Non-Markov Stationary Time Correlation in Complex Systems with Discrete Current Time

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The statistical non-Hamiltonian theory of fluctuation in the complex systems with a discrete current time is presented. Quasidynamic Liouville equation for the state vector of the complex system serves as a initial point of the discrete analysis. The projection operator in a vector state space of finite dimension allows to reduce Liouville equations to a closed non-Markov kinetic equation for a discrete time correlation function (TCF). By the subsequent projection in the space of orthogonal variables we found a discrete analoguos of famous Zwanzig-Mori's equations for the nonphysical non-Hamiltonian systems. The main advantage of the finite-difference approach developed is served with two moments. At first, the method allows to receive discrete memory functions and statistical spectrum of non-Markovity parameter for the discrete complex systems. At second, the given approach allows to plot a set of discrete dynamic information Shannon entropies. It allows successively to describe non-Markov properties and statistical memory effects in discrete complex systems of a nonphysical nature.

Key words: non-Markov discrete processes, finite-discrete kinetic equations, memory functions, dynamical Shannon entropy

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1 Introduction

Basic feature of complex systems of a substantial nature is their nonphysical and non-Hamiltonian nature. The absence of a Hamiltonian and precise Hamilton equations of motion does not allow sequentially to use a statistical physics of Hamiltonian systems for the description of complex systems in psychology, cardiology, finance, ecology as well as in seismic phenomena etc. Therefore rather actual the problem is the development of non-Hamiltonian statistical method of testings of dynamic properties of the complex systems.

Other relevant feature of the complex systems is their discretization. As a rule, for the description of properties of complex systems the continuous and sliding functions will be utilized. In a substantial nature the complex processes always take place as discrete events. However statistical theory of discrete stochastic processes now misses. On the other hand, the discretization contains the relevant and valuable information, both about random, and about regular components of complex process.

In the present article we develop the new concept of the description of random discrete processes in composite systems. Due to the registration of a discretization it allows sequentially to take into account for non-Markov properties and effects of statistical memory in random behavior of complex systems.

2 Non-Markov Kinetic Equation for Discrete Time Correlation of Random Fluctuation

To consider a random process like a sequence of random variables defined at successive times we shall denote the random variable by

$$X = \{x(T), x(T+\tau), x(T+2\tau), \\ \ldots, x(T+k\tau), \cdots, x(T+\tau N-\tau)\}, \quad (1)$$

which corresponds to signal during the time period $t = (N-1)\tau$ where τ is time interval of signal discretization. The normalized time correlation function (TCF) [1-3] depending on current time $t = m\tau, N-1 \geq m \geq 1$ can be conveniently used for the analysis of dynamic properties of complex systems

$$a(t) = \frac{1}{(N-m)\sigma^2} \\ \times \sum_{j=0}^{N-1-m} \delta x(T+j\tau)\delta x(T+(j+m\tau)). \quad (2)$$

TCF usage means that developed method is just for complex systems, when correlation function exist. In forthcoming papers we intend to apply developed method for discrete random processes analysis in complex systems in practical psychology, cardiology (for the development of diagnosis method of cardiovascular diseases), financial and ecological systems, seismic phenomena and etc. The properties of TCF $a(t)$ are easily determined by Eqs. (**??**)

$$\lim_{t\to 0} a(t) = 1, \ \lim_{t\to\infty} a(t) = 0. \quad (3)$$

We have to recognize that the second property in Eqs. (**??**) is not always satisfied for the real systems even with arbitrary big values of time t or number $(N-1) = t/\tau$. Taken into account fact that the process is discrete, we must rearrange all standard operation of differentiation and integration

$$\frac{dx}{dt} \to \frac{\triangle x(t)}{\triangle t} = \frac{x(t+\tau)-x(t)}{\tau}, \\ \int_a^b x(t)dt = \sum_{j=0}^{n-1} x(T_a + j\tau)\triangle t \\ = \tau \sum_{j=0}^{n-1} x(T_a+j\tau) = n\tau < X > , \\ b-a = c, c = \tau n. \quad (4)$$

The first derivative on the right is recorded in Eqs.(**??**). The second derivative on the right is also derived easily

$$\frac{d^2x(t)}{dt^2} \to \frac{\triangle}{\triangle t}\left(\frac{\triangle x}{\triangle t}\right) \\ = \frac{[x(t+2\tau)-x(t+\tau)]-[x(t+\tau)-x(t)]}{\tau^2} \\ = \tau^{-2}\{x(t+2\tau)-2x(t+\tau)+x(t)\}. \quad (5)$$

Now let us proceed to the description of the dynamics of the process. For real systems values $x_j = x(T+j\tau)$ and $\delta x_j = \delta x(T+j\tau)$ result from the experimental data. Thus we can introduce in Shannon's manner the evolution operator $U(T+t_2, T+t_1)$ in the following manner $(t_2 \geq t_1)$

$$x(T+t_2) = U(T+t_2, T+t_1)x(T+t_1). \quad (6)$$

For brevity let us present Eqs.(**??**) in the form

$$x(j) = U(j,k)x(k), \ j \geq k; \ j,k = 0,1,2,\cdots, N-1. \quad (7)$$

Now let us present a set of values of random variables $\delta x_j = \delta x(T+j\tau), j = 0,1,\cdots,N-1$ as a k-component vector of system state

$$\mathbf{A}_k^0(0) = (\delta x_0, \delta x_1, \delta x_2, \cdots, \delta x_{k-1}) \\ = (\delta x(T), \delta x(T+\tau), \cdots, \delta x(T+(k-1)\tau). \quad (8)$$

So we can introduce the scalar product operation

$$< \mathbf{A}\cdot\mathbf{B} >= \sum_{j=0}^{k-1} A_j B_j \quad (9)$$

with or whithout indication of obvious time dependence of vectors **A** and **B** respectively in the set of vectors $\mathbf{A}_k^0(0)$ and $\mathbf{A}_{m+k}^m(t)$ where $t = m\tau$ and

$$\mathbf{A}_{m+k}^m(t) = \{\delta x_m, \delta x_{m+1}, \delta x_{m+2}, \cdots, \delta x_{m+k-1}\} \\ = \{\delta x(T+m\tau), \delta x(T+(m+1)\tau), \\ \delta x(T+(m+2)\tau), \cdots, \delta x(T+(m+k-1)\tau\}. \quad (10)$$

