Coherence Stabilization of a Two-Qubit Gate by ac Fields

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We consider a CNOT gate operation under the influence of quantum bit-flip noise and demonstrate that ac fields can change the qubit Hamiltonian in such a way that it approximately commutes with the bath coupling. Then the noise effectively acts as phase noise which improves coherence up to several orders of magnitude while the gate operation time remains unchanged. Within a high-frequency approximation, both purity and fidelity of the gate operation are studied analytically. The numerical treatment with a Bloch-Redfield master equation confirms the analytical results.

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Despite the remarkable experimental realization of qubits [1–3] and two-qubit gates [4] in condensed matter systems, the construction of a working quantum computer remains an elusive goal, not only due to deficiencies of the control circuitry, but also due to the unavoidable coupling to the environment. Several proposals to overcome the ensuing decoherence have been put forward, such as the use of decoherence free subspaces [5–9], coherence-preserving qubits [10], quantum Zeno subspaces [11], dynamical decoupling [12–15], and coherent destruction of tunneling [16].

A single qubit under the influence of bit-flip noise can be modeled by the spin-boson Hamiltonian $H_0 = -\frac{\Delta}{2}\sigma^z + \sigma^x\xi$, where $\sigma^{x,z}$ denotes Pauli matrices and ξ is a shorthand notation for the quantum noise specified below. The influence of the noise is governed by its spectral density at the tunneling frequency $\Delta/\hbar$. A possible driving field may couple to any projection $\vec{n}$ of the (pseudo) spin operator $\vec{\sigma}$, i.e., be proportional to $\vec{n}\cdot\vec{\sigma}$. In Ref. [16], two particular choices have been studied and compared against each other: A driving of the form $H(t) = A\sigma^z\cos(\Omega t)$ commutes with the static qubit Hamiltonian but not with the bath coupling $\sigma^x\xi$. For a proper driving amplitude, this eliminates noise with frequencies below the driving frequency. Therefore, the latter should lie above the cutoff frequency of the bath. This scheme represents a continuous-wave version of dynamical decoupling. By contrast, a driving of the type $H(t) = A\sigma^x\cos(\Omega t)$ renders the qubit-bath coupling unchanged but renormalizes the tunnel splitting Δ towards smaller values and thereby causes the so-called coherent destruction of tunneling (CDT) [17,18]. Then, decoherence is determined by the spectral density of the bath at a lower effective tunnel frequency. For an Ohmic bath being linear in the frequency, the consequence is that both decoherence and the coherent oscillations are slowed down by the same factor [16]. Therefore, the number of coherent oscillations is not enlarged and, thus, for single-qubit operations, CDT might be of limited use.

In this Letter, we propose a coherence stabilization scheme for a CNOT gate based on an isotropic Heisenberg interaction [19,20]. Our scheme does not suffer from the drawbacks mentioned above because (i) it involves only intermediately large driving frequencies that can lie well below the bath cutoff and (ii) it does not increase the gate operation time. Since we shall employ a driving field that couples to the *same* coordinate as the quantum noise, the present coherence stabilization is distinctly different from the recently measured dynamical decoupling of a spin pair from surrounding spin pairs [21].

CNOT gate with bit-flip noise.—We consider a pair of qubits described by the Hamiltonian [19,20,22,23]

$$H_{\rm gate} = \frac{1}{2}\sum_{j=1,2}(\Delta_j\sigma_j^z + \epsilon_j\sigma_j^x) + J\vec{\sigma}_1\cdot\vec{\sigma}_2, \qquad (1)$$

with a qubit-qubit coupling of the Heisenberg type, where j labels the qubits. In order to construct a quantum gate, the tunnel splittings Δ_j, the biases ϵ_j, and the qubit-qubit coupling J have to be controllable in the sense that they can be turned off and that their signs can be changed. Then, a suitable sequence of interactions yields the CNOT operation [19,20,22,23]

