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Noisy dynamics of magnetic flux in mesoscopic cylinders

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Abstract. We study magnetic fluxes and currents in mesoscopic systems of cylindrical symmetry like rings, toroids and cylinders. We analyze the time evolution of the magnetic flux and the characteristic time of a formation of the ordered state. We investigate how, starting from some symmetric initial state, the magnetic flux or the current approach their corresponding asymptotic state.

1. Introduction

Quantum phenomena manifested at the mesoscopic level have attracted much theoretical and experimental attention [1]. Phase coherence and persistent currents in mesoscopic systems of cylindrical symmetry can be mentioned as examples. Persistent currents predicted as early as in 1938 [2] have been observed experimentally only since 1990 [3]. In this paper we study the dynamics of magnetic fluxes and currents in mesoscopic cylinder under conditions when dissipation and fluctuations can play an important role [4].

2. Evolution equation for the magnetic flux

We consider a collection of rings composed of individual current channels, which form a cylinder. There are N_z channels in the direction of the cylinder axis and N_r in the direction of the cylinder radius. We assume that the thickness of the cylinder wall is small as compared with the radius. Because of the mutual inductance between rings, the current in one ring induces a flux in the other rings. In turn, the flux induces a current, and so on. We assume that the rings are not contacted. So, there is no tunneling of electrons among the channels and the charge carriers moving in the different rings are independent. It has been shown [5] that the effective interaction between the ring currents, when taken in the selfconsistent mean field approximation, results in the magnetic flux $\phi_{ind} = LI_{tot}$ felt by all electrons, where L is the cylinder inductance and I_{tot} is the total current in the cylinder. At temperature $T > 0$, the total magnetic flux ϕ consists of a sum of the external flux ϕ_{ext} and the flux ϕ_{ind} stemming from the total current, which in turn is a sum of the dissipative 'normal' Ohmic current and the non-dissipative persistent current resulting from the presence of the 'phase-coherent' electrons in the system [4], i.e. it assumes the form:

$$\phi = \phi_{ext} + LI_{tot} = \phi_{ext} + L[I_{coh}(\phi, T) + I_{nor}(\phi, T)]. \quad (1)$$

Note that ϕ_{ext} is induced by an external magnetic field and can either take a fixed value or it can be a random function. Taking into account an explicit form of the "normal" current, as it follows from the Lenz's and Ohm's rules complemented with the Johnson-Nyquist noise term [6], Eq. (1) takes the form [4]

$$\frac{1}{R}\frac{d\phi}{dt} = -\frac{1}{L}(\phi - \phi_{ext}) + I_{coh}(\phi, T) + \sqrt{\frac{2k_B T}{R}}\,\Gamma(t), \tag{2}$$

where R is the resistance of the cylinder, k_B is the Boltzmann constant and $\Gamma(t)$ denotes a zero-mean Gaussian delta-correlated white noise modeling Nyquist equilibrium current noise. This equation takes the form of a classical Langevin equation and it constitutes our basic evolution equation.

The dimensionless form of (2) reads [4]

$$\dot{x} = -V'(x) + \sqrt{2D}\,\tilde{\Gamma}(\tilde{t}), \tag{3}$$

where the dot denotes a derivative with respect to the rescaled time $\tilde{t} = t/\tau_0$ with $\tau_0 = L/R$ being the relaxation time of the averaged normal current. The prime denotes a derivative with respect to the dimensionless flux $x = \phi/\phi_0$, where the flux quantum $\phi_0 = h/e$ is the ratio of the Planck constant h and the elementary charge e. The generalized potential reads $V(x) \equiv V(x, \lambda, i_0, p, T) = \frac{1}{2}x^2 - \lambda x - i_0 F(x, p, T)$, where $\lambda = \phi_{ext}/\phi_0$ is the rescaled external flux. The prefactor is $i_0 = NLI_0/\phi_0$ with $N = N_z N_r$. The function $F(x) \equiv F(x, p, T) = \int f(x, p, T)dx$ characterizes the coherent current and $f(x, p, T) = pf_e(x, T) + (1-p)f_o(x, T)$, where $f_e(x, T) = \sum_{n=1}^{\infty} A_n(T)\sin(2n\pi x) = f_o(x - \frac{1}{2}, T)$. The functions f_e and f_o describe the coherent current flowing in the channel with an even or odd number of electrons, respectively [7]. The amplitudes $A_n(T)$ are decreasing functions of temperature. Their explicit forms are given in [4, 7]. The quantity $p \in [0, 1]$ denotes the probability of the occurrence of the single current channel with an even number of electrons. In the following we consider the symmetric case, i.e. $p = 1/2$.

