

Non-Stationary Time Correlation in Discrete Complex Systems: Applications in Cardiology and Seismology

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The full set of dynamic parameters and kinetic functions (TCF, short MF's, statistical spectra of non-Markovity parameter and statistical spectra of non-stationarity parameter) presented in previous paper has made it possible to acquire the in-depth information about discreteness, non-Markov effects, long-range memory and non-stationarity of the underlying processes. The developed theory is applied to analyze the long-time (Holter) series of RR intervals of human ECG's and seismograms of different states of the Earth crust. In both systems we observed effects of fractality, standard and restricted self-organized criticality and also a certain specific arrangement of spectral lines. The received results demonstrate that the power spectra of all orders ($n = 1, 2, \dots$) MF $m_n(t)$ exhibit the neatly expressed fractal features. We have found out that the full sets of non-Markov, discrete and non-stationary parameters can serve as reliable and powerful means of diagnostics of the cardio - vascular system states and can be used to distinct healthy data from pathologic data . Also our research demonstrates that discrete non-Markov stochastic processes and long- range memory effects play a crucial role in the behavior of seismic systems. The approaches, permitting to obtain an algorithm of strong EQ's forecasting and to differentiate TE's from weak EQ's, have been developed.

Key words: non-stationary processes, non-Markov discrete processes, finite- discrete kinetic equations, memory functions

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1 Introduction

From in-depth analysis of complex systems dynamics it becomes apparent, that the fundamental methods of statistical physics based on Hamilton formalism and exact equations of motion are inapplicable directly for its quantitative description. On the other hand, a discretization of events and long-time event-event correlations are very relevant in similar dynamics. Recently, the non-Markov theory of discrete stochastic processes was developed in paper [1]. The approach advanced in [1] made feasible calculations of wide set of non-Markov characteristics of an arbitrary

complex system from experimental data.

Used data analysis techniques can be divided into time and frequency domain ones. In the time domain we have calculated the following standard functions: phase portrait in plain projections of multidimensional space of the dynamic orthogonal variables, time correlation function on the whole observed time domain and the set of three junior memory and non-stationarity functions. Also we have determined the following power spectra (PS): of initial TCF's, first, second and third short MF, PS of the first, second

and third points of statistical spectrum of non-Markovity parameter and PS of the first four non-stationarity parameter.

2 EEG data analysis

Since the time of [2] - [7] heart rate variability (HRV) serves as one of the most reliable and authentic methods of testing the state of a human heart in norm and at pathology [8]. In particular, the analysis HRV has promoted the establishment of reliable connections between the functioning of a vegetative nervous system and a sudden heart death (SHD) [5] - [12]. In present time there are many diverse approaches by theoretical physics to the problems of HRV description. The following things should be mentioned: the fractal approach based on scaling of a frequency spectrum on power law $1/\omega^\alpha$ [13] - [15], the calculation of correlation dimension [16], the simulation by non-linear oscillators [16], [17], the calculation of the Kolmogorov entropy [16], usual [18] and dynamic [19] Shannon entropy, the use of dynamics of lattice spins as a model of arrhythmia [20], Fano-factor and Allan-factor [14], the wavelet analysis [21] and the detrended fluctuation analysis [6]. The following methods are also employed here: the multifractal analysis [23], the multiscaled randomness [24], Markov formalization of dynamics [25] and the terminal dynamics model of heartbeat [26]. In recent paper M.Teich *et al.* [27] demonstrated the manner in which various measures of fluctuations of sequence of inter-beat intervals could be used to assess the presence or likelihood of cardiovascular disease.

In this preliminary study, we have included a sample of patients subdivided into two groups. The first group consists of 30 healthy persons. In the second group there are 14 patients after myocardial infarction (MI) with weak electrical risk (arrhythmias of low degree).

