

Fluctuations and Noise in Stochastic Spread of Respiratory Infection Epidemics in Social Networks

Renat Yulmetyev*, Natalya Emelyanova*, Sergey Demin*, Fail Gafarov*, Peter Hänggi† and Dinara Yulmetyeva (M.D.)**

**Department of Physics, Kazan State Pedagogical University, Mezhlauk Street 1, 420021 Kazan, Russia*

†*Department of Physics, University of Augsburg, Universitätsstrasse 1, D-86135 Augsburg, Germany*

***Division of Therapy, Republican Clinical Hospital, Orenburgskii Trakt 79, 420064 Kazan, Russia*

Abstract. For the analysis of epidemic and disease dynamics complexity, it is necessary to understand the basic principles and notions of its spreading in long-time memory media. Here we consider the problem from a theoretical and practical viewpoint, presenting the quantitative evidence confirming the existence of stochastic long-range memory and robust chaos in a real time series of respiratory infections of human upper respiratory track. In this work we present a new statistical method of analyzing the spread of gripe and acute respiratory track infections epidemic process of human upper respiratory track by means of the theory of discrete non-Markov stochastic processes. We use the results of our recent theory (Phys. Rev. E 65, 046107 (2002)) for the study of statistical effects of memory in real data series, describing the epidemic dynamics of human acute respiratory track infections and gripe. The obtained results testify to an opportunity of the strict quantitative description of the regular and stochastic components in epidemic dynamics of social networks with a view to time discreteness and effects of statistical memory.

INTRODUCTION

Now it has been well recognized that the study of epidemic and disease dynamics in social networks is a relevant theoretical issue of spreading viruses and diseases of various nature [1]-[4]. It is connected with understanding of the phase transitions in models of agent spreading in nonsteady systems. In particular, we deal with the development of the models describing epidemic spread, forest fires, the growth of populations, the activity of catalyzers, the formation of stars and galaxies. The study of the epidemic processes related to the increase in the fraction of population with respiratory infections, represents significant interest in modern biomedicine, ecology and economy [5]-[22]. The epidemic means that a large number of people in a certain country develop a disease at the same period of time. In case of "pandemic" the people in several countries develop the same disease at the same period of time. The first cases of respiratory infections with fatal outcome in USA and Europe were described in documents at the beginning of the 20th century. About 20 million people died then of "Spanish fever" within the period of 10 months. Pandemics of "Asian" and "Hongkong" gripe with smaller quantity of fatal outcomes [5]-[8] were observed in middle of the 20th century. The period of pandemic

processes is about 30-40 years. As a rule, epidemics break out in the autumn or in the winter in Northern Hemisphere and in the spring or in the summer in Southern Hemisphere. The duration of epidemics is about 1-3 months. Seasonality is one of the typical demonstrations of the phase development of epidemic processes. Respiratory infections affect people of any age. However children aged 1-14 develop the disease four times more often than adults. Each epidemic process of respiratory infections is accompanied by the increasing death rate. The parameters of grippe death rate in the world is 0.01 – 0.2%. The greatest number of deaths caused by grippe is connected with complications after the infection. Children (up to 2 years old) and elderly people (over 65 years old) are more liable to this infection and die more often.

Expenses caused by epidemics affect the successful development of economy due to the amount of money needed for financing scientific actions to prevent epidemics. These expenses are commensurable only with the sums earmarked to prevent heart diseases [9]-[16]. In many countries of the world great money is spent on the development of vaccines and serums, as well as prevention of epidemic outbreak flashes. However viruses of grippe and some other respiratory infections have great variability. There is a great variety of grippe virus cultures. Therefore the coming grippe variant is difficult to predict. The identification of a virus presents a great problem; consequently the prevention of epidemics outbreaks is complicated. Thus new preventive measures and methods of predicting possible epidemic outbreaks [17]-[22] are of great interest.

Now various methods of description and prediction of epidemic and pandemic processes are developed in medicine [17]-[22]. As a result of these studies hypotheses about risk factors have been formulated. The standard statistics indices [22] are used in traditional epidemic studies. The basic data yield the number of diseases, their intensive parameters, and average sizes. The extensive parameters, the cumulative data, the relative number of presentation, the ratio parameters and standardized parameters are used depending on the specific features of the epidemic process. However the existing methods are of no sufficient efficiency.

In this work we offer a new statistical approach in the research of epidemic processes with the help of the statistical theory of non-Markov stochastic discrete processes [23, 24, 25]. This method has already been used by the authors to study various real objects in cardiology [23, 25], seismology [24] and neuropsychology [26, 27]. With the help of this method we intend to find the full spectrum of the dynamic characteristics of the complex system. Taking in the account the discrete behavior and nonstationarity of the dynamic and kinetic epidemic parameters we shall receive a more detailed representation of the stochastic dynamics of epidemic processes.