A k- component vector $\mathbf{A}_{m+k}^m(t)$ displaced to the distance $t = m\tau$ on the discrete time scale can be formally presented by the time evolution operator $U(t+\tau, t)$ as follows

$$\mathbf{A}_{m+k}^m(t) = U(T+m\tau, T)\mathbf{A}_k^0(0) \\ = \{U(T+m\tau, T+(m-1)\tau) \\ \times U(T+(m-1)\tau, T+(m-2)\tau) \\ \ldots U(T+\tau, T\}\mathbf{A}_k^0(0). \quad (11)$$

Normalized TCF in Eqs. (**??**) can be rewritten in a more compact form by means of Eqs.(**??**) ($t = m\tau$ is discrete current time here)

$$a(t) = \frac{< \mathbf{A}_k^0 \cdot \mathbf{A}_{m+k}^m >}{< \mathbf{A}_k^0 \cdot \mathbf{A}_k^0 >} = \frac{< \mathbf{A}_k^0(0) \cdot \mathbf{A}_{m+k}^m(t) >}{< \mathbf{A}_k^0(0)^2 >}. \quad (12)$$

Such vectors' notion is very helpful for the analysis of dynamics of random processes by means of finite-difference kinetic equations of non-Markov type.

Let us consider the projection operation in the set of vectors for different system states. It is easy to introduce it employing the above scalar product (**??**). Then it is necessary to introduce vectors $\mathbf{A} = \mathbf{A}_k^0(0)$ and $\mathbf{B} = \mathbf{A}_{m+k}^m(m\tau)$. Using simple geometrical notions we can demonstrate the following relations in terms of these symbols

$$\begin{cases} < \mathbf{A}\cdot\mathbf{B} >= |\mathbf{A}|\cdot|\mathbf{B}|\cos\vartheta,\ \cos\vartheta = a(t), \\ \mathbf{B} = \mathbf{B}_\parallel + \mathbf{B}_\perp; \\ \mathbf{B}_\parallel = |\mathbf{B}|\cos\vartheta\frac{\mathbf{A}}{|\mathbf{A}|} = \frac{\mathbf{A}}{|\mathbf{A}|}|\mathbf{B}|a(t), |\mathbf{B}_\parallel|^2 \\ = |\mathbf{A}|^2\{a(t)\}^2, \\ |\mathbf{B}_\perp| = |\mathbf{B}|\sin\vartheta = |\mathbf{B}|\{1-[a(t)]\}^{1/2}, \end{cases} \quad (13)$$

where symbol $|\mathbf{A}|$ denotes the vector **A** length. Geometrical distance $R(\mathbf{A},\mathbf{B})$ between two vectors **A** and **B** can also be found

$$R(\mathbf{A},\mathbf{B}) = \{|\mathbf{A}-\mathbf{B}|^2\}^{1/2} = \{\sum_{j=0}^{k-1}(\mathbf{A}_j - \mathbf{B}_j)^2\}^{1/2}.$$

Using the latter and taking into account Eqs. (**??**), (**??**) we can find

$$R(\mathbf{A}_k^0(0), \mathbf{A}_{m+k}^m(t)) = \{|\mathbf{A}_{m+k,\perp}^m(t)|^2\}^{1/2} \\ = \sqrt{2}|\mathbf{A}_{m+k}^m(t)|\{1-a(t)\}^{1/2}.$$

The equation above immediately shows that the distance is determined by the dynamics of evolution of correlation process. Owing to the property (**??**) the following relation $\lim_{t\to\infty} R(\mathbf{A}_k^0(0)), \mathbf{A}_{m+k}^m(t)) = \sqrt{2k\sigma^2}$, where σ^2 is the variance can be developed. With regard to Eqs. (**??**) the correlation decay in limit $t \to \infty$ may result in complete annihilation of parallel component of state $\mathbf{A}_{m+k}^m(t)$ vector . Then the state of the system at the moment $t \to \infty$ is entirely determined by the perpendicular component $\mathbf{A}_{m+k,\perp}^m(t)$ of the full vector $\mathbf{A}_{m+k}^m(t)$.

It follows from Eqs. (**??**) that in the set of state $\{\mathbf{A}_k^0(0), \mathbf{A}_{m+k}^m(t)\}$ vectors at different values of t, m and k, TCF of random processes $a(t)$ plays a crucial role as an indicator of two interrelated states of a complex system. One of them deals with the creation of correlation and is specified by the $\mathbf{B}_\parallel$ component, whereas the second one is related to the annihilation of correlation and determined by the component $\mathbf{B}_\perp$. It results in the fact that in the limit of great $t \to \infty$ the following relation

$$\lim_{t\to\infty} \mathbf{A}_{m+k,\parallel}^m(t) = 0, \quad \lim_{t\to\infty} \mathbf{A}_{m+k,\perp}^m(t) = \mathbf{A}_{m+k}^m(t). \quad (14)$$

is immediately fulfilled in correspondence with to Bogolubov's principle of correlation attenuation.

From the physical point of view this fact means that TCF $a(t)$ represents two interrelated states determined by creation and annihilation of correlation. Hence it follows that such consideration must be given to both processes in an explicit form for stochastic dynamics of random processes' correlation.

It is obvious from Eqs.(13) that TCF $a(t)$ is originated by projection of vector $\mathbf{A}^m_{m+k}(t)$ (**??**) where time $t = m\tau$ on the initial vector of state $\mathbf{A}^0_k(0)$ (see, for example, formula (**??**)). The following construction of projection operator

$$\Pi \mathbf{A}^m_{m+k}(t) = \mathbf{A}^0_k(0) \frac{< \mathbf{A}^0_k(0) \mathbf{A}^m_{m+k}(t) >}{< |\mathbf{A}^0_k(0)|^2 >} = \mathbf{A}^0_k(0) a(t) \qquad (15)$$

results from here . It is turn projection operator Π from Eqs. (**??**) has the following properties

$$\Pi = \frac{\mathbf{A}^0_k(0) >< \mathbf{A}^0_k(0)}{< |\mathbf{A}^0_k(0)|^2 >}, \ \Pi^2 = \Pi, \qquad (16)$$
$$P = 1 - \Pi, \ P^2 = P, \ \Pi P = 0, \ P\Pi = 0.$$

A pair of projection operators Π and P are idempotent and mutually - supplementary. Therefore, projector Π projects on the direction $\mathbf{A}^0_k(0)$, whereas the orthogonal operator P transfers all vectors to the orthogonal direction.