$$U^H_{\rm CNOT} \sim U_H(\pi/8)U_{1z}(\pi)U_H(\pi/8), \qquad (2)$$

where $U_{1z}(\varphi) = \exp(-i\varphi\sigma_1^z/2)$ represents a rotation of qubit 1 around the z axis and $U_H(\varphi) = \exp(-i\varphi\vec{\sigma}_1\cdot\vec{\sigma}_2)$ describes the time evolution due to the qubit-qubit interaction. The symbol $\sim$ denotes equality up to local unitary transformations, i.e., transformations that act on only one qubit. For single-qubit operations, it has been shown that pulse sequences [12–15] and harmonic driving fields [16] can suppress decoherence. Therefore, we focus in the present work on decoherence during the stage of the qubit-qubit interaction and, thus, take as a working hypothesis that the coherence of one-qubit operations can be stabilized ideally. Then the remaining decoherence takes place while $\Delta_j = \epsilon_j = 0$ and $J > 0$ during the required total qubit-qubit interaction time $t_J = \pi\hbar/4J$, which corresponds to twice the angle $\varphi = \pi/8$.

The bit-flip noise is specified by the system-bath Hamiltonian $H = H_{\rm gate} + H_{\rm coupl} + H_{\rm bath}$ where $H_{\rm coupl} = \frac{1}{2}\sum_{j=1,2}\sigma_j^x\sum_\nu\hbar c_\nu(a^\dagger_{j\nu} + a_{j\nu})$ describes the coupling of qubit j to a bath of harmonic oscillators with frequencies ω_ν, $H_{\rm bath} = \sum_{j\nu}\hbar\omega_\nu a^\dagger_{j\nu}a_{j\nu}$, and the spectral density $I(\omega) = \pi\sum_\nu c_\nu^2\delta(\omega - \omega_\nu)$. Within the present work, we consider the so-called Ohmic spectral density $I(\omega) = 2\pi\alpha\omega e^{-\omega/\omega_c}$ with the dimensionless coupling strength α and the cutoff frequency ω_c. In order to complete the model, we specify the initial condition of the Feynman-Vernon type; i.e., initially, the bath is in thermal equilibrium and uncorrelated with the system, $\rho_{\rm tot}(t_0) = \rho(t_0) \otimes R_{\rm bath,eq}$, where ρ is the reduced density operator of the two qubits and $R_{\rm bath,eq} \propto \exp(-H_{\rm bath}/k_BT)$ is the canonical ensemble of the bath.

If the dissipation strength is sufficiently small, $\alpha \ll 1$, the dissipative system dynamics is well described within a Born-Markov approach. There, one starts from the Liouville–von Neumann equation $i\hbar\dot\rho_{\rm tot} = [H, \rho_{\rm tot}]$ for the total density operator and obtains by standard techniques the master equation [24]

$$\dot\rho = \frac{1}{i\hbar}[H_{\rm gate}, \rho] - \sum_j[\sigma_j^x, [Q_j(t), \rho]] - \sum_j[\sigma_j^x, \{P_j(t), \rho\}] \equiv \frac{1}{i\hbar}[H_{\rm gate}, \rho] - \Lambda(t)\rho \qquad (3)$$

with the anticommutator $\{A, B\} = AB + BA$ and

$$Q_j(t) = \frac{1}{4\pi}\int_0^\infty d\tau \int_0^\infty d\omega S(\omega)\cos(\omega\tau)\tilde\sigma_j^x(t - \tau, t). \qquad (4)$$

Here, $S(\omega) = I(\omega)\coth(\hbar\omega/2k_BT)$ is the Fourier transformed of the symmetrically ordered equilibrium correlation function $\frac{1}{2}\langle\{\xi_j(\tau), \xi_j(0)\}\rangle_{\rm eq}$ of the collective bath coordinate $\xi_j = \sum_\nu c_\nu(a^\dagger_{j\nu} + a_{j\nu})$. The notation $\tilde X(t, t')$ is shorthand for the Heisenberg operator $U^\dagger(t, t')XU(t, t')$ with U being the propagator of the coherent system dynamics. Note that $S(\omega)$ and $I(\omega)$ are independent of j due to the assumption of two identical environments. Replacing in Eq. (4) the term $S(\omega)\cos(\omega\tau)$ by $I(\omega)\sin(\omega\tau)$ yields the operator $P_j(t)$. We emphasize that the particular form (3) of the master equation is valid also for an explicitly time-dependent qubit Hamiltonian.