This work deals with the study of statistical properties of epidemic dynamics. The key distinctions of grippe and respiratory infections epidemic processes are defined with the help of the long-range memory effects and statistical effects of non-Markovity in these processes. In particular, one of the key problems consists in revealing the role of randomness, regularity and predictability of epidemic processes.

More detailed statement of the theory and the description of quantities used here can be found in the works published earlier [23]-[27]. Below we present the basic equations of our theory necessary for calculation of time correlation function (TCF) and other spectral characteristics only.

For the nonstationary discrete processes the TCF has the form ($t = m\tau, N - 1 > m > 1$)

$$a(t) = \frac{1}{(N-m)\sigma(0)\sigma(t)} \sum_{j=0}^{N-1-m} \delta x(T+j\tau)\delta x(T+(j+m\tau)), \quad (1)$$

where $\sigma(0)$ and $\sigma(t)$ is the variances of the initial ($t = 0$) and final (at moment t) states of a systems, correspondingly. The kinetic equation for the TCF $a(t)$ has the form of a closed set of the finite-difference equations of the non-Markov type

$$\frac{\Delta a(t)}{\Delta t} = \lambda_1 a(t) - \tau \Lambda_1 \sum_{j=0}^{m-1} M_1(j\tau) a(t-j\tau). \quad (2)$$

Introducing the corresponding projection operators we come to the following chain of connected non-Markov finite-difference kinetic equations ($t = m\tau, n = 1, 2, \dots$)

$$\frac{\Delta M_n(t)}{\Delta t} = \lambda_{n+1} M_n(t) - \tau \Lambda_{n+1} \sum_{j=0}^{m-1} M_{n+1}(j\tau) M_n(t-j\tau). \quad (3)$$

EXPERIMENTAL DATA AND DATA PROCESSING

The registration and accounting materials, as well as the results of certain studies are the initial data for epidemic research in medicine. We have made use of the accounting materials of Kazan sanitary - epidemiological stations located in industrial districts. The data were given by the sanitary - epidemiological station of the Privolzhskii district of Kazan covering the period from 10.27.1995 to 03.05.2002. The experimental data involve the total population of the Privolzhskii district (about 300 thousand people). The first group of the data represents the six-year dynamics of gripe, the second group describes the six-year dynamics of acute respiratory track infections in this district. The obtained data were processed with the help of the above introduced technique. The set of three memory functions was calculated for each sequence of the data. The power spectra for each of these functions are obtained by the fast Fourier transform (FFT). We will also show the phase portrait in plan projections of the multidimensional space of the dynamic orthogonal variables. For a more detailed diagnostics of the system we will consider the frequency spectrum of the first three points of the statistical spectrum of the non-Markovity parameter. In this study we will use the frequency spectrum of non-Markovity parameter [23]-[25] $\varepsilon_i(\omega) = \{\mu_{i-1}(\omega)/\mu_i(\omega)\}^{\frac{1}{2}}$. Here $i = 1, 2, \dots$ and $\mu_i(\omega)$ is a power spectrum of the i th level. The statistical spectrum of non-Markovity parameter $\varepsilon_i(\omega)$ constitutes an information measure of Markovity and non-Markovity or randomness and regularity in time evolution of underlying systems.

DISCUSSION OF THE RESULTS

In this section the quantitative and comparative analysis of the six-year period of gripe chaotic dynamics (Fig.1a) and of acute respiratory track infection (Fig.1e) will be

