

## Mean first-passage times for an ac-driven magnetic moment of a nanoparticle

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**Abstract.** – The two-dimensional backward Fokker-Planck equation is used to calculate the mean first-passage times (MFPTs) of the magnetic moment of a nanoparticle driven by a rotating magnetic field. It is shown that a magnetic field that is rapidly rotating in the plane *perpendicular* to the easy axis of the nanoparticle governs the MFPTs just in the same way as a static magnetic field that is applied *along* the easy axis. Within this framework, the features of the magnetic relaxation and net magnetization of systems composed of ferromagnetic nanoparticles arising from the action of the rotating field are revealed.

*Introduction.* – The mean first-passage time (MFPT), *i.e.*, the average time that elapses until a stochastic process reaches a prescribed domain, is an important characteristic of the considered process. It is widely used for describing various dynamic features such as exit problems, activation rates, lifetimes of metastable states and other noise-induced phenomena [1, 2]. The class of stochastic processes for which the MFPT can be calculated explicitly is rather limited [1]. In fact, the most general analytical results were obtained for continuous *one-dimensional* Markov processes within an approach based on the backward Fokker-Planck equation [3–7]. It is important to note that most of the exactly known results are restricted to Markov processes that are homogeneous in time.

However, Markov processes that describe *time-dependent* systems usually cannot be approximated by homogeneous processes. Well-known examples are the phenomenon of Stochastic Resonance [8–10] and directed transport in ratchet-like systems [11] where external time-dependent fields play a crucial role. Since the above-mentioned method is no longer applicable, the development of new approaches for calculating MFPTs in periodically driven systems presents an important challenge. For slowly varying external driving forces adiabatic and semiadiabatic approximations are available [12, 13]. The path integral formulation of the conditional probability provides a convenient basis for approximations if the noise is weak [14].

In this letter we present an analytical approach to the *two-dimensional* MFPT problem for a rapidly driven magnetic moment of a ferromagnetic nanoparticle. It is based on the backward

Fokker-Planck equation in a *rotating* coordinate system, which describes the *homogeneous* dynamics of the magnetic moment, and on an averaging procedure [15], which is similar to that used for describing the Kapitza pendulum with oscillating point of attachment [16]. The analytical results favorably compare with numerical simulations of the equivalent Langevin equation.

*Basic equations.* – In spherical coordinates, the Landau-Lifshitz equation [17] for the magnetic moment  $\mathbf{m}(t) = m(\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta)$  of a single-domain ferromagnetic nanoparticle assumes the form

$$\dot{\theta} = -\frac{\gamma}{m \sin \theta} \left( \lambda \sin \theta \frac{\partial}{\partial \theta} + \frac{\partial}{\partial \varphi} \right) W, \quad \dot{\varphi} = \frac{\gamma}{m \sin^2 \theta} \left( \sin \theta \frac{\partial}{\partial \theta} - \lambda \frac{\partial}{\partial \varphi} \right) W, \quad (1)$$

where  $\theta = \theta(t)$  and  $\varphi = \varphi(t)$  are the polar and azimuthal angles of  $\mathbf{m}(t)$ , respectively,  $\gamma (> 0)$  is the gyromagnetic ratio,  $\lambda (> 0)$  is a dimensionless damping parameter,  $m = |\mathbf{m}(t)|$  denotes the conserved total magnetic moment, and  $W = W(\theta, \varphi, t)$  is the magnetic energy of the nanoparticle. Besides the damping, the interaction with a heat bath also generates stochastic elements in the dynamics of the magnetic moment. These effects lead to a random contribution to the effective magnetic field which is assumed to be given by isotropic, Gaussian and white noise [18,19]. The conditional probability density  $P(\theta, \varphi, t | \theta', \varphi', t')$  of the resulting Markovian process satisfies the forward and the backward Fokker-Planck equations which are equivalent to each other [20]. At equal times  $P(\theta, \varphi, t | \theta', \varphi', t') = \delta(\theta - \theta') \delta(\varphi - \varphi')$ , where  $\delta(x)$  denotes the Dirac  $\delta$  function. Here we will use the backward equation, which propagates  $t'$  from  $t$  on backward in time. In the present case it takes the form [21]

