Limitation of entanglement due to spatial qubit separation

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Abstract. – We consider spatially separated qubits coupled to a thermal bosonic field that causes pure dephasing. Our focus is on the entanglement of two Bell states which for vanishing separation are known as robust and fragile entangled states. The reduced two-qubit dynamics is solved exactly and explicitly. Our results allow us to gain information about the robustness of two-qubit decoherence-free subspaces with respect to physical parameters such as temperature, qubit-bath coupling strength and spatial separation of the qubits. Moreover, we clarify the relation between single-qubit coherence and two-qubit entanglement and identify parameter regimes in which the terms robust and fragile are no longer appropriate.

In recent years, we witnessed a fast development of the experimental realisation of quantum bits and the maintenance of their quantum coherence. Despite this progress, decoherence stemming from a coupling of the qubits to a macroscopic environment remains a major obstacle for the implementation of a quantum computer. Several strategies are pursued to beat decoherence. One is dynamical decoupling: single qubits [1] or two-qubit gates [2] are effectively isolated from their environment by driving them with ac fields. Another strategy is quantum error correction, which requires a redundant encoding of a logical qubit by several physical qubits. Standard error correction protocols presuppose that all physical qubits couple to uncorrelated baths [3–5], which can be realized by putting qubits far apart [6]. A third, more direct strategy is the use of a decoherence-free subspace (DFS). There, one logical qubit is encoded by several physical qubits in such a way that the logical qubit states do not couple to the environment [7–9]. Ideal DFSs occur when physical qubits couple via a collective coordinate to a common bath and are fairly robust against perturbations [10,11].

In view of the above, it is an important question whether different qubits are exposed to spatially correlated or uncorrelated noise. For example, for charge qubits in quantum dots [12], a relevant source of decoherence is the coupling to substrate phonons [13,14]. Widely separated charge qubits at high temperatures will experience uncorrelated noise due to the phonon bath. The identification of a crossover regime at lower temperatures and smaller separations in which spatial correlations become important, requires a model in which the qubit separation and the phonon bath are taken explicitly into account.

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In this letter, we study the decoherence of spatially separated qubits which are coupled to a bosonic field that causes phase noise. On short time scales, phase noise is the main source of decoherence in solid-state environments [15]; on longer time scales, bit-flip noise becomes relevant as well [16], but we focus on the former herein. Phase noise is characterised by a qubit-bath coupling that commutes with the qubit Hamiltonian and, thus, allows an exact solution of the dissipative quantum dynamics. Still, the evaluation of the resulting exact expressions can be rather complex and often relies on approximations [3,6,7,17–22]. In contrast to those works, we here evaluate the exact solution in explicit form. We focus on the entanglement of two qubits prepared in particular Bell states and derive explicit expressions showing how their entanglement changes upon increasing their separation. For a wide range of parameters, we find that the dynamics is highly non-Markovian and that the entanglement can converge to relatively large values, even at high temperatures. We will discuss the robustness of the two-qubit DFS and clarify the relation between two-qubit entanglement and single-qubit coherence.

Robust and fragile entangled states. – Yu and Eberly [18] studied the entanglement dynamics for two qubits coupled to the same heat bath at *identical* positions, in particular for a preparation of the (maximally entangled) Bell states,

$$|\psi_{\text{robust}}\rangle = |\psi_-\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}, \qquad |\psi_{\text{fragile}}\rangle = |\psi_+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}, \tag{1}$$

where $|\boldsymbol{n}\rangle = |n_1, n_2, \ldots, n_N\rangle$ denotes the N-qubit state with $\sigma_{\nu z}|\boldsymbol{n}\rangle = (-1)^{n_\nu}|\boldsymbol{n}\rangle$, $n_\nu = 0, 1$ and $\sigma_{\nu z}$ a Pauli matrix for qubit ν. They found that the "robust state" lives in a DFS and consequently its entanglement is preserved, whereas the entanglement of the qubits initially in the "fragile state" decays to zero. In our study, we will also consider these initial states. The notation $|\psi_\pm\rangle$ has been introduced for writing equations more efficiently.