Let us consider quasidynamic finite-difference Liouville's Equation for the vector of fluctuations

$$\frac{\Delta}{\Delta t} \mathbf{A}^m_{m+k}(t) = i\hat{L}(t,\tau) \mathbf{A}^m_{m+k}(t). \qquad (17)$$

The vectors $\mathbf{A}^m_{m+k}(t)$ generate the vector finite-dimensional space $A(k)$ with scalar product in which (according to Eqs.(**??**), (**??**)) the orthogonal projection operation is expressed by Eqs. below

$$\begin{cases} A(k) = A'(k) + A''(k), \quad \mathbf{A}^m_{m+k}(t) \in A(k), \\ A'(k) = \Pi A(k), \qquad (18) \\ A''(k) = PA(k) = (1 - \Pi) A(k). \end{cases}$$

Operators Π and P split Euclidean space $A(k)$ into two mutually-orthogonal subspaces. This permits to split dynamical equation (**??**) into two Equations within two mutually-supplementary subspaces as follows

$$\begin{cases} \frac{\Delta A'(t)}{\Delta t} = i\hat{L}_{11} A'(t) + i\hat{L}_{12} A''(t), \\ \frac{\Delta A''(t)}{\Delta t} = i\hat{L}_{21} A'(t) + i\hat{L}_{22} A''(t). \end{cases} \qquad (19)$$

In the Eqs. above we crossout for short space elements indices A, A' and A'' and matrix elements arguments $\hat{L}_{ij}$, $\hat{L}_{ij} = \Pi_i \hat{L} \Pi_j$, $\Pi_1 = \Pi$, $\Pi_2 = P = 1 - \Pi$, $i = 1, 2$. We write down Liouville's operator in matrix form

$$\hat{L} = \begin{pmatrix} \hat{L}_{11} & \hat{L}_{12} \\ \hat{L}_{21} & \hat{L}_{22} \end{pmatrix},$$
$$\hat{L}_{11} = \Pi \hat{L} \Pi, \quad \hat{L}_{12} = \Pi \hat{L} P,$$
$$\hat{L}_{21} = P \hat{L} \Pi, \quad \hat{L}_{22} = P \hat{L} P. \qquad (20)$$

Operators $\hat{L}_{ij}$ act in the following way: $\hat{L}_{11}$ - from A' to A', $\hat{L}_{22}$ - from A'' to A'', $\hat{L}_{21}$ - from A' to A'', and $\hat{L}_{12}$ operates from A'' to A'.

To simplify Liouville's Eqs. (**??**) we exclude the irrelevant part $A''(t)$ and construct closed Equation for relevant part $A'(t)$. For this purpose let us solve Eqs.(**??**) step by step

$$\frac{\Delta}{\Delta t} \{\mathbf{A}^m_{m+k}(t)\}'' = i\hat{L}_{21} \{\mathbf{A}^m_{m+k}(t)\}' + i\hat{L}_{22} \{\mathbf{A}^m_{m+k}(t)\}''. \qquad (21)$$

Considering Eqs.(**??**) we arrive at finite-difference solution of this Eqs. in the following form

$$\frac{\Delta A''(t)}{\tau} = \tau^{-1}[A''(t+\tau) - A''(t)]$$
$$= i\hat{L}_{21} A'(t) + i\hat{L}_{22} A''(t), A''(t+\tau)$$
$$= \{1 + i\tau \hat{L}_{22}\} A''(t) + i\tau \hat{L}_{21} A'(t). \qquad (22)$$

In general case we find

$$A''(t + m\tau) = \{1 + i\tau \hat{L}_{22}\}^m A''(t) + \sum_{j=0}^{m-1} \{1 + i\tau \hat{L}_{22}\}^j \{i\hat{L}_{21} A'(t + (m-1-j)\tau)\} \qquad (23)$$

for the arbitrary number of m-steps. Then after the substitution of right side of Eqs.(**??**) for Eqs.(**??**) we obtain the closed finite-difference kinetic equation for the relevant parts of vectors

$$\frac{\Delta}{\Delta t} A'(t + m\tau) = i\hat{L}_{11} A'(t + m\tau) + i\hat{L}_{12} \{1 + i\tau \hat{L}_{22}\}^m A''(t) - \hat{L}_{12} \sum_{j=0}^{m+1} \{1 + i\tau \hat{L}_{22}\}^j \hat{L}_{21} A'(t + (m-1-j)\tau). \qquad (24)$$

To simplify this Eqs, let us consider the idempotentity property, and then determine ($0 \leq k \leq m-1$)

$$A''(t) = 0, \quad \{1 + i\tau\hat{L}_{22}\}^k A''(t) = 0. \qquad (25)$$

Transfering from vectors $\mathbf{A}^m_{m+k}$ in Eqs.(**??**) to a scalar value of TCF $a(t)$ by means of suitable projection we come to the closed finite-difference discrete Equation for the initial TCF

$$\frac{\Delta a(t)}{\Delta t} = i\omega_0^{(0)} a(t) - \tau\Omega_0^2 \sum_{j=0}^{m-1} M_1(j\tau) a(t - j\tau). \quad (26)$$

Here Ω_0 is the general relaxation frequency whereas frequency $\omega_0^{(0)}$ describes the eigenspectrum of the Liouville's quasioperator $\hat{L}$

$$\omega_0^{(0)} = \frac{< A^0_k(0)\hat{L}A^0_k(0) >}{< |A^0_k(0)|^2 >},$$
$$\Omega_0^2 = \frac{< A^0_k \hat{L}_{12}\hat{L}_{21}\mathbf{A}^0_k(0) >}{< |\mathbf{A}^0_k(0)|^2 >}. \qquad (27)$$

Function $M_1(j\tau)$ in the right side of Eqs.(26) is the first order memory function

$$M_1(j\tau) = \frac{< \mathbf{A}^0_k(0)\hat{L}_{12}\{1 + i\tau\hat{L}_{22}\}^j \hat{L}_{21}\mathbf{A}^0_k(0) >}{< \mathbf{A}^0_k(0)\hat{L}_{12}\hat{L}_{21}\mathbf{A}^0_k(0) >}, \qquad (28)$$
$$M_1(0) = 1.$$

Equation (26) alongside with Eqs.(27),(28) present first order discrete non-Markov kinetic equation for the discrete time correlation function $a(t)$.