$$-\frac{1}{\Delta \gamma^2} \frac{\partial P(\theta, \varphi, t | \theta', \varphi', t')}{\partial t'} = L^+(t') P(\theta, \varphi, t | \theta', \varphi', t'), \quad (2)$$

where  $\Delta = \lambda k_B T / \gamma m$  denotes the thermal noise intensity,  $k_B$  the Boltzmann constant,  $T$  the temperature of the heat bath,  $L^+(t')$  the backward Fokker-Planck operator,

$$L^+(t') = \frac{\partial^2}{\partial \theta'^2} + \frac{1}{\sin^2 \theta'} \frac{\partial^2}{\partial \varphi'^2} + (\cot \theta' + f(\theta', \varphi', t')) \frac{\partial}{\partial \theta'} + g(\theta', \varphi', t') \frac{\partial}{\partial \varphi'},$$

and the functions

$$\begin{aligned} f(\theta, \varphi, t) &= -\frac{1}{\Delta \gamma m \sin \theta} \left( \lambda \sin \theta \frac{\partial}{\partial \theta} + \frac{\partial}{\partial \varphi} \right) W, \\ g(\theta, \varphi, t) &= \frac{1}{\Delta \gamma m \sin^2 \theta} \left( \sin \theta \frac{\partial}{\partial \theta} - \lambda \frac{\partial}{\partial \varphi} \right) W \end{aligned} \quad (3)$$

are proportional to the drift components of the angles  $\theta$  and  $\varphi$ , respectively, see eq. (1). Note that if the energy  $W$  depends on time, the backward operator is also time dependent.

For the calculation of first-passage times a domain  $\Omega(t)$  of the magnetization state space, which is a sphere with radius  $|m|$ , has to be specified. In general this domain may change upon evolving time. Starting out in  $\Omega(t')$  the magnetization eventually will leave this domain. The instant  $t_f$  of first crossing the boundary  $\partial\Omega(t)$  defines the first-passage time  $t_f - t'$ . In order to prevent the trajectory to recross this boundary, absorbing boundary conditions must be imposed at  $\partial\Omega(t)$ . For the backward equation they read

$$P(\theta, \varphi, t | \theta', \varphi', t') = 0 \quad \text{for } (\theta', \varphi') \in \partial\Omega(t'). \quad (4)$$

The probability  $P_\Omega(t|\theta', \varphi', t')$  that the magnetization has been in the prescribed domain for all times, starting out at  $t'$  up to time  $t$ , *i.e.*,  $\mathbf{m}(s) \in \Omega(s)$  for all  $t' \leq s \leq t$ , follows from the conditional probability in the standard way:

$$P_\Omega(t|\theta', \varphi', t') = \int_{\Omega(t)} d\theta d\varphi P(\theta, \varphi, t|\theta', \varphi', t'). \quad (5)$$

The integral of  $P_\Omega(t|\theta', \varphi', t')$  over all times  $t$  determines the MFPT  $T_\Omega = \langle t_f - t' \rangle$ ,

$$T_\Omega(\theta', \varphi', t') = \int_{t'}^{\infty} dt P_\Omega(t|\theta', \varphi', t'). \quad (6)$$

Using all, the definition (6), the condition that  $P_\Omega(t'|\theta', \varphi', t') = 1$ , and the backward Fokker-Planck equation (2), we find that the MFPT satisfies the following equation:

$$\Delta\gamma^2 L^+(t')T_\Omega(\theta', \varphi', t') + \frac{\partial}{\partial t'} T_\Omega(\theta', \varphi', t') = -1. \quad (7)$$

Accordingly, one obtains as boundary condition from eq. (4),

$$T_\Omega(\theta', \varphi', t') = 0 \quad \text{for } (\theta', \varphi') \in \partial\Omega(t'). \quad (8)$$

For a periodic time dependence (with period  $\mathcal{T}$ ) of the backward operator and the  $\Omega$ -domain, *i.e.*, for  $L^+(t + \mathcal{T}) = L^+(t)$  and  $\Omega(t + \mathcal{T}) = \Omega(t)$ , the asymptotic solution of eq. (7) is also periodic in time with the same period  $\mathcal{T}$ . For time homogeneous processes and domains it is constant with respect to time and fulfills the well-known Pontryagin equation  $\Delta\gamma^2 L^+ T_\Omega = -1$ .