The dimensionless intensity D of rescaled Gaussian white noise $\tilde{\Gamma}(\tilde{t}) \equiv \sqrt{\tau_0}\;\Gamma(\tau_0\tilde{t})$ is $D = k_B T/2\epsilon_0$, where the characteristic magnetic energy $\epsilon_0 = \phi_0^2/2L$. Let us notice that the resistance R does not enter into the rescaled equation (3). Moreover, the prefactor i_0 depends on the geometry and material of the sample [8]. Although formally the above equations can also be applied to a single mesoscopic ring or toroids, we consider a cylinder because for such a system the prefactor i_0 in the effective potential can take sufficiently large value (because $N = N_z N_r$). We choose the parameters of the system in such a way that the diffusion coefficient $D \sim 0.001T/T^*$ and $i_0 = 1$. The characteristic temperature T^* is defined by the relation $k_B T^* = \Delta_F/2\pi^2$, where Δ_F marks the energy gap at the Fermi surface. The values of parameters which occur in the amplitudes $A_n(T)$ are the same as in [4, 8].

As it follows from (1), the total current I_{tot} is linearly related to the magnetic flux ϕ (and to the rescaled flux x). As a consequence, the properties and behavior of the current are identical to the properties and behavior of the magnetic flux. Therefore, below we use equivalently these two characteristics of the system. ¿From now on, we will use only the dimensionless variables and omit the 'tilde' for the rescaled time, $\tilde{t} \equiv t$. Temperature will be measured in units of T^*.

3. Dynamics of the magnetic flux

First, let us consider the deterministic case of the Langevin stochastic equation (3) formally neglecting the Nyquist noise term $\tilde{\Gamma}(\tilde{t})$, i.e.,

$$\dot{x} = -V'(x). \tag{4}$$

The stationary solutions x_s of (4), for which $\dot{x}_s = 0$, correspond to extrema of the generalized potential $V(x)$. The solutions x_s of the gradient differential equation (4) are stable provided they correspond to a minimum of the generalized potential and they are unstable in the case of a maximum. For the case when the externally induced flux is zero we have $\lambda = 0$ and when $x_s \neq 0$, the stable stationary states of the dynamical system (4) correspond to the so called *self-sustaining currents* which can flow without any external driving [9].

The Langevin equation (3) defines a Markov diffusion process. Its probability density $p(x,t)$ obeys the Fokker-Planck equation [10]

$$\frac{\partial}{\partial t} p(x,t) = \frac{\partial}{\partial x} V'(x) p(x,t) + D \frac{\partial^2}{\partial x^2} p(x,t) \tag{5}$$

with the natural boundary condition, i.e. $\lim_{|x|\to\infty} p(x,t) = 0$.

The stationary solution $p_s(x)$ is asymptotically stable and ergodic, i.e. it does not depend on the initial condition. Its explicit form is

$$p_s(x) = N_0 \mathrm{e}^{-V(x)/D} \tag{6}$$

with the normalization constant N_0. The properties of steady-states were investigated in [4, 8]. In the following we study the time evolution of the probability density $p(x,t)$ in the absence of the external flux, $\lambda = 0$, and for the symmetric initial conditions. As an example, we assume that the initial flux probability distribution is Gaussian, i.e. $p(x,0) = \exp(-x^2/2\sigma)/\sqrt{2\pi\sigma}$ with $\sigma = 0.005$. The numerical solutions of the Fokker-Planck equation (5) are depicted below.

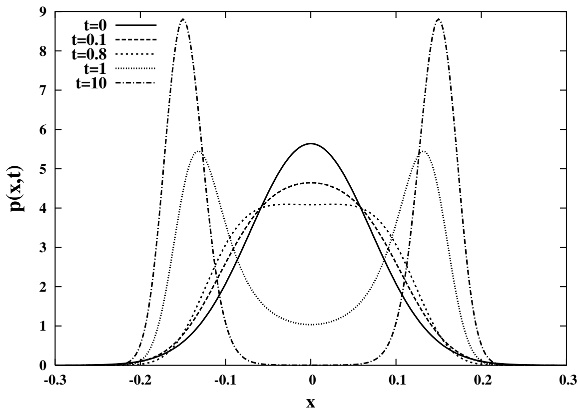

Figure 1. The probability density of the magnetic flux at several instants of time measured in units of τ_0. The temperature is set at $T = 1$. The graph at t=10 and the asymptotic probability density are indistinguishable.

