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The study of dynamic singularities of seismic signals by the generalized Langevin equation

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ABSTRACT

Analytically and quantitatively we reveal that the generalized Langevin equation (GLE), based on a memory function approach, in which memory functions and information measures of statistical memory play a fundamental role in determining the thin details of the stochastic behavior of seismic systems, naturally leads to a description of seismic phenomena in terms of strong and weak memory. Due to a discreteness of seismic signals we use a finite–discrete form of the GLE. Here we studied some cases of seismic activities of Earth ground motion in Turkey with consideration of the complexity, nonergodicity and fractality of seismic signals.

1. Introduction

Specific stochastic dynamics occur in a large variety of systems, such as supercooled liquids, seismic systems, the human brain, finance, meteorology and granular matter. These systems are characterized by an extremely rapid increase or a slowdown of relaxation times and by a non-exponential decay of time-dependent correlation functions [1,2].

The canonical theoretical framework for stochastic dynamics of complex systems is the time-dependent generalized Langevin equation (GLE) [3–7,14,15]. It successfully describes the phenomenon of statistical memory, whereby the relaxation time for order parameter fluctuations scales as a power of the correlation length. An obvious question to ask would be whether this framework can be adapted to describe seismic phenomena. Analytically and quantitatively we show that the GLE, based on a memory function approach, where the memory functions and information measures of statistical memory play a fundamental role in determining the thin details of the stochastic behavior of seismic systems, naturally leads to a description of seismic phenomena in a terms of a strong and weak memory. Due the discreteness of a seismic signals we use a finite–discrete form of the GLE. Here we study some cases of seismic activities of Earth ground motion in recent years in Turkey with consideration of the complexity, irregularity and metastability of seismic signals.

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2. Some extraction from the theory of discrete stochastic processes

The GLE analytical model was originally proposed for displaying the stochastic behavior of signals in complex systems [3–7].

Here we consider the data of seismic signal recording as a time series ξ:

$$\xi = \{\xi_0, \xi_1, \xi_2, \ldots, \xi_{N-1}\} = \{\xi(0), \xi(\tau), \xi(2\tau), \ldots, \xi([N-1]\tau)\}. \tag{1}$$

Here τ is the discretization time of seismic signals, and N is the total number of signals. The set of fluctuations $\delta\xi$ is an initial dynamic variable W_0:

$$W_0 = \{\delta\xi_0, \delta\xi_1, \delta\xi_2, \ldots, \delta\xi_{N-1}\}, \quad \delta\xi_j = \xi_j - \langle\xi\rangle, \quad \langle\xi\rangle = \frac{1}{N}\sum_{j=0}^{N-1}\xi_j. \tag{2}$$

The Gram–Schmidt orthogonalization procedure

$$\langle W_n, W_m\rangle = \delta_{n,m}\langle|W_n|^2\rangle \tag{3}$$

leads to the set of the following orthogonal dynamic variables:

$$\begin{cases} W_0 = \delta\xi, \\ W_1 = \mathcal{L}W_0 = \dfrac{\mathrm{d}}{\mathrm{d}t}\delta\xi, \\ W_2 = \mathcal{L}W_1 - \Lambda_1 W_0, \\ \ldots, \\ W_{n+1} = \mathcal{L}W_n - \Lambda_n W_{n-1}, \quad n \geq 1, \end{cases} \tag{4}$$

where $\mathcal{L} = (\Delta - 1)/\tau$ is the Liouville quasioperator and Λ_n is the relaxation parameter of the nth order (where Δ is the shift operator $\Delta x_j = x_{j+1}$ and τ is the discretization time).

