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INFLUENCE OF THERMAL FLUCTUATIONS ON MACROSCOPIC QUANTUM TUNNELLING

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The quantum decay of a system which interacts with an environment at temperature T is considered. It is found that heat enhances the tunnelling probability at T = 0 by a factor $\exp[A(T)M\omega_0 q_0^2/\hbar]$, where M is the mass of the system, ω_0 is the frequency of samll oscillations about the metastable state, and q_0 is the tunnelling distance. For an undamped system A(T) is exponentially small, $A(T) \propto \exp(-\hbar\omega_0/k_BT)$, whereas for a dissipative system A(T) grows algebraically with temperature, $A(T) \propto (k_BT/\hbar\omega_0)^n$. The exponent n is a distinctive feature of the dissipative mechanism and is 2 in the case of Ohmic dissipation.

There has been recent experimental and theoretical work on the question of whether a macroscopic system can be shown to tunnel out of a metastable state (1). In macroscopic systems, the tunnelling probability is strongly influenced by the interaction with the environment. Caldeira and Leggett (2) have shown that damping suppresses the tunnelling rate at T = 0. Their results are in qualitative agreement with recent experiments on Josephson systems (3). A more detailed comparison should have regard to the temperature dependance of the decay rate. We have found that the thermal enhancement of the decay rate at low temperatures sensitively depends on the details of the coupling to the environment (4).

Macroscopic quantum tunnelling in Josephson systems is equivalent to the escape problem of a particle of mass M moving in a potential V(q) with a metastable minimum; we choose the axes so that this lies at q = 0, V = 0. To tunnel out of the metastable state, the particle has to penetrate a potential barrier of width q_0 (that is, $V(q_0) = 0$) before reaching the region of lower potential. The system is assumed to be coupled linearly to its environment which at low temperatures can be replaced by a bath of harmonic oscillators (1). Feynman's method (5) of integrating away the environmental modes leaves a one-dimensional problem, the partition function of which is given in terms of an effective action (1,6)

$$S[q(\tau)] = \int_{-\Theta/2}^{\Theta/2} d\tau \left(\frac{1}{2}M\dot{q}^2 + V(q)\right) + \frac{1}{2}\int_{-\Theta/2}^{\Theta/2} d\tau \int_{-\Theta/2}^{\Theta/2} d\tau'\, k(\tau-\tau')q(\tau)q(\tau') \qquad (1)$$

where $q(\tau)$ is a path in "imaginary time" τ with period Θ, $q(\tau+\Theta) = q(\tau)$, where $\Theta = \hbar/k_BT$. The final term in (1) introduces dissipation. $k(\tau)$ is a Θ-periodic kernel given by (1,6)

$$k(\tau) = \Theta^{-1}\sum_m K(\nu_m)\exp(i\nu_m\tau)$$

where $\nu_m = 2\pi m/\Theta$, and

$$K(\nu) = \frac{1}{\pi}\int_0^\infty d\omega \frac{2\nu^2}{\nu^2+\omega^2}\frac{J(\omega)}{\omega}$$

Here and in the sequel sums over the index m run from $-\infty$ to $+\infty$. The spectral density $J(\omega)$ is proportional to the density of environmental modes at frequency ω and proportional to the square of the strength of their coupling to the tunnelling system (1).

To determine the tunnelling probability we employ the "bounce" technique originally used by Langer (7) and popularized by Coleman (8). The "bounce" trajectory is a saddlepoint of the action (1) which starts from the metastable region at $\tau = -\Theta/2$, traverses the potential barrier (which is a valley in imaginary time) and returns to the metastable region at $\tau = \Theta/2$. As long as k_BT is small compared with $\hbar\omega_0$, where ω_0 is the frequency of small oscillations about the metastable equilibrium, the WKB-approximation applies, and the tunnelling probability may be written

$$\Gamma = N \exp(-S_B/\hbar) \qquad (2)$$

where S_B is the action (1) evaluated along the "bounce" trajectory, and N is a prefactor which can be calculated from the small fluctuations about this path. Since the temperature dependence of the prefactor N is negligible, we find from (2) that to a good approximation the tunnelling probability $\Gamma(T)$ at low temperatures T may be written