3 Orthogonal Random Variables and Finite-Difference non-Markov Kinetic Equations for Discrete Memory Functions

The discrete memory function $M_1(j\tau)$ (**??**) in Eqs. (**??**) is in its turn the normalized TCF, evolution of which is defined by the deformed (compressed) Liouvillian's ($\hat{L}^{(0)} = \hat{L}$)

$$\hat{L}^{(1)} = \hat{L}^{(0)}_{22} = \hat{L}_{22} = (1 - \Pi)\hat{L}(1 - \Pi) \qquad (29)$$

for a new dynamical variable $B^{(1)} = i\hat{L}_{21}\mathbf{A}^0_k(0)$. Thus, we can completely repeat for $M_1(j\tau)$ the whole procedure within Eqs. (**??**)-(**??**), and obtain the following non-Markov kinetic equation for the normalized TCF. The infinite chain of equations for the initial TCF and memory functions of increasing order results from multiple repetition of similar procedure.

However this chain of equations can be obtained differently, i.e. much shorter and less costly. For this purpose let us employ the method developed earlier for the physical Hamilton systems with the continuous current time . Moreover the lack of Hamiltonian and the time discreteness must be taken into account.

Let us remember that natural equation of motion is the finite-difference Liouville's equation

$$\frac{\Delta}{\Delta t} x(t) = i\hat{L}x(t) \qquad (30)$$

where Liouville's quasioperator is $\hat{L} = \hat{L}(t, \tau) = (i\tau)^{-1}\{U(t + \tau, t) - 1\}$.

Successively applying the quasioperator $\hat{L}$ to the dynamic variables $\mathbf{A}^m_{m+k}(t)$ ($t = m\tau$, where τ is a discrete time step) we obtain the infinite set of dynamic functions

$$\mathbf{B}_n(0) = \{\hat{L}\}^n \mathbf{A}^0_k(0), \quad n \geq 1. \qquad (31)$$

Using variables $\mathbf{B}_n(0)$ one can find the formal solution of evolution Eqs.(30) in the form of

$$\mathbf{A}^m_{m+k}(m\tau) = \{1 + i\tau\hat{L}\}^m \mathbf{A}^0_k(0) = \sum_{j=0}^{m} \frac{m!(i\tau)^{m-j}}{j!(m-j)!} \mathbf{B}^0_{m-j}(0). \qquad (32)$$

However, the similar form of dynamic variables is deficient. That is why we prefer the use the orthogonal variables as vectors $\mathbf{W}_n$ given below. Employing Gram-Schmidt orthogonalization procedure for the set of variables $\mathbf{B}_n(0)$ one can obtain the new infinite set of dynamical orthogonal variables, i.e. vectors $\mathbf{W}_n$

$$< \mathbf{W}^*_n(0), \mathbf{W}_m(0) >= \delta_{n,m} < |\mathbf{W}_n(0)|^2 >, \qquad (33)$$

where the mean $< \cdots >$ should be read in terms of Eqs. (**??**)-(**??**) and $\delta_{n,m}$ is Kronecker's symbol. Now we may easily introduce the recurrence formula in which the senior values $\mathbf{W}_n = \mathbf{W}_n(t)$ are connected with the juniour values

$$\mathbf{W}_0 = \mathbf{A}_k^0(0), \quad \mathbf{W}_1 = \{\hat{L} - \omega_0^{(0)}\}\mathbf{W}_0,$$
$$\mathbf{W}_n = \{\hat{L} - \omega_0^{(n-1)}\}\mathbf{W}_{n-1} - \Omega_{n-1}^2 W_{n-2}, \quad n > 1.$$

Here we used the equation, given earlier in (27) for number n=0

$$\omega_0^{(n)} = \frac{< \mathbf{W}_n \hat{L} \mathbf{W}_n >}{< |\mathbf{W}_n|^2 >},$$
$$\Omega_n^2 = \frac{< |\mathbf{W}_n|^2 >}{< |\mathbf{W}_{n-1}|^2 >}, \tag{34}$$

where Ω_n is the general relaxation frequency, and frequency $\omega_0^{(n)}$ completely describes the eigen spectrum of Liouville's quasioperator $\hat{L}$. Now the arbitrary variables $\mathbf{W}_n$ may be expressed directly through the initial variable $\mathbf{W}_0 = \mathbf{A}_k^0(0)$ by following

$$\mathbf{W}_{n+1} \tag{35}$$
$$= \begin{vmatrix} \hat{L} - \omega_0^{(0)} & \Omega_1 & \dots & 0 \\ \Omega_1 & \hat{L} - \omega_0^{(1)} & \dots & 0 \\ 0 & \Omega_2 & \dots & 0 \\ 0 & 0 & \dots & \hat{L} - \omega_0^{(n)} \end{vmatrix} \mathbf{W}_0.$$

The physical sense of $\mathbf{W}_n$ variables (vectors of state) can be cleared up in the following way. For example, in the continuous matter physics, the local density fluctuations may be considered as initial variables. So the local flow density, energy density and energy flow density fluctuations are the dynamic variables $\mathbf{W}_n$ where numbers $n \geq 1$. The careful usage of the above mentioned variables within the long-wave limits creates the basis for the condensed matter theory in hydrodynamic approximation. The set of the orthogonal variables can be connected with the set of projection operators.The later projects the arbitrary dynamic variable (i.e.,vector of state) Y on the corresponding vector of the set

$$\Pi_n = \frac{\mathbf{W}_n >< \mathbf{W}_n^*}{< |\mathbf{W}_n|^2 >},$$
$$\Pi_n^2 = \Pi_n, \ \Pi_n \Pi_m = \delta_{n,m} \Pi_n, \tag{36}$$
$$P_n = 1 - \Pi_n, \ P_n^2 = P_n, \quad \Pi_n P_n = 0,$$
$$P_n P_m = \delta_{n,m} P_n, \quad P_n \Pi_n = 0.$$

