ELSEVIER

Chemical Physics 235 (1998) 81–92

Chemical Physics

Suppression of quantum coherence: Noise effect [1]

Jiushu Shao, Christine Zerbe, Peter Hänggi *

Institut für Physik, Universität Augsburg, Memminger Straße 6, D-86135 Augsburg, Germany

Received 20 November 1997

Abstract

The influence of noise on the tunneling dynamics in a periodically driven two-level system is discussed. To describe the noisy quantum dynamics the stochastic Schrödinger equation is used. It accounts for the mutual influence of both phase damping and circularly polarized field driving with a noisy amplitude. It is shown that in absence of deterministic driving, a random dynamic drive stabilizes the localized state while a random, non-demolition drive (see below) destabilizes the localized state. In contrast, localization which can be induced by deterministic driving fields is destroyed by either phase damping or fluctuations of the driving fields. The delocalization rate for intermediate-to-strong field strength turns out to be universal. Approximate results can be obtained by a perturbation treatment. © 1998 Elsevier Science B.V. All rights reserved.

1. Introduction

The dynamics of a dissipative two-level system or spin–boson model has been extensively studied for more than 25 years and it remains to be an active field to present days [1–5]. These studies were driven by various motivations that range from fundamental physics to practical chemistry. For instance, the spin–boson model can be used to understand macroscopic quantum coherence [6,7], chemical reactions in condensed media [4], hydrogen tunneling in metals [8], to name only a few. It is well known that the major effect of dissipation on quantum systems is the destruction of coherence. The current interest in dissipative two-level models concentrates on the understanding how the indispensable environment destroys quantum coherence, which is the bottleneck for the implementation of quantum computers [9,10].

There are two main distinct approaches, microscopic and phenomenological so to speak, to study dissipative dynamics of quantum systems. The first method uses the total Hamiltonian describing the system, environment (or heat bath) and their mutual couplings. Given an initial condition, the dynamics of the total system can in principle be derived by solving the equation of motion — the Schrödinger equation or the corresponding Liouville–von Neumann equation. Because only the dynamics of the system is of interest, it is sufficient to determine the reduced density matrix by tracing over the environmental degrees of freedom. One often assumes

* Corresponding author.

[1] Dedicated to the memory of Professor V.I. Mel'nikov.

PII: S0301-0104(98)00074-3

as the initial condition a factorized total density matrix in which the heat bath is in thermal equilibrium. Therefore, in order to derive the properties of the system it is necessary to know the dynamics of the total system, which is a difficult many-body problem. In contrast, the phenomenological approach is based on the stochastic Schrödinger equation of the *system*. This latter approach does not invoke bath degrees of freedom but rather, the influence of the heat bath is implicitly represented by a stochastic process. Generally, this approach is more convenient to use. The two approaches are expected to yield qualitatively similar results [11].

Resorting to the stochastic Schrödinger equation, we shall investigate how noise affects the control of quantum coherence, especially the *driving induced suppression of quantum coherence* by external fields. Suppression of quantum coherence for a two-level system — i.e. *coherent suppression of tunneling* — can be induced by monochromatic fields when the strengths and frequencies assume specific values [12–20]. This intriguing effect has witnessed many applications in physics and chemistry, from trapping an electron in a quantum double-well structure to controlling chemical reactions [21–29]. It has been shown that strong suppression of quantum coherence can be realized by applying both linearly [21–29] and circularly polarized driving fields [30]. The underlying physics of this phenomenon is related to the level-crossing of the corresponding two dressed states [15,16,30]. Many investigations concerning the influence of the environment on quantum coherence have also been carried out within the spin–boson model with and without driving fields [1–5,23–28,31–33]. Without external driving fields suppression of quantum coherence, i.e. localization, has been predicted at zero temperature with strong dissipation [6,7]. In presence of an external driving, field induced localization does not emerge if there is dissipation [24,25,31,32]. Naturally noise inherent in the driving field should also exert an impact on the driven dynamics. This effect was first explored in [34] where the influence of a noisy driving has been described by random periodic δ-kicks. In this case, tunneling is also hampered by the noise [34].

To gain more physical insight into the noise effect we shall put forward an exactly solvable model. Not only noise from the perturbations by the surroundings, termed background noise, but also noise inherent in the applied field is taken into account in the model system. A circularly polarized field will be utilized as the driving force. In Section 2, we describe the driven stochastic system and use a gauge transformation to change the time-dependent Hamiltonian into a time-independent one. Exact results are shown in Section 3 in which the influence of the two kinds of noise sources (phase damping noise and noisy driving), taken separate or combined, is included. An analytical closed-form approximation is developed in Section 4. A summary and conclusions are presented in Section 5.

2. Stochastic Schrödinger equation

The Hamiltonian of an open two-level system driven by a noisy circularly polarized field can be written as

$$H(t) = -\tfrac{1}{2}\Delta_0\sigma^z + \alpha\xi(t)\sigma^z + \left[\tfrac{1}{2}V_0 + \gamma\eta(t)\right]\left[\sigma^+\exp(\mathrm{i}\omega t) + \sigma^-\exp(-\mathrm{i}\omega t)\right], \tag{1}$$

where Δ_0 is the energy difference between the excited state $|2\rangle$ and the ground state $|1\rangle$ of the isolated two-level system, σ are the Pauli matrices, $\sigma^{\pm} = \sigma^x \pm \mathrm{i}\sigma^y$ and $\xi(t)$ and $\eta(t)$ are realizations of uncorrelated *Gaussian* white noise with strength α and γ, i.e. $\langle\xi(t)\xi(s)\rangle = 2\delta(t-s)$, $\langle\eta(t)\eta(s)\rangle = 2\delta(t-s)$. The strength of the circularly polarized external field is given by V_0. This Hamiltonian can exactly or at least approximately describe many physical systems, e.g. a magnetic spin–1/2 particle. One notices that the background noise operator, i.e. the second term in (1), commutes with the Hamiltonian of the isolated system. There is no energy exchange between the system and the heat bath. This kind of noise only destroys the quantum phase of the system. This will be termed phase damping. Fluctuations of the driving field are accounted for by the term $\gamma\eta(t)$. One may also include other kinds of noise which are induced by the dynamic interaction. For instance, one could consider random (magnetic) transition dipole moments proportional to σ^x. Because we focus here on decoherence rather than on the loss of energy, we only study the phase damping and field fluctuations, with the former resulting from the system–environment couplings of non-demolition type [2]. As shown below, the non-demolition perturbation of the background has a stronger effect on quantum coherence than a random coupling to σ^x. Besides, one advantage of choosing a noise structure as in (1) is that average physical quantities for this noisy Schrödinger dynamics become exactly solvable.

