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# CONTROL OF TUNNELING IN THE OHMIC TWO-STATE SYSTEM BY STRONG DRIVING FIELDS

MILENA GRIFONI, PETER HÄNGGI
*Institut für Physik, Universität Augsburg, Memminger Straße 6, 86135 Augsburg, Germany*

The dissipative dynamics of a two-state system driven by a monochromatic driving field is formulated in terms of an exact master equation. Applications of these results to the possibility of influencing quantum tunneling via quantum stochastic resonance or dynamical localization are discussed.

## 1 Introduction

The problem of a quantum particle coupled to a thermal bath and tunneling through the barrier of a slightly asymmetric double-well potential is ubiquitous in many physical and chemical systems. It can model for example long range electron-transfer reactions [1], the tunneling of atoms between an atomic-force microscope tip and a surface [2], the low-temperature excitations of the disordered lattice of amorphous solids [3] or the magnetic flux in a superconducting quantum interference device (SQUID) [4]. At sufficiently low temperatures the dynamics only involves the ground states of the potential minima, and the system can be effectively restricted to the two dimensional Hilbert space spanned by the two ground states. This two-level-system (TLS), when isolated from the thermal bath, is the simplest system exhibiting quantum interference effects, as it can be prepared to oscillate clockwise between the eigenstates in the left and right well. Quite generally, the stochastic influence results in a reduction of the coherent tunneling motion by incoherent processess [5,6], and may even lead to a transition to self-trapping at zero temperature [7]. An important question is to which degree the tunneling dynamics is influenced by externally applied time-dependent fields. In particular, a complete destruction of tunneling can be induced by a coherent driving field of appropriate frequency and strength [8]. This effect can be stabilized in the presence of dissipation [9,10]. The transition temperature, above which quantum coherence is destroyed by a stochastic environment, is modified by a driving field [10]. A novel non-markovian dynamics may arise due to driving induced correlations between tunneling transitions[11,12,13] (cf. Fig. 1). A strong driving field can even succeed in inverting the populations of the localized states, hence inverting the direction of the tunneling motion [14]. Moreover, the phenomenon of *quantum stochastic resonance* can be employed to substantially enhance [15,16] or suppress [16] the nonlinear response of the dissipative TLS, hence allowing a distortion-free amplification of signals in quantum systems. Finally, the dissipative dynamics under ac-modulation of the TLS asymmetry *and* coupling energy has recently been addressed in [17].

In this work we report on the most recent advances on the transient and asymptotic dynamics of the dissipative TLS driven by an external AC-field modulating the asymmetry energy between the localized states.

In the following we formulate the driven dissipative dynamics in terms of an *exact* non-Markovian master equation. The kernel is expressed as a power series in the intersite coupling, in which the lowest order corresponds to the familiar noninteracting-blip approximation (NIBA) for the stochastic forces. This approach also allows to compute the dynamics systematically for weak damping in the parameter regime where NIBA fails [13], and to obtain exact results for the case $\alpha = 1/2$ of the Ohmic viscosity [11].