Let us take into consideration the fact that both sets (31) and (35) are infinite. If we execute the operations in the Euclidean space of dynamic variables then the formal expressions (36) must be understood as follows

$$\Pi_n \mathbf{Y} = \mathbf{W}_n \frac{< \mathbf{W}_n^* \mathbf{Y} >}{< |\mathbf{W}_n|^2 >}, \quad \mathbf{Y}\Pi_n = \mathbf{W}_n^* \frac{< \mathbf{Y} W_n >}{< |\mathbf{W}_n|^2 >}. \tag{37}$$

Now according to (19)-(20), (36),(37) we can introduce the following notation for the splitting of the Liouville's quasioperator into the diagonal ($\hat{L}_{ii}^{(n)}$) and non-diagonal ($\hat{L}_{ij}^{(n)}$) matrix elements with $i \neq j, \ \ n \geq 1$

$$\hat{L}^{(n)} = P_{n-1} \hat{L}^{(n-1)} P_{n-1},$$
$$\hat{L}_0 = \hat{L}, \hat{L}_{ij}^{(n)} = \Pi_i^{(n-1)} \hat{L} \Pi_j^{(n-1)}, \quad i,j = 1,2,$$
$$\Pi_1^{(n)} = \Pi_n, \quad \Pi_2^{(n)} = P_n = 1 - \Pi_n. \tag{38}$$

For example, we come to the following Eqs.

$$\hat{L}_{22}^{(0)} = \hat{L}_0 = \hat{L}, \quad \hat{L}_{22}^{(n)}$$
$$= P_{n-1} P_{n-2} \dots P_0 \hat{L} P_0 \dots P_{n-2} P_{n-1}. \tag{39}$$

for the second diagonal matrix elements. Successively applying projection operators Π_n and P_n for the discrete equation (30) in the set of normalized TCF ($t = m\tau$)

$$M_n(t) = \frac{< \mathbf{W}_n [1 + i\tau \hat{L}_{22}^{(n)}]^m \mathbf{W}_n >}{< |\mathbf{W}_n(0)|^2 >} \tag{40}$$

we obtain the infinite hierarchy of connected non-Markov finite-difference kinetic equations ($t = m\tau$)

$$\frac{\Delta M_n(t)}{\Delta t} = i\omega_0^{(n)} M_n(t)$$
$$-\tau \Omega_{n+1}^2 \sum_{j=0}^{m-1} M_{n+1}(j\tau) M_n(t - j\tau), \tag{41}$$

where $\omega_0^{(n)}$ is the eigen frequency and Ω_n is the general relaxation frequency as follows

$$\omega_0^{(n)} = \frac{< W_n^* L_n W_n >}{< |W_n|^2 >},\ L_n = L_{22}^{(n)}, \qquad \Omega_n^2 = \frac{< |W_n|^2 >}{< |W_{n-1}|^2 >}. \tag{42}$$

A set of functions $M_n(t)$ (40), (41) except $n = 0$ can be considered as functions characterizing the statistical memory of time correlation in the complex systems with discrete current time. The initial TCF a(t) and the set of discrete memory functions $M_n(t)$ in Eqs. (41) are of crucial role for the further consideration. It is convenient to rewrite the set of discrete kinetic Eqs.(41) as the infinite chain of coupled non-Markov discrete equations of nonlinear type for the initial discrete TCF a(t) (discrete time $t = m\tau$ everywhere)

$$\begin{aligned}\frac{\Delta a(t)}{\Delta t} &= -\tau\Omega_1^2 \sum_{j=0}^{m-1} M_1(j\tau)a(t-j\tau) + i\omega_0^{(0)} a(t),\\ \frac{\Delta M_1(t)}{\Delta t} &= -\tau\Omega_2^2 \sum_{j=0}^{m-1} M_2(j\tau)M_1(t-j\tau) + i\omega_0^{(1)} M_1(t),\\ \frac{\Delta M_2(t)}{\Delta t} &= -\tau\Omega_3^2 \sum_{j=0}^{m-1} M_3(j\tau)M_2(t-j\tau) + i\omega_0^{(2)} M_2(t).\end{aligned} \tag{43}$$

These finite-difference Eqs. (**??**) and (**??**) are very similar to famous Zwanzig'-Mori's chain (ZMC) of kinetic equations [**?**]- [**?**], which plays the fundamental role in modern statistical physics of nonequilibrium phenomena with the smooth current time. It should be noted that ZM'sC is true only for the physical quantum and classical systems with current smooth time governed by Hamiltonian. Our finite-difference kinetic equations (**??**), (**??**) are valid for complex systems lacking Hamiltonian, the time being discrete and the exact equations of motion being absent. However, the "dynamics" and "motion" in the real complex systems are undoubtedly abundant and are immediately registered during the experiment.

The first three of those Eqs.(**??**) in the whole infinite chain (**??**) form the basis for the quasihydrodynamic description of random processes in complex systems.

Now let's find the matrix elements $\hat{L}_{ij}$ of complex systems Liouvillian's quasioperator. Employing Eqs. (**??**), (**??**), (**??**), (**??**) and (**??**) we successively found

$$\begin{aligned} i\hat{L}_{11}^{(0)} &= \Pi\frac{a(\tau)-a(0)}{\tau} = a'(0)\Pi,\\ i\hat{L}_{21}^{(0)} &= \{\tau^{-1}[U(t+\tau,t)-1] - a'(0)\}\Pi,\\ i\hat{L}_{12}^{(0)} &= \Pi\{\tau^{-1}[U(t+\tau,t)-1] - a'(0)\},\\ i\hat{L}_{22}^{(0)} &= i\hat{L} - i\{\hat{L}_{11}^{(0)} + \hat{L}_{12}^{(0)} + \hat{L}_{21}^{(0)}\}\\ &= \tau^{-1}[U(t+\tau,t)-1] - \tau^{-1}\Pi\{U(t+\tau,t)-1\}\\ &\quad -\tau^-1\{U(t+\tau,t)-1\}\Pi + a'(0)\Pi.\end{aligned} \tag{44}$$

A diagonal matrix element $\hat{L}_{22}^{(0)}$ is the part of "compressed" evolution quasioperator, which in its turn is equal to

$$\begin{aligned}1 + i\tau\hat{L}_{22} &= U(t+\tau,t) + \tau a'(0)\Pi\\ &\quad -\{\Pi, U(t+\tau,t)-1\}_+\end{aligned} \tag{45}$$

where the anticommutator of appropriate operator is designate by the brackets $\{A,B\}_+ = AB + BA$.One can see from the Eqs.(**??**) that the "compressed" evolution operator differs from the natural operator $U(t+\tau,t)$ because of the presence of contributions, associated with the first and the following derivatives of TCF the initial TCF $a(t)$.