Two-qubit entanglement can be measured by the concurrence $C[\rho] = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}$, where λ_i denotes the eigenvalues of the matrix $\rho\sigma_{1y}\sigma_{2y}\rho^*\sigma_{1y}\sigma_{2y}$ in decreasing order; ρ^* is the complex conjugate of the density matrix ρ [23]. For maximally entangled states, one finds $C = 1$, while C vanishes for incoherent mixtures of product states. It was noticed [18] that for phase noise and two qubits at vanishing separation initially in either of the states (1), the concurrence is given by the absolute values of particular density matrix elements $\rho_{\boldsymbol{m},\boldsymbol{n}} = \langle \boldsymbol{m}|\rho|\boldsymbol{n}\rangle$, namely $C_- = 2|\rho_{01,10}|$ and $C_+ = 2|\rho_{00,11}|$, respectively. These relations hold for spatially separated qubits as well.

Qubits coupled to a bosonic field. – For modelling N identical qubits coupled to a homogeneous bosonic environment, we employ a spin-boson Hamiltonian

$$H = \frac{\Delta}{2} \sum_{\nu=1}^{N} \sigma_{\nu z} + \sum_{\boldsymbol{k}} \hbar\omega_k b_{\boldsymbol{k}}^\dagger b_{\boldsymbol{k}} + H_{\text{q-b}}. \tag{2}$$

The first term denotes N qubits with energy splittings Δ and the second term represents a bosonic field with isotropic dispersion relation $\omega_{\boldsymbol{k}} = \omega_k = c|\boldsymbol{k}|$ and sound velocity c. The qubit-bath interaction

$$H_{\text{q-b}} = \hbar \sum_{\nu=1}^{N} \sigma_{\nu z}\xi_\nu, \qquad \text{with} \quad \xi_\nu = \sum_{\boldsymbol{k}} g_k(b_{\boldsymbol{k}}\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}_\nu} + b_{\boldsymbol{k}}^\dagger\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}_\nu}), \tag{3}$$

introduces phase noise due the coupling of qubit ν via $\sigma_{\nu z}$ to the field at position $\boldsymbol{x}_\nu$. We assumed identical and isotropic coupling strengths for each qubit, *i.e.* $g_{\boldsymbol{k}\nu} = g_k$.

Furthermore, we assume that at initial time $t = 0$, the density matrix of qubit plus bath, $R(0)$, is of the Feynman-Vernon type, *i.e.* the bath is in thermal equilibrium and uncorrelated with the qubits, $R(0) = \rho(0) \otimes \rho_{\mathrm{b}}^{\mathrm{eq}}$, where $\rho_{\mathrm{b}}^{\mathrm{eq}} \propto \exp[-\sum_{\boldsymbol{k}} \hbar\omega_{\boldsymbol{k}} b_{\boldsymbol{k}}^{\dagger} b_{\boldsymbol{k}} / k_{\mathrm{B}} T]$ is the canonical ensemble of the bosons. Since we are exclusively interested in the behaviour of the qubits, we trace out the bath and obtain the qubits' reduced density operator $\rho(t) = \mathrm{tr}_{\mathrm{b}} R(t)$. From the Liouville-von Neumann equation in the usual interaction picture, $i\hbar \, \mathrm{d}\widetilde{R}/\mathrm{d}t = [\widetilde{H}_{\mathrm{q\text{-}b}}(t), \widetilde{R}]$, we obtain after some algebra for the density matrix elements the closed expression [6, 7, 20]

$$\tilde{\rho}_{\boldsymbol{m},\boldsymbol{n}}(t) = \rho_{\boldsymbol{m},\boldsymbol{n}}(0) \, \mathrm{e}^{-\Lambda_{\boldsymbol{m},\boldsymbol{n}}(t) + \mathrm{i}\phi_{\boldsymbol{m},\boldsymbol{n}}(t)} \, . \tag{4}$$

