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Statistical Theory of Non-Stationary Time Correlation in Complex Systems with Discrete Time

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We develop the statistical theory of discrete non-stationary non-Markov random processes in complex systems. The objective of this paper is to find the chain of finite-difference non-Markov kinetic equations for time correlation functions (TCF) in terms of non-stationarity effects. The developed theory starts from careful analysis of time correlation through non-stationary dynamics of vectors of initial and final states and non-stationary normalized TCF. Using of projection operators technique we find the chain of finite-difference non-Markov kinetic equations for discrete non-stationary TCF and for the set of non-stationary discrete memory functions (MF). The last contains supplementary information about non-stationary properties of complex system in a whole. Another relevant result of our theory is a construction of a set of dynamic parameters of non-stationarity, which contains information on non-stationarity effects. The full set of dynamic parameters and kinetic functions (TCF, short MF, statistical spectra of non-Markovity parameter and statistical spectra of non-stationarity parameter) has made possible in-depth information about discreteness, non-Markov effects, long-range memory and non-stationarity of underlying processes.

Key words: non-Markov discrete processes, non-stationary time correlation, finite-discrete kinetic equations, memory functions.

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1 Introduction

Study of information processing in life systems is one of the central problems in modern science. It is now well known that in some of natural sequences the elements are not arranged randomly, but exhibit long-range correlations. Over a long time ago it was suggested that many complex systems observed in nature can be described by low dimensional nonlinear dynamic models. It was anticipated that the properties of these systems are expressible by Lyapunov exponents, unique fractal dimensions or Kolmogorov-Sinai entropy. However considerable recent attention has been focused on the wide variety of complex systems reveal convincingly that such a low dimensionality can be expected for rather coherent phenomena like as observed in laser systems. Alive data seems to have a more complicated structure largely due to high - dimensional and many-factor processes and due to pronounced effects of random fluctuations and long-time memory effects.

Let us remind, that one of the key moments of spectral approach in the analysis of stochastic processes is the usage of normalized time correlation function (TCF)

$$a_0(t) = \frac{\langle \mathbf{A}(T)\mathbf{A}(T+t)\rangle}{\langle |\mathbf{A}(\mathbf{T})|^2\rangle}\ , \qquad (1.1)$$

where the time T is the initial time, angular brackets indicate a scalar product of vectors, vector $\mathbf{A}(\mathbf{t})$ represents a state vector of a complex system, $|\mathbf{A}(t)|$ is the length of vector $\mathbf{A}(t)$.

The above-stated designation is true only for stationary systems. In non-stationary case, Eqn.(1.1) should be changed. The concept of TCF can be generalized for a case of discrete non-stationary sequence of signals. For this purpose the standard definition of a correlation coefficient in probability theory for two random signals X and Y

$$\rho = \frac{\langle \mathbf{XY} \rangle}{\sigma_X \sigma_Y},\ \sigma_X = |\mathbf{X}|,\ \sigma_Y = |\mathbf{Y}|, \tag{1.2}$$

must be taken into account. In Eqn. (1.2) the multicomponent vectors $\mathbf{X}, \mathbf{Y}$ are produced by fluctuations of signals $\mathbf{x}$ and $\mathbf{y}$ accordingly, σ_x^2, σ_y^2 represent dispersions of signals $\mathbf{x}$ and $\mathbf{y}$, and values $|\mathbf{X}|, |\mathbf{Y}|$ represents lengths of vectors $\mathbf{X}, \mathbf{Y}$, correspondingly. Therefore the function

$$a(T,t) = \frac{\langle \mathbf{A}(T)\mathbf{A}(T+t) \rangle}{|\mathbf{A}(T)||\mathbf{A}(T+t)|} \tag{1.3}$$

can be considered as the generalization of the concept of TCF (1.1) for non-stationary processes $\mathbf{A}(T+t)$. Non-stationary TCF (1.3) obeys to the conditions of the normalization and attenuation of correlation

$$a(T,0) = 1,\ \lim_{t\to\infty} a(T,t) = 0\ .$$

For the quantitative description of non-stationarity in accordance with Eqns. (1.3), (1.1) it is convenient to introduce a function of non-stationarity

