Dinara's Crosses, Chaoticity and Robustness in Stochastic Dynamics of Solar Activity

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The dynamics of stochastic processes in real complex systems is very complicated and entangled. At the examination of various dynamic states of similar systems one of the central points is consisted in finding a quantitative measure of chaoticity and regularity in its evolution. In this work we represent the results of the study of solar activity from the point of view of complexity, discreteness, non-stationarity and non-Markovity of dynamic evolution of atmosphere of the Sun. The study was carried out by using of means of the statistical theory of discrete non-Markov stochastic processes [1]-[3]. The statistical non-Markov effects in time series of solar activity are considered thoroughly. For realization of correlation analysis as an initial time series we use a time series of Wolf number (one of solar indexes). In this work the effects of regularity and chaoticity connected with dynamics of various cycles of solar activity come to light. For the finding of local time dependence the kinetic and relaxation parameters and obtaining of the additional information about physical nature of the phenomena going on the Sun we offers local averaging operation. In this paper the comparative analysis of the various parameters connected with minima and maxima of solar activity has been implemented. Specific features in behavior of phase clouds at a minimum of solar activity are characterized by the occurrence of obtuse angles and symbolical "Dinara's Crosses" in distribution of phase points. The phase points at a maximum of solar activity form a nucleus in the form of the oval curve. The dynamics of solar spots is connected to specific alternation of the effects of chaoticity and robustness. The peculiarities of the frequency dependence of non-Markovity parameter $\varepsilon_1(\nu)$ which is the original indicator of chaoticity and regularity reveal a complicated competition of noise and separate modes of the Sun mobility.

Key words: solar activity, chaoticity, random processes, discreteness, non-Markovity

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1 Introduction

Solar activity is one of the interesting astronomical processes which accessible for the study within various methods. In astronomical bibliography is being considered more than 20000 references on this subject. Not only astrophysicists but also geophysicists, meteorologists, physicians, telecommunications workers have a big interest to solar activity. Various formations, such as magnetohydrodynamic processes in Sun, fluctuation and undulation motions of plasma in solar atmosphere, interactions of charged particles with substance and magnetic fields are very important for the development of theoretical physics. Comprehension of solar activity phenomena and its manifestation on Earth one can allow to explain the various processes and make forecasting of behavior complex dynamical systems like solar atmosphere.

Discovered in 1610-1611 by Galilei, sunspots are the most famous and the most accessible feature among solar activity events. They represent magnetic structures located within active regions distinctly darker than the normal solar photosphere. The formations of sunspots concern the instability-driven processes in convective tubes on the Sun surface areas. The time scale for the formation of large sunspots range between a few hours and several days. They exist on surface during decades and hundreds hours and move on solar disk due its rotation. The temperature in solar spots is 1000-1900 K less than in the quiet Sun fields. It depend on the high value of the magnetic field strength in sunspots (1800 3700 G, more information see [4]).

So, sunspots are indicators of magnetic activity of the Sun. They envelop all solar atmosphere and display also filaments and prominences, flares in chromosphere, coronal holes, which are sources of high-speed charged particles. Sunspots groups and particular sunspots form the heart of an active region where dynamical energetic processes following by moving of the gas and variation of sizes and shapes of spots take place.

In middle of XIX century H. Schwabe and R. Wolf the 11-years periodicity in changing of number of sunspots on a seen disk of the Sun have established. Since then "Wolf numbers" are used as a key parameter of an estimation of solar activity and for the characteristic of condition of the Sun. In Fig. 1 Wolf numbers dependence from time by values published by the Royal observatory of Belgium (http://www.oma.be/ KSB-ORB/SIDC/) is presented. The full physical cycle of solar activity is connected with dynamics of a global magnetic field and contains two 11-years periods. The first period has smaller amplitude, during a maximum of the second period occurs change of poles of magnetic field. Today magnetic cycles and all large-scale structure of a magnetic field of the Sun are described in dynamo model in convective zone [5]. This model is adjusted with observed characteristics of solar activity. Recently the theory of a nonlinear dynamo, based on magnetic helicity conservation, was offered in Ref. [6].

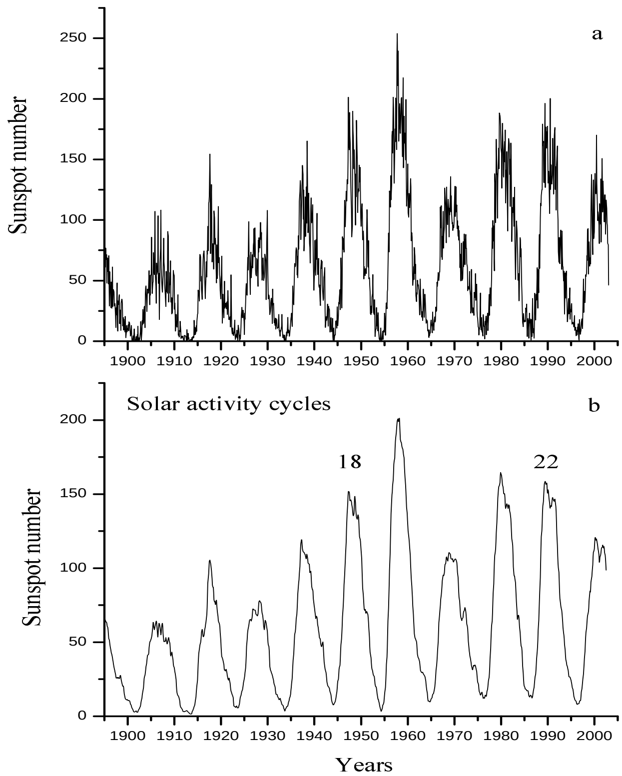

FIG. 1. The diagram of change of daily Wolf numbers (a) and the smoothed curve of monthly Wolf numbers (b) from 1895 to 2003. The 18 and 22 cycles of solar activity are marked.

Emergence of effects of nonlinearity is not random [7]. First, interaction of the magnetic field with moving plasma of the rotating star itself is the nonlinear process. Secondly, a daily fluctuations of Wolf numbers have noise character, therefore it is rather difficult to apply the standard methods of the statistical analysis to them. The basic period of the solar activity is quasi-periodic (from 9 to 13 years), with irregular phase and amplitude variations. The each cycle has an asymmetric kind (the section of growth on average is more short for 2-3 years, than the section of recession, only in three of 22 cycles was observed contrary). Moreover within the processes of decreasing and increasing of activity level are observed the short-range and long-range changes (for example, a Maunder Minimum in XVII century).

The 27-day period (it is connected with period of rotation of the Sun) is most authentic one among the short periods. Also there are periods 12-14, 50-52, 154 days which are caused by the certain physical processes in the solar atmosphere [8]. Quasi-yearly variability of a global magnetic field of the Sun was established on the basis of satellite data [9]. Various statistical methods of the analysis of indexes of solar activity (it is, first of all, the daily and monthly Wolf numbers) for search of periodicity in activity of the Sun are used. So, the standard correlation and Fourier-analysis specify the 27-day, 11-year and secular periods, but these methods are intended for the description of regular processes only. Therefore other specific techniques are developed. For example, a Singular Spectrum Analysis method easily discriminate a secular component, the period of Schwabe and quasi-biennial variations of Wolf numbers and other indexes of activity [10].

The review of the long-period variations of solar activity on the basis of solar, astrochemical (isotopes), geophysical, and also the historical data is given in work [11]. Here the Gleissberg (50-80 and 190-140 years) cycles and the period of Suess by duration of 170-260 year are considered. They also have physical basis and slowly change, a moreover last cycle is more stable one. The mathematical procedure of wavelet-transformation and the Fourier-analysis are used for definition of these periods.