The heat baths, whose influence is described by the second and third terms of the master equation (3), lead to decoherence, i.e., the evolution from a pure state to an incoherent mixture. Decoherence can be measured by the decay of the purity $\mathrm{tr}(\rho^2)$ from the ideal value 1. The gate purity (later referred to as "purity") $\mathcal{P}(t) = \overline{\mathrm{tr}(\rho^2(t))}$, which characterizes the gate independently of the specific input, results from the ensemble average over all pure initial states [25]. For weak dissipation, the purity is determined by its decay rate at initial time

$$\dot{\mathcal{P}}(t)|_{t=0} = -2\,\overline{\mathrm{tr}(\rho\Lambda\rho)} = -\frac{4}{d(d+1)}\sum_j \mathrm{tr}(\sigma_j^x Q_j), \qquad (5)$$

where $d = 4$ is the dimension of the system Hilbert space.

In order to evaluate the purity decay, we need explicit expressions for the operators Q_j and, thus, have to compute the Heisenberg operators $\tilde\sigma_j^x(t - \tau, t)$ for the Hamiltonian $H_0 = J\vec\sigma_1 \cdot \vec\sigma_2$. This calculation is most conveniently performed in the basis of the total (pseudo) spin $\vec L = \frac{1}{2} \times (\vec\sigma_1 + \vec\sigma_2)$. We finally arrive at $\dot{\mathcal{P}} = -\frac{2}{5}\{S(0) + S(4J/\hbar)\}$, where we have ignored Lamb shifts and defined $S(0) \equiv \lim_{\omega\to 0}S(\omega) = 4\pi\alpha k_BT/\hbar$. In particular, we find that for low temperatures, $k_BT \lesssim J$, decoherence is dominated by $S(4J/\hbar)$ such that $\dot{\mathcal{P}} \approx -16\pi\alpha J/5\hbar$. This part reflects the influence of the so-called quantum noise, which is temperature independent.

ac driving field.—In order to manipulate the coherence properties, we act upon qubit 1 by an ac field that causes a time-dependent level splitting according to

$$H_{\rm ac}(t) = f(t)\sigma_1^x, \qquad (6)$$

where $f(t)$ is a $2\pi/\Omega$-periodic function with zero time average. Since the driving acts only during the finite time t_J while the Heisenberg coupling is switched on, its spectrum has a dispersion $\Delta\omega \approx 1/t_J$. To keep its influence small, we have to choose $\Omega \gg 1/t_J$.

Next, we derive within a high-frequency approximation analytical expressions for both the coherent propagator $U(t, t')$ and the purity decay (5). We start out by transforming the total Hamiltonian into a rotating frame with respect to the driving via the unitary transformation

$$U_{\rm ac}(t) = e^{-i\phi(t)\sigma_1^x}, \qquad \phi(t) = \frac{1}{\hbar}\int_0^t dt' f(t'). \qquad (7)$$

This yields the likewise $2\pi/\Omega$-periodic interaction-picture Hamiltonian $\tilde H(t) = U^\dagger_{\rm ac}(t)H_{\rm gate}U_{\rm ac}(t)$ and the S-matrix $S(t, t') = U_0^\dagger(t)U(t, t')U_0(t')$. For large driving frequencies $\Omega \gg J/\hbar$, it is possible to separate time scales and thereby replace $\tilde H(t)$ by its time average

$$\bar H = (J - J_\perp)\sigma_1^x\sigma_2^x + J_\perp\vec\sigma_1 \cdot \vec\sigma_2, \qquad (8)$$

where the constant $J_\perp = J\langle\cos[2\phi(t)]\rangle_{2\pi/\Omega}$ denotes an effective interaction "transverse" to the driving and $\langle\ldots\rangle_{2\pi/\Omega}$ the time average over the driving period. Consequently, we find $S(t, t') = \exp\{-i\bar H(t - t')/\hbar\}$, such that the propagator of the *driven* system reads

$$U_{\rm eff}(t, t') = e^{-i\phi(t)\sigma_1^x}e^{-i\bar H(t-t')/\hbar}e^{i\phi(t')\sigma_1^x}. \qquad (9)$$