$$\gamma(T,t) = \frac{|\mathbf{A}(T+t)|}{|\mathbf{A}(T)|} = \left\{ \frac{\sigma^2(T+t)}{\sigma^2(T)} \right\}^{1/2}\ , \tag{1.4}$$

which is equal to the ratio of lengths of vectors of final and initial states. In the case of stationary process the dispersion does not vary with the time(or its variation is very weak). Therefore the following relations

$$\sigma(T+t) = \sigma(T),\ \gamma(T,t) = 1 \tag{1.5}$$

are true for stationary process. Due to condition (1.5) the following function

$$\Gamma(T,t) = 1 - \gamma(T,t) \tag{1.6}$$

is convenient to consider as a dynamic parameter of non-stationarity. This dynamic parameter can serve as quantitative measure of non-stationarity of process under investigation. According to Eqns. (1.4) -(1.6) it is reasonable safe to suggest the existence of three different classes of non-stationarity

$$|\Gamma(T,t)| = |1-\gamma(T,t)| \left\{ \begin{array}{cc} \ll 1, & \text{weak} \\ \sim 1, & \text{intermediate} \\ \gg 1, & \text{strong.} \end{array} \right\} . \tag{1.7}$$

The existence of dynamic parameter of non-stationarity permits to determine , in-principle, the type of non-stationarity of investigated process and to find its spectral characteristics from experimental data.

2 Statistical theory of non-stationary discrete non-Markov processes in complex systems

Here we extend original results of the statistical theory of discrete non-Markov processes in complex systems, developed recently by us in paper [1], for the case of non-stationary processes. The theory is modelled on the basis of the first principles and represents a discrete finite-difference analogy for complex systems of well known Zwanzig'-Mori's kinetic equations [2], [3] in statistical physics of condensed matter.

We consider a discrete stochastic process

$$X = \{x(T), x(T+\tau), x(T+2\tau), \dots\ x(T+k\tau), \dots x(T+(N-1)\tau\}, \tag{2.1}$$

where τ is a discretization of time. The normalized time correlation function reads (TCF)

$$a(t) = \frac{1}{(N-m)\sigma^2} \times \sum_{j=0}^{N-1-m} \delta x(T+j\tau)\delta(T+(j+m)\tau). \tag{2.2}$$

The dispersion σ^2, fluctuation $\delta x(T+j\tau)$ and mean value $\langle x\rangle$ are written as

$$\delta x_j = \delta x(T + j\tau) = x(T + j\tau) - \langle x\rangle,$$
$$\sigma^2 = \frac{1}{(N-m)} \sum_{j=0}^{N-1-m} \{\delta x(T + j\tau)\}^2 , \qquad (2.3)$$

$$\langle x\rangle = \frac{1}{(N-m)} \sum_{j=0}^{N-1-m} x(T + j\tau) \qquad (2.4)$$

and discrete time t is equal to $t = m\tau$.

In general, mean, dispersion and TCF in (2.2), (2.3) and (2.4) are dependent on numbers m and N. Similar situation exists with regard to the case of non-stationary processes. All indicated values cease to depends on numbers m and N for stationary processes for $m \ll N$. Definition of TCF in Eqn. (2.2) is also valid for stationary processes only.

It is necessary to consider this important dependence to take into account effects of non-stationarity. For this purpose let us form two k dimensional vectors of state

$$\mathbf{A}_k^0 = (\delta x_0, \delta x_1, \delta x_2 ... \delta x_{k-1}),\ \mathbf{A}_{m+k}^m$$
$$= (\delta x_m, \delta x_{m+1}, \delta x_{m+2} ... \delta x_{m+k-1}) . \qquad (2.5)$$

by the process (2.1). In Euclidean space of vectors of state (2.5) TCF $a(t)$ describes correlation of two different states of the system ($t = m\tau$)

$$a(t) = \frac{\langle \mathbf{A}_{N-1-m}^0 \mathbf{A}_{N-1}^m \rangle}{(N-m)\{\sigma(N-m)\}^2} = \frac{\langle \mathbf{A}_{N-1-m}^0 \mathbf{A}_{N-1}^m \rangle}{\langle \{\mathbf{A}_{N-1-m}^0\}^2 \rangle} . \qquad (2.6)$$