Suppose that $|\psi(t)\rangle$ is the wavefunction state of $H(t)$, i.e. $\mathrm{i}\partial|\psi(t)\rangle/\partial t = H(t)|\psi(t)\rangle$. For the following we set $\hbar = 1$ and all frequencies are measured in units of Δ_0. Employing the gauge transformation $|\tilde{\psi}(t)\rangle = \mathscr{U}|\psi(t)\rangle$, where $\mathscr{U} = \exp(-\mathrm{i}\,\omega t \sigma^z/2)$, we obtain [20]

$$\mathrm{i}\frac{\partial|\tilde{\psi}(t)\rangle}{\partial t} = \mathrm{i}\frac{\partial \mathscr{U}}{\partial t}|\psi(t)\rangle + \mathrm{i}\mathscr{U}\frac{\partial|\psi(t)\rangle}{\partial t} = \tilde{H}(t)|\tilde{\psi}(t)\rangle . \tag{2}$$

In the basis of the superposed localized states, namely the left one $|L\rangle = (|1\rangle + |2\rangle)/\sqrt{2}$ and the right one $|R\rangle = (|1\rangle - |2\rangle)/\sqrt{2}$ — usually employed in *tunneling* problems — the transformed Hamiltonian $\tilde{H}(t)$ reads

$$\tilde{H}(t) = -\tfrac{1}{2}\delta\sigma^x + \alpha\xi(t)\sigma^x + \left[V_0 + \gamma\eta(t)\right]\sigma^z . \tag{3}$$

Here $\delta = \Delta_0 - \omega$ denotes the detuning. Note, that in this form the Hamiltonian has the structure of a two-level system driven by a linearly polarized random field and a randomly varying tunnel splitting. One can establish the relationship between the eigenvectors of $H(t)$ and $\tilde{H}(t)$, i.e. with $|\psi(t)\rangle \equiv c_L(t)|L\rangle + c_R(t)|R\rangle$ and $|\tilde{\psi}(t)\rangle \equiv \tilde{c}_L(t)|L\rangle + \tilde{c}_R(t)|R\rangle$ we find

$$|\psi(t)\rangle = \left[\cos\left(\frac{\omega t}{2}\right)\mathbf{1} + \mathrm{i}\sin\left(\frac{\omega t}{2}\right)\sigma^x\right]|\tilde{\psi}(t)\rangle . \tag{4}$$

Here, $\mathbf{1}$ is the 2×2 identity matrix. Denoting the stochastic influence of the noise sources by ζ the quantity related to quantum coherence is the probability of the system occupying one of the two localized states, say $P_L^\zeta(t) = |\langle L|\psi(t)\rangle|^2$. To determine $P_L^\zeta(t)$ it is convenient to define a time-dependent three-component vector $\mathbf{S}^\zeta(t) \equiv (S_x^\zeta(t), S_y^\zeta(t), S_z^\zeta(t)) = \langle\tilde{\psi}(t)|\boldsymbol{\sigma}|\tilde{\psi}(t)\rangle$, which accounts for the quantum expectation at fixed stochastic realization ζ of $\eta(t)$ and $\xi(t)$. If discussing a magnetic spin$-1/2$ system, one immediately recognizes that $\mathbf{S}^\zeta(t)$ is nothing but a stochastic Bloch vector. The evolution of $\mathbf{S}^\zeta(t)$ describes the trajectory of a unit vector. One can calculate many properties of the system described by either $\tilde{H}$ or H with a given $\mathbf{S}^\zeta(t)$. For instance, the quantity of interest $P_L^\zeta(t)$ turns out to be

$$P_L^\zeta(t) = \tfrac{1}{2}\left[1 - \sin(\omega t)S_y^\zeta(t) + \cos(\omega t)S_z^\zeta(t)\right] . \tag{5}$$

We should stress that the quantity $P_L^\zeta(t)$ stands for the probability in the localized state which is different from the probabilities to find the system in the eigenstates denoted by $P_g^\zeta(t)$ and $P_e^\zeta(t)$, respectively. Note that $P_L^\zeta(t)$ is associated with quantum coherence, which is represented by the oscillating population transport between the localized states. However, the probability in the ground state $P_g^\zeta(t)$ (or in the excited state $P_e^\zeta(t)$ as well) is related to quantum transition, which does not take place for a closed system in absence of the driving field. One can show that

$$P_g^\zeta(t) = \tfrac{1}{2}\left[1 + S_x^\zeta(t)\right] .$$

Noting that the vector $\mathbf{S}^\zeta(t)$ is determined by $|\tilde{\psi}(t)\rangle$, we only need to deal with the transformed stochastic Hamiltonian $\tilde{H}(t)$. There are two ways to calculate physical quantities of the systems described by stochastic Hamiltonians. One is the direct perturbation treatment [41,42] and the other is based on the stochastic calculus [43–45]. The dynamics of the noisy Schrödinger equation is not unique in the sense that one can evaluate its

[2] The non-demolition interaction was first introduced in quantum measurement theory. See [35,36]. The effect induced by non-demolition interaction in magnetic tunneling is known as topological decoherence. See for instance [37,38]. See also[39,40].