Within the framework of statistical theory and Zwanzig–Mori's theoretical–functional procedure of projection operators, one can obtain the following recurrent relation as a finite-difference kinetic equation:

$$\Delta M_n(t) = \tau\lambda_{n+1}M_n(t) - \tau^2\Lambda_{n+1}\sum_{j=0}^{m-1}M_{n+1}(t-j\tau)M_n(j\tau), \quad n = 0, 1, 2, \ldots. \tag{5}$$

Here we introduce a Liouville quasioperator eigenvalue λ_{n+1}, a relaxation parameter Λ_{n+1} and a memory function $M_n(t)$ of the nth order, respectively:

$$\lambda_n = \frac{\langle W_{n-1}\mathcal{L}W_{n-1}\rangle}{\langle|W_{n-1}|^2\rangle}, \qquad \Lambda_n = \frac{\langle|W_n|^2\rangle}{\langle|W_{n-1}|^2\rangle}, \qquad M_n(t) = \frac{\langle W_n(t)W_n\rangle}{\langle|W_n|^2\rangle}. \tag{6}$$

For analysis of the relaxation time scales of the underlying processes we use the frequency-dependent statistical non-Markovity parameter $\varepsilon_n(\omega)$:

$$\varepsilon_n(\omega) = \left\{\frac{\mu_{n-1}(\omega)}{\mu_n(\omega)}\right\}^{1/2}. \tag{7}$$

Here $\mu_n(\omega)$ is a frequency power spectrum for the memory function of the nth order:

$$\mu_n(\omega) = \left|\tau\sum_{j=0}^{N-1}M_n(j\tau)\cos(j\tau\omega)\right|^2. \tag{8}$$

Using Eqs. (1)–(8) we can study all specific singularities of the statistical memory effects in an underlying system. The non-Markovity parameter and its statistical spectrum were introduced by Yulmetyev et al. in Ref. [8]. It is worth mentioning that the non-Markovian character of seismic data was discussed by Varotsos et al. [9]. One of the first proofs of non-Markovity of empirical random processes was given in Refs. [10]. Stochastic origins of the long-range correlations of ionic current fluctuations in membrane channels with non-Markovian behavior were studied in Ref. [11].

3. An analysis of results

Fig. 1 presents the initial time series of seismic signals for seven seismic origins: *grsn*, *kelt*, *mack*, *sgkt*, *uldt*, *seyt*, and *gdz*. The discretization time is $\tau = 0.02$ s. We can see that all the time series have distinctive features.

Fig. 2 demonstrates the frequency dependence of the first point of the non-Markovity parameter $\varepsilon_1(\omega)$ for seven seismic origins from Turkey: *grsn*, *kelt*, *mack*, *sgkt*, *uldt*, *seyt*, and *gdz*. Since the nature of each seismic source is unknown to us, it would be interesting to establish its character. It seems possible that the signals can be distributed into three groups: **group A** (*kelt*, *gdz*), **group B** (*grsn*, *sgkt*, *uldt*, *seyt*), and **group C** (*mack*).