Dynamic ECG recording has been done on three channels. The bipolar orthogonal channel X is channel N1, Y is channel N2 and Z is channel N3. Our analysis is executed on the channel N1. The RR recordings were drawn from the Division of Cardiac Surgery of 6th Kazan city Hospital (Kazan, Tatarstan) congestive heart-failure database comprising 30 records from normal patients (age:18-31 years; mean 22 year) and 14 records from severe congestive heart disease patients (age:32-67years, mean 55 years). The recording, which form a standard database for evaluating the merits of various measures for identification of heart disease, were made with a standard Holter Monitor (Astrocard Holter system-2F), digitized at a fixed value of 250 Hz. We use the long time series to $2^{16} = 65536$ beats to eliminate spurious effects due to variations in data to nonsinus beats associated with artifacts

In Figs. 1 the phase clouds are submitted for healthy (Kshf., Figs. 1 a-c) and patient after MI (Sibg., Figs. 1 d-f) in three plane projections (W_0, W_1) , (W_0, W_3) and (W_1, W_2) of four first orthogonal dynamic variables $W_i, i = 0, 1, 2, 3$. In phase portraits of the healthy (Figs. 1 a-c) in a plane (W_i, W_j) is some asymmetry of a phase cloud along a variable W_0 . But projection of a phase cloud in planes (W_i, W_j) with $i, j = 1, 2, 3$ are characterized by symmetrical distribution of a phase cloud. In the case of (Figs.1 d-f) patients after MI some features rush sharply in eyes. The basic feature is a fingerlike scattering of a phase cloud in planes (W_0, W_1) and (W_0, W_3) with numbers. This scattering is so specific, that its occurrence is exclusively characteristic as indicator of MI. The next feature is octopus-like distribution of phase clouds in three plane (W_1, W_2) (see, Fig.1f).

In Figs. 2 a, b the power spectra of TCF $a(t) = m_0(t)$ (Fig. 1a), first (Fig. 1b) MF of dynamics of RR-intervals of the ECG for healthy (Kshf., y) are signify. Fractal nature is exhibited for spectra of all memory function (zero (TCF) and first, second and third orders). There ap-