$$\Gamma(T) = \Gamma_0 \exp[\Delta S_B(T)/\hbar] \qquad (3)$$

where Γ_0 is the tunnelling probability at T = 0 (including the influence of dissipation) and $\Delta S_B(T) = S_B(0)-S_B(T)$.
We have evaluated (3) for various potentials. Our principal findings for the behavior at low temperatures of the thermal enhancement of the tunnelling probability are as follows.
(i) The thermal enhancement factor may be written

$$\Gamma(T)/\Gamma_0 = \exp[A(T)M\omega_0 q_0^2/\hbar] \qquad (4)$$

where A(T) is a dimensionless quantity characterizing the influence of thermal fluctuations.
(ii) For an undamped system, A(T) is exponentially small, $A(T) = a \exp(-\hbar\omega_0/k_BT)$, where a is a numerical factor which depends on the potential. This is in agreement with results of Affleck (9) and of Weiss and Haeffner (10) which have been obtained on different lines.
(iii) For a system with linear frequency-independent damping whose classical equation of motion is $M\ddot{q} + \eta\dot{q} + \partial V/\partial q = 0$, the spectral density $J(\omega)$ must have the form $J(\omega) = \eta\omega$. Then A(T) increases quadratically with temperature, $A(T) = a(\alpha)(k_BT/\hbar\omega_0)^2$, where $a(\alpha)$ is a function of the dimensionless damping parameter $\alpha = \eta/2M\omega_0$. This function depends on the form of the potential. The low temperature power law $A \propto T^2$, however holds for all metastable potentials and is a distinctive feature of "Ohmic dissipation".
(iv) For tunnelling centers in solids, the spectral density $J(\omega)$ is typically proportional to ω^3 for small frequencies (11). Then A(T) grows as fourth power of temperature.
(v) If the environmental spectrum has a low-frequency cut-off, as in the oxide junction model of Ambegaokar, Eckern and Schön (12), the thermal enhancement is exponentially small like in undamped systems.
Let us consider the practically important case of a cubic potential, $V(q) = \frac{1}{2}M\omega_0^2q^2 - \frac{1}{3}Muq^3$, more closely. From the equation of motion obeyed by the extremal paths of the action (1), one finds

$$(\nu_m^2 + \omega_0^2 + \zeta_m)Q_m = (2\pi u/\Theta)\sum_n Q_{m+n}Q_n . \qquad (5)$$

Further, by virtue of (5), the "bounce" action may be written

$$S_B(T) = \frac{1}{3}\pi Mu(2\pi/\Theta)^2 \sum_{m,n} Q_m Q_n Q_{m+n} \qquad (6)$$

At zero temperature, the sums in (5) and (6) are replaced by corresponding integrals. The Euler-Maclaurin expansions of these sums yields the asymptotic expansions of both the form of the saddlepoint trajectory and the "bounce" action $S_B(T)$ for large $\Theta = \hbar/k_BT$, where the coefficients are given in terms of the Fourier representation $Q^0(\nu) = (1/2\pi)\int d\tau \exp(-i\nu\tau)q(\tau)$ of the "bounce" at T = 0. Here, we restrict ourselves to the important case of frequency-independent damping, $J(\omega) = \eta\omega$. We then have

$$S_B(T) = S_B(0) - \frac{2}{3}\pi^3\eta[Q^0(0)k_BT/\hbar]^2 \qquad (7)$$

where terms of the fourth order in T have been disregarded. From (7) we obtain a thermal enhancement factor of the form (4) where

$$A(T) = \frac{1}{3}\pi\alpha(k_BT/\hbar\omega_B)^2 \qquad (8)$$

Here we have introduced a characteristic frequency $\omega_B = \tau_B^{-1}$ associated with the "bounce length" $\tau_B = q_0^{-1}\int d\tau\, q(\tau)$.The zero temperature "bounce" trajectory is known explicitely for very weak and for very strong damping (1). This yields $\omega_B = \omega_0/4$ for $\alpha \ll 1$ and $\omega_B = 3\omega_0/8\pi\alpha$ for $\alpha \gg 1$.
These results may be applied to the physically interesting problem of quantum tunnelling in SQUID s or current-biassed Josephson junctions(1). An increase of A(T) proportional to T^2 would indicate that these systems are adequately described by the RSJ model.

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