The irregularity is emerged not only in change of solar activity, but also in the processes occurring on the Sun. For example, the asymmetry "north - south" in distribution of sunspots in different hemispheres is observed on short time scales [12, 13]. The irregular components separating at the analysis of the spots activity, are caused by random noise, instead of elements of chaos [8]. Carbonell et al. [14] have carried out the study of possible chaotic behavior of Sun activity from the sampling of Wolf numbers with the help of method of correlation integral. The conclusion was made, that due to the insufficient completeness of the observed data is obstructing detection of this behavior. Meanwhile, the aperiodic character and separate processes testify in favor of the description of solar activity as deterministic chaos. Until now a modern observations testifies, that modulation of a solar cycle submits to laws of the deterministic chaos [5].

It was earlier established, that chaotic character of processes of solar activity is emerged on rather large time intervals. The short-period processes (for example, solar flares, [15]) submit to stochastic laws. Use of the algorithm of Grassberger-Procaccia shows, that it is possible to consider a time interval of 8 years [16, 17] as the time boundary of transition stochastic processes into chaotic.

As it was well established by various authors, dynamic behavior of solar activity displays various multi-fractal properties. On intervals of order of a few days up to 2 months fractal dimension corresponds to stochastic changes of parameters submitting to Gaussian distribution. Spatially these stochastic structures form the small groups of spots. Global indexes of solar activity are demonstrate irregular behavior and on the ranges of times from 1-2 months to 2 years they are described by Poisson distribution. Fractal structure of magnetic fields on a surface of the Sun is characterized by the big groups of spots and active areas [18, 19]. Other the big time scales (2-13 years) are correspond to the quasi-periodic variations with the periods 1-2 years which are elements of a 11-years cycle of solar activity [20]. Similar structures are occupy giant and supergiant cells which create large-scale structure of magnetic field in space. Thus, the behavior of temporal and spatial structures will be quasi-regular with attributes of chaos in these scales. Intimate connection between structures of different scales and times is observed. So, on the basis of study Wolf numbers by methods of nonlinear dynamics an elements of the deterministic chaos [21] in solar activity even on small time intervals were found. The long-time changes in the fractal properties of of solar activity were found also [11].

Daily measurements of Wolf numbers during enough large time (since 1749) have allowed to collect the great database. It allows to carry out a various statistical estimation of behavior of sunspots and accordingly activity of the Sun. A study of the stochastic processes arising in solar atmosphere, estimations of regularity and chaoticity of dynamics of the phenomena occurring during change of activity of the Sun, generates the big interest in modern physics and astrophysics. Forecasting of solar activity based on results of studying of these processes is also rather important problem.

Many works are devoted to the prediction of conduct of the solar activity (especially per years, previous to the next maximum in the cycle). Various methods of analysis of the observational data on the previous cycles of activity or their geomagnetic manifestation are used for this purpose. Here we choose only some examples. The energetic parameters of a global magnetic field, their connection with activity of spots are received in Ref. [22]. Analytical and graphic connections between them are found on this basis. It allows to predict amplitude of the following maxima of activity with the certain accuracy.

The forecasting of a maximum for the 23 cycles of activity is carried out in work Li [23] on the basis of behavior of all known cycles by observation of growth of activity within the first 3-4 years of a new cycle (over analogy to one of observed maxima). But this method is appropriated for prediction of behavior of one cycle only.

Khramova et. al [24] have offered "method of phase average" in which predicted values of Wolf numbers are defined by extrapolation from the certain technique on the basis of unsmoothed average monthly Wolf numbers. On the grounds of this principle the long-term forecast is given, for example, on 23 and 24 cycles.

Complexities in forecasting a maximum of activity of the Sun are caused first of all by deficiency of connection between the period and amplitude of cycles of activity. The dependence between Wolf number and the period, and also duration of a growth phase of the given cycle was observed in Ref. [25]. Probably, the amplitude of cycle and time derivative of number of sunspots also are connected with each other. It allows to calculate dynamics of solar activity in model of dynamo.

Nonlinearity of processes of solar activity require the application of non-standard and nonconventional forecasting methods. To them it is necessary to attribute the method of the nonlinear forecast [8], as well as method of nonlinear dynamics [26]. But nonlinear and chaotic nature of processes imposes the certain restrictions on these methods, therefore exact prediction of behavior of solar activity are possible only for some years forward.

Excepting of the foregoing methods, the wavelets-transformation and auto-correlation methods described in works [27, 28] are applied intensively to the study of the future behavior of solar activity. For example, with the help of wavelet entropy method [29] the contribution to solar activity of the certain measure of the disorder was found out. Existence of such contribution results in evolution of magnetic cycles. There is an assumption, that there is a certain connection between qualitative behavior of wavelet entropy and excursion phases of solar dipoles.

In this paper the method based on the statistical theory of discrete non-Markov stochastic processes is offered. We use the set of Wolf numbers as observational sampling. Here we perform the analysis with the help of our technique which allows to extract the properties of regularity and chaoticity from the dynamics of different phases of solar activity. In the second section we present the basic points of our statistical theory non-stationary non-Markov processes in complex systems (of main base of the method developed by some of the authors in last years). In section 3 we describe the basic observational data and a technique of their processing. In fourth section the received results are discussed. In last part the basic conclusions from the done work are submitted.

2 Basic concepts and definition of the statistical theory of nonstationary discrete non-Markov processes in complex systems

The obtained data were processed by means of the below introduced technique. We use results of our last theory of discrete non-Markov random processes for the quantitative description of chaotic and regular components in stochastic alteration of registered data. The set of three memory functions was calculated for each sequence of data. Frequency power spectra for each of these functions are obtained using the fast Fourier transform. For a more detailed analysis of properties of the system we examine also the frequency spectrum of the first three points of the statistical spectrum of non-Markovity parameter. The spectrum of non-Markovity parameter was entered earlier in following articles: Refs. [1, 2, 30, 31] and was also used in statistical physics of liquids [32, 33]. In this study we use frequency spectrum of non-Markovity parameter

$$\varepsilon_i(\nu) = \left\{ \frac{\mu_{i-1}(\nu)}{\mu_i(\nu)} \right\}^{\frac{1}{2}}, \mu_i(\nu) = |Re\tilde{M}_i(\nu)|^2,$$

as an information measure of chaoticity and robustness of the studied process. Here $i = 1, 2, 3...$, $\tilde{M}_i(\nu)$ and $\mu_i(\nu)$ there is Fourier transform and power spectrum of ith level memory function $M_i(t)$ (see, Eqs. (20), (22) below). The parameters ε_i allow to receive quantitative estimation of long-term memory effects in experimental time series of the data as shown in Refs. [32]-[35]. From the physical point of view the value of parameter ε_i allows to mark out the three most important cases [32]-[35]. The Markov and completely randomized processes correspond to the values $\varepsilon \to \infty$, quasi-Markov processes (elements of memory can be noticed there) correspond to values $\varepsilon > 1$. The limiting case $\varepsilon \sim 1$ concerns the case of non-Markov processes, i.e., processes, where exist the effects of the long-range memory.

The matter is that at analysis of the complex system we obtain the discrete equidistant series of the experimental data, the so-called random variable

$$X = \{x(T), x(T+\tau), x(T+2\tau), \cdots, x(T+k\tau), \cdots, x(T+\tau N-\tau)\}. \quad (1)$$

This set corresponds to the signal measured within time $t = (N-1)\tau$, where τ is a temporary sampling interval of the signal. The mean value $< X >$, fluctuation δx_j, absolute (σ^2) and relative (δ^2) dispersion for the set of random variables in Eq. (1) are defined as follows

$$< X > = \frac{1}{N} \sum_{j=0}^{N-1} x(T+j\tau), \quad (2)$$

$$x_j = x(T+j\tau), \delta x_j = x_j - < X >, \quad (3)$$

$$\sigma^2 = \frac{1}{N} \sum_{j=0}^{N-1} \delta x_j^2, \quad (4)$$

$$\delta^2 = \frac{\sigma^2}{< X >^2} = \frac{\frac{1}{N}\sum_{j=0}^{N-1} \delta x_j^2}{\{\frac{1}{N}\sum_{j=0}^{N-1} x(T+j\tau)\}^2}. \quad (5)$$

The above-mentioned set of values determine the static (independent from time) property of the considered system. For the dynamical analysis it is more convenient to use the normalized time correlation function (TCF). For the discrete processes the TCF has the regular form ($t = m\tau, N-1 \geq m \geq 1$)

$$a(t) = \frac{1}{(N-m)\sigma^2} \sum_{j=0}^{N-1-m} \delta x(T+j\tau) \delta x(T+(j+m\tau)). \quad (6)$$

The properties of TCF $a(t)$ are determined by the condition of normalization and attenuation of correlation

$$\lim_{t\to 0} a(t) = 1, \lim_{t\to\infty} a(t) = 0. \quad (7)$$

For real systems the values $x_j = x(T+j\tau)$ and $\delta x_j = \delta x(T+j\tau)$ represent the experimental data.