dynamics within the Stratonovitch or Îto calculus, respectively. This is rooted in the fact that the Schrödinger equation corresponding to the stochastic Hamiltonian in (3) involves the stochastic products $\xi(t)|\tilde{\psi}$ and $\eta(t)|\tilde{\psi}\rangle$. If we interpret these products in the sense of Stratonovitch, the corresponding Schrödinger equation reads

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}|\tilde{\psi}\rangle = -\tfrac{1}{2}\delta\sigma^x|\tilde{\psi}\rangle + V_0\sigma^z|\tilde{\psi}\rangle + \alpha\sigma^x\xi(t)|\tilde{\psi}\rangle + \gamma\sigma^z\eta(t)|\tilde{\psi}\rangle. \tag{6}$$

To perform explicit calculations it is more convenient to utilize the martingale property of Îto-calculus (i.e. this property yields vanishing expectations for the noisy products in (6)). The Schrödinger equation in (6) is *stochastically equivalent* recast as the Îto–Schrödinger equation, namely

$$\mathrm{d}|\tilde{\psi}\rangle = \tfrac{\mathrm{i}}{2}\delta\sigma^x|\tilde{\psi}\rangle\mathrm{d}t - \mathrm{i}V_0\sigma^z|\tilde{\psi}\rangle\mathrm{d}t - \mathrm{i}\alpha\sigma^x|\tilde{\psi}\rangle\cdot\mathrm{d}W_1 - \mathrm{i}\gamma\sigma^z|\tilde{\psi}\rangle\cdot\mathrm{d}W_2 - \tfrac{1}{2}(\alpha^2+\gamma^2)|\tilde{\psi}\rangle\mathrm{d}t, \tag{7}$$

where $\cdot$ denotes multiplication in the Îto sense, and where $W_1(t)\equiv\int_0^t\xi(s)\mathrm{d}s$ and $W_2(t)\equiv\int_0^t\eta(s)\mathrm{d}s$ are two uncorrelated Brownian motions (Wiener noise). From (7) we find the equation of motion for the quantum expectation $\mathbf{S}^\zeta$ of the Pauli operators σ's, i.e.

$$\mathrm{d}\mathbf{S}^\zeta = A\mathbf{S}^\zeta\mathrm{d}t + B\mathbf{S}^\zeta\cdot\mathrm{d}W_1 + C\mathbf{S}^\zeta\cdot\mathrm{d}W_2, \tag{8}$$

where

$$A=\begin{pmatrix} -2\gamma^2 & -2V_0 & 0 \\ 2V_0 & -2(\alpha^2+\gamma^2) & \delta \\ 0 & -\delta & -2\alpha^2 \end{pmatrix},\quad B=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -2\alpha \\ 0 & 2\alpha & 0 \end{pmatrix},\quad C=\begin{pmatrix} 0 & -2\gamma & 0 \\ 2\gamma & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{9}$$

Next, we investigate the statistical average of $\mathbf{S}^\zeta(t)$ with respect to the fluctuations $W_1(t)$ and $W_2(t)$, which we denote by $\bar{\mathbf{S}}$. From (8) we obtain

$$\frac{\mathrm{d}\bar{\mathbf{S}}}{\mathrm{d}t} = A\bar{\mathbf{S}}, \tag{10}$$

because within the Îto-calculus the two Brownian noise terms do not contribute to the statistical average. Its solution formally reads

$$\bar{\mathbf{S}}(t) = \exp(At)\bar{\mathbf{S}}(0). \tag{11}$$

3. Results

3.1. Background noise ($\alpha \neq 0$)

Let us first consider the case without driving, i.e. $V_0 = 0$ and $\gamma = 0$. In this case, of course, it is not necessary to employ the gauge transformation. However, for the consistency of our discussion we follow the procedure described above. The coefficient matrix in (10) becomes

$$A=\begin{pmatrix} 0 & 0 & 0 \\ 0 & -2\alpha^2 & \delta \\ 0 & -\delta & -2\alpha^2 \end{pmatrix}. \tag{12}$$

Upon inserting (12) into (11) we find

$$\overline{S_x(t)} = \overline{S_x(0)}, \tag{13}$$

$$\overline{S_y(t)} = \exp(-2\alpha^2 t)\left[\cos(\delta t)\overline{S_y(0)} + \sin(\delta t)\overline{S_z(0)}\right], \tag{14}$$

$$\overline{S_z(t)} = \exp(-2\alpha^2 t)\left[-\sin(\delta t)\overline{S_y(0)} + \cos(\delta t)\overline{S_z(0)}\right]. \tag{15}$$

Note that $\overline{S_x(t)}$ being a constant means that the population densities in the eigenstates do not change with time. This is the specific feature of the non-demolition interaction between the two-level system and the reservoir [40], characterizing this case.

Suppose that the system evolves from the left state: $\overline{S_x(0)} = \overline{S_y(0)} = 0$, $\overline{S_z(0)} = 1$. With this initial condition we obtain $\overline{S_y(t)} = \exp(-2\alpha^2 t)\sin(\delta t)$ and $\overline{S_z(t)} = \exp(-2\alpha^2 t)\cos(\delta t)$. Replacing $S_y^{\xi}(t)$ and $S_z^{\xi}(t)$ in (5) by $\overline{S_y(t)}$ and $\overline{S_z(t)}$, respectively, we obtain the statistical average of $P_L^{\xi}(t)$, namely

$$\begin{aligned}\overline{P_L(t)} &= \tfrac{1}{2} + \tfrac{1}{2}\exp(-2\alpha^2 t)\left[\cos(\delta t)\cos(\omega t) - \sin(\delta t)\sin(\omega t)\right] \\ &= \tfrac{1}{2} + \tfrac{1}{2}\exp(-2\alpha^2 t)\cos(\Delta_0 t)\end{aligned} \tag{16}$$

because $\delta = \Delta_0 - \omega$. Obviously, $\overline{P_L}$ is generally a damped oscillating function of time. Even with $\Delta_0 = 0$, we find $\overline{P_L} \to 1/2$ as $t \to \infty$. Therefore, *in this case the degeneracy of the bare system is split by the phase damping*.