*General results.* – We now consider the magnetic moment of a nanoparticle with uniaxial anisotropy and assume that a static magnetic field  $\mathbf{H}$  is applied along the easy axis of magnetization which we choose as  $z$ -axis, *i.e.*,  $\mathbf{H} = (0, 0, H)$ . Additionally, a rotating magnetic field  $\mathbf{h}(t)$  acts perpendicular to this axis, *i.e.*,  $\mathbf{h}(t) = h(\cos \omega t, \rho \sin \omega t, 0)$ , where  $\omega$  is the angular frequency and  $\rho = -1, +1$  corresponds to clockwise and counterclockwise rotation, respectively. Hence, the magnetic energy of the nanoparticle is given by

$$W = \frac{1}{2}mH_a \sin^2 \theta - mH \cos \theta - mh \sin \theta \cos \psi \quad (9)$$

with  $H_a$  denoting the anisotropy field and  $\psi = \varphi - \rho\omega t$ . We note that in this case a detailed analysis of the deterministic dynamics of  $\mathbf{m}(t)$  is presented in [22]. According to eq. (9), the functions  $f$  and  $g$  in eq. (3) depend on  $\varphi'$  and  $t'$  only through the single variable  $\psi' = \varphi' - \rho\omega t'$ :

$$\begin{aligned} f &= -2a(\cos \theta' + \tilde{H}) \sin \theta' + \frac{2a\tilde{h}}{\lambda} (\lambda \cos \theta' \cos \psi' - \sin \psi'), \\ g &= \frac{2a}{\lambda} (\cos \theta' + \tilde{H}) - \frac{2a\tilde{h}}{\lambda \sin \theta'} (\lambda \sin \psi' + \cos \theta' \cos \psi'), \end{aligned} \quad (10)$$

where  $a = mH_a/2k_B T$  is the anisotropy barrier height in units of thermal energy  $k_B T$ ,  $\tilde{H} = H/H_a$ , and  $\tilde{h} = h/H_a$ . By introducing a rotating frame, in which  $\mathbf{h}(t) = h(1, 0, 0)$  and the azimuthal angle  $\varphi$  is replaced by  $\psi$ , the time derivative  $\partial/\partial t'$  goes over into  $\partial/\partial t' - \rho\omega\partial/\partial\psi'$ , and consequently the backward equation for the MFPT reads

$$\frac{\partial^2 T_\Omega}{\partial \theta'^2} + \frac{1}{\sin^2 \theta'} \frac{\partial^2 T_\Omega}{\partial \psi'^2} + (\cot \theta' + f) \frac{\partial T_\Omega}{\partial \theta'} + (g - \rho a t_r \omega) \frac{\partial T_\Omega}{\partial \psi'} = -a t_r \left( 1 + \frac{\partial T_\Omega}{\partial t'} \right), \quad (11)$$

where  $t_r = 2/(\lambda\gamma H_a)$  is the characteristic relaxation time of the precessional motion of the magnetic moment. Moreover, we assume that the domain  $\Omega(t)$  is stationary in the rotating frame, *i.e.*,  $\Omega(t)$  is bounded by a curve  $\phi_\Omega(\psi)$  on the sphere. Then, the steady-state solution of (11) is independent of time and can be found as the solution of the stationary Pontryagin equation

$$\frac{\partial^2 T_\Omega}{\partial \theta'^2} + \frac{1}{\sin^2 \theta'} \frac{\partial^2 T_\Omega}{\partial \psi'^2} + (\cot \theta' + f) \frac{\partial T_\Omega}{\partial \theta'} + (g - \rho a t_r \omega) \frac{\partial T_\Omega}{\partial \psi'} = -a t_r. \quad (12)$$

This presents already a major simplification of the original problem. However, still a partial differential equation in two spatial dimensions remains to be solved.