To account the dynamics of a system we shall define the evolution operator $U(T+t_2, T+t_1)$ as follows $(t_2 \geq t_1)$

$$x(t+\tau) = U(t+\tau, t)x(t). \tag{8}$$

Then the equation of the motion becomes discrete

$$\frac{dx}{dt} = \frac{\triangle x(t)}{\triangle t} = i\hat{L}(t,\tau)x(t),$$
$$\hat{L}(t,\tau) = (i\tau)^{-1}[U(t+\tau,t) - 1]. \tag{9}$$

Let us introduce the vector of the initial and final states of the system

$$\begin{aligned}\mathbf{A}_k^0(0) &= (\delta x_0, \delta x_1, \delta x_2, \cdots, \delta x_{k-1}) = \\ (\delta x(T), &\quad \delta \quad x(T+\tau), \cdots, \delta x(T+(k-1)\tau).\end{aligned} \tag{10}$$

$$\begin{aligned}\mathbf{A}_{m+k}^m(t) &= \{\delta x_m, \delta x_{m+1}, \delta x_{m+2}, \cdots, \delta x_{m+k-1}\} \\ &= \{\delta x(T+m\tau), \delta x(T+(m+1)\tau), \ldots, \\ &\quad \delta x(T+(m+k-1)\tau\}.\end{aligned} \tag{11}$$

The normalized TCF can be presented as a scalar product of the state's vectors ($t = m\tau$ is discrete time):

$$a(t) = \frac{< \mathbf{A}_k^0 \cdot \mathbf{A}_{m+k}^m >}{< \mathbf{A}_k^0 \cdot \mathbf{A}_k^0 >} = \frac{< \mathbf{A}_k^0(0) \cdot \mathbf{A}_{\mathbf{m+k}}^{\mathbf{m}}(\mathbf{t}) >}{< \mathbf{A}_k^0(0)^2 >}. \tag{12}$$

The initial TCF can be received by projection of the vector of the final state on the direction of vector of the initial state by the use of the following projection operator

$$\begin{aligned}\Pi \mathbf{A}_{m+k}^m(t) &= \mathbf{A}_k^0(0) \frac{< \mathbf{A}_k^0(0)\mathbf{A}_{m+k}^m(t) >}{< |\mathbf{A}_k^0(0)|^2 >} \\ &= \mathbf{A}_k^0(0)a(t).\end{aligned} \tag{13}$$

The projection operator Π possesses the following properties

$$\Pi = \frac{|\mathbf{A}_k^0(0) >< \mathbf{A}_k^0(0)|}{< |\mathbf{A}_k^0(0)|^2 >}, \Pi^2 = \Pi,$$
$$P = 1 - \Pi, \quad P^2 = P, \quad \Pi P = 0, P\Pi = 0. \tag{14}$$

The vector of the fluctuation obeys the finite-difference Liouville's equation

$$\frac{\Delta}{\Delta t}\mathbf{A}_{m+k}^m(t) = i\hat{L}(t,\tau)\mathbf{A}_{m+k}^m(t). \tag{15}$$

The projection operators Π and P split the Euclidean space of states $A(k)$ into two mutually - orthogonal subspaces which allows us to split the dynamic Eq. (15) into two equations in two subspaces

$$\frac{\Delta A'(t)}{\Delta t} = i\hat{L}_{11}A'(t) + i\hat{L}_{12}A''(t), \tag{16}$$
$$\frac{\Delta A''(t)}{\Delta t} = i\hat{L}_{21}A'(t) + i\hat{L}_{22}A''(t). \tag{17}$$

Extracting from Eq. (17) the irrelevant part $\Delta A''(t)$ we obtain the closed finite-difference equation of a non-Markov type for TCF $a(t)$:

$$\frac{\Delta a(t)}{\Delta t} = \lambda_1 a(t) - \tau\Lambda_1 \sum_{j=0}^{m-1} M_1(j\tau)a(t-j\tau). \tag{18}$$

Here Λ_1 is a relaxation parameter with the dimension of square of frequency and parameter λ_1 describes an eigen-spectrum of Liouville's quasi-operator $\hat{L}$

$$\lambda_1 = i\frac{< \mathbf{A}_k^0(0)\hat{L}\mathbf{A}_k^0(0) >}{< |\mathbf{A}_k^0(0)|^2 >}, \tag{19}$$
$$\Lambda_1 = \frac{< \mathbf{A}_k^0\hat{L}_{12}\hat{L}_{21}\mathbf{A}_k^0(0) >}{< |\mathbf{A}_k^0(0)|^2 >}.$$

The function $M_1(j\tau)$ in the rhs of Eq. (18) represents the first memory function

$$M_1(j\tau) = \frac{< \mathbf{A}_k^0(0)\hat{L}_{12}\{1 + i\tau\hat{L}_{22}\}^j\hat{L}_{21}\mathbf{A}_k^0(0) >}{< \mathbf{A}_k^0(0)\hat{L}_{12}\hat{L}_{21}\mathbf{A}_k^0(0) >},$$
$$M_1(0) = 1. \tag{20}$$

It is easy to notice, that except for the initial TCF in Eq. (20) we consider the time correlation of the new orthogonal dynamic variable $\hat{L}_{21}\mathbf{A}_k^0(0)$.

Eq. (18) represents the first equation of the chain of finite-difference kinetic equations with memory for the discrete TCF $a(t)$. The memory function $M_1(t)$ takes into account the time memory about the all previous states of the system. Acting similarly to the above-stated procedure one can receive kinetic equations for subsequent memory functions. However a more convenient way is making use of the Gram-Schmidt orthogonalization procedure. Because of this it is easy to obtain the recurrence formula, in which senior variables $\mathbf{W}_n = \mathbf{W}_n(t)$ with higher index are expressed in terms of the junior variables with lower indices

$$\mathbf{W}_0 = \mathbf{A}_k^0(0), \quad \mathbf{W}_1 = \{i\hat{L} - \lambda_1\}\mathbf{W}_0, \ldots$$
$$\mathbf{W}_n = \{i\hat{L} - \lambda_{n-1}\}\mathbf{W}_{n-1} \quad (21)$$
$$+\Lambda_{n-1}\mathbf{W}_{n-2} + ..., \quad n > 1.$$

Using the above mentioned procedure and introducing the corresponding projection operators, we come to the following chain of connected non-Markov finite-difference kinetic equations ($t = m\tau$)

$$\frac{\Delta M_n(t)}{\Delta t} = \lambda_{n+1} M_n(t) \quad (22)$$
$$-\tau\Lambda_{n+1} \sum_{j=0}^{m-1} M_{n+1}(j\tau) M_n(t - j\tau).$$

Here parameters λ_{n+1} represent an eigenvalues of the Liouville's quasioperator and the relaxation parameters of Λ_{n+1} are determined as follows

$$\lambda_n = i\frac{< \mathbf{W}_n \hat{L} \mathbf{W}_n >}{< |\mathbf{W}_n|^2 >},$$
$$\Lambda_n = -\frac{< \mathbf{W}_{n-1}(i\hat{L} - \lambda_{n+1})\mathbf{W}_n >}{< |\mathbf{W}_{n-1}|^2 >}.$$