Let us now switch on an external noise-free field, i.e. $\gamma = 0$. If the two-level system is free from external perturbations except coherent driving, localization takes place if and only if ω and V_0 satisfy [30]

$$\omega = \frac{\Delta_0^2 + 4V_0^2}{2\Delta_0} \tag{17}$$

and $\omega \gg \Delta_0$ for given Δ_0. The probability in the left state $P_L(t)$ under localization condition reads [30]

$$P_L(t) = \tfrac{1}{2}\left\{1 + \frac{x^2-1}{x^2+1}\sin^2(\omega t) + \cos(\omega t)\left[\frac{4x^2}{(x^2+1)^2} + \frac{(x^2-1)^2}{(x^2+1)^2}\cos(\omega t)\right]\right\}, \tag{18}$$

where $x = 2V_0/\Delta_0$. Strong localization is achieved already for $x > 4$, which corresponds to a driving field of intermediate-to-strong strength V_0.

We are aware that even for $\alpha \neq 0$ the formal solution Eq. (11) can be written down analytically by solving a cubic equation via use of Cardani formulas, but the results are formidably complicated. Of course, we can perform exact numerical calculations if all parameters assume given values. For instance, Fig. 1 displays the behavior of $\overline{P_L(t)}$ for $\alpha = 0$, $\alpha = 0.2$ and $\alpha = 1.0$ from exact calculations. The localization condition with $V_0 = 2$ and $\omega = 17/2$ in units of $\Delta_0 = 1$ is used.

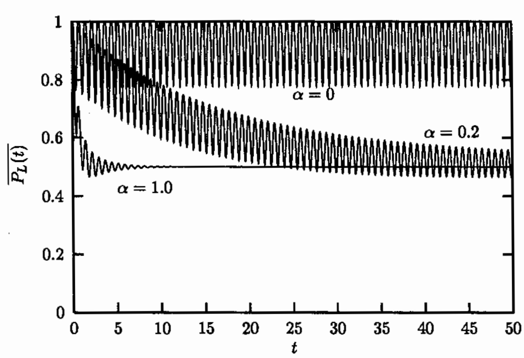

Fig. 1. Time evolution of $\overline{P_L(t)}$, with $\overline{P_L(0)} = 1$ under the localization condition $V_0 = 2\Delta_0$. The remaining parameters are $\omega = 17/2\,\Delta_0$: $\alpha = 0.0$, $\alpha = 0.2$, $\alpha = 1.0$ and $\gamma = 0$. The dimensionless time is measured in units of Δ_0; i.e. $t \to t(\Delta_0/\hbar) \equiv t$.

Without background noise, $\alpha = 0$, it is seen that the system is localized in the left state. For a small noise strength $\alpha = 0.2$, the system sustains quantum coherence (the oscillatory behavior shown in the curve) for a very long time (beyond the time range depicted in the figure). However, if the noise strength becomes large, e.g. $\alpha = 1.0$, quantum coherence will be destroyed already on short-time scales, such as for $t \approx 15$.

3.2. Field fluctuations ($\gamma \neq 0$)

We consider the effect of noise inherent in the applied field only, i.e. $\alpha = 0$. In this case the matrix A becomes

$$A = \begin{pmatrix} -2\gamma^2 & -2V_0 & 0 \\ 2V_0 & -2\gamma^2 & \delta \\ 0 & -\delta & 0 \end{pmatrix}.$$

For $V_0 = 0$, we solve this equation exactly to obtain

$$\overline{S_x(t)} = \exp(-2\gamma^2 t)\overline{S_x(0)}, \tag{19}$$

$$\overline{S_y(t)} = \tfrac{1}{2}\exp(-\gamma^2 t)\left[C_1(t) - \frac{\gamma^2}{D}C_2(t)\right]\overline{S_y(0)} + \tfrac{1}{2}\frac{\delta}{D}\exp(-\gamma^2 t)C_2(t)\overline{S_z(0)}, \tag{20}$$

$$\overline{S_z(t)} = \tfrac{1}{2}\frac{\delta}{D}\exp(-\gamma^2 t)C_2(t)\overline{S_y(0)} + \tfrac{1}{2}\exp(-\gamma^2 t)\left[C_1(t) + \frac{\gamma^2}{D}C_2(t)\right]\overline{S_z(0)}, \tag{21}$$

where $D = \sqrt{\gamma^4 - \delta^2}$, $C_1(t) = \exp(Dt) + \exp(-Dt)$ and $C_2(t) = \exp(Dt) - \exp(-Dt)$. A significant fact is that while $\overline{S_x}$ always depicts an exponential damping, $\overline{S_y}$ and $\overline{S_z}$ may exhibit two distinct behaviors, depending on whether D is real or imaginary. If D is real, i.e. $\gamma^2 > |\delta|$, $\bar{\mathbf{S}}$ will be an exponentially decaying function of time. In contrast, $\bar{\mathbf{S}}$ will display damped oscillations if D is imaginary, i.e. $\gamma^2 < |\delta|$. For a very strong fluctuation we have the approximate relations $D = \gamma^2 - \delta^2/2\gamma^2$ and $C_1(t) = C_2(t) = \exp[(\gamma^2 - \delta^2/2\gamma^2)t]$. As a consequence, Eq. (20) and Eq. (21) become

$$\overline{S_y(t)} = -\tfrac{1}{4}\frac{\delta^2}{\gamma^4}\exp\left(-\frac{\delta^2}{2\gamma^2}t\right)\overline{S_y(0)} + \tfrac{1}{2}\frac{\delta}{\gamma^2}\exp\left(-\frac{\delta^2}{2\gamma^2}t\right)\overline{S_z(0)}, \tag{22}$$

and

$$\overline{S_z(t)} = -\tfrac{1}{2}\frac{\delta}{\gamma^2}\exp\left(-\frac{\delta^2}{2\gamma^2}t\right)\overline{S_y(0)} + \left(1 + \tfrac{1}{4}\frac{\delta^2}{\gamma^4}\right)\exp\left(-\frac{\delta^2}{2\gamma^2}t\right)\overline{S_z(0)}. \tag{23}$$

As $\gamma^2 \to \infty$ and δ remains finite, we find $\overline{S_y(t)} = 0$ and $\overline{S_z(t)} = \overline{S_z(0)}$. In other words, with an extreme fluctuation strength, $\overline{S_y(t)}$ vanishes instantaneously while $\overline{S_z(t)}$ does not vary with time.