Here the brackets $\langle\ \ \rangle$ indicate scalar product of two vectors. Also the dimension dependence of the corresponding vectors is taken into account in the dispersion $\sigma = \sigma(N-m)$. As a matter of fact, TCF $a(t)$ represents $cos\vartheta$, where ϑ is the angle between two vectors from Eqn. (2.5). Let us introduce a unit vector of dimension $(N-m)$ as follows

$$\mathbf{n}_0 = \frac{\mathbf{A}_{N-1-m}^0}{\sqrt{(N-m)\sigma^2}} . \qquad (2.7)$$

Then TCF $a(t)$ (2.2) can be represented as

$$a(t) = \langle \mathbf{n}_0 \mathbf{n}'_m \rangle = \langle \mathbf{n}_0(0) \mathbf{n}'_m(t) \rangle . \qquad (2.8)$$

From the above discussion it is clear that Eqns. (2.6) -(2.8) are valid for stationary processes only. In the case of non-stationary processes, along with Eqns.(1.2) -(1.7) it is necessary to redefine TCF to take into account the non-stationarity in the dispersion σ^2. For this purpose we redefine a unit vector of final state as follows

$$\mathbf{n}_m(t) = \frac{\mathbf{A}_{N-1}^m(t)}{\sqrt{|\mathbf{A}_{N-1}^m(t)|^2}} . \qquad (2.9)$$

Then for non-stationary processes it is convenient to write TCF as the scalar product of two unit vectors of states

$$a(t) = \langle \mathbf{n}_0(0) \mathbf{n}_m(t) \rangle$$
$$= \frac{\langle \mathbf{A}_{N-1-m}^0(0) \mathbf{A}_{N-1}^m(t) \rangle}{\sqrt{\{\mathbf{A}_{N-1-m}^0(0)\}^2 \{\mathbf{A}_{N-1}^m(t)\}^2}} . \qquad (2.10)$$

Now let us consider dynamics of non-stationary stochastic process. The equation of motion of a random variable x_j can be written in a finite- difference form for $0 \le j \le N-1$ as follows

$$\frac{dx_j}{dt} \Rightarrow \frac{\triangle \delta x_j}{\triangle t} = \frac{\delta x_j(t+\tau) - \delta x_j(t)}{\tau} . \qquad (2.11)$$

The discrete evolution single step operator is conveniently expressed by:

$$x_{j+1}(T + (j+1)\tau) = U(\tau) x_j(T + j\tau). \qquad (2.12)$$

In the case of stationary process we can rewrite the equation of motion (2.11) in a more simple form

$$\frac{\triangle \delta x_j}{\triangle t} = \tau^{-1}\{U(\tau) - 1\}\delta x_j . \qquad (2.13)$$

The invariance of mean $\langle x\rangle$

$$\langle x\rangle = U(\tau)\langle x\rangle,\ \{U(\tau) - 1\}\langle x\rangle = 0 . \qquad (2.14)$$

is taken into account in Eqn.(2.13). In the case of non-stationary process let us address to the equation of motion for vector of final state $\mathbf{A}_{m+k}^m(t)$ ($k = N-1-m$)

$$\frac{\triangle \mathbf{A}_{m+k}^m(t)}{\triangle t} = i\hat{L}(t,\tau)\mathbf{A}_{m+k}^m(t) , \qquad (2.15)$$

where the Liouville's quasioperator reads

$$\hat{L}(t,\tau) = (i\tau)^{-1}\{U(t+\tau,t) - 1\} \ . \qquad (2.16)$$

The matrix representation of the Liouville's quasioperator and evolution operator allows to take into account carefully non-stationary features of the dynamics of multidimensional vector of the final state of a system.

So, due to Eqns. (2.10), (2.15) it is possible to take into account the non-stationarity of stochastic process. Now let us introduce projection operator Π in Euclidean space of state vectors

$$\Pi \mathbf{A}(t) = \frac{\mathbf{A}(0)\rangle\langle\mathbf{A}(0)}{\sqrt{|\mathbf{A}(0)|^2|\mathbf{A}(t)|^2}}\mathbf{A}(t)\rangle \ , \qquad (2.17)$$

where the angular brackets reflect the boundaries of action of scalar product.