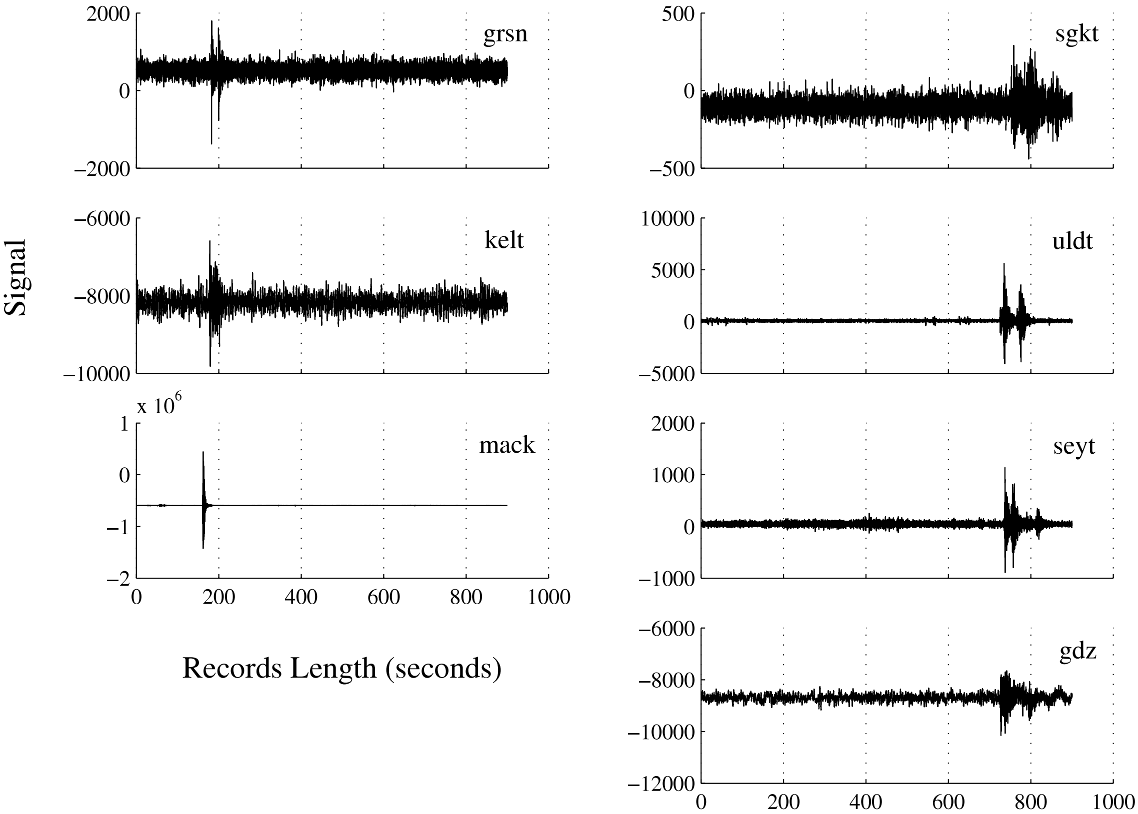

Fig. 1. An initial row of data of seismic signals from seven areas of seismic signals: *grsn*, *kelt*, *mack*, *sgkt*, *uldt*, *seyt*, and *gdz*. The discretization time is $\tau = 0.02$ s.