With the initial condition $\overline{S_x(0)} = \overline{S_y(0)} = 0$ and $\overline{S_z(0)} = 1$, i.e., $|\psi(0)\rangle = |L\rangle$, Eqs. (20) and (21) become

$$\overline{S_y(t)} = \tfrac{1}{2}\frac{\delta}{D}\exp(-\gamma^2 t)C_2(t), \tag{24}$$

and

$$\overline{S_z(t)} = \tfrac{1}{2}\exp(-\gamma^2 t)\left[C_1(t) + \frac{\gamma^2}{D}C_2(t)\right]. \tag{25}$$

Replacing $S_y^\zeta(t)$ and $S_z^\zeta(t)$ in (5) by the above expressions, respectively, we obtain the statistical average of $P_L^\zeta(t)$, namely

$$\overline{P_L(t)} = \tfrac{1}{2} - \tfrac{1}{4}\exp(-\gamma^2 t)\left\{\frac{\delta}{D}C_2(t)\sin(\omega t) - \left[C_1(t) + \frac{\gamma^2}{D}C_2(t)\right]\cos(\omega t)\right\}. \tag{26}$$

Obviously, $\overline{P_L}$ generally exhibits damped oscillations. If $\gamma^2 < |\delta|$, we find the relations $C_1 = 2\cos(|D|t)$ and $C_2 = 2\mathrm{i}\sin(|D|t)$, arriving at

$$\overline{P_L(t)} = \tfrac{1}{2} - \tfrac{1}{2}\exp(-\gamma^2 t)\left\{\frac{\sin(|D|t)}{|D|}\left[\delta\sin(\omega t) - \gamma^2\cos(\omega t)\right] - \cos(|D|t)\cos(\omega t)\right\}. \tag{27}$$

For a large γ we find

$$\overline{P_L(t)} \approx \tfrac{1}{2} + \tfrac{1}{2}\cos(\omega t). \tag{28}$$

The system, therefore, seems to acquire an effective energy splitting ω from the coupling to the environment. If $\omega = 0$, then $\overline{P_L(t)} = 1$. In this case quantum coherence is entirely suppressed. This is the effect of 'localization stabilized by noise' [34,46].

Let us consider a specific case where $V_0 = 0$ and $\omega = 0$. The original Hamiltonian now becomes

$$H(t) = -\tfrac{1}{2}\Delta_0\sigma^z + 2\gamma\eta(t)\sigma^x, \tag{29}$$

which describes an undriven system coupled to a stochastic field through *dynamic* interaction. This model first appeared in van Kampen's book [41,42]. Its detailed dynamics was revealed only recently by Blanchard et al. [46]. For a *degenerate* two-level system $\Delta_0 = 0$, Eq. (20) and Eq. (21) reduce to

$$\overline{S_y(t)} = \exp(-2\gamma^2 t)\overline{S_y(0)} \tag{30}$$

and

$$\overline{S_z(t)} = \overline{S_z(0)}. \tag{31}$$

Therefore, $\overline{P_L(t)}$ will always be unity if the system evolves from the left state. In other words, unlike in the case of non-demolition noise, *dynamic noise* — the very noise in the driving field at $\omega = 0$ — *does not have impact on quantum coherence of a degenerate two-level system.*

Again, although we are able to derive an analytical solution in the case of $V_0 \neq 0$, its explicit solution contains too many terms to be of insightful value. The results of numerical calculations for $V_0 = 2$, $\omega = 17/2$, $\gamma = 0.2$ and $\gamma = 1.0$ are shown in Fig. 2.

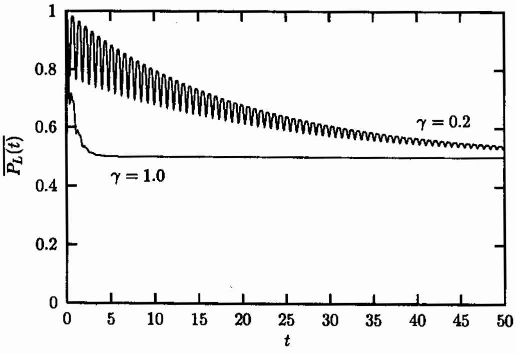

Fig. 2. Time evolution of $\overline{P_L(t)}$ with $\overline{P_L(0)} = 1$ under the localization condition $V_0 = 2\Delta_0$ and $\omega = 17/2\Delta_0$: $\gamma = 0.2$, $\gamma = 1.0$, and $\alpha = 0$. Time is measured in units of Δ_0, i.e. $t \to t(\Delta_0/\hbar) \equiv t$.

If the fluctuations are weak, e.g. $\gamma = 0.2$, the system can maintain its coherence for a very long time. With increasing fluctuation strength, however, e.g. $\gamma = 1.0$, one can hardly resolve the oscillations of $\overline{P_L(t)}$. In this case, the dynamics is dominated by an exponential decay.

We can take both the background and the field noises into account together. Since there is no interference between the two noises, one expects that these yield only additive but not cooperative effects. Hence, we do not work out the exact dynamics here; instead we will discuss the additivity by an approximate approach.