In the following we will consider two types of domains which we distinguish by the index  $\Omega := \pm 1$ . The domain with  $\Omega = +1$  (denoted as up domain) contains the up magnetization,  $\theta = 0$ , and is bounded in the rotating frame by a curve  $\phi_{+1}(\psi)$ . Accordingly, the  $\Omega = -1$  domain (down domain) contains the down magnetization,  $\theta = \pi$ , and is bounded by a curve  $\phi_{-1}(\psi)$ . It is convenient to present the respective MFPTs out of these domains in the form  $T_\Omega = \overline{T}_\Omega(\theta') + S_\Omega(\theta', \psi')$ , where the overbar denotes an average over  $\psi'$ , *i.e.*  $\overline{(\cdot)} = (1/2\pi) \int_0^{2\pi} d\psi'(\cdot)$ . Substituting the expansions  $f = \overline{f} + f_1$  and  $g = \overline{g} + g_1$  into eq. (12), one obtains an equation for  $\overline{T}_\Omega$ , reading

$$\frac{d^2 \overline{T}_\Omega}{d\theta'^2} + (\cot \theta' + \overline{f}) \frac{d\overline{T}_\Omega}{d\theta'} + \overline{f_1} \frac{\partial S_\Omega}{\partial \theta'} + \overline{g_1} \frac{\partial S_\Omega}{\partial \psi'} = -a t_r \quad (13)$$

and an equation for  $S_\Omega$ , reading

$$\frac{\partial^2 S_\Omega}{\partial \theta'^2} + \frac{1}{\sin^2 \theta'} \frac{\partial^2 S_\Omega}{\partial \psi'^2} + (\cot \theta' + f) \frac{\partial S_\Omega}{\partial \theta'} + f_1 \frac{d\overline{T}_\Omega}{d\theta'} + (g - \rho a t_r \omega) \frac{\partial S_\Omega}{\partial \psi'} - f_1 \frac{\partial S_\Omega}{\partial \theta'} - g_1 \frac{\partial S_\Omega}{\partial \psi'} = 0. \quad (14)$$

We emphasize that these equations are exact, *i.e.*, they follow from the backward Fokker-Planck equation (3) for stationary domains in the rotating frame.

*High-frequency limit.* – In the case of an arbitrary rotating field we are not able to solve eq. (12) analytically. But in the asymptotic limit of a fast rotating field, *i.e.* for  $\omega \gg \omega_r \equiv \gamma H_a$ , eqs. (13) and (14) can be solved readily. The key observation leading to their solution is that  $S_\Omega \rightarrow 0$  for  $\omega \rightarrow \infty$ . Assuming also that the derivatives of  $S_\Omega$  tend to zero as  $1/\omega$  for  $\omega \rightarrow \infty$ , eq. (14) simplifies to read in this high-frequency limit

$$\rho a t_r \omega \frac{\partial S_\Omega}{\partial \psi'} - f_1 \frac{d\overline{T}_\Omega}{d\theta'} = 0. \quad (15)$$

With  $f_1 = (2a\tilde{h}/\lambda)(\lambda \cos \theta' \cos \psi' - \sin \psi')$ , see eq. (10), the solution of this equation that satisfies the condition  $\overline{S}_\Omega = 0$  can be written in the form

$$S_\Omega = \rho \frac{\tilde{h}}{\tilde{\omega}} (\lambda \cos \theta' \sin \psi' + \cos \psi') \frac{d\overline{T}_\Omega}{d\theta'}, \quad (16)$$

where  $\tilde{\omega} = \omega/\omega_r$ . Note, that according to the above assumptions, this solution is valid if  $\tilde{h}/\tilde{\omega} \ll 1$ . The  $\omega$ -dependence assumed in the derivation of eq. (15) is now confirmed self-consistently. Using eq. (16) and the relation  $g_1 = -(2a\tilde{h}/\lambda)(\lambda \sin \psi' + \cos \theta' \cos \psi')/\sin \theta'$ , we proceed to calculate the averages