The zero order memory function $M_0(t)$ in Eq. (22)

$$M_0(t) = a(t) = \frac{< \mathbf{A}_k^0(0)\mathbf{A}_{m+k}^m(t) >}{< |\mathbf{A}_k^0(0)|^2 >}, \quad t = m\tau$$

describes the statistical correlation in complex systems with discrete time. The initial TCF a(t) and the set of discrete memory functions $M_n(t)$ in Eq. (22) are important for further consideration. The first three equations of this chain ($t = m\tau$ is discrete time) can be presented as follows

$$\frac{\Delta a(t)}{\Delta t} = -\tau\Lambda_1 \sum_{j=0}^{m-1} M_1(j\tau) a(t - j\tau) + \lambda_1 a(t),$$
$$\frac{\Delta M_1(t)}{\Delta t} = -\tau\Lambda_2 \sum_{j=0}^{m-1} M_2(j\tau) M_1(t - j\tau)$$
$$+\lambda_2 M_1(t),$$

$$\frac{\Delta M_2(t)}{\Delta t} = -\tau\Lambda_3 \sum_{j=0}^{m-1} M_3(j\tau) M_2(t-j\tau) + \lambda_3 M_2(t).$$

These systems of finite-difference Eqs. (22) and (23) are a discrete analogue of the well-known chain of kinetic Zwanzig'-Mori's equations. The latter plays a fundamental role in modern statistical physics of non-equilibrium phenomena with continuous time. It is necessary to note that the chain of Zwanzig'-Mori's equations is valid only for quantum and classical Hamiltonian systems with continuous time. The finite-difference chain of kinetic Eqs. (22),(23) is valid for complex systems, in which there is not any Hamiltonian, but time is discrete, and the exact equations of motion are absent . However, the "dynamics" and "motion" in real complex systems undoubtedly exist and can be directly registered in the experiment. The first three equations in the chain (23) form the basis for quasihydrodynamic description of stochastic discrete processes in complex systems.

The obtained relations allows to find the all necessary memory function $M_s(t)$ of any order $s = 1, 2...$ on the basis of the experimental data, using only the initial TCF $a(m\tau)$. Relaxation parameters λ_i and Λ_i, $i = 1, 2, 3...$, in Eqs. (23) can be calculated from experimental data. The application of Eqs. (23) opens new possibilities in the detailed analysis of the statistical properties of the correlations in complex systems. The fact of the existence of a finite- difference Eqs. (22), (23) allows us to evaluate unknown memory functions directly from the experimental data.

3 Observational data and data processing

By the analogy with other studies we use in this study daily sunspot numbers in time interval 1940-2001. This database includes five cycles of the solar activity from 18 to 22th and 22065 data points. Analyzed sample of sunspot observations was provided by the Sunspot Index Data Center of Belgian Royal Observatory (http://www.oma.be/KSB-ORB/SIDC/).

It is necessary to note that Wolf numbers as time series have the high noise level therefore in our future papers we will use other physical parameters of the solar activity. Data processing is carried out on the basis of the above-stated statistical theory of non-stationary discrete non-Markov processes. The received experimental data are used as the initial data.

4 Discussion of results

In this section we present the quantitative and comparative analysis of chaotic dynamics of solar activity in the range from 1940 to 2001 on the basis of the theory submitted in Section 3. The time series of the initial signals and the dynamic variables, the phase portraits of the four first dynamic variables, the power spectra of the TCF and the junior memory functions as well as frequency dependence of the first three points of the statistical non-Markov parameter, the local time dependence the kinetic and the relaxation parameters λ_1, λ_2, λ_3, Λ_1 and Λ_2 are submitted in Figs. 2-6. Here we offer the new method of definition of chaoticity and regularity of the stochastic processes taking place on the Sun.

4.1 The study of chaotic dynamics of the solar activity submitted for the full-scale time interval

The time series of the orthogonal variables W_0 (Fig. 2(a)), W_1 (Fig. 2(b)), W_2 (Fig. 2(c)), W_3 (Fig. 2(d)) for chaotic dynamics of solar activity are submitted in Fig. 2. As the initial time series the variable W_0 forms a dynamic noise. The analysis of the time series of a variable W_0 shows, that the most appreciable fluctuations of this dynamic variable relate on the maxima of solar activity. The bursts of solar activity are located on quasiequal intervals from each other. It testifies about the quasi-periodicity changes of Sun activity during all research time interval. The time series of the first three variables W_1, W_2, W_3 are symmetric in regard to the axis of abscissa. The five dynamic peaks falls at five full cycles (for the period with 1940 on 2001). We can note the bifurcation in the each dynamic peak and this indicate on the existence of two peaks during maximum of the solar activity in each cycle (Fig. 1(b)). The kind of each separate maximum resembles outwardly a sea wave. At first, the wave gathers force, then reaches a point of the maximal height, weakens a little, then again gathers force and falls down finally.

In Fig. 3 phase clouds in six plan projections of the four first dynamic variables W_i, $i = 0, \dots, 3$ are submitted for cycles of solar activity. The asymmetry of phase clouds concerning the center of coordinates on the three first phase portraits is observed. The phase points form an oval nucleus and settle down in such a manner that remind "a flame which is taking off for nozzle rocket". The most cluster in this case fall on a place of an output of a flame. The diversity of phase points is enhanced at removal from a place of flameout. On the last three phase portraits are appreciable central nucleuses which are symmetric about center of coordinates.

In Fig. 4 the power spectra of TCF $\mu_0(\nu)$ and the three junior functions memory $\mu_i(\nu), i = 1, 2, 3$ for the chaotic dynamics of solar activity are submitted. The frequency spectra are given in doubly logarithmic scale for a more detailed analysis of the data. The power spectra of TCF $\mu_0(\nu)$ has fractal dependence on the middle and high frequencies. The small burst of power on frequency $\nu = 4 * 10^{-2} f.u.$ $(1f.u. = 1/\tau)$ divides area of these frequencies where τ there is time of discretization, $\tau = 1$ day. A distinct peak with period 25-27 days (the frequency $\nu = 4{*}10^{-2} f.u.$)

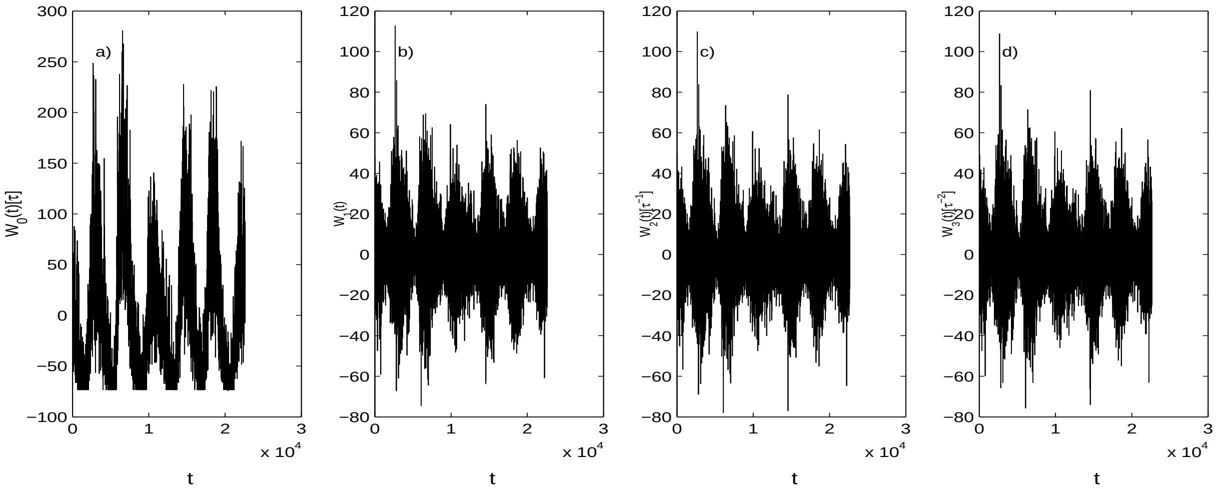

FIG. 2. The time series (for the period with 1940 on 2001) of the orthogonal variables W_0 (a), W_1 (b), W_2 (c), W_3 (d) for chaotic dynamics of solar activity. The most fluctuations of a time series of a variable W_0 fall at maxima of solar activity. The bursts of solar activity are located on quasiequal intervals from each other, that speaks about quasi-periodicity changes of activity of the Sun during all studied time interval.