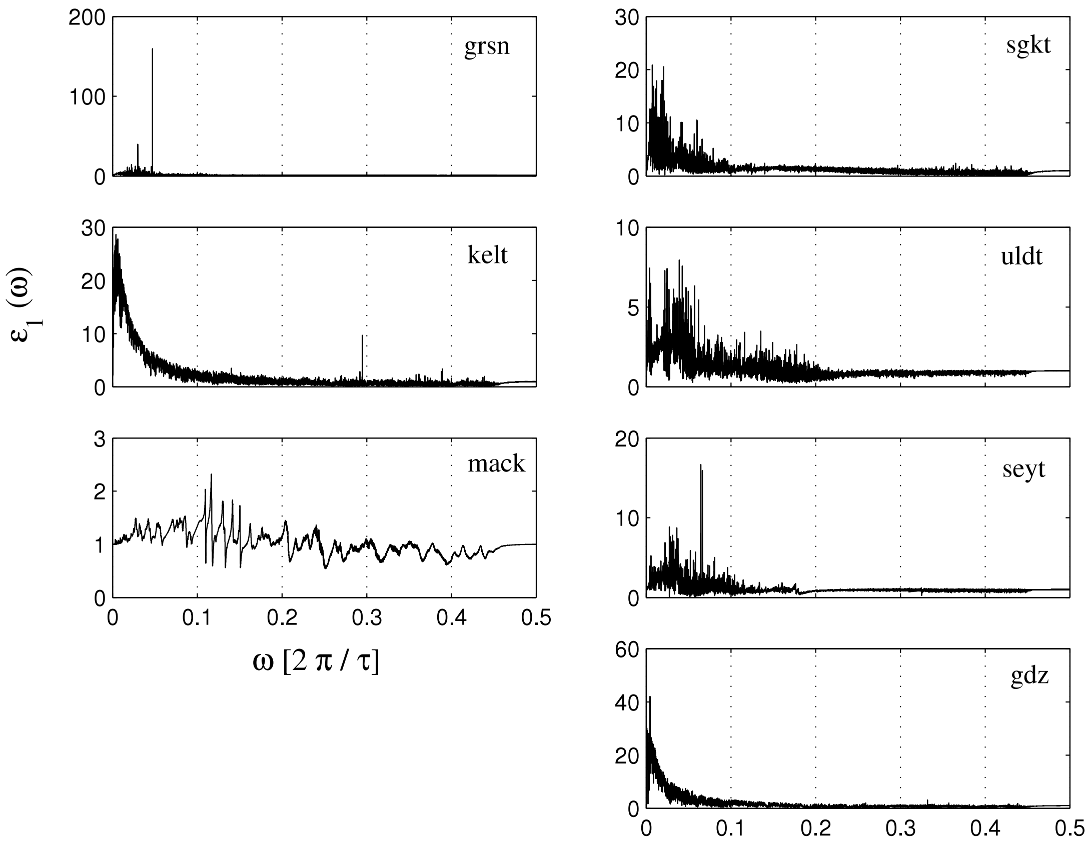

Fig. 2. The frequency dependence of the first point of the non-Markovity parameter $\varepsilon_1(\omega)$ for each seismic origin: *grsn*, *kelt*, *mack*, *sgkt*, *uldt*, *seyt*, and *gdz*.

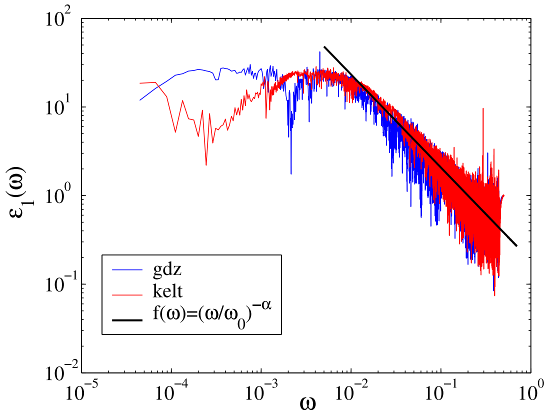

Fig. 3. (Color online) The frequency dependence of $\varepsilon_1(\omega)$ in double log–log scale for two seismic origins: *gdz* and *kelt*. The discretization time is $\tau = 0.02$ s. The general power dependence of $\varepsilon_1(\omega) = (\omega/\omega_0)^{-\alpha}$ is shown by a continuous line with parameters $\omega_0 = 0.2, \alpha = 1.05$.

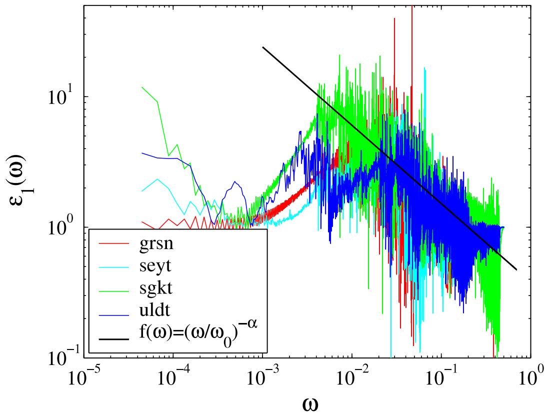

Fig. 4. (Color online) The frequency dependence of $\varepsilon_1(\omega)$ in double log–log scale for four seismic origins: *grsn*, *sgkt*, *uldt*, and *seyt*. The discretization time is $\tau = 0.02$ s. The general power dependence of $\varepsilon_1(\omega) = (\omega/\omega_0)^{-\alpha}$ is shown by a continuous line with parameters $\omega_0 = 0.2, \alpha = 0.6$.

Signals of group A are characterized by the more regular structure and smooth decay of the function $\varepsilon_1(\omega)$.

There is a frequency dependence of the non-Markovity parameter for seismic origins *gdz* and *kelt*, presented in Fig. 3. The general power dependence of $\varepsilon_1(\omega) = (\omega/\omega_0)^{-\alpha}$ is shown by a continuous line with parameters $\omega_0 = 0.2, \alpha = 1.05$.