$$\overline{f_1 \frac{\partial S_\Omega}{\partial \theta'}} = \overline{g_1 \frac{\partial S_\Omega}{\partial \psi'}} = -a \tilde{h}_{\text{eff}} \sin \theta' \frac{d\overline{T}_\Omega}{d\theta'} \quad (17)$$

with  $\tilde{h}_{\text{eff}} = -\rho\tilde{h}^2/\tilde{\omega}$ . Finally, substituting eq. (17) and  $\tilde{f} = -2a(\cos\theta' + \tilde{H})\sin\theta'$  into eq. (13), we obtain the desired equation for  $\overline{T}_\Omega$  in the high-frequency limit:

$$\frac{d^2\overline{T}_\Omega}{d\theta'^2} + [\cot\theta' - 2a(\cos\theta' + \tilde{H} + \tilde{h}_{\text{eff}})\sin\theta']\frac{d\overline{T}_\Omega}{d\theta'} = -at_r. \quad (18)$$

This equation exhibits the remarkable result that a magnetic field that is rapidly rotating in the plane *perpendicular* to the easy axis of the nanoparticle acts on the nanoparticle's magnetic moment precisely as a static magnetic field  $\tilde{h}_{\text{eff}}$  (in units of  $H_a$ ) which is applied *along* the easy axis. The direction of the effective field  $\tilde{h}_{\text{eff}}$  and the direction of the field rotation follow the left-hand rule, and the value of  $\tilde{h}_{\text{eff}}$  is the same for up and down domains. It is important to emphasize that although the condition  $\tilde{h}/\tilde{\omega} \ll 1$  holds, the effective field can be large if  $\tilde{h} \gg 1$ .

We note that  $\theta' = 0, \pi$  are singular points of eq. (18) for  $\Omega = +1, -1$ , respectively. At these points the general solution of eq. (18) exhibits logarithmic singularities. To prevent this non-physical behavior of  $\overline{T}_\Omega$  the regularity condition  $d\overline{T}_\Omega/d\theta'|_{\theta'=\pi(1-\Omega)/2} = 0$  must hold [23]. In order to derive the boundary condition for eq. (18), we decompose the function  $\phi_\Omega(\psi)$  into its average and its periodic parts, *i.e.*,  $\phi_\Omega(\psi) = \overline{\phi}_\Omega + \vartheta_\Omega(\psi)$ . Assuming that  $|\vartheta_\Omega(\psi)| \ll \overline{\phi}_\Omega$ , from the absorbing boundary condition of the full two-dimensional problem,  $T_\Omega(\phi_\Omega(\psi'), \psi') = 0$ , we find the periodic part of  $\phi_\Omega(\psi)$ ,  $\vartheta_\Omega(\psi) = -\rho\tilde{h}(\lambda \cos\overline{\phi}_\Omega \sin\psi + \cos\psi)/\tilde{\omega}$ , and the desired boundary condition for the averaged MFPT,  $\overline{T}_\Omega(\overline{\phi}_\Omega) = 0$ . The solution of eq. (18) with the specified regularity and boundary conditions becomes

$$\overline{T}_\Omega(\theta') = at_r \int_{\cos\overline{\phi}_\Omega}^{\cos\theta'} dx \frac{e^{-a(x+\tilde{H}+\tilde{h}_{\text{eff}})^2}}{1-x^2} \int_x^\Omega dy e^{a(y+\tilde{H}+\tilde{h}_{\text{eff}})^2}, \quad (19)$$

where  $\theta' \in [0, \overline{\phi}_{+1}]$  if  $\Omega = +1$ ,  $\theta' \in [\overline{\phi}_{-1}, \pi]$  if  $\Omega = -1$ , and the angles  $\overline{\phi}_\Omega$  can be chosen depending on physical situation.

