Influence of thermal fluctuations on macroscopic quantum tunnelling

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The quantum decay of a system which interacts with an environment at a temperature $T$ is concomitant, it is found that heat enhances the tunnelling probability at $T = 0$ by a factor exp$[\Delta G(T)/kT]$, where $\Delta G$ is the change in the energy, $k$ is the frequency of small oscillations about the metastable state, and $kT$ is the thermodynamically small, $\Delta G = E_1 - E_2 = \frac{1}{2} kT \ln \frac{n}{m}$. The exponent $n$ is a distinctive feature of the dissipative mechanism and is $\frac{1}{2}$ in the case of classical dissipation.

There has been recent experimental and theoretical work on the question of whether a macroscopic system cannot be shut out of a metastable state [1]. In macroscopic systems, the tunnelling probability is strongly influenced by the interaction with the environment. Callen and Langer [2] have shown that damping suppresses the tunnelling rate at $T = 0$. Their results are in qualitative agreement with recent experiments on Josephson systems [3]. A more detailed comparison should have regard to the temperature dependence of the decay rate. We have found that the thermal enhancement of the decay rate at low temperatures sensitively depends on the details of the coupling to the environment [4].

Macroscopic quantum tunnelling in Josephson systems is equivalent to the escape problem of a particle of mass $m$ moving in a potential $V(\phi)$ with a metastable barrier; we choose the axes so that this line at $\phi = 0, \phi = 0$. To turn our of the metastable state, the particle has to penetrate a potential barrier of which $\phi = 0, V(\phi) = 0$: before reaching the region of lower potential. The system is assumed to be clamped statically to its environment and at low temperatures can be replaced by a set of harmonic oscillators [1]. Feynman's method [5] for integrating over the environmental modes leaves us with a one-dimensional problem, the partition function of which is given in terms of an effective action [1,6]

$$s[q(\tau)] = \int \frac{d[\delta q(\tau)]}{\sqrt{2\pi}} e^{-\int d\tau \frac{1}{2} \delta q(\tau)^2 - V(q(\tau))}$$

where $\delta q(\tau)$ is a path in "imaginary time" $\tau$ with period $0, T(\phi = 0) = 0$, where $0, 0$ is a path. The final term in (1) introduces dissipation. $T(\phi)$ is a periodic potential given by [1,6]

$$T(\phi) = E_0 s[\phi(\omega)] e^{\exp[i\omega \phi]}$$

where $\omega = 2\pi k T m$, and

$$K[T] = \frac{1}{T} \int \frac{d\omega}{2\pi} \frac{1}{\omega} \frac{\delta}{\delta \phi(\omega)}$$

$\phi(\omega)$ and the second term over the integrand is run from $-\infty$ to $+\infty$. The spectral density $K[T]$ is proportional to the density of environmental modes at frequency $\omega$ and proportional to the square of the strength of their coupling to the tunnelling system [7].

To determine the tunnelling probability we employ the "bounce" technique originally used by Langevin [7] and popularized by Coleman [12]. The "bounce" trajectory is a saddlepoint of the action (1) which starts from the metastable region at $\phi = 0$, traverses the potential barrier (which is a valley in imaginary time) and returns to the metastable region at $\phi = 0$. As long as $\frac{1}{2} T$ is small compared with $\frac{1}{2} M$,
where \( n \) is the frequency of small oscillations about the metastable equilibrium, the RR-correlation factor, and the tunnelling probability may be written
\[
\gamma = \exp(-S_0/M)
\]
where \( S_0 \) is the action evaluated along the 'bounce' trajectory, and \( M \) is a prefactor which can be calculated from the small fluctuations about this path. Since the temperature dependence of the prefactor \( M \) is negligible, we find from (2) that in a good approximation the tunnelling probability \( \gamma(T) \) at low temperatures \( T \) may be written
\[
\gamma(T) = \exp(-S_0(0)/T)
\]
where \( S_0(0) \) is the tunnelling probability at \( T = 0 \) (including the influence of dissipation) and \( S_0(0) = S_0 - S_0(T) \). We have evaluated (3) for various potentials. Our principal findings for the behaviour at low temperatures of the thermal enhancement of the tunnelling probability are as follows.