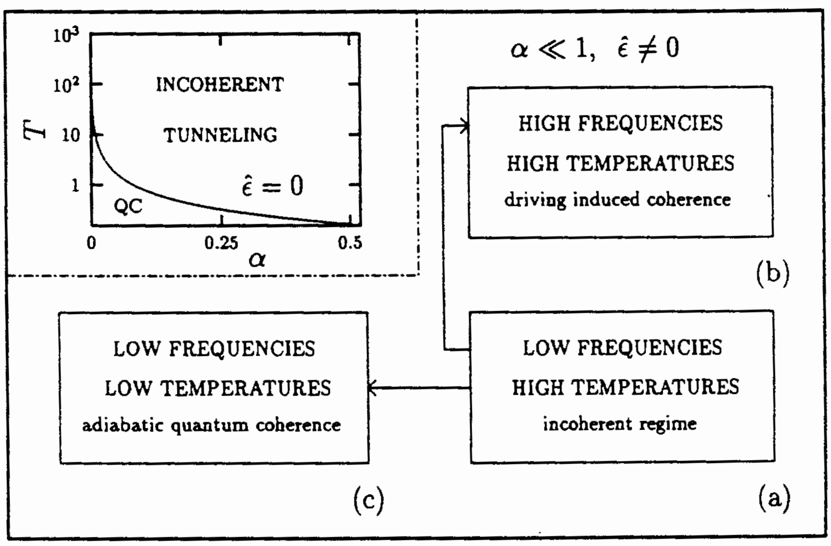


Figure 1: Dynamics of the *driven* ($\hat{\epsilon} \neq 0$) TLS for weak Ohmic coupling $\alpha$. As the temperature $T$ or frequency $\Omega$ are varied, different tunneling regimes are encountered. For strong coupling, the regimes (a) and (b) extend down to the lowest temperatures. For comparison, the static case ($\hat{\epsilon} = 0$) is considered in the inset, where the parameter regions in the $(\alpha, T)$ plane for incoherent or quantum coherent tunneling (QC) are drawn.

It is found that the dynamics encompasses a dissipative-dominated incoherent tunneling regime occurring at "high" temperatures and adiabatic driving, as well as coherent tunneling regimes reached either at "low" temperatures or/and nonadiabatic frequencies (see Figure 1).

Finally, we discuss the applications of our results to the possibility to influence quantum processes via quantum stochastic resonance or dyamical localization.

## 2 The dynamics under driving

As a working model we consider the time-dependent spin-boson Hamiltonian where the bath is described by an ensemble of harmonic oscillators with a bilinear coupling in the TLS-bath coordinates, i.e.,

$$\begin{aligned} H(t) &= -\frac{\hbar}{2}\Big(\Delta\sigma_x + \varepsilon(t)\sigma_z\Big) \\ &+\frac{1}{2}\sum_\alpha\left(\frac{p_\alpha^2}{m_\alpha} + m_\alpha\omega_\alpha^2 x_\alpha^2 - c_\alpha x_\alpha d\sigma_z\right) . \end{aligned} \tag{1}$$

Here the $\sigma$'s are Pauli matrices, and the eigenstates of $\sigma_z$ are the basis states in a localized representation where $d$ is the tunneling distance. The tunneling splitting energy is given by $\hbar\Delta$ while the asymmetry energy is $\hbar\varepsilon(t) = \epsilon_0 + \hat{\epsilon}\cos\Omega t$, where $\epsilon_0$ represents the asymmetry energy in the absence of the driving field.

Suppose now that at times $t < 0$ the particle is held at the site $\sigma_z = 1$ with the bath having a thermal distribution. We then compute the probability $\langle\sigma_z(t)\rangle \equiv P(t)$ at times $t \geq 0$ for this factorizing initial state. After tracing out the thermal bath, all environmental effects are captured by the twice-integrated bath correlation function[5,6] ($\beta = 1/k_B T$)

$$Q(t) = \frac{d^2}{\pi}\int_0^\infty d\omega\,\frac{J(\omega)}{\omega^2}\,\frac{\cosh[\omega\beta/2] - \cosh[\omega(\beta/2 - it)]}{\sinh[\omega\beta/2]} ,$$

where $J(\omega) = \frac{\pi}{2}\sum_\alpha \frac{c_\alpha^2}{m_\alpha\omega_\alpha}\delta(\omega - \omega_\alpha)$ is the spectral density of the heat bath. To make quantitative predictions, we consider the case of Ohmic dissipation where the spectral density takes the form $J(\omega) = (2\pi\hbar^2/d^2)\alpha\omega e^{-\omega/\omega_c}$, with $\alpha$ the dimensionless coupling strength and $\omega_c$ a cut-off frequency. Then, for times $\omega_c\tau \gg 1$, $Q(\tau) = Q'(\tau) + iQ''(\tau)$ reads[5,6]

$$Q'(\tau) = 2\alpha\ln[(\hbar\beta\omega_c/\pi)\sinh(\pi\tau/\hbar\beta)] , \qquad Q''(\tau) = \pi\alpha . \tag{2}$$