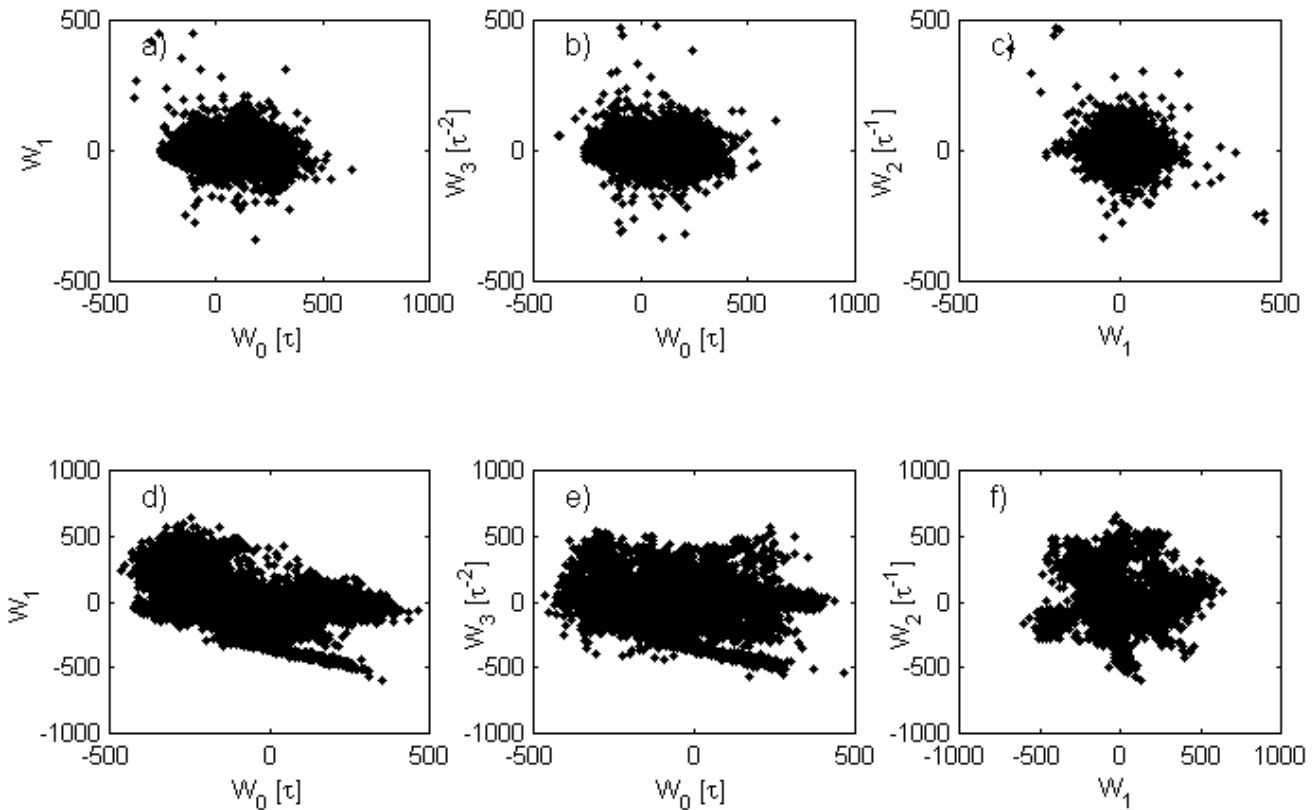


FIG. 1. Phase portrait of RR-intervals dynamics from human ECG's in a plain of two various orthogonal variables (W_i, W_j) for healthy (Kshf), a, b, c and for patient Sibg. on 20th day after MI, d, e, f.

pears a frequency dependence such as $\mu_i(\omega) \sim \omega^{-\alpha}$, $i = 0, 2$. Fractal behavior exists in the full frequency range only for initial TCF (see, Fig. 2a). Power spectra of first three junior MF's $\mu_i(\omega)$, $i = 1, 2, 3$ depict the non-fractal behavior in frequency domain $10^{-2} < \omega < 0,5$ f.u., where set of peaks is coupled with fast alteration of the three first orthogonal variables W_1 , W_2 and W_3 , which describe a human cardiovascular system (CVS) state.

Thus, in contrast to the commonly established point [8], [17], [21] the surprising occasion of group of high frequency peaks in a spectrum of the healthy for a function μ_1 can provide evidence to a latent pathology in human CVS activity.

Let go again to fractal behavior in Fig. 2a and 2b. Self-similar behavior of spectra $\mu_i(\omega)$ for healthy is accompanied with set of effects. Effects

of a respiratory arrhythmia (RA) are noticeable in both Figs. 2a and 2b. In the spectrum of initial TCF (Fig. 2a) the influence of RA can be seen on frequency 0,11 f.u. in the form of weak spectral splash. The fractal behavior of all spectra is associated also with the phenomenon of self-organized criticality (SOC) [42]. Nevertheless, the length of the linear segment in Figs. 2a and 2b is different. For example, for initial TCF (see, Fig. 2a) it extends from 0,5 f.u. up to 5×10^{-4} f.u., and SOC is characteristic for all registered frequency area. Vice-versa, SOC in the short MF's (see, Figs. 1b) is watched only in restricted frequency area from 10^{-2} f.u. up to frequency 5×10^{-4} f.u. As a result the restricted self-organized criticality (RSOC) is significant in the spectra of all short MF's.

The power spectra for patients after MI visibly differ a little from a case for healthy. Fractality

and SOC are manifested themselves in this case also, but contain already especially limited character. SOC is seen lying on the linear region for initial TCF (see, Fig. 2d) in a frequency interval from 0,4 f.u. up to frequency $0,9 \times 10^{-4}$ f.u., for all short MF's (Fig. 2e) in frequency interval from 2×10^{-2} f.u. $< \omega < 6 \times 10^{-5}$ f.u. Fractality in behavior of all spectra is characterized by sharp rupture of a linear region. For short MF's the power spectra (Fig. 2e) are accompanied with packets of spectral lines in analogous high-frequency region: from 2×10^{-2} f.u. up to 0,5 f.u.