(1) The thermal enhancement factor may be written
\[
\gamma(T)/\gamma_0 = \exp(S_0(0)/kT)
\]
where \( \gamma(T) \) is a dimensionless quantity characterizing the influence of thermal fluctuations. 

(2) For an undamped system, \( \gamma(T) \) is exponentially small, \( \gamma(T) \approx \exp(-S_0/2kT) \), whereas it is a non-negligible factor which depends on the potential. This is in agreement with results of Eliezer (8) and of Weiss and Haufler (10) who have obtained it under different lines.

(3) For a system with linear frequency-independent damping whose classical equation of motion is \( \ddot{\chi} + \nu \dot{\chi} + \chi = 0 \), the spectral density \( j(\omega) \) must have the form \( j(\omega) \propto \omega \). Then \( \gamma(T) \) increases quadratically with temperature, \( \gamma(T) \approx (S_0(0)/T)^2 \omega_0 \), where \( \omega_0 \) is a function of the dimensionless damping parameter \( \nu = 2\pi/\omega_0 \). This function depends on the form of the potential. The low-temperature phase \( T \rightarrow 0 ^+ \) however holds for all metastable potentials and is a distinctive characteristic of quantum tunnelling.

(4) For tunnelling centers in solids, the spectral density \( j(\omega) \) is typically proportional to \( \omega \) for small frequencies \( \omega \). Then \( \gamma(T) \) grows as fourth power of temperature. 

(5) In the environmentless spectrum has a low-frequency cutoff, as in the same function model of Abeles, Eckert and Schön (12), the thermal enhancement is exponentially small in the environmentless spectrum.

Let us consider the practically important case of a cubic potential, \( \Phi(x) = M(x^2 - \nu x) \), more closely. From the equation of motion obeyed by the expectation value of the action \( S_0 \), one finds
\[
\langle \dot{x}^2 \rangle = \frac{\langle x^4 \rangle - 3 \langle x^2 \rangle^2}{\langle x^2 \rangle} M
\]
Further, by virtue of (5), the "bounce" action may be written
\[
S_0(T) = S_0(0) + \frac{kT}{2} \left( 3 - \frac{1}{2} \frac{\langle x^2 \rangle}{\langle x^4 \rangle} M \right)
\]
At zero temperatures, the signs in (5) and (6) are reversed by corresponding integrals. The Euler-Maclaurin expansions of these sums yield the asymptotic expansions of both the classical trajectory and the "bounce" action \( S_0(T) \) for large \( T < \nu \omega_0 \), where the coefficients are given in terms of the Fourier representation \( \Phi_n(x) = (2\pi)^{-1} \int \exp(i\omega_n x) \Phi(x) \) of the "bounced" at \( T = 0 \). Here, we restrict ourselves to the important case of frequency-independent damping, \( \nu \omega \approx \omega_0 \). Then one has
\[
\gamma(T) = \exp \left( \frac{S_0(0)}{kT} \right) \left( 1 + \frac{3}{2} \frac{\langle x^2 \rangle}{\langle x^4 \rangle} M \right)
\]
where terms of the fourth order in \( T \) have been disregarded. From (i) we obtain a thermal enhancement factor of the form \( \gamma(T) \) where
\[
\gamma(0) = \exp(S_0(0)/kT)
\]
Here we have introduced a characteristic frequency \( T = \nu \omega_0 \) associated with the "bounce length" in \( \nu \omega_0 \). The frequency "bounce" frequency is known explicitly for very weak and very strong damping (1). This yields \( \nu = \nu_0/T \) for small and \( \nu = \omega_0 \) for large \( T \).

These results may be applied to the physically interesting problem of quantum tunnelling in \( SO(4) \) or current-biased Josephson junctions (1) in the case of AMF perturbing. In (1) it would indicate that these systems are adequately described by the RSJ model.

REFERENCES