Having this propagator at hand, we are in the position to derive explicit expressions for the operators $\sigma_j^x(t - \tau, t)$, Q_j, and P_j and, therefore, also for the generator of the dissipative dynamics Λ. Again, the calculation is conveniently done in the basis of the total spin $\vec L$ and L_x which, owing to the relation $\sigma_1^x\sigma_2^x = \frac{1}{2}(\sigma_1^x + \sigma_2^x)^2 - 1$, is an eigenbasis of the Hamiltonian (8). We insert the resulting expression for Λ into Eq. (5) and finally obtain the manipulated purity decay

$$\dot{\mathcal{P}} = -\frac{2}{5}\{S(0) + S(4J_\perp/\hbar)\}. \quad (10)$$

For $f(t) \equiv 0$, we find $J_\perp = J$ such that Eq. (10) agrees with what we found in the static case; otherwise, the inequality $|J_\perp| < J$ holds and, thus, the bath correlation function S in Eq. (10) has to be evaluated at a lower frequency. For an Ohmic or a super-Ohmic bath, $S(\omega)$ is a monotonically increasing function and, consequently, the ac field reduces $\dot{\mathcal{P}}$ (unless $J > \omega_{\text{cutoff}}$).

The purity decay assumes its minimum for $J_\perp = 0$. This condition marks the working points on which we shall focus henceforth. For an Ohmic spectral density $I(\omega) = 2\pi\alpha\omega$, the purity decay at the working points becomes $\dot{\mathcal{P}} = -\frac{4}{5}S(0) = -8\pi\alpha k_B T/5$. This value has to be compared to the purity decay in the absence of driving: An analysis reveals that for $k_B T > J$, decoherence is essentially independent of the driving. By contrast for low temperatures, $k_B T < J$, the driving reduces the decoherence rate by a factor $k_B T/2J$. This low-temperature behavior results from the fact that for $J_\perp = 0$, the effective Hamiltonian (8) commutes with the qubit-bath coupling operators σ_j^x, which are not affected by the transformation (7). As a consequence, dissipative transitions become impossible and for the effective Hamiltonian, the bath acts as pure phase noise whose influence is proportional to the temperature. In that sense, the present scheme is complementary to coherence-preserving qubits [10], for which heating errors are the only source of decoherence.

For a rectangular driving for which $f(t)$ switches between the values $\pm A/2$, the condition $J_\perp = 0$ yields $A = \hbar\Omega$, which corresponds to two π pulses per period. For a harmonic driving, $f(t) = A\cos(\Omega t)/2$, one obtains $J_\perp = JJ_0(A/\hbar\Omega)$, where J_0 denotes the zeroth-order Bessel function of the first kind. Then, at the working points $J_\perp = 0$, the ratio $A/\hbar\Omega$ assumes a zero of J_0, i.e., one of the values 2.405…, 5.520…, 8.654…, ….

So far, we ignored that the driving also affects the coherent dynamics and, thus, the pulse sequence of the CNOT operation needs a modification: At the working points of the driven system, the propagator (9) becomes $U_{\text{eff}}(t, t') = \exp[-iJ\sigma_1^x\sigma_2^x(t - t')/\hbar]$; i.e., it represents the time evolution caused by a so-called Ising interaction $J\sigma_1^x\sigma_2^x$. This allows one to implement the alternative CNOT operation $U_{\text{CNOT}}^I \sim \exp(-i\pi\sigma_1^x\sigma_2^x/4) = U_{\text{eff}}(t + t_J, t)$ [22,26]. Note that the interaction time $t_J = \pi\hbar/4J$ is the same as for the original gate operation U_{CNOT}^H. Since $U_{\text{ac}}(2\pi/\Omega)$ is the identity [cf. Eq. (7)], we assume for convenience that t_J is an integer multiple of the driving period $2\pi/\Omega$, i.e., $\Omega = 8kJ/\hbar$ with integer k.