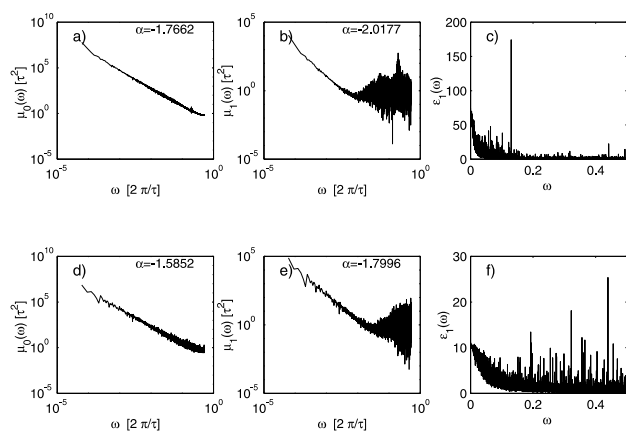


FIG. 2. Power spectra $\mu_i(\omega)$, $i = 0, 1$ and of the first point of non-Markovity parameter $\epsilon_1(\omega)$ for healthy (Kshf.,) (a, b, c) and patient (Sibg.,) after MI (d, e, f) from time dynamics of RR-intervals of human ECG's.

Behavior of spectra of the first point $\epsilon_1(\omega)$ of a statistical spectrum of NMP for the healthy (Fig. 2c) and patient after MI (Figs. 1f) appears as more dramatic. Behavior of first non-Markovity parameter $\epsilon_1(\omega)$ (see Fig. 1c) for healthy is representative for quasi-Markov relaxation scenario. Behavior of $\epsilon_1(\omega)$ for patient after MI in whole frequency region (see, Fig. 1f) most closely corresponds to non-Markov relaxation scenario. Dramatic change of $\epsilon_1(0)$ value from healthy ($\epsilon_1(0) \sim 71,6$) to patient after MI ($\epsilon_1(0) \sim 11,0$) (almost 6,5 times!) is valuable for pathologic data sets based in difference of these non-Markov properties. Close inspection of

these data shows that dynamics of RR intervals is non-Markovian specially for second and third relaxation levels for healthy and patient after MI. Careful analysis of Figs. 1c and 1f reveals a less prominent non-Markov behavior for patient after MI rather than for healthy. CVS of healthy represents the system that is more chaotic whereas CVS of patient after MI show evidence of more ordered system. Therefore, breaking of chaos and order forming is reliable predictor for myocardial infarction.

3 Seismology data analysis

Earthquakes (EQ's) are among the most dramatic phenomena in nature. We propose here a discrete stochastic model for possible solution of a problem of strong EQ's forecasting and differentiation of technogenic explosions (TE's) from the weak EQ's. For the study of basic mechanisms underlying its nature modern numerical and statistical methods are used now in modelling and understanding of the EQ phenomenon. In papers [28],[29] the modified renormalization group theory with complex critical exponents has been studied for implications of EQ's predictions. Long-periodic corrections found fit well the experimental data. Then universal long-periodic corrections based on the modified renormalization group theory have been used successfully [30] for possible predictions of the failure stress phenomenon foregoing an EQ. The failure stress data are in good reliability with acoustic emission measurements. In paper [31] it has been shown that the log-periodic corrections are of general nature, they are related to the discrete scale invariance and complex fractal dimension. This idea has been checked in [32], [33] for diffusion-limited-aggregate clusters. The paradox of the expected time until the next EQ with an attempt of finding of acceptable distribution is discussed in [34]. New explanation of Guttenberg-Richter power law related to the roughness of the frac-

tured solid surfaces has been outlined in [35]. Recent achievements and progress in understanding of complex EQ phenomena from different points of view are discussed in recent review [36]. New numerical methods like wavelets and multi-scale singular-spectrum analysis in treatment of seismic data are considered in [37].