For the analysis of dynamics of stochastic process $\mathbf{A}(t)$ the vector $\mathbf{A}_k^0(0)$ from (2.5) can be considered as a vector of an initial state $\mathbf{A}(0)$, and the vector $\mathbf{A}_{m+k}^m(t)$ from (2.5) for $m+k = N-1$ can be considered as a vector of a final state $\mathbf{A}(t)$.

It is necessary to note, that the projection operator (2.17) has necessary property of idempotentity $\Pi^2 = \Pi$. The presence of operator Π allows to introduce mutually supplementary projection operator P as follows

$$P = 1 - \Pi, \quad P^2 = P, \quad \Pi P = P\Pi = 0 \ . \qquad (2.18)$$

It is necessary to mark, that both projectors Π and P are non-linear and can be recorded only for fulfillment of operations in particular space.

Due to the property (2.10) it is easy to obtain required TCF as follows

$$\Pi\mathbf{A}(t) = \Pi\mathbf{A}_{m+k}^m(t) \\ = \mathbf{A}_k^0(0)\langle\mathbf{n}_k^0(0), \mathbf{n}_{k+m}^m(t)\rangle = \mathbf{A}_k^0(0)a(t) \ . \qquad (2.19)$$

By the property (2.10), the projector Π will generate a unit vector along a vector of final state $\mathbf{A}(t)$ and creates its projection to an initial state vector $\mathbf{A}(0)$.

Existence of a pair of mutually supplementary projection operators Π and P allows to carry out splitting of the Euclidean space of vectors $A(\mathbf{A}(0), \mathbf{A}(t) \in A)$ into a direct sum of two mutually supplementary subspaces as follows

$$A = A' \overset{\bullet}{+} A'', \ A' = \Pi A, \ A'' = PA \ . \qquad (2.20)$$

Liouville's quasioperator $\hat{L}$ in Eqn. (2.16) is splitted as follows

$$\hat{L} = \hat{L}_{11} + \hat{L}_{12} + \hat{L}_{21} + \hat{L}_{22} \ , \qquad (2.21)$$

where the matrix elements

$$\hat{L}_{11} = \Pi\hat{L}\Pi, \ \hat{L}_{12} = \Pi\hat{L}P, \ \hat{L}_{21} = P\hat{L}\Pi, \ \hat{L}_{22} = P\hat{L}P \qquad (2.22)$$

are introduced. Euclidean space of values $W = \hat{L}A$ of Liouville's quasioperator can be generated by vectors $\mathbf{W}$ of dimension $k-1$

$$(\mathbf{W}(0) \in W, \ \mathbf{W}(t) \in W) \\ W = W' \overset{\bullet}{+} W'', \ W' = \Pi W, \ W'' = PW \ . \qquad (2.23)$$

Matrix elements $\hat{L}_{ij}$ of contracted description

$$\hat{L} = \begin{pmatrix} \hat{L}_{11} & \hat{L}_{12} \\ \hat{L}_{21} & \hat{L}_{22} \end{pmatrix} \qquad (2.24)$$

act as follows: $\hat{L}_{11}$ - from a subspace A' to subspace W', $\hat{L}_{12}$ - from A'' to W', $\hat{L}_{21}$ - from W' to W'' and $\hat{L}_{22}$ - from A'' to W''. The projection operators Π and P allow to execute the contracted description of stochastic process. Splitting the dynamic equation (2.15) on two equations in two mutually supplementary subspaces (see, for example [1]),one gets

$$\frac{\triangle\mathbf{A}'(t)}{\triangle t} = i\hat{L}_{11}\mathbf{A}'(t) + i\hat{L}_{12}\mathbf{A}''(t) \ , \qquad (2.25)$$

$$\frac{\triangle\mathbf{A}''(t)}{\triangle t} = i\hat{L}_{21}\mathbf{A}'(t) + i\hat{L}_{22}\mathbf{A}''(t) \ . \qquad (2.26)$$

Following [1], for simplification of the Liouville's equation (2.15) firstly it is necessary to eliminate irrelevant part $\mathbf{A}''(t)$, and then to write a closed equation for relevant part $\mathbf{A}'(t)$. According to [1] this can be realized by series of successive steps (for example, see Eqns. (32) - (36) in Ref. [1]).

