Electron bunching in stacks of coupled quantum dots

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We study the transport properties of two double quantum dots in a parallel arrangement at temperatures of a few kelvin. Thereby, we show that decoherence entailed by the substrate phonons affects the shot noise. For asymmetric coupling between the dots and the respective lead, the current noise is sub-Poissonian for resonant tunneling, but super-Poissonian in the vicinity of the resonances. Our results indicate that the interaction between different channels together with phonon emission and absorption are responsible for the shot noise characteristics. The observed asymmetry of the peaks at low temperatures stems from spontaneous emission.

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I. INTRODUCTION

The aim of controlling and manipulating nanoscale devices requires good knowledge of the processes involved in the electronic transport through open quantum systems. The increasing success in accessing single electron states in semiconductor quantum dots and the unavoidable presence of lattice vibrations in such devices obliges one to consider dissipation caused by electron-phonon interaction.[1,2] The study of the electronic current fluctuations provides further information about the system.[3,4] E.g., from the investigation of shot noise—a consequence of the charge discreteness—we know about deviations from Poissonian statistics indicating correlations between tunneling events.

A particular example for Poissonian statistics is the electron transport through a point contact, for which all tunneling events are statistically independent. For resonant tunneling through single quantum dots, this is no longer the case: As long as an electron populates the quantum dot, no further electron can enter and, consequently, tunneling events are antibunched. However, when several of such transport channels conduct in parallel and are coupled capacitively, the current noise becomes super-Poissonian, as has been demonstrated experimentally.[5,6] This means that electrons tend to be transferred in bunches, which at first sight is counterintuitive if one thinks in terms of the Pauli exclusion principle. The phenomenon can be understood in terms of Coulomb interactions between electrons in different channels, so that an electron in one channel suppresses the transport through the other,[7–9], and one observes *dynamical channel blockade* (DCB). Consequently the electron transport through one dot occurs in bunches during lapses of time when the other dots are empty.

In a recent experiment[10] with transport channels that consist of double quantum dots (see Fig. 1), intriguing noise properties have been observed: By slightly modifying the source-drain voltage, the levels of a double quantum dot can be tuned across a resonance which yields a current peak at whose center, the noise is sub-Poissonian. In the vicinity of such resonances, by contrast, the noise is super-Poissonian such that the Fano factor assumes values up to 1.5. This structure becomes washed out with increasing temperature, indicating the suspension of DCB by the interaction with substrate phonons. In this work, we show that a model with a single transport channel qualitatively reproduces this behavior. For a quantitative agreement with the experimentally observed Fano factor and temperature dependence, however, the capacitive coupling to a second, almost identical channel is found to be essential.

II. MODEL

We start out by modeling a single transport channel of the setup sketched in Fig. 1. The double quantum dot coupled to fermionic leads and substrate phonons is described by the Hamiltonian[11,12]

$$H = H_0 + H_{\text{leads}} + H_T + H_{e\text{-ph}} + H_{\text{ph}}, \tag{1}$$

where $H_0 = \Sigma_{l=L,R}\varepsilon_l n_l + U n_L n_R - \Omega(c_L^\dagger c_R + c_R^\dagger c_L)/2$ describes the coherent dynamics inside the double dot and n_l denotes the population of dot $l=L,R$. Henceforth, we will assume that the Coulomb repulsion U is so strong that only the zero-electron state $|0\rangle$ and the states with one electron in the left or the right dot, $|L\rangle$ and $|R\rangle$, play a role. The leads and the phonons are described by $H_{\text{leads}} = \Sigma_{l,k}\varepsilon_{lk} n_{lk}$ and $H_{\text{ph}} = \Sigma_\nu \hbar\omega_\nu a_\nu^\dagger a_\nu$, respectively, where n_{lk} is the electron number in state k in lead l and a_ν is the annihilation operator of the νth phonon mode. The interaction with the double dot is given by the tunneling Hamiltonian

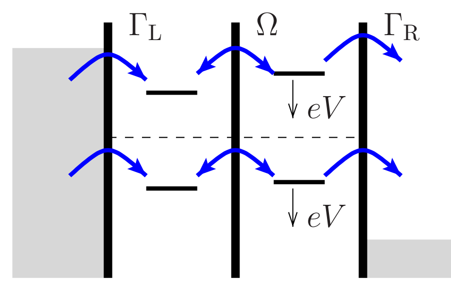

FIG. 1. (Color online) Sketch of the transport through two parallel double quantum dots measured in Ref. 10. Both transport channels are capacitively coupled. The source-drain voltage shifts the relative position of the levels by eV; albeit it is so large that all levels lie within the voltage window.