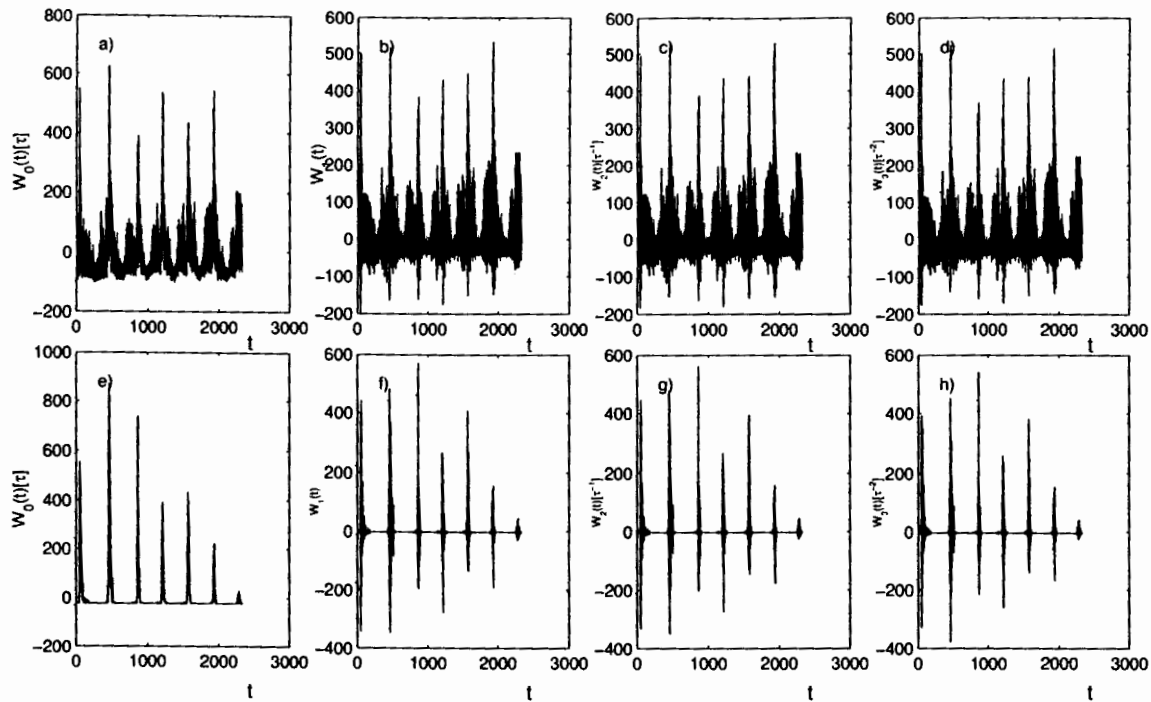


FIGURE 1. Time series of orthogonal variables W_0 (a, e), W_1 (b, f), W_2 (c, g), W_3 (d, h) of chaotic dynamics of gripe and acute respiratory track infections (ARTI)

carried out. The time series of the initial signals, the phase portraits of the dynamic variables, the power spectra of TCF and the junior functions of memory as well as frequency dependence of the first three points of the statistical non-Markov parameter are submitted in Figs. 1-4. The six-year sampling of gripe and acute respiratory track infections (ARTI) dynamics are submitted in Fig. 1 (Figs.1 a-d for gripe , Figs.1 e-h for ARTI). The time series of the orthogonal variable W_0 (Fig. 1a), W_1 (Fig. 1b), W_2 (Fig. 1c), W_3 (Fig. 1d) for the chaotic dynamics of gripe and ARTI have appreciable symmetry relative to straight line $W_i = 0$, where $i = 1, 2, 3$. The analysis of the time series of variable W_0 (Fig.1a) for a gripe epidemic shows that the moment of the gripe outburst or epidemic corresponds to the most significant fluctuations of this dynamic variable. Epidemic outbursts of the disease are located at equal distances from each other. This is the evidence of periodicity and interval constancy between gripe epidemics within the time of observation. The time series for three orthogonal variables W_1 (Fig. 1b), W_2 (Fig. 1c), W_3 (Fig. 1d) of gripe epidemics are almost perfectly symmetrical. The time series of dynamic variables W_0 (Fig. 1e), W_1 (Fig. 1f), W_2 (Fig. 1g), W_3 (Fig. 1h) of ARTI epidemiological processes present a different picture. The visible fluctuations of the dynamic variables for ARTI are observed in the autumn-winter period same as in case of gripe epidemics. However dynamic noises can be observed between the outbursts of ARTI (epidemics). They are represented by monotonously inequable parabolas. One of the half-parabola corresponds to one of the epidemics and another corresponds to the subsequent one. The time development of orthogonal variables W_1 (Fig. 1f), W_2 (Fig. 1g), W_3 (Fig. 1h) of the ARTI epidemiological process represents symmetric series about straight line $W_i = 0$. There are seven outbursts in the time behaviour of the dynamic