Signals of group B are characterized by the irregular frequency structure and by the frequency bursts on the distinct frequencies. The spectra have a noisy character.

There is a frequency dependence of $\varepsilon_1(\omega)$ for the four seismic origins *grsn*, *sgkt*, *uldt*, and *seyt* in double log–log scale, presented in Fig. 4. The discretization time is $\tau = 0.02$ s. The general power dependence of $\varepsilon_1(\omega) = (\omega/\omega_0)^{-\alpha}$ is shown by a continuous line with parameters $\omega_0 = 0.2, \alpha = 0.6$.

Signals of group C cannot be attributed to one of the above groups. The parameter $\varepsilon_1(\omega)$ fluctuates strongly between 1 and 10 in the full frequency scale. That testifies the existence of strong memory effects in the long-range time correlation. A possible origin of the similar signals is due to a strong earthquake. More careful and detailed analysis of the signal structure on the various time scales and relaxation levels (with the taking into account of the long-range correlation, memory effects, nonergodicity and metastability of the underlying system) is required.

Fig. 5 displays the frequency dependence of $\varepsilon_1(\omega)$ in double log–log scale for *mack*. The discretization time is $\tau = 0.02$ s. The general power dependence of $\varepsilon_1(\omega) = (\omega/\omega_0)^{-\alpha}$ is shown by a continuous line with parameters $\omega_0 = 0.2, \alpha = 0.4$.

The analysis of all spectra shows that all the signals can be classified into three different groups in the order of the breaking of fractal behavior of the high-frequency dependence of $\varepsilon_1(\omega)$. The signals for **group A** can be characterized by stronger fractality with the exponent $\alpha = 1.05$. A linear trend with small fluctuation has been simultaneously observed in the spectrum. We can see range of the diversity $10 < \varepsilon_1(\omega) < 100$. The signals for **group B** are characterized by the breaking of fractality with the exponent $\alpha = 0.6$. A nonlinear oscillating trend with big fluctuation has been observed here. The spectra of signals for **group C** are characterized by a weak fractality with the exponent $\alpha = 0.4$ and $\varepsilon_1(\omega) \sim 1$.

The auto-correlation functions (ACFs)

$$C(t) = \frac{\langle \xi(0)\xi(t)\rangle}{\langle |\xi(0)|^2\rangle} \tag{9}$$

for the signals of different groups have been presented in Fig. 6.

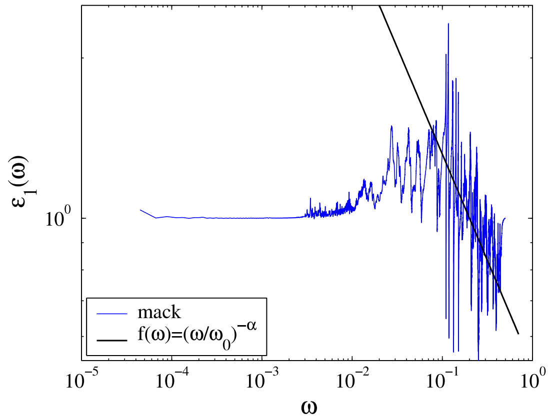

Fig. 5. (Color online) The frequency dependence of non-Markovity parameter $\varepsilon_1(\omega)$ for seismic origin *mack* in the double log–log scale. The discretization time is $\tau = 0.02$ s. The general power dependence of $\varepsilon_0(\omega) = (\omega/\omega_0)^{-\alpha}$ is shown by a continuous line with parameters $\omega_0 = 0.2, \alpha = 0.4$.