When considered projection resulting to appearance of TCF $a(t=m\tau)$ we derive a finite-difference kinetic equation of non-Markov type

$$\frac{\triangle a(t)}{\triangle t} = \lambda_1 a(t) - \tau\Lambda_1 \sum_{j=0}^{m-1} M_1(t-j\tau)a(j\tau) \ . \quad (2.27)$$

Here eigenvalue (λ_1) and relaxation (Λ_1) parameters of the Liouville's quasioperator $\hat{L}$ are introduced by the relations

$$\lambda_1 = i\frac{\langle \mathbf{A}_k^0(0)\hat{L}\mathbf{A}_k^0(0)\rangle}{\langle|\mathbf{A}_k^0(0)|^2\rangle}, \quad (2.28)$$

$$\Lambda_1 = \frac{\langle \mathbf{A}_k^0(0)\hat{L}_{12}\hat{L}_{21}\mathbf{A}_k^0(0)\rangle}{\langle|\mathbf{A}_k^0(0)|^2\rangle} = -\frac{\langle \mathbf{A}_k^0(0)i\hat{L}(i\hat{L}-\lambda_1)\mathbf{A}_k^0(0)\rangle}{\langle|\mathbf{A}_k^0(0)^2|\rangle}, \quad (2.29)$$

where angular brackets indicate scalar product of new vectors of state. Function $M_1(t-j\tau)$ in the right side of Eqn. (2.27) represents a modified memory function (MF)

$$M_1(t-j\tau) = \gamma_1(t-j\tau)m_1(t-j\tau). \quad (2.30)$$

The equation is the first kinetic finite-difference equation for TCF. It is remarkable that the non-Markovity, discretization and non-stationarity of stochastic process can be considered explicitly. Due to the consideration of non-stationarity both in TCF and first memory function this equation generalizes our results obtained recently in Ref. [1]. We have introduced the following notions for functions in Eqns. (2.27), (2.30)

$$m_1(t-j\tau) = \frac{\langle \mathbf{W}_1(0)\hat{V}(T+m\tau,T+(j+1)\tau)\mathbf{W}_1(j\tau)\rangle}{\{\langle|\mathbf{W}_1(0)|^2\rangle\}^{1/2}\{|\mathbf{W}_1(t-j\tau)|^2\}^{1/2}}, \quad (2.31)$$

$$\gamma_1(t-j\tau) = \left\{\frac{\langle|\mathbf{W}_1(t-j\tau)|^2\rangle}{\langle|\mathbf{W}_1(0)|^2\}}\right\}^{1/2}, \quad (2.32)$$

$$\mathbf{W}_1(t-j\tau) = \hat{V}(T+m\tau,T+(j+1)\tau) \times\hat{L}_{21}(T+(j+1)\tau,T+j\tau)\mathbf{A}_k^0(0). \quad (2.33)$$

The modified evolution operator in Eqn. (2.31) has the property $\hat{V}(T+t,T+t)=1$. It should be mentioned, that new dynamic parameter of non-stationarity $\gamma_1(t-j\tau)$, Eqn. (2.31) appears first in MF, $M_1(t-j\tau)$. On the one hand the short MF $m_1(t-j\tau)$, as indicated by Eqn. (2.31), represent an ordinary memory function of the first order, normalized with regard to effects of non-stationarity. On the other hand, the short TCF $m_1(t-j\tau)$ is normalized non-stationary TCF for new orthogonal dynamic variable $\mathbf{W}_1(t-j\tau)$ in Eq. (2.33). This variable is a result of action of matrix $\hat{L}_{21}$ of complex system's Liouvillian quasioperator. The last related to the projection operator $P=1-\Pi$, which generates the orthogonal supplement to vector $\mathbf{A}_k^0(0)$.

It is important to mark, that the memory function $m_1(j\tau)$ is normalized TCF for a new random variable $\mathbf{W}_1$

$$\mathbf{W}_1(j\tau) = U_{22}(T+j\tau,T)\hat{L}_{21}\mathbf{A}_k^0(0) \ . \quad (2.34)$$

Time evolution is determined now by contracted Liouville's quasioperator

$$L^{(1)} = L_{22} = P\hat{L}P \ . \quad (2.35)$$

Through of discreteness of time series the dimension $(k-1)$ of new state vector $\mathbf{W}_1$ is less than dimension (k) of initial vector $\hat{W}_0 = \mathbf{A}_k^0$ on unit. Consideration must be given to this circumstance in numerical calculations.