The phases $\phi_{\boldsymbol{m},\boldsymbol{n}}(t)$ correspond to Lamb shifts which are brought about by time ordering. Since all quantities considered in the following are given by absolute values of single density matrix elements, these phases will not be relevant. The amplitude damping depends on the qubit separations $\boldsymbol{x}_{\nu\nu'} = \boldsymbol{x}_\nu - \boldsymbol{x}_{\nu'}$ and reads

$$\Lambda_{\boldsymbol{m},\boldsymbol{n}}(t) = \sum_{\boldsymbol{k}} g_{\boldsymbol{k}}^2 \frac{1 - \cos(\omega_k t)}{\omega_k^2} \coth\left(\frac{\hbar\omega_k}{2k_{\mathrm{B}} T}\right) \left| \sum_{\nu,\nu'=1}^{N} [(-1)^{m_\nu} - (-1)^{n_\nu}] \mathrm{e}^{-\mathrm{i}\boldsymbol{k} \cdot \boldsymbol{x}_{\nu\nu'}} \right|^2 . \tag{5}$$

For the evaluation of these damping factors, it is convenient to introduce the spectral density $J^{(d)}(\omega) \equiv \sum_{\boldsymbol{k}} g_{\boldsymbol{k}}^2 \delta(\omega - ck)$. For a homogeneous isotropic d-dimensional environment, it reads $J^{(d)}(\omega) = \alpha\omega(\omega/\omega_{\mathrm{c}})^{d-1} \exp[-\omega/\omega_{\mathrm{c}}]$ [24], featuring the damping strength α and the cutoff frequency ω_{c} which for phonons is the Debye frequency. Then, the summation over the wave vectors $\boldsymbol{k}$ can be replaced by a frequency integration plus an integration over the solid angle. Evaluating the latter, we obtain for the concurrence the exact expression

$$C_{\pm}^{(d)}(t) = \exp\left[-8 \int_0^\infty \mathrm{d}\omega \, J^{(d)}(\omega) \left[1 \pm G^{(d)}\left(\frac{\omega x_{12}}{c}\right)\right] \frac{1 - \cos(\omega t)}{\omega^2} \coth\left(\frac{\hbar\omega}{2k_{\mathrm{B}} T}\right)\right], \tag{6}$$

where $G^{(d)}(x)$ denotes a dimension-dependent geometrical factor which reads $G^{(1)}(x) = \cos(x)$, $G^{(2)}(x) = J_0(x)$, and $G^{(3)}(x) = \sin(x)/x$, with J_0 the zeroth-order Bessel function of the first kind. Note that $G^{(d)}(0) = 1$ for all dimensions, which causes the robust and the fragile behaviour of the Bell states (1) for vanishing separation. For large argument, $G^{(d)}$ decays for $d = 2, 3$, but not for $d = 1$.

Below, we will compare the concurrence of a qubit pair with the single-qubit coherence $|\rho_{0,1}(t)/\rho_{0,1}(0)| = \exp[-\Lambda_{0,1}^{(d)}(t)]$, which we define for $N = 1$, *i.e.* when only one qubit is present. It is formally given by the r.h.s. of eq. (6) but with the replacement $1 \pm G^{(d)} \to \frac{1}{2}$. Thus it is the geometrical factor $G^{(d)}$ which determines the difference between single-qubit decoherence and entanglement decay of a qubit pair. In particular, the dimension dependence of $G^{(d)}$ will turn out to be crucial but has been ignored in prior studies [20]. For qubits coupled to a three-dimensional bath, we find in the remote limit $x_{12} \to \infty$ for both concurrences the relation $C_{\pm}^{(3)}(t) = \exp[-2\Lambda_{0,1}^{(3)}(t)]$, which was also obtained for a model consisting of independent baths [21]. More generally, an intriguing corollary to eq. (6) is the exact relation $C_+^{(d)}(t) \, C_-^{(d)}(t) = \exp[-4\Lambda_{0,1}^{(d)}(t)]$, which for arbitrary separation links the concurrences to the single-qubit coherence. It implies that if one of the concurrences vanishes, the single-qubit coherence must vanish as well. A finite single-qubit coherence, in turn, requires non-vanishing concurrences.