We will apply the discrete non-Markov procedure for the analysis of real seismic data. The basic problems, which we are trying to solve in this analysis, are the following. The first problem relates to possibility of seismic activity description by statistical parameters and functions of non-Markov nature. The second problem relates to distinctive parameters and functions for differentiation of weak EQ's (with small magnitudes) from TE's. The third problem is the most important one and relates to strong EQ's forecasting. With this aim in mind we analyzed three parts of real seismogram: before the event (EQ and TE), during the event and after the event. A typical seismogram contains 4000 registered points. The complete analysis includes the following information: phase portraits of junior dynamical variables, power spectra of four junior memory functions and three first points of statistical spectrum of non-Markovity parameter. We took into account also the values of numerical parameters characterizing seismic activity. To analyze time functions we used also the power spectra obtained by the fast Fourier transform. The complete analysis exhibits great variety of data.

We used 4 types of available experimental data courteously given by Laboratory of Geophysics and Seismology (Amman, Jordan) for the following seismic phenomena: strong EQ in Turkey (I) (summer 1999), a weak local EQ in Jordan (II) (summer 1998). As TE we had the local underground explosion (III). The case (IV) corresponds to the calm state of the Earth before the explosion. All data correspond to transverse seismic displacements. The real temporal step of digitization τ between registered points of seismic activity has the following values, viz., $\tau = 0,02s$ for

the case I, and $\tau = 0,01s$ for the cases II-IV.

Figs. 3 demonstrates the power spectra of the first three points of the statistical spectrum of non-Markovity parameter for calm state of the Earth (a, b, c), and the states before (d, e, f) and during (g, h, i) strong EQ. The frequency behavior of three points of non-Markov parameters $\epsilon_1(\omega)$, $\epsilon_2(\omega)$ and $\epsilon_3(\omega)$ appeared to be practically the same. The behavior of functions $\epsilon_i(\omega)$ exhibits typical non-Markov character with small oscillations of random nature at LFR. The spectral characteristics of the system IV are very useful in comparison to the results obtained for the system I (before strong EQ).

One can make the following conclusions from Figs. 3 d, e, f. On the first level of relaxation process (see, Fig. 3d) the strained state of the Earth crust before EQ can be associated with Markov and quasi-Markov behavior in ULFR and LFR, correspondingly. The influence of non-Markov effects is reinforced in MFR with $5 \cdot 10^{-2} f.u. < \omega < 10^{-1} f.u.$, ($1 f.u. = 2\pi/\tau$). Strong non-Markovity of the processes considered for $\epsilon_1(\omega)$ takes place in HFR with $10^{-1} f.u. < \omega < 0.5 f.u.$. Simultaneously we have the numerical values $\epsilon_2(\omega), \epsilon_3(\omega) \sim 1$ in the whole frequency region (see, Figs. 3 e, f). However, this behavior implies that strong non-Markovity effects are observed in these cases.

The similar picture becomes unrecognizable for seismic state during the strong EQ (see, Figs. 3 g, h, i). Firstly, it is immediately obvious that $\epsilon_1(\omega) \sim 1$ on first relaxation level.

Secondly, the second and third relaxation levels are non-Markovian (see, Figs. 3 h, i). Thus, the behavior of seismic signals during the strong EQ is characterized by strong non-Markovity on the whole frequency region. Our observation shows that zero point values of non-Markovity parameters for calm Earth state are equal $\epsilon_1^{IV}(0) : \epsilon_2^{IV}(0) : \epsilon_3^{IV}(0) \approx 4.99 : 0.947 : 0.861$. These values are convenient for the comparison with similar values for the Earth seismic state before the strong EQ: $\epsilon_1^I(0) : \epsilon_2^I(0) : \epsilon_3^I(0) \approx 214.3 : 0.624 : 0.727$. The change of ratio of two first