$H_T=\Sigma_{l,k}(\gamma_l d_{lk}^{\dagger}c_l+\text{H.c.})$ and the electron-phonon coupling[2] $H_{e\text{-ph}}=\Sigma_\nu(n_L-n_R)\lambda_\nu(a_\nu^{\dagger}+a_\nu)$. By tracing out the leads and the bath within a Born-Markov approximation, we obtain for the reduced density matrix the equation of motion

$$\dot{\rho}=\mathcal{L}\rho=(\mathcal{L}_0+\mathcal{L}_T+\mathcal{L}_{e\text{-ph}})\rho. \tag{2}$$

Introducing for the density matrix the vector notation $\rho=(\rho_{00},\rho_{LL},\rho_{LR},\rho_{RL},\rho_{RR})^T$, the Liouvillian reads

$$\mathcal{L}=\frac{1}{\hbar}\begin{pmatrix} -\Gamma_L & 0 & 0 & 0 & \Gamma_R \\ \Gamma_L & 0 & -\frac{i}{2}\Omega & \frac{i}{2}\Omega & 0 \\ 0 & -\frac{i}{2}\Omega+A_+ & i\delta-B & 0 & \frac{i}{2}\Omega-A_- \\ 0 & \frac{i}{2}\Omega+A_+ & 0 & -i\delta-B & -\frac{i}{2}\Omega-A_- \\ 0 & 0 & \frac{i}{2}\Omega & -\frac{i}{2}\Omega & -\Gamma_R \end{pmatrix}, \tag{3}$$

where the detuning $\delta=\varepsilon_R-\varepsilon_L-eV$ depends on the source-drain voltage (or on the gate voltages in lateral quantum dots) and $E^2=\delta^2+\Omega^2$. For the phonons, we assume an Ohmic spectral density[1] $J(\omega)=\pi\Sigma_\nu\lambda_\nu^2\delta(\omega-\omega_\nu)=2\pi\alpha\omega$, so that their influence is determined by the coefficients

$$A_\pm=2\pi\alpha\Omega\pm2\pi\alpha\delta\Omega\left(\frac{2k_BT}{E^2}-\frac{1}{E}\coth\frac{E}{2k_BT}\right), \tag{4}$$

$$B=4\pi\alpha\left(\frac{2\delta^2k_BT}{E^2}+\frac{\Omega^2}{E}\coth\frac{E}{2k_BT}\right)+\gamma, \tag{5}$$

where $\gamma=\Gamma_R/2$ stems from the additional decoherence associated with the tunneling to the leads. In consistency with the experiment of Ref. 10, we have assumed that the voltage is so large that the Fermi level of the left (right) lead is well above (below) the energy of the left (right) dot level. Therefore, it is sufficient to consider only unidirectional transport from the left lead to the right lead[13] described by the effective tunneling rates Γ_l which are proportional to $|\gamma_l|^2$.

Within the same approximation, one can derive for the current, defined as the time derivative of the charge in the right lead, the expression $I=e\,\text{tr}_{\text{sys}}[(\mathcal{J}_+-\mathcal{J}_-)\rho_0]$, where ρ_0 denotes the stationary solution of the master equation (2) and $\mathcal{J}_\pm$ are the superoperators describing the tunneling of an electron from the right dot to the right lead and back, respectively. For unidirectional transport, they read $\mathcal{J}_-=0$ and $\mathcal{J}_+\rho=(\Gamma_R/\hbar)\rho_{RR}|0\rangle\langle0|$.

The noise will be characterized by the variance of the transported net charge which at long times grows linear in time, $\langle\Delta Q_R^2\rangle=St$. For its computation, we introduce the operator $\text{tr}_{\text{leads+ph}}(N_R\rho_{\text{total}})$ which resembles the reduced density operator[14,15] and obeys

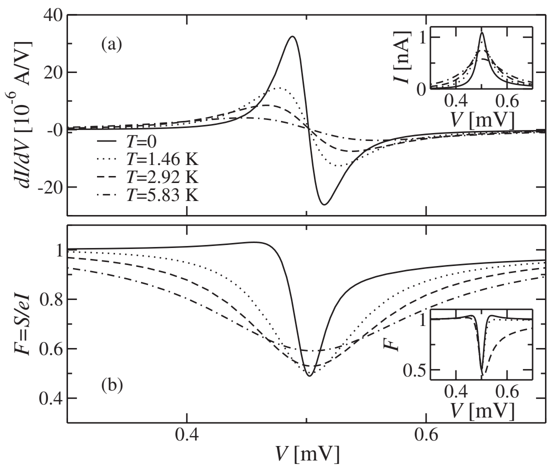

FIG. 2. (a) Differential conductance, current (inset), and (b) Fano factor through a single channel for various temperatures and $\Gamma_L=0.025$, $\Gamma_R=0.0125$, $\Omega=0.025$, $\varepsilon=0.5$, and $\alpha=0.005$ (in meV). The inset of panel (b) shows the Fano factor for dissipation strength $\alpha=0$ (solid), 10^{-3} (dotted), and 10^{-2} (dashed) at zero temperature.