Let us write now in an obvious form the equation of motion for $\mathbf{W}_1(j\tau)$ having regard to Eqn. (2.15)

$$\frac{\triangle\mathbf{W}_1(t)}{\triangle t} = \frac{\mathbf{W}_1(t+\tau)-\mathbf{W}_1(t)}{\tau} \quad (2.36)$$

$$= i\hat{L}_{22}\mathbf{W}_1(t) = i\hat{L}^{(1)}\mathbf{W}_1(t) \ . \quad (2.37)$$

For new random dynamic variable $\mathbf{W}_1(t)$ it is possible to repeat all above mentioned arguments, which one used finding the kinetic equation. Then it is possible to obtain the second equation for a short normalized memory function $m_1(j\tau)$.

However is more convenient to use the Gram-Schmidt orthogonalization procedure [4], [5] for the set of new dynamical orthogonal variables

$$\langle \mathbf{W}_n, \mathbf{W}_m\rangle = \delta_{n,m}\langle|\mathbf{W}_n|^2\rangle \ , \quad (2.38)$$

where $\delta_{n,m}$ is Kronecker's symbol. It is possible to find the recurrence formula for the orthogonal variable of different orders

$$\begin{aligned} \mathbf{W}_0 = \mathbf{A}_k^0(0),\ \mathbf{W}_1 = \left\{ i\hat{L} - \lambda_1 \right\} \mathbf{W}_0 \ , \\ \mathbf{W}_n = \left\{ i\hat{L} - \lambda_{n-1} \right\} \mathbf{W}_{n-1} + \Lambda_{n-1} \mathbf{W}_{n-2} \end{aligned} \tag{2.39}$$

with notations for eigenvalues (ω_n) and relaxation frequencies (Ω_n)

$$\begin{aligned} \lambda_n = i \frac{\langle \mathbf{W}_n \hat{L} \mathbf{W}_n \rangle}{\langle |\mathbf{W}_n|^2 \rangle}; \\ \Lambda_n = - \frac{\langle \mathbf{W}_{n-1} (i\hat{L} - \lambda_{n+1}) \mathbf{W}_n \rangle}{\langle |\mathbf{W}_{n-1}|^2 \rangle} \ . \end{aligned} \tag{2.40}$$

As the initial stochastic process $\mathbf{W}_0(t) = \mathbf{A}(t)$ is non-stationary, all subsequent orthogonal dynamic variables $\mathbf{W}_n(t)$ (see, Eqn. (2.34)) describe non-stationary processes as well. It is necessary to take into account, that in the considered case nor of eigenvalues vanish, $\lambda_n \neq 0$. As noted above, by the simple, but cumbersome calculations it is possible to show, that the first short memory function $m_1(t)$ represents a normalized TCF of the first dynamic variable W_1

$$\left. \begin{aligned} \mathbf{m}_1(t) = \langle \mathbf{n}_W(0) \mathbf{n}_W(t) \rangle \\ = \frac{\langle \mathbf{W}_1(0) \mathbf{W}_1(t) \rangle}{\sqrt{\langle |\mathbf{W}_1(0)|^2 \rangle \langle |\mathbf{W}_1(t)|^2 \rangle}} \\ \mathbf{W}_1(t) = U_{22}(T+t, T) \mathbf{W}_1(0) \end{aligned} \right\} \tag{2.41}$$

where $\mathbf{n}_W(0)$ and $\mathbf{n}_W(t)$ represent unit vectors in Euclidean space $W_1(\mathbf{W}_1(0), \mathbf{W}_1(t) \in W_1)$ of new orthogonal $(k-1)$ - dimensional vectors of state.

