Erratum: Relativistic Brownian motion: From a microscopic binary collision model to the Langevin equation [Phys. Rev. E 74, 051106 (2006)]

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In Sec. II of this paper we have discussed how one can obtain a nonrelativistic Langevin equation (NRLE) from a simple microscopic collision model by means of several approximation steps. In the paragraph below Eqs. (15) it was stated that, given the fluctuation-dissipation theorem (15d), only the post-point discretization rule [1–4] yields a Maxwellian as the stationary distribution of the Brownian particle. This is correct, but for the post-point discretization rule the mean value of the fluctuating force \( \xi(P,t) \) will be nonzero in general, i.e., \( \langle \xi(P,t) \rangle \neq 0 \). A vanishing mean value \( \langle \xi(P,t) \rangle = 0 \), as indicated in Eq. (15b), is obtained only if one adopts the Ito pre-point discretization rule [5–7]. Therefore, in order to make the dependence on the discretization rule more explicit, one should rewrite the NRLE (15a) in terms of an explicit multiplicative coupling (with post-point discretization), i.e.,

\[
\dot{P} = -v_0(P)P + \sqrt{2D_0(P)}\xi(t), \tag{15a}
\]

where now \( \xi(t) \) is a normalized, momentum-independent Gaussian white noise, characterized by

\[
\langle \xi(t) \rangle = 0, \quad \langle \xi(t)\xi(s) \rangle = \delta(t-s).
\]

In the limit where the Brownian particle is much heavier than the heat bath particles, the momentum-dependent noise amplitude \( D_0(P) \) is determined by the fluctuation-dissipation theorem \( D_0(P) = MP v_0(P)kT \) with friction coefficient \( v_0(P) \) given by Eq. (11). Then, in accordance with the above remarks, one finds \( \langle \sqrt{2D_0(P)}\xi(t) \rangle = 0 \) only for the Ito stochastic integral interpretation [5–7], but \( \langle \sqrt{2D_0(P)}\xi(t) \rangle \neq 0 \) for any other discretization rule [1–4].

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