$$\dot{\zeta}(t)=\mathcal{L}\zeta(t)+(\mathcal{J}_+-\mathcal{J}_-)\rho(t). \tag{6}$$

One can show[15] that ζ has a divergent component which is proportional to ρ_0 and does not contribute to the zero-frequency noise S. Thus S is fully determined by the traceless part $\zeta_\perp=\zeta_0-\rho_0\,\text{tr}\,\zeta$. In terms of ρ_0 and $\zeta_\perp$, the zero-frequency noise reads[15]

$$S=e^2\text{tr}_{\text{sys}}[2(\mathcal{J}_+-\mathcal{J}_-)\zeta_\perp+(\mathcal{J}_++\mathcal{J}_-)\rho_0]. \tag{7}$$

A proper dimensionless measure for the noise is the Fano factor $F=S/eI$ which equals unity for a Poisson process, while a larger value reflects electron bunching.

III. TRANSPORT THROUGH A SINGLE CHANNEL

Figure 2(a) shows the differential conductance and the current for various temperatures as a function of the internal bias. In contrast to the dissipationless case ($\alpha=0$),[13,16] the shape of the curve is no longer Lorentzian but exhibits an asymmetry. At higher temperatures, the peak becomes broader and more symmetric. This behavior is also reflected by the noise. In the absence of dissipation, the Fano factor deviates from the Poissonian value $F=1$: For $\Gamma_L>\Gamma_R$ (as in the experiment) and $\alpha=0$, we observe an antiresonant behavior with a dip ($F\approx0.5$), which is accompanied by two maxima with values slightly above 1. This double peak structure does not appear if $\Gamma_L\leqslant\Gamma_R$. With increasing dissipation strength α and increasing temperature, the maxima vanish and the Fano factor eventually tends to the Poissonian value $F=1$.

Although this behavior resembles the experimental findings reported in Ref. 10, there are significant quantitative differences. For the maximal peak value of the Fano factor, which is assumed in the dissipationless limit $\alpha\to0$ for $\delta=\Omega/\sqrt{2}$, we find the analytic expression

$$F_p(\alpha=0) = 1 + \frac{\Omega^2(\Gamma_L - \Gamma_R)^2}{2\Omega^2(\Gamma_L\Gamma_R + 2\Gamma_L^2 - \Gamma_R^2) + 8\Gamma_L^2\Gamma_R^2}. \quad (8)$$

It implies $F_p \leqslant 5/4$, with the maximum assumed for $\Gamma_R \ll \Gamma_L, \Omega$. This means that for a single channel, the theoretical prediction for the maximal Fano factor is clearly smaller than the value observed in the experiment even at finite temperature and in the presence of dissipation;[10] cf. inset in Fig. 2(b). Therefore, we must conclude that the one-channel model does not fully capture the experimentally observed shot noise enhancement.

IV. TRANSPORT THROUGH TWO COUPLED CHANNELS

The natural assumption is now that the shot noise must be influenced also by the interaction with a second transport channel; cf. Fig. 1 and Ref. 10. Thus we now consider two capacitively coupled channels, so that the system Hamiltonian reads $H_0 = \sum_{i,l}(\varepsilon_{il}n_{il} + \frac{1}{2}\sum_{i',l'}U_{ii'll'}n_{il}n_{i'l'})$, where $i = 1,2$ labels the different transport channels. Note that without interchannel interaction ($U_{ii'll'}=0$ for $i \neq i'$), both channels are statistically independent. Thus, the behavior observed in the one-channel case (see Fig. 2) is repeated at a different voltage, but still the Fano factor cannot exceed the value 5/4.