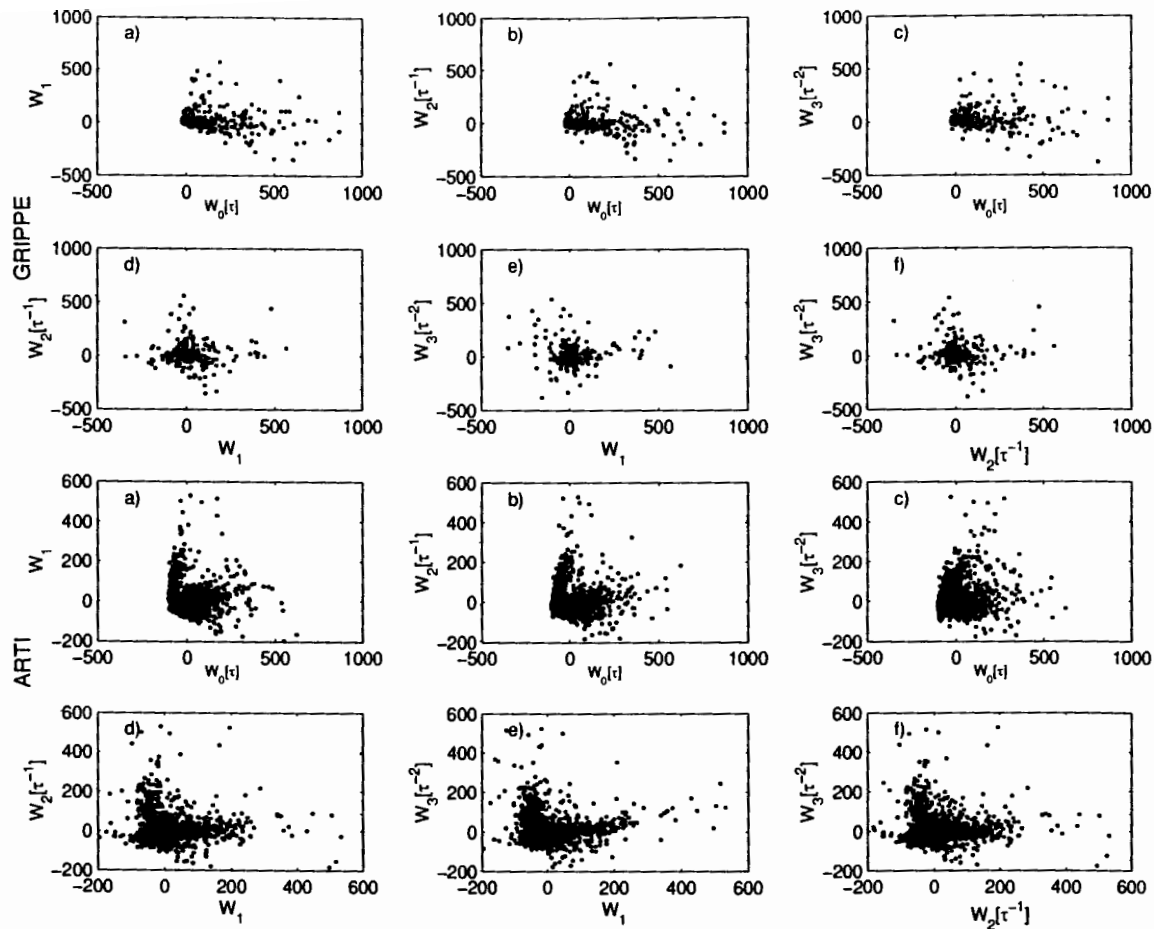


FIGURE 2. The phase clouds for epidemic processes of grippe and acute respiratory track infections (ARTI) in plane projections for the various combinations of orthogonal variables W_i, W_j where $i, j = 0...3$.

variables in the seven autumn - winter periods (from 10.27.1995 till 03.05.02). They corresponds to the seven cycles of grippe and ARTI epidemics. However the time series of orthogonal variables W_i ($i = 1, 2, 3$) for ARTI differ from grippe by the strongly expressed asymmetry and the existence of a clearly expressed noise.

In Fig. 2 the phase clouds in the six plane projections of the four first dynamic variables $W_i, i = 0...3$ for epidemic processes of grippe and ARTI are submitted. In case of grippe the phase clouds have a well defined asymmetry about the centre of coordinates. All the phase clouds contain a centralized nucleus and an aggregate of points scattered on perimeter in a fan-shaped way. The interval of the dispersal makes up 900τ . Owing to such asymmetric stratification the phase clouds get an elongated shape. The phase portraits of the ARTI epidemiological process have smaller symmetry about the centre of coordinates. The nucleus of the portrait gets the shape of a right angle and resembles a "boot". Apical emissions of separate points are appreciable on each side of the "boot". The interval of the dispersal of the points decreases up to 700τ . The stratification of the phase clouds of a grippe epidemiological process is more essential, than in case of an ARTI epidemic. The analysis of the data in other Kazan districts reveals a prominent phase portrait of a grippe epidemic. It is the elongation of