Following (2.17) - (2.22) in sequence of Euclidean spaces $W_n(\mathbf{W}_n(0), \mathbf{W}_n(t) \in W_n)$ with $n \geq 1$ it is possible to introduce a sequence of the projection operators Π_n

$$\Pi_n \mathbf{W}_n(t) = \frac{\mathbf{W}_n(0) \rangle \langle \mathbf{W}_n(0)}{\sqrt{\langle |\mathbf{W}_n(0)|^2 \rangle \langle |\mathbf{W}_n(t)|^2}} \mathbf{W}_n(t) = \mathbf{W}_n(0) m_n(t) \ . \tag{2.42}$$

Alongside with a set of projectors Π_n it is possible to introduce a set of mutually supplementary projectors P_n

$$\left. \begin{aligned} P_n = 1 - \Pi_n \ , \quad P_n \Pi_n = \Pi_n P_n = 0 \ , \\ \Pi_n \Pi_m = \delta_{n,m} \Pi_n \ , \quad P_n P_m = \delta_{m,n} P_n \ . \end{aligned} \right\} \tag{2.43}$$

Each pair of projection operators Π_n, P_n split the corresponding Euclidean space W_n of vectors of state $\mathbf{W}_n, \mathbf{W}_n(t) \in W_n$ into two mutually supplementary subspaces

$$W_n = W_n' \dot{+} W_n'' \quad W_n' = \Pi_n W_n \quad W_n'' = P_n W_n \ . \tag{2.44}$$

Correspondingly, discrete equation of motion of a variable $\mathbf{W}_n(t)$ can be written as

$$\begin{aligned} \frac{\triangle \mathbf{W}_n(t)}{\triangle t} = \frac{1}{\tau} \left\{ \mathbf{W}_n(t+\tau) - \mathbf{W}_n(t) \right\} \\ = \frac{1}{\tau} \left\{ U_{22}^{(n)}(\tau) - 1 \right\} \mathbf{W}_n(t) = i\hat{L}^{(n)} \mathbf{W}_n(t) \end{aligned} \tag{2.45}$$

with new Liouville's quaisioperator

$$\hat{L}^{(n)} = \hat{L}_{22}^{(n)} = (i\tau)^{-1} \left\{ U_{22}^{(n)}(\tau) - 1 \right\} = P_n \hat{L}_{22}^{(n-1)} P_n \ . \tag{2.46}$$

Following to projection technique outlined above, for normalized short memory functions $m_n(t)$ in Euclidean space of state vectors of dimension $(k-n)$ we find a chain of connected kinetic finite- difference equations of non-Markov type

$$\begin{aligned} \frac{\triangle m_n(t)}{\triangle t} = \lambda_{n+1} m_n(t) - \tau \Lambda_{n+1} \\ \times \sum_{j=0}^{m-1} m_{n+1}(j\tau) \gamma_{n+1}(j\tau) m_n(t - j\tau) \ , \end{aligned} \tag{2.47}$$

$$\gamma_n(j\tau) = \left\{ \frac{\langle |\mathbf{W}_n(j\tau)|^2 \rangle}{\sqrt{\langle |\mathbf{W}_n(0)|^2 \rangle}} \right\}^{1/2} \ , \tag{2.48}$$

where $\gamma_n(j\tau)$ is nth order non-stationarity function.

Set of all memory functions $m_1(t), m_2(t), m_3(t)$... allows to describe non-Markov processes and statistical memory effects in considered non-stationary system. For the particular case, when all eigenvalues $\lambda_n = 0$, for the first three short memory functions, the set of equations (2.38) get a more simple form

$$\left.\begin{aligned} \frac{\triangle a(t)}{\triangle t} &= -\tau\Lambda_1 \\ \times \sum_{j=0}^{m-1} m_1(j\tau)\gamma_1(j\tau)a(t-j\tau), \\ \frac{\triangle m_1(t)}{\triangle t} &= -\tau\Lambda_2 \\ \times \sum_{j=0}^{m-1} m_2(j\tau)\gamma_2(j\tau)m_1(t-j\tau), \\ \frac{\triangle m_2(t)}{\triangle t} &= -\tau\Lambda_3 \\ \times \sum_{j=0}^{m-1} m_3(j\tau)\gamma_3(j\tau)m_2(t-j\tau), \end{aligned}\right\} \tag{2.49}$$

where relaxation parameters Λ_1, Λ_2 and Λ_3 are determined by Eqn. (2.40), the non-stationarity functions $\gamma_n(t)$ are introduced in Eqns. (2.32), (2.48). Now by analogy with Eqn. (1.6), for arbitrary nth relaxation level we can introduce the set of dynamic parameters of non-stationarity (PNS)

$$\Gamma_n(T,t) = 1 - \gamma_n(t) = 1 - \gamma_n(T,t) \ . \tag{2.50}$$

3 Conclusion

In the present paper we built the kinetic theory of discrete non-stationary non-Markov processes in complex systems of various nature. From the very beginning we developed the theory on the basis of non-stationary TCF. For finding the last we have taken the advantage of general definition of correlation coefficient of stochastic processes in the probability theory. The construction of non-stationary TCF allows to get the non-linear projection operator acting in Euclidean space of non-stationary dynamic vectors of states.