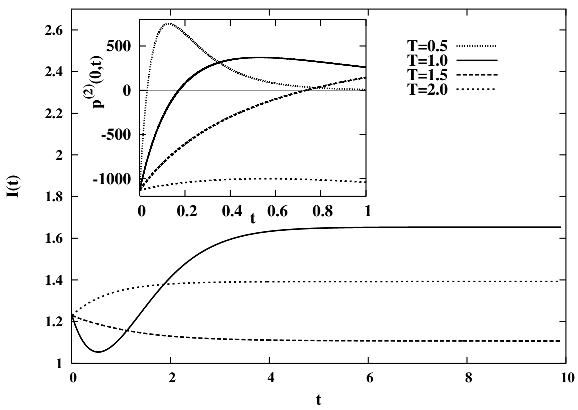

Figure 2. The Shannon information defined by the equation (7)(main graph) and the second derivative of the probability density at zero (inset) as a function of time in units of τ_0 and different temperatures. The change in sign of $p^{(2)}(0,t)$ indicates the formation of the bistability.

Because the initial condition as well as the generalized potential are symmetric with respect to the magnetic flux x, the averaged magnetic flux becomes $< x(t) >= 0$. ¿From the practical point of view, the experimental results are accumulated around the most probable values which correspond to maxima of the probability distribution $p(x,t)$. In the stationary state, they correspond to the minima of the generalized potential $V(x)$. If the maximum x_M of $p(x,t)$ is non-zero, it corresponds to the case of the self-sustaining current and an ordered state. In the stationary case and for the fixed values of parameters that we have assumed, the ordered states exist only below some critical temperature T_c, for details see [8]. For the assumed values of the parameters, $T_c \approx 1.66$. In figure 1 we depict the time evolution of $p(x, t)$ in the bistability case when in the stationary regime two symmetric ordered states coexist, i.e. when $p_s(x)$ has two symmetric maxima. One can observe the time-transition from the monostable state to the bistable state. We can introduce some characteristic time t_f of the formation of the ordered states. It can be related to the time when the maximum of $p(x, t)$ at $x = 0$ changes to the non-zero value, $x_M \neq 0$. It leads to the definition of this characteristic time t_f as the time when $p^{(2)}(0, t_f) = 0$, i.e. when the second derivative with respect to x at zero changes its sign (note that for this definition the time t_f depends on the initial condition). The initially 'concave-up' function becomes 'concave-down' at zero. We see that this time t_f is an increasing function of temperature, cf. figure 2. It tends to infinity when temperature tends to its critical value, $T \to T_c$. Above the critical temperature [4], there are no values of parameters which can guarantee the change of sign of $p(x, t)$ at $x = 0$. Let us notice that for vanishing temperature the curve representing $p^{(2)}(0, t)$ approaches asymptotically zero. It is clear since at $T = 0$ the asymptotic probability density of the flux is $p_s(x) = \delta(x - x_s)/2 + \delta(x + x_s)/2$.

The quantity of interest is the entropy or the information gained or loosed during the evolution. We investigate the time-evolution of the Shannon information defined by the relation

$$I[p](t) = \int_{-\infty}^{\infty} p(x, t) \ln(p(x, t)) dx. \tag{7}$$

The time evolution of the Shannon information belongs to one of three qualitatively different classes. At larger temperatures, the information can be either an increasing or a decreasing function of time, depending on the relation between the initial and asymptotic states. The most interesting non-monotonic behavior appears at low temperatures. The initial information lost connected with the formation of bistability is followed by the information gain originating from the quasi-localization of the flux x in maxima of $p(x, t)$. This localization is caused by a relatively long escape time at low temperatures [4].

In conclusion, we have investigated the time evolution of the probability distribution of the magnetic flux in a mesoscopic cylinder. For our chosen parameters, there is a critical temperature below which ordered states can exist in the system. This bistability phenomenon corresponds to the self-sustaining currents flowing in the system. For this case, we have defined the characteristic time of formation of the ordered states. It is an increasing function of temperature and it diverges as temperature approaches its critical value.

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