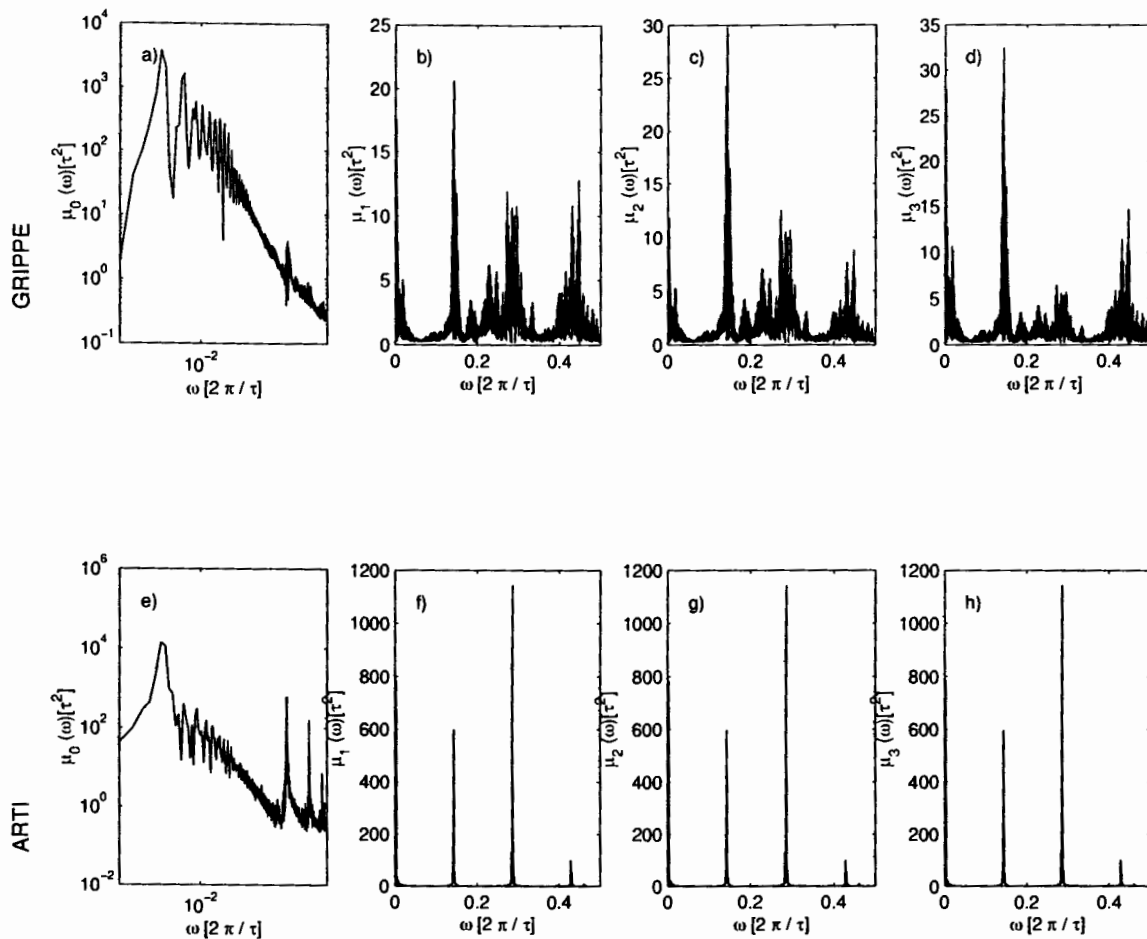


FIGURE 3. The power spectra of time correlation function (TCF) $\mu_0(\omega)$ (a) and of the three junior memory functions $\mu_i(\omega)$, $i = 1$ (b), 2 (c), 3 (d) for the epidemic processes of grippe and acute respiratory track infections (ARTI). The spectral peaks in the registered spectra are the evidence of of long-range memory in the system.

the phase portrait for an ARTI epidemic namely its "boot-type" shape.

In Fig. 3 the power spectra of TCF $\mu_0(\omega)$ and three junior memory functions $\mu_i(\omega)$, $i = 1, 2, 3$ for grippe and ARTI epidemiological processes are submitted. The frequency spectra of $\mu_0(\omega)$ are given in double logarithmic scale for a more detailed analysis of the data. In case of grippe the fractal dependence in the medium and high frequencies is found in the power spectrum of the initial TCF $\mu_0(\omega)$. The area of these frequencies is divided by a small outburst of power on frequency $\omega = 3 \cdot 10^{-1} f.u.$ ($1 f.u. = 2\pi/\tau$). In the domain of the low frequencies sharp breaks of the spectrum are observed. The frequency dependencies of the next three junior memory functions $\mu_i(\omega)$, $i = 1, 2, 3$ are accompanied by spectral outbursts at equal intervals and by condensation of spectral lines near the outbursts. A spectral noise is present in all diagrams of the frequency dependencies. The spectrum of the initial TCF $\mu_0(\omega)$ for an ARTI epidemic process differs from a grippe epidemic only in the high frequency area. Additional spectral peaks are appreciable on frequency $\omega = 10^{-1} f.u.$ and $\omega = 10^{-1/2} f.u.$ The power spectra of the three junior memory functions $\mu_i(\omega)$, $i = 1, 2, 3$ for an ARTI epidemic have bright