Numerical solution.—Our analytical results for the purity decay rely on a high-frequency approximation that is correct only to lowest order in $J/\hbar\Omega$. Thus, they should be compared to the exact numerical solution of the master Eq. (3). An efficient scheme for that purpose is a modified Bloch-Redfield formalism whose cornerstone is a decomposition into the Floquet basis of the driven qubits [24]: According to the Floquet theorem, the Schrödinger equation of a periodically driven quantum system possesses a complete set of solutions of the form $|\psi_\alpha(t)\rangle = \exp(-i\epsilon_\alpha t/\hbar)|\phi_\alpha(t)\rangle$, with the quasienergies ϵ_α and the Floquet states $|\phi_\alpha(t)\rangle = |\phi_\alpha(t + T)\rangle$, which are computed from the eigenvalue equation $[H(t) - i\hbar d/dt]|\phi_\alpha(t)\rangle = \epsilon_\alpha|\phi_\alpha(t)\rangle$. In the Floquet basis $\{|\phi_\alpha(t)\rangle\}$, the master Eq. (3) assumes the form $\dot{\rho}_{\alpha\beta} = -\frac{i}{\hbar}(\epsilon_\alpha - \epsilon_\beta)\rho_{\alpha\beta} - \sum_{\alpha'\beta'}\Lambda_{\alpha\beta,\alpha'\beta'}\rho_{\alpha'\beta'}$. Moreover, for weak dissipation, we can replace within a rotating-wave approximation $\Lambda(t)$ by its time average [24]. Finally, we integrate the master equation to obtain the dissipative propagator.

In our numerical studies, we restrict ourselves to purely harmonic driving $f(t) = A\cos(\Omega t)/2$. The resulting purity loss during the interaction time t_J is depicted in Fig. 1. We find that for $k_B T > J$, decoherence is fairly independent of the driving. This behavior changes as the temperature is lowered: Once $k_B T < J$, the purity loss is significantly reduced whenever the ratio $A/\hbar\Omega$ is close to a zero of the Bessel function J_0. Both observations confirm the preceding analytical estimates. The behavior at the first working point $A \approx 2.4\hbar\Omega$ is depicted in Fig. 2(a). For relatively low driving frequencies, we find the purity loss being proportional to J/Ω. This significant deviation from the analytical result for small Ω relates to the fact that the low-frequency regime is not within the scope of our high-frequency approximation. With increasing driving frequency, the discrepancy decreases until finally decoherence is dominated by thermal noise $\propto T$. The numerical solution confirms the analytical results.

Still, there remains one caveat: The gate operation U_{CNOT}^I relies on the fact that U_{eff} is a good approximation for the

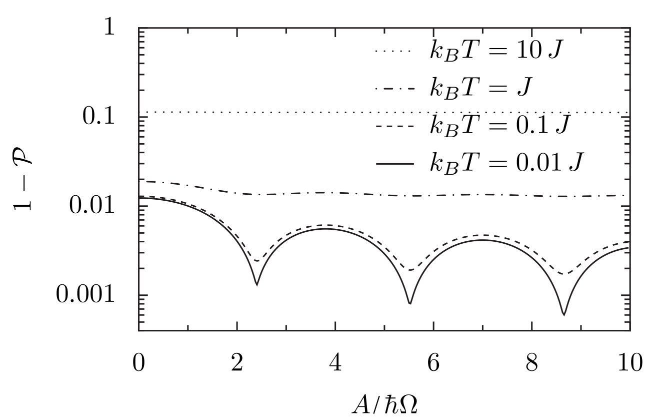

FIG. 1. Purity loss $1 - \mathcal{P}$ during the interaction time t_J as a function of the driving amplitude. The driving frequency is $\Omega = 32J/\hbar$ and the dissipation strength $2\pi\alpha = 0.01$. For $A = 0$, the undriven situation is reproduced.