Now let us move to practical realization of Eqs.(**??**), forming a basis of pseudohydrodynamic description of correlation dynamics. Thus using orthogonal dynamic variables (**??**), (**??**),(**??**), we immediately obtain

$$\begin{aligned}\hat{W}_0 = \mathbf{A}_k^0, \quad \hat{W}_1 = \{\hat{L} - \omega_0^{(0)}\}\hat{W}_0 = \hat{L}\hat{W}_0 = (i\tau)^{-1}(U_\tau - 1)\mathbf{A}_k^0(0), \\ \hat{W}_2 = \hat{L}\hat{W}_1 - \Omega_1^2\hat{W}_0 = \{\hat{L}^2 - \Omega_1^2\}\hat{W}_0 = (i\tau)^{-2}\{U_\tau - 1\}^2\mathbf{A}_k - \Omega_1^2\mathbf{A}_k^0, \\ \hat{W}_3 = \hat{L}\hat{W}_2 - \Omega_2^2\hat{W}_1 = \hat{L}(\hat{L}^2 - \Omega_1^2)\hat{W}_0 - \Omega_2^2\hat{L}\hat{W}_0 = \{\hat{L}^3 - (\Omega_1^2 + \Omega_2^2)\hat{L}\}\hat{W}_0 \\ = \{(i\tau)^3[U_\tau - 1]^3 - (i\tau)^{-1}(\Omega_1^2 + \Omega_2^2)(U_\tau - 1)\}\mathbf{A}_k^0.\end{aligned} \tag{46}$$

Simple relation for the eigen and general relaxation frequencies

$$\begin{aligned}\omega_0^{(n)} = \frac{< \hat{W}_n\hat{L}\hat{W}_n >}{< |\hat{W}_n|^2 >} = 0, \quad \Omega_n^2 = \frac{< |\hat{W}_n|^2 >}{< |\hat{W}_{n-1}|^2 >}, \\ \Omega_1^2 = |a^{(2)}(0)|, \quad \Omega_2^2 = \frac{a^{(4)}(0) - (a^{(2)}(0))^2}{|a^2(0)|}, \\ \Omega_3^2 = \frac{a^{(6)}(0) - 2a^{(4)}(0)(\Omega_1^2 + \Omega_2^2) - (\Omega_1^2 + \Omega_2^2)^2 a^{(2)}(0)}{a^{(4)}(0) - (a^{(2)}(0))^2}\end{aligned} \tag{47}$$

should be taken into consideration here. The orthogonal variables $\hat{W}_n$ can be easily rearranged as follows

$$\begin{aligned}\hat{W}_0 = \mathbf{A}_k^0, \quad \hat{W}_1 = -i\frac{\Delta}{\Delta t}\mathbf{A}_k, \\ \hat{W}_2 = \{\left(\frac{\Delta}{\Delta t}\right)^2 + \Omega_1^2\}\mathbf{A}_k^0, \\ \hat{W}_3 = i\left\{\left(\frac{\Delta}{\Delta t}\right)^3 + (\Omega_1^2 + \Omega_2^2)\frac{\Delta}{\Delta t}\right\}\mathbf{A}_k^0.\end{aligned} \tag{48}$$

Those formulas (**??**) have considerable utility inasmuch as they permit to see the structure of formation of orthogonal variables and junior orders memory functions for the numbers $n = 1, 2, 3$. Eqs. (**??**), (**??**) open up new fields of construction of quasikinetic description of random processes $\{\mathbf{A}_k^0(0), \mathbf{A}_{m+k}^m(m\tau)\}$. By analogy with hydrodynamics the variables $\hat{W}_0$, $\hat{W}_1$, $\hat{W}_2$ and $\hat{W}_3$ in Eqs. (**??**) play the role similar to that of the local density, local flow, local energy density and energy flow. It is clear that this is only formal analogy and the variables $\hat{W}_n$ don't possess any physical sense. However, such analogies can be helpful in revealing of the real sense of orthogonal variables.

To describe pseudohydrodynamics we have to use the set of first three discrete kinetic Eqs. (**??**) with frequencies Ω_i^2 $(i = 1, 2, 3)$ derived from Eqs.(**??**).

It is essential that all frequencies Ω_i^2 are connected straightly with the properties of the initial TCF $a(t)$ only. The latter can be easily derived directly from the experimental data [**?**]. Thus the system of Eqs. (**??**) has considerable utility for the experimental investigations of statistical memory effects and non-Markov processes in complex systems.

Among them it seems to us that one could propose more physical interpretation of the different terms in the right side of the three Eqs. (48). For example, term $-i\Delta A/\Delta t$ is like a dissipation, $\Delta^2 A/\Delta t^2$ is like a inertia and $\Omega^2 A(t)$ is like a restoring force. Third derivative $\Delta^3 A/\Delta t^3$ is the finite-difference generic form of the Abraham-Lorenz force corresponding to dissipation feedback due to radiative losses.