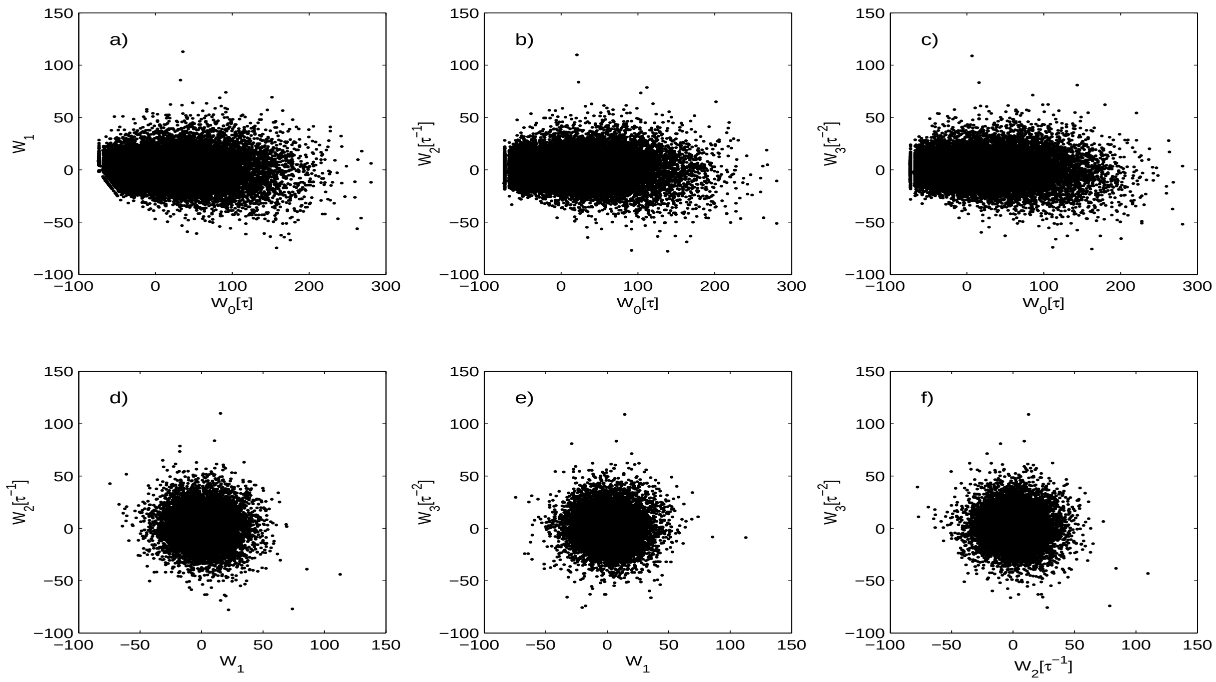

FIG. 3. The phase clouds of solar activity in six plan projections of the four first dynamic variables W_i, $i = 0...3$ for all time interval. The phase points of clouds (a, b, c) form an oval nucleus and settle down in such a manner that look as "a flame which is taking off for nozzle rocket". The most cluster in this case fall on a place of an output of a flame. The diversity of phase points is enhanced at removal from a place of flameout.

represents the rotation of the Sun as viewed from Earth [16]. In the field of low frequencies the sharp break which maximum falls to the frequency $\nu = 3 * 10^{-4} f.u.$ is observed. The large peak at frequency $0.0003 f.u.$ on the left in Fig. 4 corresponds to the decadal solar activity cycle. Between these two peaks the spectrum displays a power-law dependence on scale [16]. The frequency behavior of the three junior memory functions $\mu_i(\nu), i = 1, 2, 3$ has an identical structure. In the field of low frequencies the sharp break with a characteristic maximum is found. The burst on the certain frequency $\nu = 4 * 10^{-2} f.u.$ divides the areas of middle and high frequencies.

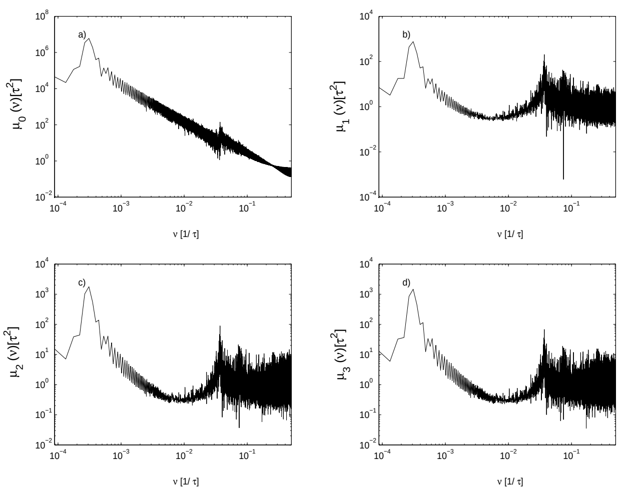

FIG. 4. The power spectra of TCF $\mu_0(\nu)$ (a) and the three junior memory functions $\mu_i(\nu), i = 1, 2, 3$ for chaotic dynamics of solar activity. The frequency spectra are given in doubly logarithmic scale for a more detailed analysis of the data. The areas of middle and high frequencies are divided by small burst of power on frequency $\nu = 4 * 10^{-2} f.u.$ on the all diagrams.

In Fig. 5 the spectra of the first three points of statistical non-Markovity parameter $\varepsilon_i(\nu)$, where $i = 1, 2, 3$ are presented. The parameter $\varepsilon_1(\nu)$ on the frequency $\nu = 0$ accept a value 95 that testifies about strong markovization and amplification of chaoticity of the process. The Markovian bursts on the frequencies $\nu = 0, 0.04, 0.07 f.u.$ are observed and they reach the values 95, 23 and 110. The second burst corresponds to a maximum (in point of peak on low frequencies) in the power spectra of initial TCF (Fig. 4(a)). The spectrum of non-Markovity parameter for the second point $\varepsilon_2(\nu)$ is symmetric about the direct line $\varepsilon_2(\nu)$ =1. In the frequency spectrum of the third point of the statistical non-Markovity parameter $\varepsilon_3(\nu)$ the characteristic minimum with peaks in the beginning and the end of the frequency dependencies is appreciable. In the spectra $\varepsilon_2(\nu)$, $\varepsilon_3(\nu)$ at the frequency $\nu = 0.04, 0.07 f.u.$ characteristic bursts are observed. The first peaks correspond to the maxima of low-frequency bursts in the power spectra $\mu_1(\nu)$, $\mu_2(\nu)$.

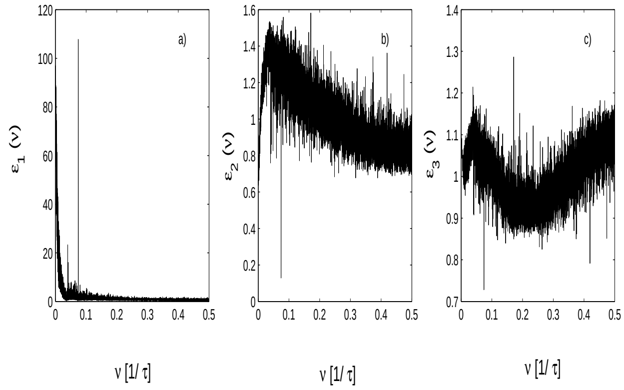

FIG. 5. The spectra of first three points of the statistical non-Markovity parameter $\varepsilon_i(\nu)$, where $i = 1, 2, 3$. At the frequency $\nu = 0$ the parameter $\varepsilon_1(\nu)$ (a) achieves value 60 that speaks about strong markovization of studied process and transition in a mode of Markov chaoticity. The Markovian bursts on the frequencies $\nu = 0.04, 0.07 f.u.$ are observed and they reach values 23, 110. The first peaks correspond to maximum of peak on low frequencies in the power spectra of the initial TCF.