In order to simplify the model, we assume that the interaction $U_{ii'll'}$ is huge whenever $i=i'$ or $l=l'$. Then, the system will accept up to two extra electrons provided that they are placed in different stacks and different layers.[8,17] This means that we have to consider the following seven states (the ith letter refers to channel i): The empty state $|00\rangle$, the one-electron states $|L0\rangle$, $|R0\rangle$, $|0L\rangle$, $|0R\rangle$, and the two-electron states $|RL\rangle$, $|LR\rangle$. We assume that both dots on the right-hand side couple to the same lead, while each channel couples to an individual phonon bath. Then we derive for the coupled channels a master equation of the form (2) with a Liouvillian given by an 11×11 matrix. A closer inspection of this Liouvillian reveals that—formally—it can be obtained also in the following way: One writes the reduced density operator of the double channel as a direct product of each channel, $\rho = \rho^{(1)} \otimes \rho^{(2)}$, and the Liouvillian accordingly as $\mathcal{L} = \mathcal{L}^{(1)} + \mathcal{L}^{(2)}$, where $\mathcal{L}^{(i)}$ is the Liouvillian (3) with the parameters replaced by those of channel i. In this case, $\gamma_i = (\Gamma_{jL} + \Gamma_{iR})/2$, $j \neq i$. Finally, one removes all lines and columns that contain one of the "forbidden" states $|LL\rangle$, $|RR\rangle$.

For self-assembled quantum dots, a realistic assumption is that all barriers are almost identical, so that $\Gamma_{L/R}$ and Ω do not depend on the channel index i. By contrast, for the internal bias $\varepsilon_i = \varepsilon_{iR} - \varepsilon_{iL}$, we will find that already small differences play a role, so that we have to maintain the channel index i in the effective detunings $\delta_i = \varepsilon_i - eV$. For unidirectional transport, the current operators now read $\mathcal{J}_- = 0$, while $\mathcal{J}_+ = \mathcal{J}_+^{(1)} + \mathcal{J}_+^{(2)}$ acts on the reduced density operator as $\mathcal{J}_+\rho = (\Gamma_R/\hbar)[(\rho_{R0} + \rho_{0R})|00\rangle\langle 00| + \rho_{RL}|0L\rangle\langle 0L| + \rho_{LR}|L0\rangle\langle L0|]$.

In the absence of the phonons, we find the scenario discussed already in Ref. 8: The Fano factor exhibits two peaks, but their origin is now different than in the one-channel case. If both double quantum dots become resonant at different source-drain voltages, an electron in the double dot that is out of resonance has only a small probability to tunnel through the central barrier. Therefore, the nonresonant double dot will mostly be occupied with one electron and thereby block the other channel, so that the current peaks becomes smaller than in the one-channel case; cf. insets of Figs. 2(a) and 3(a). Whenever the nonresonant channel is empty, however, the resonant channel will transmit a bunch of electrons, so that eventually the noise is super-Poissonian.

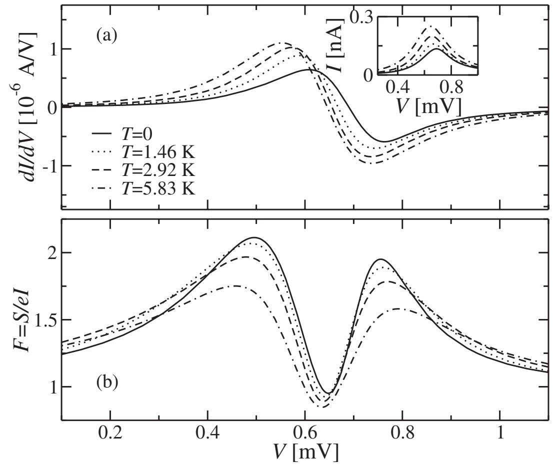

FIG. 3. (a) Differential conductance and (b) Fano factor for two coupled channels for various temperatures and the parameters $\Gamma_L=0.025$, $\Gamma_R=0.0125$, $\Omega=0.025$, $\alpha=0.005$, $\varepsilon_1=0.5$, and $\varepsilon_2=0.75$ (in meV). The inset shows the temperature broadening of the current peak.

Figures 3 and 4 show the corresponding current and the Fano factor in the presence of dissipation for two different configurations. We observe two striking features which are in accordance with the experimental results of Ref. 10: First, dynamical channel blocking is less pronounced at higher temperatures and, second, the structure of the Fano factor exhibits a clear asymmetry.