4. Approximate results

4.1. Background noise ($\alpha \neq 0$)

For a weak background noise, i.e. $\alpha \ll |\delta|$, V_0, we can resort to the perturbation theory to obtain a simple approximation to $\overline{P_L}$ that reads

$$\overline{P_L(t)} = \tfrac{1}{2}\left\{1 - \frac{\delta}{\Omega}\exp\left[\frac{\left(-2+4V_0^2\right)\alpha^2 t}{\Omega^2}\right]\sin(\omega t)\sin(\Omega t) + \cos(\omega t)\left[\frac{4V_0^2}{\Omega^2}\exp\left(\frac{-8V_0^2\alpha^2 t}{\Omega^2}\right)\right.\right.$$
$$\left.\left. + \frac{\delta^2}{\Omega^2}\exp\left[\frac{\left(-2+4V_0^2\right)\alpha^2 t}{\Omega^2}\right]\cos(\Omega t)\right]\right\}. \qquad (32)$$

Here, $\Omega = \sqrt{\delta^2 + 4V_0^2}$ is the Rabi frequency. Inserting the dynamic localization condition in (17) into (32), we obtain

$$\overline{P_L(t)} = \tfrac{1}{2}\left\{1 + \tfrac{1}{2}(\rho^2 - 3\rho + 2)\exp\left[(2\rho - \rho^2 - 2)\alpha^2 t\right] + \rho(2-\rho)\exp\left[-2\rho(2-\rho)\alpha^2 t\right]\cos(\omega t)\right.$$
$$\left. + \tfrac{1}{2}\rho(\rho - 1)\exp\left[(2\rho - \rho^2 - 2)\alpha^2 t\right]\cos(2\omega t)\right\}, \qquad (33)$$

where $\rho = \Delta_0/\omega$. Comparing this result, depicted in Fig. 3, with the exact one in Fig. 1, we see that the approximation indeed leads to accurate results even for strong noise strength $\alpha = 1$.

Asymptotically, the oscillating factors $\cos(\omega t)$ and $\cos(2\omega t)$ are damped out. Therefore, we obtain

$$\overline{P_L(t)}_{t\to\infty} = \tfrac{1}{2} - \tfrac{1}{4}(\rho^2 - 3\rho + 2)\exp\{[\rho(2-\rho) - 2]\alpha^2 t\}. \qquad (34)$$

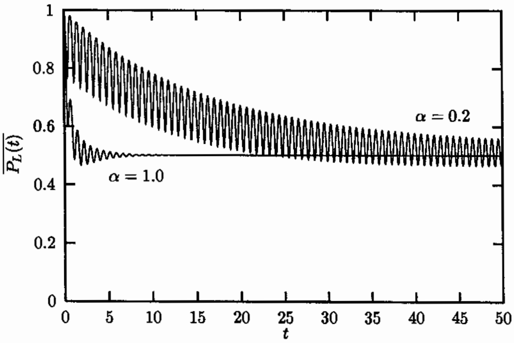

Fig. 3. Perturbation result for the time evolution of $\overline{P_L(t)}$ with $\overline{P_L(0)} = 1$ under the localization condition. The same parameter set is used as in Fig. 1.

Thus, $\overline{P_L(t)}$ will eventually decay towards 1/2. The decay rate Γ which measures the lifetime of the localized state is

$$\Gamma = [2 - \rho(2-\rho)]\alpha^2. \tag{35}$$

Because $\rho = 2/(1+x^2)$ with $x = 2V_0/\Delta_0$, one finds

$$\Gamma = \left[2 - \frac{4x^2}{(1+x^2)^2}\right]\alpha^2. \tag{36}$$

One readily shows that $\alpha_2 \leqslant \Gamma \leqslant 2\alpha^2$. The lower bound of Γ thus corresponds to $x=1$, i.e. $\omega = \Delta_0 = 1$, which is the resonance condition. In this case, localization cannot take place. The upper bound of Γ corresponds to $x \to \infty$, which leads to perfect localization. Therefore, the lifetime of an initially localized state under good localization condition ($x > 4$, for instance) is about $1/(2\alpha^2)$, remaining almost a constant. This observation is similar to that of a spin–boson model driven by a linearly polarized field [33].

4.2. Field fluctuations ($\gamma \neq 0$)

For the case of weak noise for the driving field amplitude, we again use the perturbation approach to obtain

$$\overline{P_L(t)} = \tfrac{1}{2}\left\{1 - \frac{\delta}{\Omega}\exp\left[\frac{-(1+4V_0^2)\gamma^2 t}{\Omega^2}\right]\sin(\omega t)\sin(\Omega t) + \cos(\omega t)\left[\frac{4V_0^2}{\Omega^2}\exp\left(\frac{-2\delta^2\gamma^2 t}{\Omega^2}\right) + \frac{\delta^2}{\Omega^2}\exp\left[-\frac{(1+4V_0^2)\gamma^2 t}{\Omega^2}\right]\cos(\Omega t)\right]\right\}. \tag{37}$$

With the localization condition in (17), Eq. (37) with $\rho = 2/(1+x^2)$, $x = 2V_0/\Delta_0$ becomes

$$\overline{P_L(t)} = \tfrac{1}{2}\left[1 + \tfrac{1}{2}(\rho^2 - 3\rho + 2)\exp\left[-(1+2\rho-\rho^2)\gamma^2 t\right] + \cos(\omega t)\rho(2-\rho)\exp\left[-2(\rho-1)^2\gamma^2 t\right] + \tfrac{1}{2}\rho(\rho-1)\exp\left[-(1+2\rho-\rho^2)\gamma^2 t\right]\cos(2\omega t)\right]. \tag{38}$$

This result is depicted with Fig. 4.

Compared to Fig. 2, Fig. 4 displays the second harmonic oscillation more distinctly in the case of weak fluctuations $\gamma = 0.2$, but exhibits barely resolvable oscillations in the case of strong fluctuations $\gamma = 1.0$, even during the initial period. However, on the whole, the perturbation approximation indeed reproduces *well* all characteristic features of the exact dynamics.