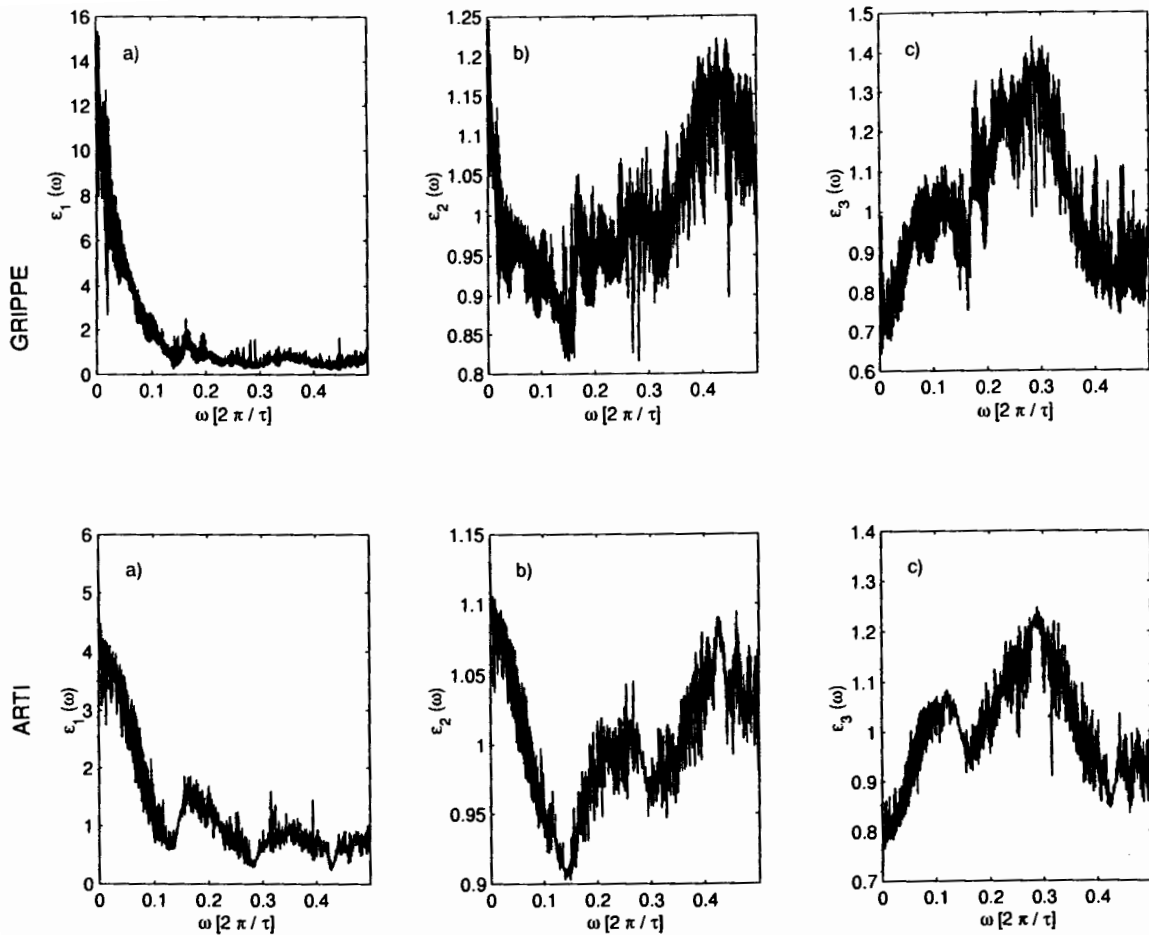


FIGURE 4. The frequency dependence of the first three points of statistical non-Markov parameter $\varepsilon_i(\omega)$ for epidemic processes of grippe and acute respiratory track infections (ARTI). This reflects the existence of long-range memory order in the system.

fractal dependence with several spectral peaks at every $0.15 f.u.$ In Fig. 4 the spectra of the first three points of statistical non-Markov parameter $\varepsilon_i(\omega)$, where $i = 1, 2, 3$ are presented. Fractalness is appreciable only in the behaviour of $\varepsilon_1(\omega)$. With approximation to frequency $\omega = 0$ the monotonous amplification of markovization is observed. The spectra of the non-Markovity parameter for the second point of $\varepsilon_2(\omega)$ for ARTI and grippe epidemics accept value $\varepsilon_2(\omega) = 1$ and are almost identical but have small fluctuations. It testifies to strong non-Markovity. The sinuosity with two frequency crests in area $\omega = 0.1 f.u.$ and $\omega = 0.3 f.u.$ is appreciable in frequency spectra of the third point of statistical non-Markovity parameter $\varepsilon_3(\omega)$.