Recently the correlation analysis has experienced a marked lack of information concerning the object under study. Procedure of local averaging of various parameters allows to examine the separate hidden properties of objects studied. The characteristic feature of the usual correlation analysis is the fact that the greatest possible set of signals is required for the qualitative analysis of the properties of the object of the research. At longer sample of such signals it is possible to receive more exact information with the help of the correlation analysis. Let us take a random non-Markov process as an example. This process consists of sequence of alternating random states. Thus, the problem of extraction of more information not only about the common process but about various single dynamic states of a system arises. In this case the use of the correlation analysis for the whole time series will be inefficient. The processing of the signals is necessary for separate local sites of the full time series. It will allow to consider the properties of separate dynamic states of the system.

Hereinafter a new method of data processing based on the local averaging of kinetic and relaxation parameters is offered. This method allows to consider the properties of separate non-stationary states of the systems. The idea of a method is the following: there exists an initial data set. Let's take a sampling in length N of signals and to calculate its kinetic and relaxation parameters. Then the operation of "step-by-step shift to the right" for one time interval is carried out. The kinetic and relaxation parameters are calculated again. The "step-by-step shift to the right" will continued to the end of time series. Such locally averaged parameters have high sensitivity to the effects of intermittency and non-stationarity. If the initial time series has some irregularity, it is instantly reflected in the behavior of the locally averaged parameters.

The use of this method requires the choice of the optimal length of a sampling which enables to receive the most trustworthy information. If a sampling is too short, so noise effects does not allow to receive qualitative information. Besides with a short length sampling we have significant errors. On the other hand at great length of a sampling locally averaged parameters lose "sensitivity" necessary for the study. As a result of the study of different lengths of local sampling we have received the optimal length compose 100-120 points. Further proofs of all aforesaid will be given below.

In Fig. 6 the time dependence located kinetic (λ_1, λ_2, λ_3) and relaxation (Λ_1, Λ_2) parameters is submitted. Procedure of localization allows to receive more detailed representation about a physical nature of researched object. The located parameters have "increase"of sensitivity to features of local states. The local parameters reflect separate local changes which occur in investigated object. The detailed analysis of the time dependence of the local kinetic parameter $\lambda_1(t)$ allows to show the six most significant changes. The behavior of parameter changes sharply in the case of occurrence of minima of solar activity. Equidistance and periodicity of similar changes follows from here. The similar picture is observed and for other local parameters. The parameters λ_1 and Λ_1 have an maximal sensitivity among all local parameters. The parameters λ_1, λ_2, λ_3 possess negative values on all the time interval whereas Λ_1, Λ_2 have an both positive and negative numerical values.

4.2 The study of some features of separate solar cycles

The analysis of qualitative results of data processing for chaotic dynamics of separate cycles of solar activity (18) allows to reveal the following regularity. In the most cases the maximum of solar activity has an complex structure on which is emerged no one, but two peaks. The first peak achieves the greatest amplitude in addition. The amplitude of the first peak in a maximum of each cycle is defined by special "indicator" which is the first point of the non-Markovity parameter $\varepsilon_1(\nu)$. This parameter constitute an informative measure of chaoticity or regularity of the processes in real object.

In Fig. 7 the power spectra of TCF $\mu_0(\nu)$ and the three junior memory functions $\mu_i(\nu), i = 1, 2, 3$ for chaotic dynamics for one of cycles of solar activity are submitted. The frequency spectra of initial TCF $\mu_0(\nu)$ are submitted in doubly logarithmic scale. Fractal dependence of the power spectra has been collapsed by the small burst on the frequency $\nu = 4 * 10^{-2} f.u.$ (the 25-27 day period of the rotation of the Sun as viewed from Earth). The power spectra of three junior memory functions $\mu_i(\nu), i = 1, 2, 3$ are submitted in a

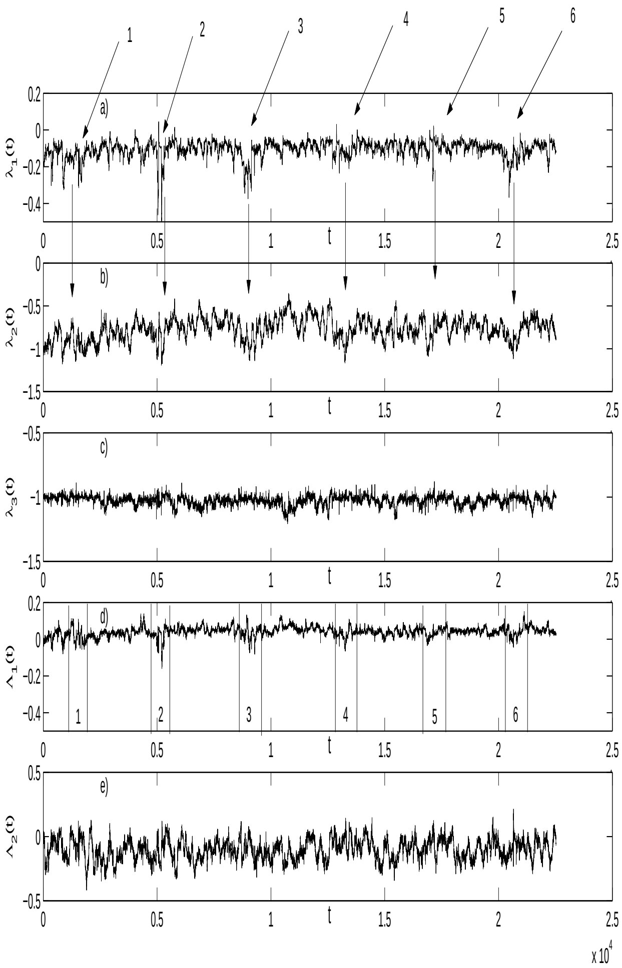

FIG. 6. The time dependence of local kinetic (λ_1, λ_2, λ_3) and relaxation (Λ_1, Λ_2) parameters. The analysis of the time dependence of the located kinetic parameter $\lambda_1(t)$ allows to show the six most significant changes. The behavior of the parameters sharply changes in case of an ascertainment of minima of solar activity. Equidistance and periodicity of similar changes follows from here. The behavior of other parameters is similar.

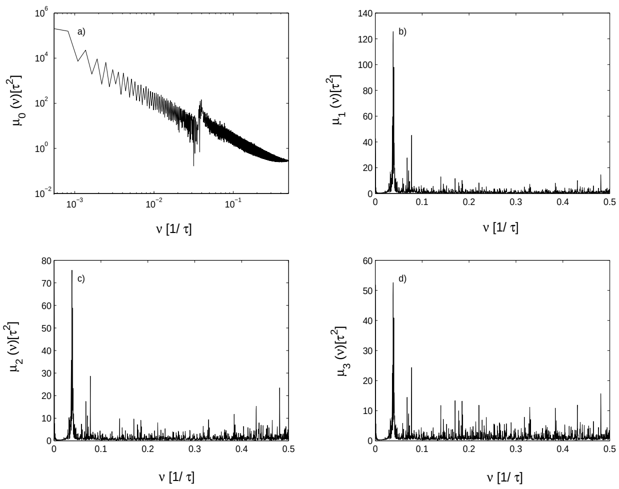

FIG. 7. The power spectra of the TCF $\mu_0(\nu)$ and the first three junior memory functions $\mu_i(\nu), i = 1, 2, 3$ for chaotic dynamics for one of cycles of solar activity - 18. The frequency spectra initial TCF $\mu_0(\nu)$ are given in doubly logarithmic scale. Fractal dependence of the power spectra is broken small burst on the frequency $\nu = 4 * 10^{-2} f.u.$ In the field of high frequencies a condensation of spectral lines is appreciable. The power spectra of three junior memory functions $\mu_i(\nu), i = 1, 2, 3$ are submitted in a usual frequency scale. In the field of low frequencies a few consistently going bursts are discovered (among which the greatest attention is necessary for first burst at frequency $0.04 f.u.$). The amplitude of the zero burst (at zero frequency) displays amplitude of the first peak of a maximum of solar activity.

usual frequency scale. In the field of low frequencies a few consistently going bursts are discovered, among which the greatest attention is necessary for giving of the first burst at frequency $0.04 f.u.$ The amplitude of the zero burst (at zero frequency) displays amplitude of the first peak of a maximum of solar activity.