In order to evaluate the concurrences (6), we introduce the scaled time $\tau = \omega_{\mathrm{c}} t$ and transit time $\tau_{12} = \omega_{\mathrm{c}} x_{12}/c$, and the scaled temperature $\theta = k_{\mathrm{B}} T/\hbar\omega_{\mathrm{c}}$. After inserting a Taylor expansion for $\cos(\omega\tau)$, we accomplish the frequency integrals in the resulting series [25]. Then

we obtain for the single-qubit coherence and likewise for the concurrence an infinite product which can be combined into Gamma functions and their derivatives. We restrict ourselves to the cases of one and three dimensions and find for the former case the single-qubit coherence

$$\mathrm{e}^{-\Lambda_{0,1}^{(1)}(\tau)} = \left| \frac{\Gamma(\theta[1+\mathrm{i}\tau])\Gamma(\theta[1-\mathrm{i}\tau])}{\Gamma^2(\theta)} \right|^{4\alpha} \left(1+\tau^2\right)^{2\alpha} , \tag{7}$$

and the concurrence

$$C_\pm^{(1)}(\tau) = \mathrm{e}^{-2\Lambda_{0,1}^{(1)}(\tau)} \left| \frac{\Gamma(\theta[1-\mathrm{i}(\tau_{12}-\tau)])\Gamma(\theta[1-\mathrm{i}(\tau_{12}+\tau)])}{\Gamma^2(\theta[1-\mathrm{i}\tau_{12}])} \right|^{\pm 8\alpha} \left| 1 + \frac{\tau^2}{(1-\mathrm{i}\tau_{12})^2} \right|^{\pm 4\alpha} , \tag{8}$$

where Γ denotes the Euler Gamma function. The corresponding expressions for a three-dimensional environment read

$$\mathrm{e}^{-\Lambda_{0,1}^{(3)}(\tau)} = \exp\left[-4\alpha\left(2\theta^2 \,\mathrm{Re}\left[\Psi_1(\theta) - \Psi_1(\theta[1-\mathrm{i}\tau]) \right] - \frac{\tau^2(\tau^2+3)}{(1+\tau^2)^2} \right) \right] , \tag{9}$$

$$
\begin{aligned}
C_\pm^{(3)}(\tau) = \mathrm{e}^{-2\Lambda_{0,1}^{(3)}(\tau)} \exp\Bigg[&\pm 8\alpha\bigg(\frac{\theta}{\tau_{12}}\,\mathrm{Im}\left[2\Psi_0(\theta[1-\mathrm{i}\tau_{12}]) - \Psi_0(\theta[1-\mathrm{i}(\tau-\tau_{12})]) - \right. \\
&\left. - \Psi_0(\theta[1-\mathrm{i}(\tau+\tau_{12})]) \right] + \frac{\tau^2(\tau^2-\tau_{12}^2+3)}{(1+\tau_{12}^2)(\tau^4-2\tau^2\left[\tau_{12}^2-1\right]+\left[1+\tau_{12}^2\right]^2)} \bigg) \Bigg] ,
\end{aligned}
\tag{10}
$$

where Ψ_0 and Ψ_1 are Di-Gamma and Tri-Gamma functions, respectively. The importance of eqs. (8) and (10) lies in the fact that they explicitly yield the concurrences at all times for arbitrary spatial separations τ_{12}, from a perfect DFS ($\tau_{12} = 0$) to uncorrelated noise ($\tau_{12} \to \infty$). In both expressions, the respective single-qubit coherences (7) and (9) appear.