Judging by the behaviour of the parameters the distinction between the epidemic of grippe (I) and that of ARTI (II) comes to the following. The initial values of the intensity $\mu_i(\omega = 0)$ for all $i = 0, 1, 2, 3$ differ for systems I and II by more than one order. It is the evidence of serious distinction in relaxation (correlation) times. For system I these times are almost 20-50 times shorter, than for system II. The intensity scales also differ almost by 50 times. Thus the initial values of $\varepsilon_1(\omega = 0)$ differ almost by 5 times for systems I and II.

On the basis of the aforesaid it is possible to come to certain conclusions about a great randomness of grippe dynamics in comparison with ARTI dynamics. The last one has greater regularity and predictability.

Another feature of the investigated epidemics is the existence of the three additional well - defined spectral splashes except the basic annual. From figs. 3 b-d and 3 f-h one can see that these groups of spectral peaks correspond to the characteristic frequencies equal accordingly to $0.14f.u.$, $0.28f.u.$ and $0.43f.u.$ The intensity of the high-frequency peak is almost an order lower, than the two other peaks of lower frequencies.

The strong distinction in the dynamic behaviour of grippe and ARTI epidemics is investigated in local time dependence of relaxation constants λ_1 , λ_2 , λ_3 , Λ_1 and Λ_2 . It is necessary to remind, that parameters λ_i , $i = 1, 2, 3...$ remind Luapunov's exponents for various relaxation levels. For system II great regularity is observed. All parameters of λ_1 , λ_2 , λ_3 are negative, which is the evidence of the stability of the system. For system I (grippe) owing to the general instability the parameter $\lambda_1(t)$ gets small positive values on tail. This corresponds to the slump of epidemic.

CONCLUSIONS

In this paper we have shown that the chaotic dynamics of epidemic processes of grippe and ARTI can be considered as a stochastic discrete non-Markov process. It is more convenient to use the statistical theory of the discrete non-Markov processes for real objects and alive systems [23] - [25] to study the similar processes. The used theory allows to define essential distinctions between the epidemic processes of grippe and acute respiratory track infections by degrees of randomness, regularity and predictability. By processing the experimental data about patient's quantity we obtained the time series of dynamic variables $W_i(t)$ and calculated the memory functions and the first three points of the statistical non-Markov parameter. We used the power spectra, which were obtained with help of the fast Fourier transform to analyze the time functions (correlation and memory functions).

It is possible to come to the following conclusions on the basis of the conceptions about long-range memory and localization of parameters. The epidemic process of grippe causes a great danger to mankind because of its randomness and unpredictability. Each epidemic process of grippe is characterized by a sudden outburst and sudden attenuation. The peak of an epidemic lasts a comparatively short period of time (60 days). This means efforts to liquidate the consequences of a grippe epidemic should be made during this period of time. The epidemic process of ARTI is characterized by a greater regularity and predictability. It is accompanied by a small outburst at the beginning of the epidemic, then the number of patients diminishes. After this, a drastic outburst corresponding to the epidemic peak is observed. The process ends with small short-term splashes which later quickly fade. Unlike grippe, an ARTI epidemic is characterized by a less drastic outburst and attenuation. During the studied interval insignificant outbursts of respiratory infections are observed followed by periodic outbursts of ARTI.

We have revealed and explained essential distinctions in phase portraits, frequency spectra, and quantitative estimations of memory functions and statistical non-Markov

parameters between the epidemic processes of grippe and ARTI. Relative distinctions of the quantitative data on epidemics of grippe and ARTI are the evidence of the distinction in the long-range memory, order, and organization of each process in the same system. These distinctions allow us to define the degree of importance of each of the epidemics. In particular, we have shown that the epidemic process of grippe and respiratory infections have essentially different quantitative measure of randomness and regularity.

The obtained results can be of practical value while researching other diseases and comparatively estimating epidemic danger.

In this paper we have demonstrated that the set of relaxation, kinetic, and spectral parameters and characteristics of discrete non-Markov stochastic processes are valuable for the description of the role of randomness, regularity, and predictability of epidemic processes, the spread of grippe and ARTI in the world.

The results obtained here generally relate to non-stationary and non-ergodic processes in stochastic systems. Therefore, they are certainly of value for the physics of the disordered matter in which similar processes can occur on molecular or structural levels.