The complex structure of power spectra appears in frequency dependence of the first three points of the statistical spectrum non-Markovity parameter $\varepsilon_i(\nu)$, $i = 1, 2, 3$, see Fig. 8. On the zero frequency appreciable amplification of Markov effects is observed in non-Markovity parameter $\varepsilon_1(\nu)$. The amplitude of this burst defines the greatest amplitude of the first maximum. The greater is the amplitude of the first point, the great becomes amplitude of the first maximum of solar activity. At the greater value of amplitude of the first point of non-Markovity parameter on zero frequency the amplitude of the first maximum of solar activity also accept greater value. On the frequency $\nu = 0.04 f.u.$ the small peak which reflects the appropriate low-frequency contribution to the power spectra of initial TCF appears (the 25-27 day period). The structure of the spectrum of non-Markovity parameter for the second point $\varepsilon_2(\nu)$ reflects the structure of the power spectrum of the first memory function $\mu_1(\nu)$. In the field of low frequencies there are peaks with maxima which are coincided with the appropriate maxima of bursts in the power spectra of the memory function $\mu_1(\nu)$. In the frequency behavior of the third point of non-Markovity parameter $\varepsilon_3(\nu)$ one can discover the crest with amplitude corresponding to the amplitude of burst in the power spectrum of the second memory function $\mu_2(\nu)$.

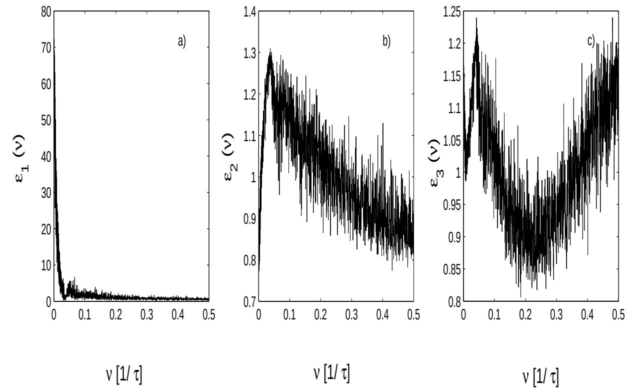

FIG. 8. The spectra of the first three points of the non-Markovity parameter $\varepsilon_i(\nu)$, $i = 1, 2, 3$ for chaotic dynamics of one of the cycles of solar activity. From the dependence of the non-Markovity parameter $\varepsilon_1(\nu)$ (a) near to zero frequency the appreciable amplification of Markov effects is observed. The amplitude of this burst defines the most significant amplitude of the first peak of a maximum of solar activity. The greater amplitude of the first point of non-Markovity parameter on zero frequency corresponds to the greater amplitude of the first peak of solar activity. The frequency behavior of the second and third points of non-Markovity parameter reflects frequency structure of power spectra of the first and second memory functions.

4.3 The definition of chaoticity and regularity of the processes proceeding on the Sun

Phase clouds in six plane projections of dynamic orthogonal variables for typical year of maximum (1947) (Figs. 9 (A-F)) and year of minimum (1986) (Figs. 9 (a-f)) of solar activity are submitted on Fig. 9. The represented diagrams are characteristic for all years with the maximal and minimal solar activity. Thus further analysis will be is made only for these typical cases. The phase points in the case of a maximum forms a nucleus in the form of the oval curve(Fig. 9 (A)). In comparison with other years of the cycle the interval of scattering of the phase points along the horizontal axis is maximal and equal to 280 τ. The phase points per year with a minimum of solar activity (Fig. 9 (a)) are dissipated from the center. On the left side of the phase cloud in a plane (W_0, W_1) the phase points are built along two lines which form the certain blunt angle. These lines appear in the phase clouds two - three years prior to a minimum of solar activity. On the following phase clouds of a point are built clearly along a direct line. The rest of phase points are distributed in the right half-plane concerning this direct line. On the last three phase portraits the phase points form symbolic "Dinara's Crosses" with the center in the beginning of coordinates. These crosses, the distinctive for the one year with the minimal solar activity, has received the name "Dinara's Crosses", in honor of the girl - student who was the first to discover this phenomena. We have revealed that the such kind of the phase clouds is characteristic for any year with the minimum of solar activity.

At observation of the phase clouds in the same scale the following picture appears. For any year with the greatest solar activity phase points are scattered on a phase plane as much as possible. In the following year the phase points gather closer to the center and occupies a more correct circular area of a plane. The next year points are grouped even more around the center. Per year of the minimum of solar activity the phase points create an almost ideal circle as much as possible compressed to the center.

In Fig. 10 the first three points of the non-Markovity parameter $\varepsilon_i(\nu)$, where $i = 1, 2, 3$ on the zero frequency for all time interval is shown. Thus parameter $\varepsilon_1(0)$ gets special physical value. This point is the original "indicator" of manifes-