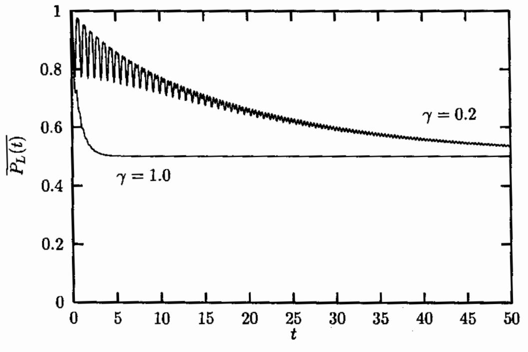

Fig. 4. Result obtained within perturbation theory for the time evolution of $\overline{P_L(t)}$ with $\overline{P_L(0)} = 1$, under the localization condition, see (17). The same parameter set is used as in Fig. 2.

Employing the foregoing discussion for the weak background noise, we can show that the decay rate of the localized state under the localization condition is given by

$$\Gamma=\left[1+\frac{4x^2}{\left(1+x^2\right)^2}\right]\gamma^2. \tag{39}$$

Therefore, for an intermediate-to-strong field drive, i.e. $x>4$, the decay rate assumes an universal value of $\Gamma\sim\gamma^2$.

4.3. Combined noise ($\alpha\neq 0$, $\gamma\neq 0$)

Finally, we consider the situation when both random sources act simultaneously. In the regime of weak-to-intermediate noise the perturbation approach is also applicable. Extending the discussion above to this case, we obtain from (17)

$$\begin{aligned}\overline{P_L(t)}=\tfrac{1}{2}\Big\{1&+\tfrac{1}{2}(\rho^2-3\rho+2)\exp\left\{-\left[(2+\rho^2-2\rho)\alpha^2+(1+2\rho-\rho^2)\gamma^2\right]t\right\}+\cos(\omega t)\rho(2-\rho)\\&\times\exp\left\{-2\left[(2\rho-\rho^2)\alpha^2+(\rho^2-1)2\gamma^2\right]t\right\}+\cos(2\omega t)\rho(\rho-1)\exp\left\{-\left[(2+\rho^2-2\rho)\alpha^2\right.\right.\\&\left.\left.+(1+2\rho-\rho^2)\gamma^2\right]t\right\}\Big\}.\end{aligned} \tag{40}$$

It becomes clear that the noise behaves additive in the sense that the damping rate is the sum of the two contributions. As a check, we investigate the asymptotical behavior of $\overline{P_L(t)}$ as $t\to\infty$, i.e.

$$\overline{P_L(t)}=\tfrac{1}{2}+\tfrac{1}{4}(\rho^2-3\rho+2)\exp\left\{-\left[(2+\rho^2-2\rho)\alpha^2+(1+2\rho-\rho^2)\gamma^2\right]t\right\}. \tag{41}$$

The decay rate thus reads

$$\Gamma=2\alpha^2+\gamma^2+\frac{4x^2}{\left(1+x^2\right)^2}(\gamma^2-\alpha^2). \tag{42}$$

Therefore, the total decay rate for intermediate-to-strong field strength, i.e. $x>4$, is $2\alpha^2+\gamma^2$, being the sum of the individual two decay rates induced by the background noise and the field noise.

5. Discussion

Starting with the stochastic Schrödinger equation in (6), we have been able to present an exactly solvable model of an open two-level system driven by noisy circularly polarized fields. Both the background noise due to non-demolition perturbation, which induces phase damping, and the amplitude fluctuations of the driving circularly polarized field are investigated on the same footing. Several physical consequences including the localization stabilized by noise, the degeneracy breaking by the phase damping and the universal delocalization rate are displayed within this model.

When there acts *no driving* field, i.e. $V_0=0$, we find that dynamic noise, i.e. $\omega=0$, still can stabilize the localized state while the background noise (i.e. phase damping) assists delocalization even for the degenerate two-level system. The delocalization process can be regarded as a hopping dynamics of a particle between two classical (localized) states. Dynamic noise increases the energy barrier between the two localized states; in contrast, phase dissipation decreases the barrier.

If the driving field is switched on, i.e. $V_0\neq 0$, however, both kinds of noise play a similar role, i.e. they destabilize the localized state. One obvious outcome is that the action of noise and driving is to destroy the field-controlled localization of the noise-free deterministic driven two-level system.

The lifetime (or delocalization rate) of the localized state under localization condition has been calculated. It assumes a universal value for intermediate-to-strong field strength V_0. This is so, because the dynamics of the noise-free system, represented by the average probability in the localized state, does not exhibit an appreciable variation in this parameter regime [30]. Besides, there is no interference between the background and the amplitude noise for the driving field. The two noise sources exert additive effects on the system.

This topic of field and noise-driven coherence and decoherence is closely related in spirit to prominent contributions by Professor V.I. Mel'nikov to both field-driven escape [47,48] and field-driven tunneling through a barrier [49,50]. One of us (PH) remembers him as a kind and compassionate friend. He benefited greatly from his extraordinary knowledge and skills in doing analytic work. The scientific community surely will remember him for his many insightful key contributions to reaction rate theory; he will be missed very much by those who knew him personally.

Acknowledgements

This work was supported by the Alexander von Humboldt-Stiftung, the National Natural Science Foundation of China (JS), the Hochschulsonderprogramm III (CZ) and the Deutsche Forschungsgemeinschaft (PH) via the Schwerpunktprogramm, 'Zeitabhängige Phänomene und Methoden in Quantensystemen der Physik und Chemie' (Az. HA 1517/14-2).