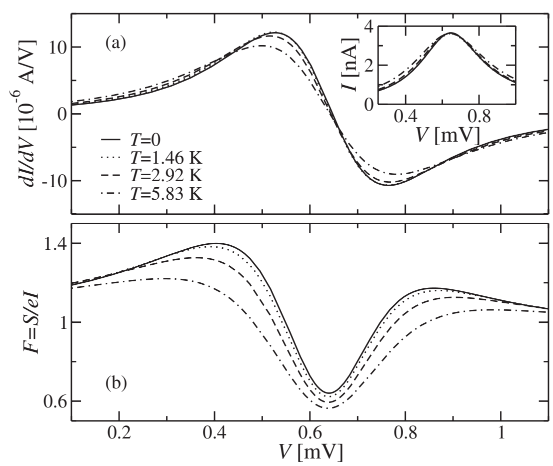

FIG. 4. (a) Differential conductance and (b) Fano factor for two coupled channels for various temperatures and the parameters $\Gamma_L=0.11$, $\Gamma_R=0.055$, $\Omega=0.11$, $\alpha=0.005$, $\varepsilon_1=0.5$, and $\varepsilon_2=0.75$ (in meV). The inset shows the temperature broadening of the current peak.

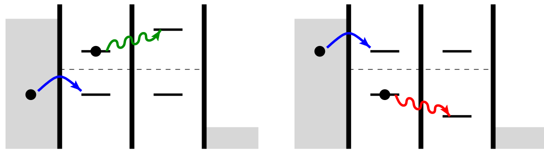

FIG. 5. (Color online) Phonon-assisted channel opening: The blocking electron in the off-resonant channel can tunnel through the interdot barrier after phonon absorption (left) or emission (right) and thereby open the resonant channel.

This behavior can be explained within the following picture: Let us consider, for instance, the situation sketched in Fig. 5 where $\varepsilon_1 < \varepsilon_2$. When the source-drain voltage puts the first double quantum dot in resonance, i.e., $\delta_1 = 0$, the second double dot is still *above resonance* ($\delta_2 > 0$), thus blocking the resonant channel. If now the electron in channel 2 absorbs a phonon, the blockade is lifted. On the other hand, when $\delta_2 = 0$, double dot 1 one is already *below resonance* ($\delta_1 < 0$) and phonon emission can resolve the blockade. Both processes are more frequent the higher the temperature, so that dynamical channel blockade is eventually resolved. The fact that emission is more likely than absorption, explains the observed asymmetry and its reduction with increasing temperature. We emphasize that this does not rely on differences in the interdot hoppings Ω_i, in contrast to the mechanism of Ref. 8.

This *phonon-induced channel opening* is also manifested in the enhancement of the current shown in the inset of Fig. 3(a). The current peak becomes larger with increasing temperature and experiences a slight shift away in its location. At low temperatures it tends to be around the larger resonance voltage (ε_2/e) which is driven by phonon relaxation. As phonon emission becomes important with temperature, the current peak becomes larger and shifts towards $eV_0 = (\varepsilon_1 + \varepsilon_2)/2$ coinciding with the maximal current voltage in the absence of dissipation.

This effect is weaker when considering stronger tunneling couplings: The Fano factor is reduced by DCB lifting and the current peak remains centered at eV_0 when increasing temperature, which only affects to its broadening; cf. Fig. 4.

The observed behavior reproduces rather well the measurements reported in Ref. 10, but there is still one difference: In the experiment, the Fano factor far from resonance is clearly smaller than 1, while for the two-channel model, it tends to be Poissonian. This can be explained by leakage currents I_k that inevitably flow through the whole sample, but have been ignored so far. We assume that the leakage currents are statistically independent of each other and of the coupled double dots considered. Then we can write both the current and the noise of the complete sample as a sum of the independent channels: $I_{\rm sample} = I_{\rm sys} + \Sigma_k I_k$ and $S_{\rm sample} = S_{\rm sys} + \Sigma_k F_k I_k$, where F_k is the Fano factor associated to I_k. If the leakage currents stem from resonant tunneling through single quantum dots or double dots far from resonance, $F_i < 1$ (Ref. 18) and, thus, the total Fano factor is decreased: $F_{\rm sample} = S_{\rm sample}/I_{\rm sample} < F_{\rm sys}$. However, since there are about 10^6 leakage channels,[10] it is not possible to estimate their effect more precisely.

V. CONCLUSIONS

To summarize, we have studied the effect of electron-phonon interaction in the transport through double quantum dots systems, predicting super-Poissonian shot noise whenever the source-drain voltage tunes a double dot close to resonance. The corresponding Fano factor exhibits an asymmetric double-peak structure which becomes less pronounced with increasing temperature. In order to obtain the experimentally observed values[10] for the Fano factor, we have assumed that two transport channels are so close that they can block each other. The temperature dependence of the double peaks have been explained by the suspension of dynamical channel blocking via phonon emission or absorption. We attribute the sub-Poissonian noise observed far from resonance to the appearance of independent leakage currents. So experiments with devices where these systems were isolated from these additional noise sources are highly desirable.

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