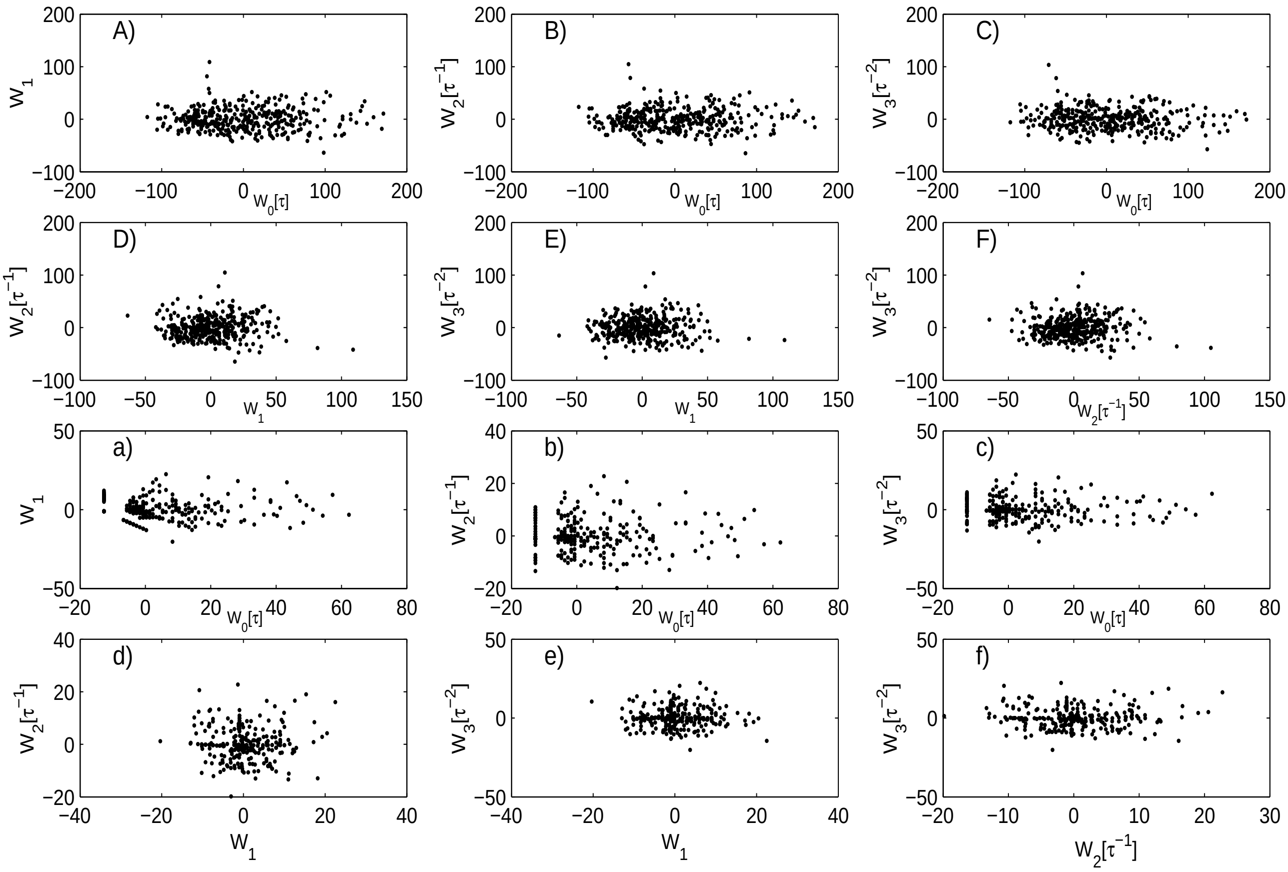

FIG. 9. The phase clouds in six plane projections of dynamic orthogonal variables for typical year with maximum of solar activity (1947) (A-F) and with minimum of solar activity (1986) (a-f). In the case of maximum the phase points are formed as a nucleus in the form of oval (A). The straggling of the phase points along abscissa axes is maximal as compared with other years of the cycle. The phase points for one year with minimum of solar activity (a) are approached to the center. On the left side of the phase cloud in a plane (W_0, W_1) the phase points are built along two lines which form the certain blunt angle. On the last three phase portraits the phase points form symbolical "Dinara's Crosses" with the center in the beginning of coordinates.

tation of chaoticity or regularity. The values of this point are minimal within maxima and minima years of solar activity and may range from 4 to 8. Per years appropriate growth phases and recession of solar activity, parameter $\varepsilon_1(0)$ achieves value 15-26. At the same time, this parameter defines the quantitative measure of the chaoticity of the processes on the Sun. The greater numerical value is of this parameter, the greater is a chaoticity. Thus, with removal from the minimum or the maximum of solar activity the randomness of the processes on the Sun amplifies. It is connected to the greater variability of set of the various processes on the Sun. Specifically, the following years: 1956 ($\varepsilon_1(0)$ =26.1), 1984 (26.3), 1988 (24.3), 1992 (20.1), 1998 (19.6) differ by the greatest chaoticity. These years correspond to years of phases of growth or slump for solar activity. It means, that most suggestive events during solar cycle occur in these phases. The greatest robustness and appreciable non-Markovity effects are characteristic for the next years: 1954 (4.78), 1958 (3.7), 1964 (4.8), 1968 (3.98), 1980 (3.8). If to compare these dates with those in table of the solar activity cycles (Ishkov, 2001) then one can note, that minimum of 18 cycle was in 1954 year,

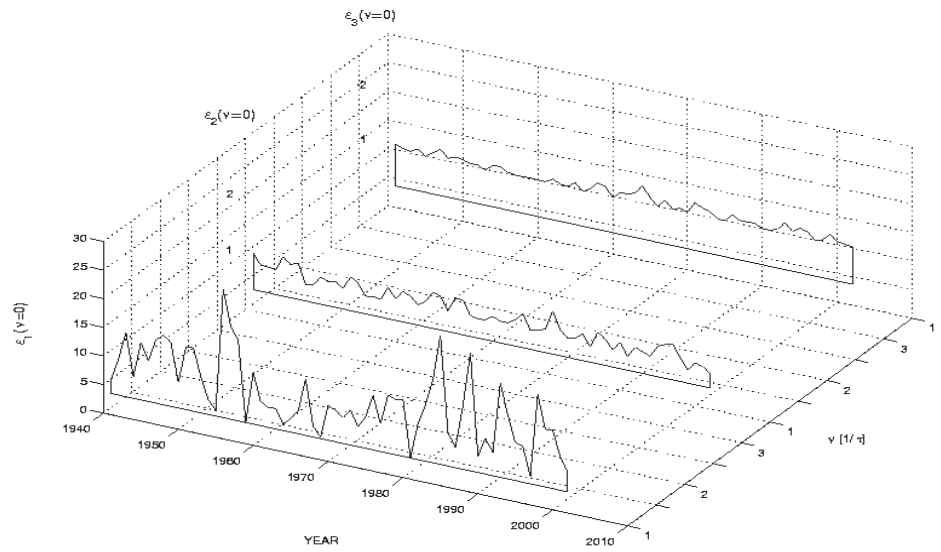

FIG. 10. The first three points of non-Markovity parameter $\varepsilon_i(\nu = 0)$ where $i = 1, 2, 3$, on zero frequency are calculated for the one year sampling and in aggregate for all time interval are given. The first point of non-Markovity parameter is original "indicator" of display of randomness or regularity. The values of this point within maxima and minima of solar activity are minimal. Per years which are sandwiched between maxima and minima of the solar activity, numerical values of size $\varepsilon_1(0)$ are maximal. The greater is numerical value of this parameter, the greater is a chaoticity. Thus, chaoticity of the processes on the Sun, with removal from minimum or maximum of solar activity intensifies.

20th maximum was in 1968 year etc. The complex frequency structure of the first point of non-Markovity parameter is reflected in frequency dependence $\varepsilon_2(0)$, $\varepsilon_3(0)$ also.

5 Conclusions

In this paper a method of the correlation analysis of dynamics of solar activity on the basis of the theory discrete non-Markov processes is offered. The developed method allows for discreteness of various processes on the Sun, effects of long-range memory and aftereffect, and also effects of dynamic alternation. It enables us to visualize and consider a series of the regularities which arise owing to periodicity and cyclicity of the solar activity. The regularities connected with cyclicity of solar activity, are reflected in the phase portraits of the first four dynamic orthogonal variables. The characteristic compression and the expansion of the phase clouds, recalling a pulsation of heart is observed at that. The phase clouds, the most dense in minimum, are increased on volume in 3-4 times at a period of the maximum of solar activity.

The physical non-Markovity parameter $\varepsilon_i(\nu)$ where $i = 1, 2, 3$ represents the quantitative measure of chaoticity and a regularity of the random processes on the Sun. Per years between minima and maxima the chaoticity of the stochastic processes connected to solar activity, is maximal. It is connected with greater variability of various dynamic states of system. It corresponds to years of reconstruction of dynamic state of the Sun. The analysis of fluctuations in the spectra of memory functions and frequency dependencies of the first three points of statistical non-Markovity parameter testifies an opportunity of use of our method for forecasting of solar activity. Points of the first phase portraits per years with the minimal solar activity generate two lines forming a blunt angle. Distinctive Dinara's Crosses which can be considered as a predictors of the appearance of minimum of solar activity appear in the phase clouds just at this period. The similar construction of the phase clouds emerges as two - three years prior to a minimum of solar activity. The dynamic peaks on the zero frequency at frequency spectrum of the time correlation function $\mu_0(\nu)$ and frequency behavior of the first point of the non-Markovity parameter $\varepsilon_1(\nu)$, determine the amplitude of the first dynamic peak per year with the maximal solar activity.

The locally averaged kinetic and relaxation parameters of chaotic dynamics of solar activity allow to study statistical features of the processes connected to solar activity in more details. The local time series reflect internal features of cyclicity of solar activity that helps to study the regularity afforded by solar activity. The offered method allow to find the features inherent in any cycle of solar activity at the big volume of experimental data. Physical feature of the local parameters is that the any irregularity arising in the initial time series, is instantly reflected in local time behavior of studied parameters. Procedure of local averaging allows to find out properties of system which are latent for the usual correlation analysis. Because of use of the given procedure the structure of any cycle of solar activity becomes more obvious. It means an opportunity of the calculation of the detailed quantitative parameters of various dynamic modes of solar activity. We plan to use this technique for the studying of other manifestations the solar activity on the Earth.

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