References

[1] A.J. Leggett, S. Chakravarty, A.T. Dorsey, M.P.A. Fisher, A. Garg, W. Zwerger, Rev. Mod. Phys. 59 (1987) 1.
[2] A.J. Leggett, S. Chakravarty, A.T. Dorsey, M.P.A. Fisher, A. Garg, W. Zwerger, Rev. Mod. Phys. 67 (1995) 725, erratum.
[3] U. Weiss, Quantum Dissipative Systems (World Scientific, Singapore) 1993.
[4] P. Hänggi, P. Talkner, M. Borkovec, Rev. Mod. Phys. 62 (1990) 251.
[5] V.A. Benderskii, D.E. Makarov, C.A. Wight, Adv. Chem. Phys. 88 (1994) 1.
[6] S. Chakravarty, Phys. Rev. Lett. 49 (1982) 681.
[7] A.J. Bray, M.A. Moore, Phys. Rev. Lett. 49 (1982) 1546.
[8] H. Grabert, H.R. Schober, Top. App. Phys. 73 (1997) 5.
[9] D.P. DiVincenzo, Science 270 (1995) 255.
[10] A. Ekert, R. Josza, Rev. Mod. Phys. 68 (1996) 733.
[11] L. Diósi, N. Gisin, J. Halliwell, I.C. Cival, Phys. Rev. Lett. 74 (1995) 203.
[12] F. Großmann, P. Jung, T. Dittrich, P. Hänggi, Z. Phys. B-Condens. Matter 84 (1991) 315.
[13] F. Großmann, T. Dittrich, P. Jung, P. Hänggi, Phys. Rev. Lett. 67 (1991) 516.
[14] F. Großmann, T. Dittrich, P. Hänggi, Physica B 175 (1991) 293.
[15] F. Großmann, P. Hänggi, Europhys. Lett. 18 (1992) 571.
[16] J.M. Gomez Llorente, J. Plata, Phys. Rev. A 45 (1992) R6958.
[17] R. Bavli, H. Metiu, Phys. Rev. A 47 (1992) 3299.
[18] L. Wang, J. Shao, Phys. Rev. A 49 (1994) R637.
[19] Y. Kayanama, Phys. Rev. A 50 (1994) 843.
[20] For a recent review, see P. Hänggi, in: T. Dittrich, P. Hänggi, L.-G. Ingold, B. Kramer, G. Schön, W. Zwerger (Eds.), Quantum Transport and Dissipation, Ch. 5 (VCH, Weinheim, 1997).
[21] R. Bavli, H. Metiu, Phys. Rev. Lett. 69 (1992) 1986.
[22] J.E. Combariza, B. Just, J. Manz, G.K. Paramonov, J. Phys. Chem. 95 (1992) 10351.
[23] M. Morillo, R.I. Cukier, J. Chem. Phys. 98 (1993) 4548.
[24] Y. Dakhnovskii, Phys. Rev. B 49 (1994) 4649.
[25] Y. Dakhnovskii, Ann. Phys. 230 (1994) 145.
[26] M. Grifoni, M. Sassetti, P. Hänggi, U. Weiss, Phys. Rev. E 52 (1995) 3596.
[27] D.E. Makarov, N. Makri, Phys. Rev. E 52 (1995) 5863.
[28] M. Grifoni, L. Hartmann, P. Hänggi, Chem. Phys. 217 (1997) 167.
[29] See also the Special Issue, in: W. Domcke, P. Hänggi, D. Tannor (Eds.), Dynamics of Driven Quantum Systems, Chem. Phys., 217, Nos. 2,3, 1997, pp. 119–416.

[30] J. Shao, P. Hänggi, Phys. Rev A 56 (1997) R4397.
[31] T. Dittrich, B. Oelschlägel, P. Hänggi, Europhys. Lett. 22 (1993) 5.
[32] T. Dittrich, B. Oelschlägel, P. Hänggi, Acta Phys. Pol. B 24 (1993) 845.
[33] N. Makri, J. Chem. Phys. 106 (1997) 2286.
[34] F. Großmann, T. Dittrich, P. Jung, P. Hänggi, J. Stat. Phys. 70 (1993) 229.
[35] C. Caves, K. Thorne, R. Drever, V. Sandberg, M. Zimmermann, Rev. Mod. Phys. 52 (1980) 341.
[36] D. Walls, G. Milburn, Quantum Optics (Springer, Berlin, 1994).
[37] N.V. Prokof'ev, P.C.E. Stamp, J. Phys. Condens. Matter 5 (1993) L663.
[38] N.V. Prokof'ev, P.C.E. Stamp, J. Low Temp. Phys. 104 (1996) 143.
[39] E. Shimshoni, Y. Gefen, Ann. Phys. 210 (1991) 16.
[40] J. Shao, M.-L. Ge, H. Cheng, Phys. Rev. E 53 (1996) 1243.
[41] N.G. van Kampen, Stochastic Processes in Physics and Chemistry (North Holland Publ., Amsterdam, 1992).
[42] N.G. Van Kampen, Phys. Rep. 24 (1976) 171.
[43] P. Hänggi, H. Thomas, Phys. Rep. 88 (1982) 207.
[44] H. Risken, The Fokker–Planck Equation: Methods of Solution and Application, Vol. 18, Springer Series in Synergetics (Springer, Berlin, 1984).
[45] C.W. Gardiner, Handbook of Stochastic Methods, 2nd edn., Vol. 56, Springer Series in Synergetics (Springer, Berlin, 1985).
[46] Ph. Blanchard, G. Bolz, M. Cini, G.F. De Angelis, M. Serva, J. Stat. Phys. 75 (1994) 749.
[47] V.I. Mel'nikov, Phys. Rev. A 45 (1992) 5474.
[48] V.I. Mel'nikov, J. Stat. Phys. 70 (1992) 77.
[49] B.I. Ivlev, V.I. Mel'nikov, Sov. Phys. JEPT 63 (1986) 1295, [Zh. Eksp. Teor. Fiz. 90, 2208 (1986)].
[50] B.I. Ivlev, V.I. Mel'nikov, Phys. Rev. Lett. 55 (1985) 1614.