=0.153;full circles: D=0.114; open circles: D=0.083 .

Likewise,we considered the model by Graham and Schenzle[11] for the complex-valued transmitted field, $E = x + iy$, in optical , dispersive bistability with cavity detuning γ and atomic detuning δ. The results are :

(i) For white Gaussian noise and $\gamma = \delta$ (no detailed balance!) we obtain analytic results for the switching rates , both for the Arrhenius factor and the prefactor[12].For $\gamma=\delta=0$ (pure absorptive case) one recovers the Kramers rate (in dimension d = 2) .

(ii) Next we represent the fluctuations of the external, driving laser field more realistically with correlated Gaussian noise. Then we can model the long-time dynamics with a novel Fokker-Planck Eq.[10b] which contains τ-induced cross-diffusion terms and nonequal diagonal diffusion strengths; i.e. $D_{xx}(\tau) \neq D_{yy}(\tau)$. For $\gamma = \delta > 0$, we again succed in obtaining analytic results for the colored noise Arrhenius factor and the colored noise prefactor. The rates are again exponentially suppressed over the white Gaussian noise results of Ref. 12.

(iii) For pure absorptive bistability ($\gamma=\delta=0$) we find that the transition rates for the transmitted amplitude $A = (x^2 + y^2)^{\frac{1}{2}}$, undergo an exponential suppression like in (2) with an Arrhenius factor ($\Delta\phi_{WG}$: White Gaussian noise result derived in Ref. 12)

$$\Delta\phi(\tau) = \Delta\phi_{WG}\{1 + \tau(1 + \Omega^2[(1+\langle y^2\rangle - \langle x^2\rangle)/(1+\langle x^2\rangle + \langle y^2\rangle)^2])\} \tag{3}$$

with Ω^2 denoting the cooperativity parameter [11,12].

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