In the case of a high potential barrier,  $a \gg 1$ , and moderately large total effective fields,  $|\tilde{H} + \tilde{h}_{\text{eff}}| < 1$ , the magnetic moment resides near one of two equilibrium directions, up or down. Since transition times between these states by far exceed the relaxation times towards these states, the averaged MFPT  $\overline{T}_\Omega(\theta')$  describing the transition from one state  $\Omega$  to the opposite state  $-\Omega$  only weakly depends on the precise location of the initial magnetization, as long as  $\theta'$  lies within the domain of attraction of the considered state  $\Omega$ . Also the precise location of the absorbing boundary  $\overline{\phi}_\Omega$  has practically no influence on  $\overline{T}_\Omega(\theta')$  if it is located well beyond the separatrix which divides the state space into domains of attraction of the up and down magnetization. Then eq. (19) yields in leading order in  $a$

$$\overline{T}_\Omega = t_r \sqrt{\frac{\pi}{a}} \frac{e^{a[1+\Omega(\tilde{H}+\tilde{h}_{\text{eff}})]^2}}{2[1-(\tilde{H}+\tilde{h}_{\text{eff}})^2][1+\Omega(\tilde{H}+\tilde{h}_{\text{eff}})]}. \quad (20)$$

Using eqs. (16), (19) and (20), it is not too difficult to demonstrate that  $|S_\Omega| \ll |\Delta\overline{T}_\Omega|$ , where  $\Delta\overline{T}_\Omega = \overline{T}_\Omega - \overline{T}_\Omega|_{\tilde{h}=0}$  is the contribution of  $\tilde{h}$  to  $\overline{T}_\Omega$ . This means that the periodic part of  $T_\Omega(\theta', \psi')$  can be neglected such that  $T_\Omega(\theta', \psi') \approx \overline{T}_\Omega$ .

In fig. 1 the theoretical prediction (19) and the asymptotic approximation (20) are compared with the results of a numerical simulation of the coupled Langevin equations for the two angles  $\theta$  and  $\phi$  which are equivalent to the process described by eq. (2).

Next we examine the most interesting case of zero static field,  $\tilde{H} = 0$ , when only the effective field influences the MFPTs. Specifically, if  $|\tilde{h}_{\text{eff}}| \ll 1$  then eq. (20) yields  $\overline{T}_\Omega = \tau_0 \exp[\Omega 2a\tilde{h}_{\text{eff}}]$ , where  $\tau_0 = (t_r/4)\sqrt{\pi/a} \exp[a]$ . Thus, the rotating magnetic field increases the MFPT for the magnetic moment  $\mathbf{m}(t)$  in the state  $\Omega = -\rho$  and decreases it for  $\mathbf{m}(t)$  in

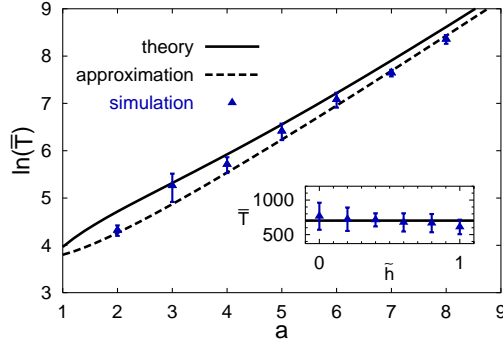


Fig. 1 – (Color online) The natural logarithm of the dimensionless MFPT,  $\bar{T} = \bar{T}_{+1} \cdot \omega_r$ , is displayed as a function of the dimensionless anisotropy barrier height  $a$ . The solid line is the result of present theory (19), and the broken line depicts the approximate relation (20). The symbols indicate results from the numerical simulation of  $4 \times 10^4$  runs of the Langevin equation which is equivalent to the backward equation (2) with absorbing boundary at  $\theta'_{\text{abs}} = 0.8\pi$  and a starting point at  $\theta' = 0.05\pi$ ,  $\phi' = 0$ . The values of the other parameters are  $\lambda = 0.1$ ,  $\rho = 1$ ,  $\omega = 10 \omega_r$ ,  $\tilde{H} = 0$  and  $\tilde{h}_{\text{eff}} = -0.1$ .  $\bar{T}$  is displayed in the inset as a function of  $\tilde{h}$ , where  $\tilde{H} + \tilde{h}_{\text{eff}} = -0.1$ ,  $a = 5$ ,  $\omega = 10 \omega_r$ ,  $\lambda = 0.1$  and  $\rho = 1$ . Only small systematic deviations from the theoretical prediction (solid line) are visible.