Time-evolution of the robust Bell state. – Let us first focus on the entanglement of a qubit pair that starts out in the robust state $|\psi_-\rangle$ and couples to a one-dimensional heat bath. Figure 1a depicts the time evolution of the concurrence for a temperature well below the Debye temperature. For vanishing separation, $\tau_{12} = 0$, the concurrence $C_-^{(1)}(\tau)$ remains at its initial value 1. This reflects the fact that then $|\psi_-\rangle$ lives in a DFS and, consequently, is robust. For $\tau_{12} > 0$, we find that the concurrence initially decays until the transit time τ_{12} is reached. At time $\tau = \tau_{12}$, the decay comes to a standstill and the concurrence remains at a finite value $C_-^{(1)}(\tau \to \infty) = (1+\tau_{12}^2)^{4\alpha} |\Gamma(\theta[1-\mathrm{i}\tau_{12}])/\Gamma(\theta)|^{16\alpha}$ which becomes $(1+\tau_{12}^2)^{-4\alpha}$ for $\theta \to 0$. The time evolution allows the interpretation that before the transit time is reached, uncorrelated noise affects the qubits and entails an entanglement decay. After the transit time, the noise at the two positions is sufficiently correlated to establish a decoherence-free subspace. In the remote limit $\tau_{12} \to \infty$, the concurrence of the robust state finally vanishes and, thus, the residual entanglement for finite τ_{12} can be attributed to spatial bath correlations. We emphasise that for a one-dimensional bath, this scenario holds true for all temperatures.

An intuitive physical picture for the observed entanglement dynamics is that at long times, decoherence is governed by the low-frequency modes of the bath. Owing to their large wavelengths, these modes act effectively as a collective bath coordinate which leaves the entanglement robust. In higher dimensions, the role of the low-frequency modes is suppressed and, thus, the long-time behaviour may be significantly different [7].

Figure 1c reveals that for a three-dimensional bath, in general the concurrence (10) decays and saturates at a finite value which stays larger for closer qubits. In the experimentally relevant limit of low temperatures and $\tau_{12}\theta^2 \lesssim 1$, the final concurrence emerges as

$$C_-^{(3)}(\tau \to \infty) = \exp\left[-8\alpha\left(1 - \frac{1}{1+\tau_{12}^2} + \frac{\pi^4\tau_{12}^2\theta^4}{45} \right) \right] . \tag{11}$$
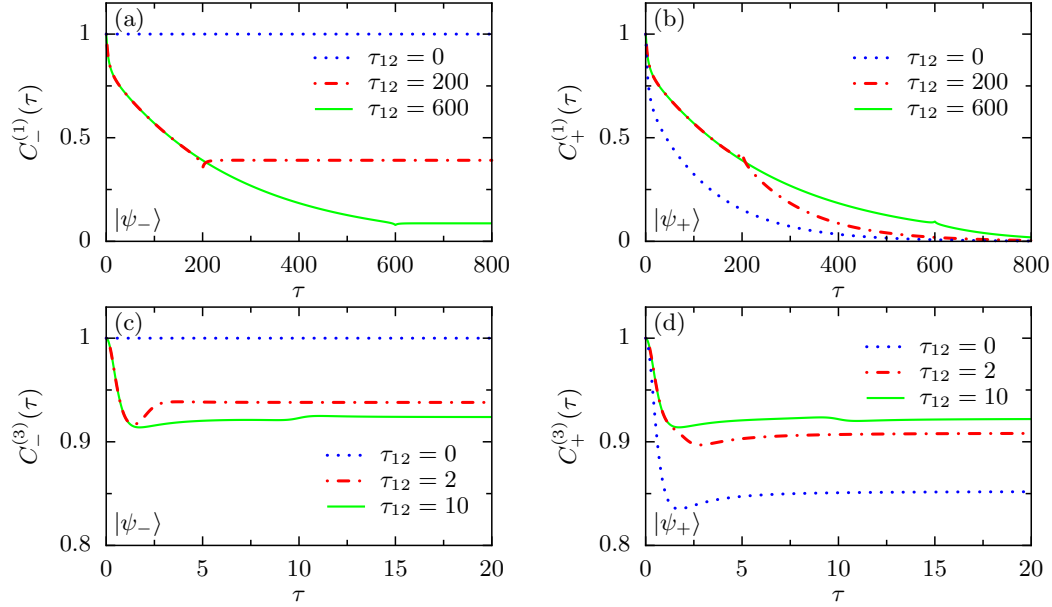
Fig. 1 – Time evolution of the concurrence for the robust (left) and the fragile (right) Bell state for various spatial separations $c\tau_{12}/\omega_c$. The qubits couple with a strength $\alpha = 0.01$ to a one-dimensional (a,b) and a three-dimensional (c,d) bath, respectively, at temperature $\theta = k_B T/\hbar\omega_c = 0.015$. For qubit energies $\Delta = 0.01\hbar\omega_c$, the time range in the upper (lower) plots corresponds to 1.3 (0.03) coherent oscillations.