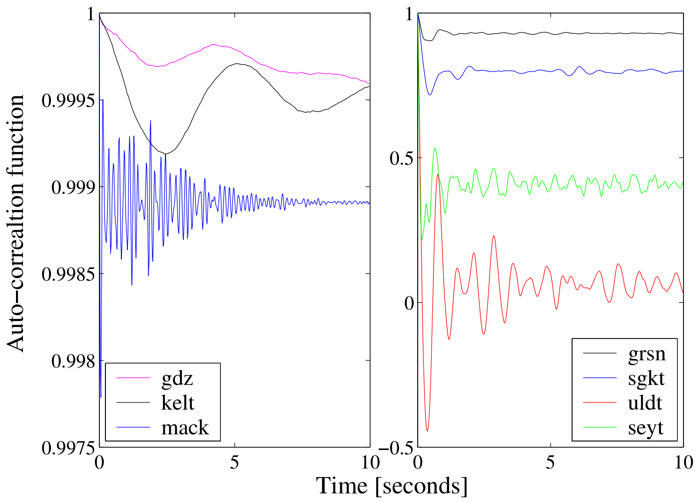

Fig. 6. (Color online) The auto-correlation function of the signals.

The left panel includes the ACFs for the signals of **group A** and **group C**, (*mack*) and (*kelt*, *gdz*), while the right panel contains the ACFs for the signals of **group B** (*grsn*, *sgkt*, *uldt*, *seyt*). The brackets here note averaging in time iterations. It is seen that auto-correlation of the signals has a pronounced nonergodic character (undamped behavior of the time correlation function at time $t \to \infty$):

$$\lim_{t\to\infty} C(t) \neq 0. \tag{10}$$

According to recent works on the ergodic hypothesis, Eqs. (9) and (10) would imply violation of ergodicity. Net results [12] suggest the breaking of ergodicity for a class of generalized, Brownian motion, obeying a non-Markovian dynamics being driven by a generalized Langevin equation. This very feature originates from vanishing of the effective friction. Khinchin's theorem of ergodicity is examined [13] by means of linear response theory. The resulting ergodic condition shows that, contrary to the theorem, irreversibility is not a sufficient condition for ergodicity.

Similar behavior of the time correlation functions is characteristic for the supercooled and glass states of condensed matter [14,15]. A higher level of nonergodicity corresponds with the signals of **group A** and **group C**, whereas the signals of **group B** are characterized by minor nonergodicity. The signals from the object *uldt* are rigorously ergodic. Therefore one can suppose that these signals cannot belong to the earthquake.

Table 1 presents a set of relaxation parameters λ_1, λ_2, λ_3, Λ_1, and Λ_2 for seismic signals from *grsn*, *kelt*, *mack*, *sgkt*, *uldt*, *seyt*, and *gdz*. The set of these parameters characterizes some peculiarities of relaxation processes on the low relaxation levels of seismic systems (1, 2 and 3). It has seen from the table that these parameters do not have a clear distinction vs. distinctions unlike those visible from the frequency dependence of $\varepsilon_1(\omega)$. Therefore thinner and more sensitive techniques are needed for studying of dynamic processes in examined signals.

Fig. 7 depicts the comparison of seismic data with results of computer simulation for the metallic glass $Al_{50}Cu_{50}$. At the top, the auto-correlation functions of the seismic events *grsn*, *sgkt*, *uldt*, and *seyt* are shown. At the bottom, auto-correlation functions for incoherent scattering of copper atoms are shown for comparison. The data are obtained with the help of statistical averaging on ensembles of statistical systems. Each curve was obtained by time averaging. The full sample consists

Table 1
The frequency relaxation parameters for seismic signals.

Object	λ_1	λ_2	λ_3	Λ_1	Λ_2
grsn	−0.0306	−0.2004	−0.8945	0.0497	0.0396
kelt	−0.0667	−1.1399	−1.0485	−0.0193	0.2954
mack	−0.3938	−0.5808	−1.0747	0.4374	−0.0342
sgkt	−0.0306	−0.7490	−1.0849	0.0156	−0.2967
uldt	−0.0349	−0.2605	−0.7609	0.0526	0.1243
seyt	−0.0745	−0.2434	−0.8427	0.1173	0.0590
gdz	−0.0646	−0.8836	−0.9650	0.0155	0.2586