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REFERENCES

1. M.E. Newman, *Phys. Rev. E* **66**, 016128 (2002).
2. R. Pastor-Satorras, and A. Vespignani, *Phys. Rev. E* **65**, 035108 (2002).
3. P. Grassberger, H. Chate, and G. Rousseau, *Phys. Rev. E* **55**, 2488 (1997); M. Ipsen, and A. Mikhailov, *Rhys. Rev. E* **66**, 046109 (2002).
4. L.A. Amaral, A. Scala, M. Barthelemy, and H.E. Stanley, *Proc. Natl. Acad. Sci. USA* **97**, 11149 (2000); M. Barthelemy, and L.A.N. Amaral, *Phys. Rev. Lett.* **82**, 3180 (1999); S. Mossa, M. Barthelemy, H. E. Stanley, and L.A.N. Amaral, *Phys. Rev. Lett.* **88**, 138701 (2002); J. Camacho, R. Guimera, and L.A.N. Amaral, *Phys. Rev. E* **65**, 030901(R) (2002).
5. R.F. Burk, W. Schaffner, M.G. Koenig, *Arch. Intern. Med.* **127**, 1122 (1971).
6. S.D. Collins, J. Lehman, Excess deaths from influenza and pneumonia and from important chronic diseases during epidemic periods 1918-51, *Public Health monographs* **10**, 1 (1953).
7. S.W. Schwarzmann, J.L. Adler, R.J. Sullivan, W.M. Marine, *Arch. Intern. Med.* **127**, 1037 (1971).
8. C.H. Stuart-Harris, *Nature* **225**, 850 (1970).
9. C.K. Peng, S.V. Buldyrev, A.L. Goldberger, S. Havlin, M. Simons, H.E. Stanley, *Phys.Rev.E* **47**, 3730 (1993).
10. C.K. Peng, S. Havlin, H.E. Stanley, A.L. Goldberger, *Chaos* **6**, 82 (1995).
11. L.A.N. Amaral, S.V. Buldyrev, S. Havlin, M.A. Salinger, H.E. Stanley, *Phys.Rev.Lett.* **80**, 1385 (1998).

12. Y. Ashkenazy, P.Ch. Ivanov, S. Havlin, C.K. Peng, A.L. Goldberger, H.E. Stanley, *Phys.Rev.Lett.* **86**, 1900 (2001).
13. L.A.N. Amaral, P.Ch. Ivanov, N. Aoyagi, I. Hidaka, S. Tomono, A.L. Goldberger, H.E. Stanley, Y. Yamamoto, *Phys.Rev.Lett.* **86**, 6026 (2001).
14. V. Schulte-Frohlinde, Y. Ashkenazy, P.Ch. Ivanov, L. Glass, A.L. Goldberger, H.E. Stanley, *Phys.Rev.Lett.* **87**, 068104 (2001).
15. Z. Chen, P.Ch. Ivanov, K. Hu, H.E. Stanley, *Phys.Rev.E* **65**, 041107 (2002).
16. P.Ch. Ivanov, L.A.N. Amaral, A.L. Goldberger, S. Havlin, M.G. Rosenblum, H.E. Stanley, Z.R. Struzik, *Chaos* **11**, 641 (2001).
17. R.G. Webster, W.J. Bean, T.O. Gorman, *Microbiol.* **56**, 159 (1992).
18. H. Scheiblaue, M. Reinacher, M. Tashiro, R. Rott, *J. Infect Dis* **166**, 783 (1992).
19. F.M. LaForce, K.L. Nichol, N.J. Cox, *Am. J. Prev. Med.* **10**, 31 (1994).
20. P.O. Hokanen, T. Keistinen and S-L. Kivela, *Public Health* **110**, 163 (1996).
21. Y. Ghendon, *Euro J Epidemiology* **10**, 485 (1994).
22. R. Snacken, J.C. Manuguerra, P. Taylor, *Method of Information in Medicine* **37**, 266 (1998).
23. R.M. Yulmetyev, P. Hänggi, F.M. Gafarov, *Phys. Rev. E* **62**, 6178 (2000).
24. R.M. Yulmetyev, F.M. Gafarov, P. Hänggi, R.R. Nigmatullin, Sh. Kayumov, *Phys. Rev. E* **64**, 066132 (2001).
25. R.M. Yulmetyev, P. Hänggi, F. Gafarov, *Phys. Rev. E* **65**, 046107 (2002).
26. R.M. Yulmetyev, F.M. Gafarov, D.G. Yulmetyeva, N.A. Emelyanova, *Physica A* **303**, 427 (2002).
27. R. Yulmetyev, N. Emelyanova, P. Hänggi, F. Gafarov, A. Prokhorov, *Physica A* **316**, 361 (2002).