However, the saturation generally occurs already at a time $\tau \ll \tau_{12}$, *i.e.* long before a field distortion can have propagated from one qubit to the other. To make this statement more quantitative, we numerically estimate the duration τ^* of the concurrence decay by the time at which 90% of the decay has happened. Figure 2 shows that $\tau^* \approx 1$. In particular, τ^* is independent of the spatial separation τ_{12}, unless the qubits are very close. Hence the saturation of $C_-^{(3)}(\tau)$ cannot be explained as a delayed build-up of a DFS. Instead, a single-qubit mechanism must be at work, since at times $\tau < \tau_{12}$, the qubits experience effectively *uncorrelated* noise. This conjecture is supported by the resemblance of $C_-^{(3)}(\tau)$ to the single-qubit coherence $\exp[-\Lambda_{0,1}^{(3)}(\tau)]$ shown in fig. 3. A second difference to the one-dimensional case concerns the remote limit of the qubits: For $\tau_{12} \to \infty$, the stationary value is still finite. In this limit, $G^{(3)}$ in eq. (6) is negligible and the concurrence is given by the square of the finite single-qubit coherence.

The concurrence (11) depends only quadratically on τ_{12} and, thus, is rather robust against variations of the separation, provided the separation is small. Such robustness was predicted [10] and confirmed experimentally [11] for symmetry-breaking perturbations. Interestingly, eq. (11) shows that the concurrence is robust against temperature variations as well, about which the theory in ref. [10] makes no predictions.

Time-evolution of the fragile Bell state. – If the qubits are initially in the fragile state $|\psi_+\rangle$, a one-dimensional bath causes an entanglement decay that becomes faster once the transit time is reached; see fig. 1b. As for the robust state $|\psi_-\rangle$, cooperative effects only set in after a time τ_{12}. Whether the qubits are spatially separated or not, for a one-dimensional
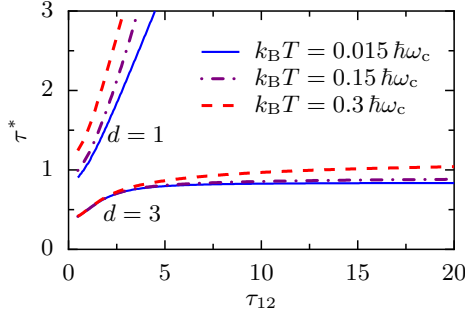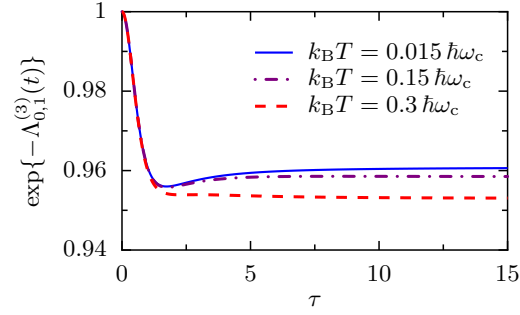
Fig. 2

Fig. 3

Fig. 2 – Duration τ^* of the concurrence decay of the robust Bell state $|\psi_-\rangle$ as a function of the transit time τ_{12} for one- and three-dimensional baths. The dissipation strength is $\alpha = 0.01$.