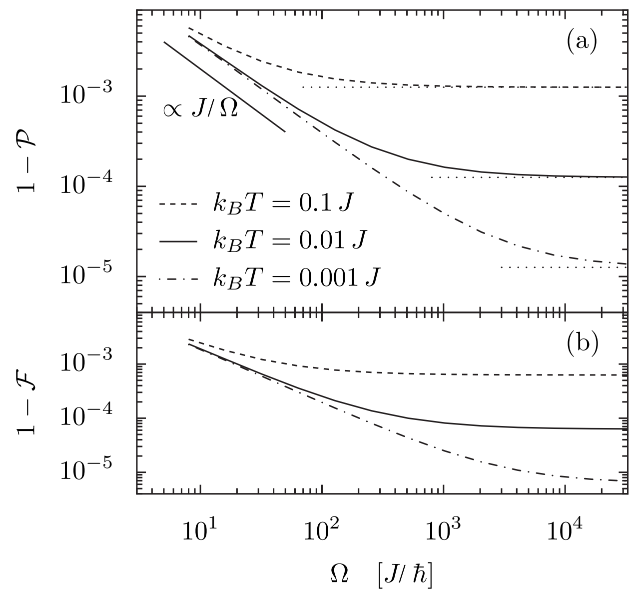

FIG. 2. (a) Purity loss during the interaction time t_J as a function of the driving frequency. The driving amplitude $A \approx 2.4\hbar\Omega$ is adjusted such that $1 - \mathcal{P}$ assumes its first minimum; cf. Fig. 1. The dotted lines mark the analytical estimate $1 - \mathcal{P}(t_J) \approx -\dot{\mathcal{P}}(0)t_J$. (b) Corresponding fidelity defect $1 - \mathcal{F}$.

dynamics of the driven gate because any deviation results in a coherent error. Therefore, we still have to justify that such coherent errors are sufficiently small. As a measure, we employ the so-called fidelity $\mathcal{F} = \overline{\mathrm{tr}[\rho_{\mathrm{ideal}}\rho(t_J)]}$ [25], which constitutes the overlap between the real outcome of the operation, $\rho(t_J)$, and the desired final state $\rho_{\mathrm{ideal}} = U_I(\pi/4)\rho_{\mathrm{in}}U_I^\dagger(\pi/4)$ in the average over all pure initial states. Here, $U_I(\varphi) = \exp(-i\varphi\sigma_1^x\sigma_2^x)$ is the propagator of the ideal Ising qubit-qubit interaction, which is characterized by $\mathcal{F} = 1$. Figure 2(b) demonstrates that the fidelity defect $1 - \mathcal{F}$ at the first working point is even smaller than the purity loss. Thus, we can conclude that coherent errors are not of a hindrance.

For spin qubits in quantum dots [19] a typical exchange coupling is $J = 0.1$ meV which for a temperature $T = 10$ mK corresponds to the solid lines in Figs. 1 and 2. These results demonstrate that a driving with frequency $\Omega = 2\pi \times 100J/\hbar \approx 10^{12}$ Hz and amplitude $A = 10$ meV already reduces the purity loss by 2 orders of magnitude while the fidelity loss stays at a tolerable level.

In summary, we have shown that for two qubits, a suited ac field turns a Heisenberg interaction into an effective Ising interaction and that, moreover, the latter is less sensitive to decoherence. For qubits with Heisenberg interaction, like, e.g., spin qubits, this suggests the following coherence stabilization protocol: Use for the CNOT operation a pulse sequence that is suitable for Ising interaction, which is realized by a Heisenberg interaction with a suited additional ac field. This coherence stabilization scheme differs from previous proposals in two respects: First, it is different from dynamical decoupling because the driving commutes with the bath coupling. By contrast, the central idea of our scheme is rather to suppress the *coherent* system dynamics transverse to this sensitive system coordinate. Thus, the bit-flip noise acts as pure phase noise, which is proportional to the temperature. Cooling, thus, enables a further coherence gain. The second difference is that the proposed scheme eliminates also the noise stemming from the spectral range above the driving frequency and, thus, is particularly suited for Ohmic noise spectra with large cutoff frequencies. Moreover, the driven system still allows one to perform the desired CNOT operation with high fidelity and within the same operation time as in the absence of the control field. Hence, the gained coherence time fully contributes to the number of feasible gate operations.

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