For the analysis of non-stationary dynamics of stochastic process we have constructed a discrete - difference stochastic Liouville's equation, corresponding Liouville's quasioperator and evolution operator in the form of diagonal matrices. We have executed careful investigation of stochastic non-stationary dynamics of multidimensional vectors of initial and final chaotic states. For extracting non-stationary TCF we have taken the advantage of the projection operators technique, developed in our previous paper [1], and specially have updated it for the analysis of non-stationary stochastic processes.

Due to splitting of stochastic Liouville's equation into two mutually supplementary Euclidian subspaces we could get the chain of connected finite-difference kinetic equations for discrete non-stationary TCF and MF's.

It is possible, in-principle, to find the relaxation parameters and discrete functions (TCF and MF's of different orders) in this set of equations from experimental time series. This circumstance opens large capabilities in application of our theory for the study of a wide class of discrete non-stationary stochastic processes with long-range memory. It is necessary to mark one more relevant feature of developed theory. Our theory has particular analogy with famous Zwanzig' - Mori's theory in statistical physics. But in it there are two key differences. At first, our results are true for non-Hamilton systems, where there is no Hamiltonian and precise equations of motion. At second, our theory made feasible the discreteness of underlying process with discretization time τ. It is easy to note, that our theory contains Zwanzig '- Mori's results as a particular case. For this purpose it is necessary to proceed to limit $\tau \to 0$ and to replace stochastic Liouville's quasioperator on physical one. The obtained equations are very similar to well known Zwanzig'-Mori's kinetic equations [2], [3] in the non-equilibrium statistical physics of condensed matters. Let us mark three essential differences of our equations (2.47), (2.49) from results of Ref. [2], [3]. In Zwanzig'-Mori's theory the key moment in the analysis of considered physical systems is the presence of a Hamiltonian and operation of statistical averaging performed by the quantum density operator or classic Gibbs distribution function. In the case under study both Hamiltonian and distribution function are unavailable. In physics there are exact classic or quantum equations of motion, subsequently Liouville's equation of motion and the Liouville's operator is useful in many applications. Motion both individual particles, and all statistic system in a whole is described by states with smooth time. It permits to use for physical systems integro-differential calculus effectively, based on mathematically habitual representation about infinitesimal values of coordinates and time. The majority of complex systems on its own nature is discrete. As well known, discretization is common to a wide variety both of classic and quantum complex systems. It constrains us to reject the concept of an infinite small values and continuity and to address to the discrete - difference schemes. And, at last, third feature is connected with incorporation the non-stationary processes in our theory. The Zwanzig'-Mori's theory is true only for stationary processes. Due to introduction of normalized vectors of states and use of appropriate projection technique our theory permits to take into account non-stationary processes, which is possible to describe by non-Markov kinetic equations with introduction of the set of non-stationarity functions, that generalize the approach our previous paper [1].

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References

[1] R. Yulmetyev, P.Hanggi, and F. Gafarov. Phys. Rev. E **62**, 6178 (2000).

[2] R. Zwanzig. Phys. Rev **124**, 1338 (1961).

[3] H. Mori, Progr. Theoret. Phys. **33**, 423 (1965); **34**, 765, (1965).

[4] M. Reed, B. Samon. *Methods of Mathematical Physics* (Academic, New York, 1972)

[5] H.Graber.*Projection Operator Technique in Nonequilibrium Statistical Mechanics. Springer Tracts in Modern Physics,* **95** (Springer Verlag, Berlin, 1982)

[6] R. M. Yulmetyev, and N.R. Khusnutdinov. J. Phys. A **27**, 5363 (1994).

[7] R. M. Yulmetyev, and N. R. Khusnutdinov. Theoret. Math. Phys. **105**, 292 (1995). (in Russian)