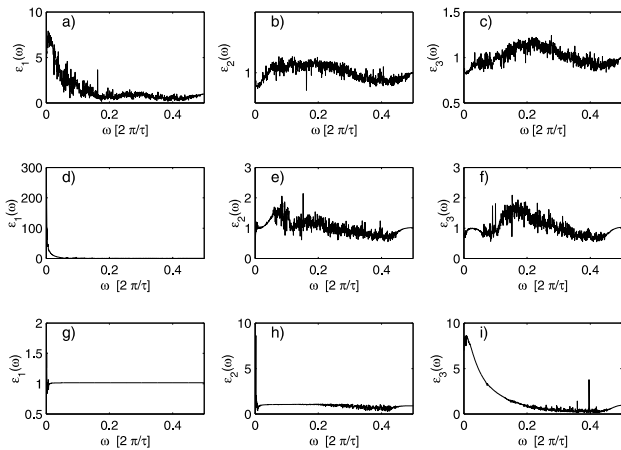


FIG. 3. Frequency spectra of the first three points of non-Markovity $\epsilon_1, \epsilon_2, \epsilon_3$: (a,b,c)- for calm state of the Earth before explosion(IV), (d,e,f)- before the strong EQ, (g,h,i)- during strong EQ. In behavior of $\epsilon_2(\omega)$ and $\epsilon_3(\omega)$ one can see a transition from quasi-Markovity (at low frequencies) to strong non-Markovity (at high frequencies).

non-Markovity parameters $\epsilon_1(0)/\epsilon_2(0)$ is particularly striking . This ratio is equal to 5.27 for the calm Earth state, then it comes into particular prominence for the state before strong EQ: $\epsilon_1^I(0)/\epsilon_2^I(0) \approx 343.4$. Thus, this ratio changes approximately in 60 times! Hence, the behavior of this numerical parameter is operable as a reliable diagnostic tool for strong EQ prediction. The foregoing proves that the indicated value drastically increases in process of nearing to strong EQ.

Figs. 4 depicts power spectra of MF M_0, M_1 and of the first two points of non-Markovity parameter $\epsilon_i(\omega), i = 1, 2$. for seismic states II and III. The preliminary results suggest that there is remarkable difference between weak EQ's and TE's especially in the area of low frequencies.

It is necessary to remark some peculiarities in power spectra of $\mu_i(\omega), i = 0, 1$ (see, Figs.4 a,b and e,f) for the cases II and III. All these spectra have distinctive similarities for memory functions $M_i(t)$ with $i = 0, 1, 2$ and 3. The character and the form of the spectra considered for the cases II and III are very similar to each other.

The same similarity is observed for the three non-Markovity parameters $\epsilon_i(\omega), i = 1, 2$ (see, Figs. 4 c,d and g,h). Nevertheless the analysis of the power frequency spectra allows to extract distinctive specific features between weak EQ's and TE's. Such quantitative criteria can be associated with frequency spectra of memory functions $\mu_i(\omega)$ characterizing long-range memory effects in seismic activity. This new criterion allows to distinguish definitely a weak EQ from a TE, viz., to differentiate case II from case III.