4 Information Shannon Entropy for the Discrete Time Correlation and Discrete Time Memory in Complex Systems

According to the results in section 3, the information measure for the description of random processes in complex systems can be expressed not only via TCF, but also by means of the certain set of time memory functions. To accomplish that let us return to section III in which we presented the geometrical picture of stochastic dynamics of correlation . In a line with Shannon in case of discrete source of information we were able to determine a definite rate of generating information, namely the entropy of the underlying stochastic information by introduction fidelity evaluation function $\nu(P(x,y))$. Here the function $P(x,y)$ is the two-dimensional distribution of random variables (x,y) and

$$\nu(P(x,y)) = \int\int dx\ dy P(x,y)\rho(x,y), \qquad (49)$$

where the function $\rho(x,y)$ has the general nature of the "distance" between x and y. As pointed by Shannon the function $\rho(x,y)$ is not a "metric" in the strict sense, however, since in general it does not satisfy either $\rho(x,y) = \rho(y,x)$ or $\rho(x,y) + \rho(y,z) \geq \rho(x,z)$.. It measures how undesirable it is according to our fidelity criterion (**??**) to receive y when x transmitted. According to Shannon any evolution of fidelity must correspond mathematically to the operation of a simple ordering of systems by the transmission of a signals within the certain tolerance. According to Shannon the following is simple example of fidelity evaluation function

$$\nu(P(x,y)) =< (x(t) - y(t))^2 > . \qquad (50)$$

In our case it is convenient to consider the initial vector $A_k^0(0)$ as a variable x and the final vector $A_{m+k}^m(t)$ at time $t = m\tau$ for a variable y. The distance function $\rho(x,y)$

$$\rho(x,y) = \frac{1}{T}\int_0^T dt\{x(t) - y(t)\}^2 \qquad (51)$$

is the most commonly used measure of fidelity.

Taking into account Eqs.(**??**), (**??**) and the results in Section 2 as the fidelity function one can use the following function of geometrical distance

$$\nu(P(\mathbf{A}_k^0(0), \mathbf{A}_{m+k}^m(t)) = 2k\sigma^2\{1 - a(t)\}, \qquad (52)$$

where distance function is

$$\rho(\mathbf{A}_k^0(0), \mathbf{A}_{m+k}^m(t)) = R^2(\mathbf{A}_k^0(0), \mathbf{A}_{m+k}^m(t)). \qquad (53)$$

According to Shannon partial solution of the general maximizing problem for determining the rate of generating information of a source can be given using Lagrange's method and considering the following functional

$$\int\int\{P(x,y)\log\frac{P(x,y)}{P(x)P(y)}$$
$$+\mu P(x,y)\rho(x,y) + \nu(x)P(x,y)\}dx\ dy, \qquad (54)$$

where the function $\nu(x)$ and μ are unknown. The following equation for the conditional probability can be obtained by variation on $P(x,y)$

$$P_y(x) = \frac{P(x,y)}{P(y)} = B(x)\exp\{-\lambda\rho(x,y)\}. \qquad (55)$$

This shows that with best encoding the conditional probability of a certain cause for various received y, $P_y(x)$ will decline exponentially with the distance function $\rho(x,y)$ between values the x and y in problem. Unknown constant λ is defined by the required fidelity, and function $B(x)$ in the case of continuous variables obeys the normalization condition

$$\int B(x)\exp\{-\lambda\rho(x,y)\}dx = 1. \qquad (56)$$

Since the distance function $\rho(x,y)$ (**??**) is dependent only on the vectors difference $\rho(x,y) = \rho(x-y)$, we get a simple solution for the special case $B(x) = \alpha$

$$P_y(x) = \alpha\exp\{-\lambda\rho(x-y)\} = \alpha\exp\{-c[1 - a(t)]\} \qquad (57)$$

instead of Eqs.(**??**). Constants α and λ result from the corresponding normalizing condition and in accordance with the required fidelity. From the physical point of view the basic value of solution (**??**) is directly related to the occurrence of the TCF $a(t)$. Therefore,the solution (**??**) describes the state of the system with certain level and scale of correlation.

Now let us employ Shannon's solution for continuous variables (**??**), (**??**) and pass to simplified discrete two-level description of the system. Then let us consider the conditional probability (**??**) which describes the state on time axis at the moment $t = m\tau$ as corresponding to the creation of correlation. Whereas the other state at the fixed moment $t = m\tau$ which accounts for the state with the absence (annihilation) of correlation will exist. Let us introduce two probabilities, which will fit normalizing condition

$$P_1(t) + P_2(t) = 1, \quad P_1(t) = P_{cc}(t), \\ P_2(t) = P_{ac}(t), \quad P_{cc}(t) + P_{ac}(t) = 1. \tag{58}$$

In the case of two lewels Shannon entropy

$$S = -\sum_{i=1}^{2} P_i \ln P_i \tag{59}$$

increases at full disorder and takes its limiting value

$$\lim_{t\to\infty} S = \lim_{t\to\infty} S(t) = \ln 2. \tag{60}$$

To find unknown parameters α and c in two-level description (creation and annihilation of correlation) in Eqs. (**??**) we should take into account normalization condition, principle of entropy increase (**??**) at $t \to \infty$ and of entropy extremality (presence of minimum) at full order when the following relationship: $\lim_{t\to o} a(t) = 1$ is true for the TCF. We obtained the following equation

$$\lim_{t\to 0} S(t) = -\{\alpha \ln \alpha + (1-\alpha)\ln(1-\alpha)\} = 0$$

for the parameters α and c ($c \geq 0,\ 0 \leq \alpha \leq 1$) having regard to these requirements. Among two solutions ($\alpha_1 = 1,\ \alpha_2 = 0$) only the first one ($\alpha_1 = 1$) has physical sense. Two probabilities calculated by means of Eqs.(**??**) will satisfy conditions (**??**), (**??**)

$$P_1(t) = P_{cc}(t) = \exp\{-\ln 2[1 - a(t)]\}, \\ P_2(t) = P_{ac}(t) = 1 - \exp\{-\ln 2[1 - a(t)]\} \tag{61}$$

respectively. In acordance with two -lewel description it would be convenient to deal with two dynamic channels of entropy (creation (cc) and annihilation (ac)) of correlation

$$S_{cc}(t) = \ln 2\{1 - a(t)\}\exp\{-\ln 2[1 - a(t)]\}, \\ S_{ac}(t) = -\{1 - \exp[(-\ln 2(1 - a(t))]\} \\ \ln\{1 - \exp[(-\ln 2(1 - a(t))]\}. \tag{62}$$

The probabilities obtained are in the line with full dynamic (time dependent) information Shannon entropy

$$S_0(t) = S_{cc}(t) + S_{ac}(t) \\ = \ln 2\{1 - a(t)\}\exp\{-\ln 2[1 - a(t)]\} \\ -\{1 - \exp[(-\ln 2(1 - a(t))]\} \\ \times \ln\{1 - \exp[(-\ln 2(1 - a(t))]\}. \tag{63}$$

The entropy introduced in to Eqs. (**??**),(**??**) characterized a quantitative measure of disorder in the system related to creation and annihilation of dynamic correlation. Owing to discreteness of the TCF $a(t)$ all functions $P_{\alpha\beta}$, $S_{\alpha\beta}$ as well as $S_0(t)$ ($\alpha = a, c;\ \beta = c$) are discrete in the real complex systems.