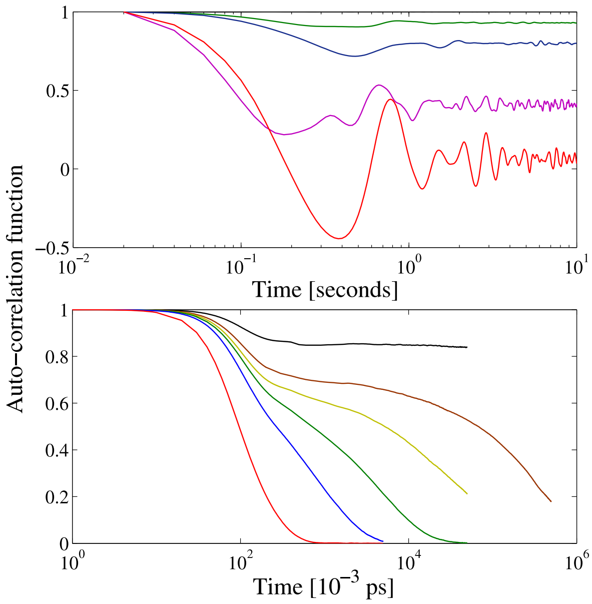

Fig. 7. (Color online) Top: auto-correlation functions of the seismic events *grsn*, *sgkt*, *uldt*, and *seyt* in time log-scale. Bottom: temperature dependence of the incoherent scattering function $F_s(k, t)$ for the Cu component in the metallic glass system $Al_{50}Cu_{50}$ alloy for the wave vector $k = 3.05$ Å^{-1} at the temperature $T = 2000$, 1000, 600, 500, 400, and 200 K (from the bottom up).

of 45 000 points. A single calculation was carried out for each separate time window of size 500 points with a time step of 0.02 s. Further, this window was displaced one step to the right up to the end of time sampling.

On comparison of the data for seismic phenomena with the results of computer simulations for the glassy system it is seen that in the behaviour of the correlation functions for an earthquake's nonergodic effects, characteristics of glass-like behaviour of dense systems are distinctly observed.

To illustrate the general picture of nonergodic singularities in chaotic seismic systems, the parameter of nonergodicity $f = \lim_{t\to\infty} c(t)$ was calculated for each set of seismic events. The values obtained are 0.9995 (*gdz*), 0.9995 (*kelt*), 0.99885 (*mack*), 0.9310 (*grsn*), 0.7241 (*sgkt*), 0.3965 (*uldt*), and 0.0603 (*seyt*). The parameter of nonergodicity for one of the sets of seismic events appeared to be equal. The resulting data are evidence of strong singularity of seismic phenomena in five sources: *gdz*, *kelt*, *mack*, *grsn*, and *sgkt*; they show moderate nonergodicity for one source (*uldt*) and weak singularity for another source (*seyt*). Taken together, the data reflect well the wide variety of effects of nonergodicity in the seismic phenomena. A similar variety of nonergodicity effects can be very useful and extremely effective for the classification of the wide variety of seismic phenomena. Note that the notions of fractality and non-Markovity have been quite extensively explored in the past; see, for example, Refs. [16–19].

4. Summary

In this work we have presented the results of statistical analysis of seismic signals in Turkey for seven objects (*grsn*, *kelt*, *mack*, *sgkt*, *uldt*, *seyt*, and *gdz*). Our study was made in the context of statistical theory on discrete non-Markovian processes, which is based on the generalized Langevin equation (GLE). It allows us to take into account the effects of the statistical memory, metastability and space-time nonlocality. We have shown with the theory that all considered signals can be divided into three groups in order of breaking of the fractal behavior in the high frequency zone of the spectrum of the non-Markovity parameter. Signals from group A (*kelt* and *gdz*) can be characterized by pronounced fractality, signals from group B (*grsn*, *sgkt*, *uldt*, and *seyt*) can be characterized by moderate fractality and signals from group C (*mack*) correspond to weak fractality and powerful non-Markovian processes. From the analysis of the time correlation function we can confidently certify the hypothesis of Abe [1] of the nonergodic "glass-like nature" of seismic signals for Earth's activity. On the other hand, the analysis also reveals a wide variety of metastability in seismic phenomena.

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