the state  $\Omega = \rho$ , where  $\rho = \pm 1$ . Physically, this difference in MFPTs follows from the natural counter-clockwise precession (if looked from above) of the magnetic moment. Therefore, for the two directions of the magnetic-field rotation,  $\rho = -1$  and  $\rho = +1$ , the forced dynamics of  $\mathbf{m}(t)$  in the up and down states is different. Hence, the rotating magnetic field breaks the degeneracy between the up and down orientations of the magnetic moment for  $\tilde{H} = 0$ .

*Magnetic relaxation.* – To illustrate the role of the effective magnetic field  $\tilde{h}_{\text{eff}}$ , we consider the relaxation of magnetization in a system composed of ferromagnetic nanoparticles whose easy axes are perpendicular to the plane of field rotation and  $a \gg 1$ . The reduced magnetization of this system can be defined as  $\mu(t) = [N_{+1}(t) - N_{-1}(t)]/N$ , where  $N (\gg 1)$  and  $N_{\Omega}(t)$  denote the total number of nanoparticles and those that are in the state  $\Omega$ , respectively. Using  $N_{-1}(t) + N_{+1}(t) = N$ , we obtain  $\dot{\mu}(t) = 2\dot{N}_{+1}(t)/N$ . On the other hand, the time dependence of  $N_{\Omega}(t)$  is governed by the kinetic equation  $\dot{N}_{\Omega}(t) = N_{-\Omega}(t)w_{-\Omega} - N_{\Omega}(t)w_{\Omega}$ , where  $w_{\Omega}$  denotes the transition rate from  $\Omega$  to  $-\Omega$ . This gives for the magnetization the well-known equation

$$\dot{\mu}(t) = -\mu(t)(w_{-1} + w_{+1}) - w_{+1} + w_{-1}. \quad (21)$$

Because the mean residence time in the state  $\Omega$  equals  $\bar{T}_{\Omega}$ , the transition rate  $w_{\Omega}$  is given by  $w_{\Omega} = 1/\bar{T}_{\Omega}$ . In this case, solving eq. (21) with the initial condition  $\mu(0) = 1$  and assuming  $\tilde{H} = 0$ , we obtain the relaxation law

$$\mu(t) = (1 - \mu_{\infty}) \exp[-t/\tau] + \mu_{\infty}, \quad (22)$$

where  $\tau = 1/(w_{-1} + w_{+1}) = \tau_0/\cosh(2a\tilde{h}_{\text{eff}})$  and  $\mu_{\infty} = (w_{-1} - w_{+1})/(w_{-1} + w_{+1}) = \tanh(2a\tilde{h}_{\text{eff}})$  are the relaxation time and steady-state magnetization, respectively. Thus, the rotating magnetic field decreases the relaxation time and magnetizes the nanoparticle system. These results are not evident because they arise from the difference between the up and down dynamical states of the magnetic moments. It is important to note in this context that even

for small values of the effective field the magnetization effect can be sizable. In particular, if  $\tilde{h} = 0.1$  and  $\tilde{\omega} = 10$  then  $|\tilde{h}_{\text{eff}}| = 10^{-3}$ , and for  $a = 50$  we get  $|\mu_{\infty}| = 0.1$ .

*Conclusion.* – We have shown that a magnetic field rapidly rotating in the plane perpendicular to the easy axis of a nanoparticle, lifts the degeneracy between the up and down orientations of the nanoparticle magnetic moment. This lifting is characterized by the effective magnetic field acting along the easy axis in a direction that is uniquely defined by the direction of the magnetic-field rotation. The effective field changes the MFPTs for the nanoparticle's magnetic moment and, as a consequence, changes the relaxation law and yields a net magnetization for a system of ferromagnetic nanoparticles.

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