The results obtained in Section III permit to present the set of entropies for the states connected with the set of orthogonal variables W_i and set of memory functions $M_i(t) = \{M_1(t), M_2(t), M_3(t), \cdots\}$.

Four corresponding entropies $S_0(t),\ S_1(t), S_2(t)$ and $S_3(t)$ and their power frequency spectra are available from the set of four time functions (TCF $a(t)$ and three memory functions $M_1(t),\ M_2(t),\ M_3(t)$). Eqs. (**??**)-(**??**) are of great value because they allow us to estimate stochastic dynamics of the real complex systems with discrete time. As a matter of principle the first three memory functions $M_i(t)$ ($i = 1, 2, 3$) are easy to find via Eqs. (**??**). Using dimensionless parameter $\varepsilon_1 = \tau^2\Omega_1^2$ and solution of the finite-difference Eqs.(**??**) we can found the recurrence relations between the memory functions of junior and higher in the following form

$$M_s(m\tau) = -\sum_{j=0}^{m-1} M_s(j\tau)M_{s-1}((m-j)\tau) \\ +\varepsilon_s^{-1}\{M_{s-1}((m+1)\tau) - M_{s-1}((m+2)\tau)\}, \\ \varepsilon_s = \tau^2\Omega_s^2,\ s = 1, 2, 3, \cdots \tag{64}$$

The relations obtained allow us to derive straightly the necessary memory functions $M_s(t)$ of any order $s = 1, 2, ...$ from experimental data using the registered TCF $a(m\tau)$ [?]. Relaxation frequencies Ω_i^2, $i = 1, 2, 3, ...$, given in Eqs. (??) are available to experimental registration. Thus, it is fair to say that the applications of Eqs.(??) will open up fresh opportunities for detailed study of statistical properties of correlations in the complex systems. The very fact of existence of finite -difference Eqs. (??), (??) enables us to develop any functions directly from the experiment. Therefore, the availability of discreteness permits to enhance substantially the capability to get information for the complex systems' state.

Equations above are useful for the discussion of the experimental data. Close inspection of these equations shows that the behaviour of derivative $\left(\frac{\partial S_0}{\partial t}\right)$ is described in many respects by the function $a'(t) = \tau^{-1}[a(t+\tau) - a(t)]$, which is in its turn can be obtained from the time series observed. Relations analogous to (??), (??) are easily available for the sequence of memory functions $M_i(t)$ (40) as well.

5 Conclusion

Present paper deals with two interrelated important results. The first one is connected with the establishment of the chain of finite-difference non-Markov kinetic equations for the discrete TCF. In this case the state of complex systems at the definite level of correlation is described by two vectors constructed over the strict determined rules. It is natural finite-difference equation of motion, being the peculiar analogue of Liouville's equation for the initial dynamic variables, that is of particular interest for our analysis. In the subsequent discussion we employ the strict deduced mathematical fact of the existence of the normalized TCF. Due to the operation of scalar product the availability of TCF makes it possible to introduce the projection operators in the space of vectors of states. Those projection operations and matrix elements of Liouville's quasioperator ensure the splitting of natural equations of motion and then they are solved in the closed finite-difference form. Using Gram-Schmidt orthogonalization procedure we find an infinite set of the orthogonal dynamic random variables. It allows us to obtain the whole infinite chain of finite-difference kinetic equations for the initial discrete TCF. These equations contain the set of all memory functions characterizing the complete spectrum of non-Markov processes and statistical memory effects in the complex system. The presence of discretness and the very fact of the existence of finite-difference structure enable, in principle, to find all memory functions solving successively kinetic equations for the TCF. Parameters of these equations can be easily obtained from the experimentally registered TCF. In chaotic dynamics of complex systems the TCF above playes the role similar to that of the statistical integral in equilibrium statistical physics.

Another important result of our work is the dynamic (time dependent) information Shannon entropy given in terms of the TCF. It allows us to use the information measure for the quantitative characteristic of two interrelated correlation channels. One of them corresponds to the creation of time correlation and the other - to the annihilation of correlation.

For that as we employ one of the classical Shannon's results, related to the introduction of fidelity evolution function and distance function between two vectors of state. The existence of new information measure opens up new fields for exploration of information characteristics of complex systems. In particular, some interesting data arise from calculations frequency spectra of power of information entropy.

The important consequence of the results obtained is the usage of power spectra of memory functions $M_j(m\tau)$, where $m = 0, 1, 2, 3, ...$ and $j = 1, 2, 3, ...$ The set of three junior memory functions with numbers $j = 1, 2, 3$ provides the basis for the pseudohydrodynamical description of the complex system. In practice, any memory function can be extracted from the experimental time sets and experimentally recorded TCF. These criteria provide the possibility to get reliable information about non-Markov processes and memory effects in natural evolution of complex systems. In principle, the new point in the analysis of complex systems arises from the opportunity to construct the dynamical information Shannon entropy for the experimental memory functions. Undoubtedly, detection of the frequency spectra of power of entropy for memory functions gives us new unique information about the statistical non-Markov properties as well as memory effects in complex systems of various nature.

In conclusion it may be said that this paper describes a first-principle derivation of a hierarchy of finite-difference equations for time correlation function of out-of-equilibrium systems without Hamiltonian. The approach developed seems to have potentials and offer few advantages over the usual Hamiltonian point of view. A similar situation are true apparently with regard to turbulence, aging for instance as in spinglasses and glasses as well as experimental time series for living, social and natural complex systems (physiology, cardiology, finance, psychology and seismology, etc.)

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