Fig. 3 – Time evolution of the single-qubit coherence for various temperatures. The qubit is coupled with the strength $\alpha = 0.01$ to a three-dimensional bath.

environment their concurrence $C_+^{(1)}(\tau)$ ultimately decays to zero. The three-dimensional case again bears more surprises, as seen in fig. 1d: the concurrence (10) decays, but the final value $C_+^{(3)}(\tau \to \infty)$ is nonzero, in contrast to earlier statements [18,19]. For low temperatures such that $\theta \lesssim 1, \tau_{12}^{-1/2}$, the long-time limit is given by

$$C_+^{(3)}(\tau \to \infty) = \exp\left[-8\alpha\left(1 + \frac{1}{1+\tau_{12}^2} + \frac{2\pi^2\theta^2}{3}\right)\right]. \tag{12}$$

This asymptotic value can be increased by reducing the temperature and by increasing the qubit separation. For a separation $\tau_{12} \gtrsim 1$, $C_+^{(3)}(\infty) = C_-^{(3)}(\infty)$, i.e. the concurrence of both the "robust" and the "fragile" state become identical; cf. the solid lines in figs. 1c, d.

Conclusions. – The explicit evaluation of the exact reduced dynamics of two qubits with a nondemolition coupling to a bosonic heat bath allowed us to investigate the consequences of a spatial qubit separation. We focused on two Bell states whose entanglement for vanishing separation is either robust or fragile. The most significant consequence of a finite spatial separation is that the entanglement of the robust Bell state no longer remains robust: it decays initially, yet after a time $t^* = \tau^*/\omega_c$, it saturates. Indeed, it is interesting to find stable finite bipartite entanglement even at high temperatures in our macroscopic system-bath model, which resembles recent results for a quite different model [26]. The duration t^* of the entanglement decay depends sensitively on the dimension of the environment: In one dimension, it equals the transit time of the field from one qubit to the other. In three dimensions, by contrast, the saturation is governed by a single-qubit effect and t^* is approximately given by the inverse of the cutoff frequency ω_c. Rather surprisingly, these durations exhibit only a weak temperature dependence.

In a typical solid-state substrate, the Debye temperature is of the order $500\,\text{K}$ which corresponds to $\hbar\omega_c = 40\,\text{meV}$. For a qubit separation of $300\,\text{nm}$ and a sound velocity $c = 3000\,\text{m/s}$, we find for a one-dimensional environment $t^* = \tau^*/\omega_c \approx 10^{-10}\,\text{s}$ while in three dimensions, this time scale is much shorter: $t^* \approx 10^{-13}\,\text{s}$. For a typical tunnel splitting $\Delta = 10\,\mu\text{eV}$, the coherent oscillation period is $2\pi\hbar/\Delta \approx 10^{-10}\,\text{s}$, so that in the three-dimensional environment the concurrence of the robust Bell state undergoes a decay only during a very

short initial stage. At later times, a decoherence-free subspace is established and, thus, the concurrence stays robust.

For the fragile Bell state, a three-dimensional environment in combination with a finite qubit separation prevents the entanglement from decaying entirely. If the qubits are sufficiently well separated, *i.e.* for $x_{12} \gtrsim c/\omega_c$, the entanglement of the "fragile" and the "robust" Bell state even assumes practically the same final value. In the above example, this is already the case if the qubit-qubit distance is larger than $1\,\mu$m, which usually holds for solid-state qubits. For typical parameters, the concurrence initially drops to and then remains at values of the order 0.9 already long before a first coherent oscillation is performed. Thus, uncorrelated phase noise creates decoherence-poor subspaces, which might be used for quantum information processing when complemented with quantum error correction protocols.

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