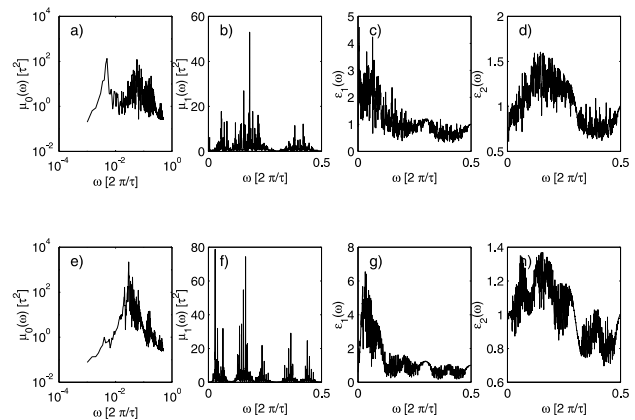


FIG. 4. The power spectra for two first memory functions μ_0, μ_1 and first two points in statistical spectrum of non-Markovity parameters $\epsilon_1, \epsilon_2, \epsilon_3$: (a,b,c,d) -during weak EQ, (e,f,g,h)- during technogenic explosion. A noticeable difference for states II and III exists in behavior $\epsilon_1(\omega)$ in point $\omega = 0$. Due to this fact, one can develop reliable approach to differentiation between weak EQ's and underground TE's.

Close examination of Figs. 4 c,d and g,h shows that this distinction appears in frequency behavior of the first point of non-Markovity parameter $\epsilon_1(\omega)$ close to zero point $\omega = 0$. Specifically, the ratio of values $\epsilon_1(0)$ for weak EQ and TE equals $\epsilon_1^{II}(0)/\epsilon_1^{III}(0) = 0.92/0.57 = 1.61$.

Seismic data are an object of careful analysis and numerous methods of their treatment are used especially for forecasting of EQ's with strong magnitudes. In spite of wide application of approaches based on nonlinear dynamics methods, the Fourier and wavelet transformations etc., we

have essential limitations, which narrows down the range of applicability of the results obtained. One of the main difficulty is that the discrete character of seismic signals registration is not taken into account. Another factor, which should be taken into account is related to the influence of local time effects. Alongside with of discreteness and local behavior of seismic signals considered here there exists the third peculiarity, viz., the influence of long-range memory effects. Theoretical analysis is performed by use of statistical theory of discrete non-Markov stochastic processes. The following Earth states have been considered among them: before (Ib) and during (I) strong EQ, during weak EQ (II) and during TE (III), and in a calm state of Earth core (IV). By the discrete non-Markov stochastic processes and the local Hurst exponent analysis we have found explicitly some features of several different states of the Earth crust: states of the Earth *before* and *during* strong and weak EQ's, during TE's. The used methods allow to present the seismogram analyzed in the form of set non-Markov variables and parameters. They contain great amount of qualitative and quantitative information about seismic activity.

The dynamic information is contained in time recordings of new orthogonal dynamic variables, different plane projections of multidimensional phase portrait. The information on the kinetic, spectral and statistical properties of the system is expressed through time dependence of the initial TCF, memory functions of junior orders, their power and frequency spectra of the first three points of statistical spectrum of non-Markovity parameter.

The main advantage of our two new methods is a great amount of supplementary information about properties of seismic signals. The problem is its correct application. What kind of possibilities can one expect? It is possible to answer as follows. Firstly, our preliminary study, convincingly demonstrates that the relevant and valuable information on non-Markov and discrete proper-

ties of the system considered is contained in seismic signals. In all the studied systems (I- IV) we have found out unique manifestations of Markov, quasi-Markov and non-Markov processes on the particular behavior of the signals in a broad range of frequencies.

The similar results cannot be obtained, in principle, by other methods used in the analysis of seismic activity.

4 Conclusion

In non-linear non-Markov characteristics some of well-known spectral effects are evident. Among them the following effects are exhibited noticeably: fractal spectra with an exponential function $\omega^{-\alpha}$, which are connected to the phenomenon of usual (SOC) and restricted (RSOC) self-organized criticality, behavior of some frequency spectra in the form of white and color noises. Thirdly, the frequency spectra introduced above are characterized by particular alternation of Markov (fractal) and non-Markov spectra (such as color or white noises). The similar alternation resembles in particular the peculiar alternation of effects of Markov and non-Markov behavior for hydrodynamic systems in statistical physics of condensed matter detected in papers [17], [18] for the first time. The fine specification of such alternation appears essentially different for studied states I-IV. These features allow to view optimistically for new HRV investigation methods development, the solution of the problem of forecasting of strong EQ's and differentiation TE's from weak EQ's.

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