Upon summing over the history of the system's visits of the four states of the reduced density matrix, we can find the formal solution for the evolution of a driven damped system in the form of a series in the number of time-ordered tunneling transitions [11,12,13]. It reads

$$
\begin{aligned}
P(t) &= 1+\sum_{n=1}^{\infty}(-\Delta^2)^n\int_0^t dt_{2n}\int_0^{t_{2n}} dt_{2n-1}\cdots\int_0^{t_2} dt_1 \\
&\quad \times 2^{-n}\sum_{\{\xi_j=\pm 1\}}\left(F_n^{(+)}C_n^{(+)}-F_n^{(-)}C_n^{(-)}\right) , \qquad (3)\\
C_n^{(+)} &= \cos\Phi_n \;; \quad C_n^{(-)}=\sin\Phi_n \,. \qquad (4)
\end{aligned}
$$

Here the $\xi$-charges label the two off-diagonal states of the reduced density matrix. The phase $\Phi_n$ describes the influence of the time-dependent biasing forces,

$$
\Phi_n=\sum_{j=1}^{n}\xi_j[g(t_{2j})-g(t_{2j-1})] , \qquad (5)
$$

where $g(t) = \int^t dt'\,\varepsilon(t')$. All the dissipative influences are in the functions $F_n^{(\pm)}$. To express them in compact form, we introduce the functions $Q_{j,k} = Q(t_j - t_k)$ and

$$
\begin{aligned}
\Lambda_{j,k} &= Q'_{2j,2k-1}+Q'_{2j-1,2k}-Q'_{2j,2k}-Q'_{2j-1,2k-1} ,\\
X_{j,k} &= Q''_{2j,2k+1}+Q''_{2j-1,2k}-Q''_{2j,2k}-Q''_{2j-1,2k+1} .
\end{aligned}
$$

Denoting as *sojourns* the periods $t_{2j} < t' < t_{2j+1}$ in which the system is in a diagonal state, and as *blips* the periods $t_{2j-1} < t' < t_{2j}$ in which the system stays in one of the two off-diagonal states (cf. Refs. [5,6]), the function $\Lambda_{j,k}$ describes the interblip correlations of the blip pair $\{j,k\}$, while the function $X_{j,k}$ describes the correlations of the blip $j$ with a preceding sojourn $k$. Then, all intra-blip and inter-blip correlations of $n$ blips are combined in the expression

$$
G_n=\exp\left(-\sum_{j=1}^{n}Q'_{2j,2j-1}-\sum_{j=2}^{n}\sum_{k=1}^{j-1}\xi_j\xi_k\Lambda_{j,k}\right) .
$$

Upon introducing the influence phases describing the correlations between the $k$'th sojourn and the $n-k$ succeeding blips, $\eta_{n,k}=\sum_{j=k+1}^{n}\xi_jX_{j,k}$, the full influence functions take the form

$$
F_n^{(+)}=G_n\prod_{k=0}^{n-1}\cos\eta_{n,k} \;; \quad F_n^{(-)}=F_n^{(+)}\tan\eta_{n,0} \,. \qquad (6)
$$

Up to now our results are exact. Further, having captured the bath and driving correlations in the influence functions $F_n^{(\pm)}$ and in the coefficients $C_n^{(\pm)}$, respectively, the exact master equation for the probability $P(t)$ can be derived from (3) as prescribed in Ref. [13]. It reads

$$\dot{P}(t) = \int_0^t dt'[K^{(-)}(t,t') - K^{(+)}(t,t')P(t')] \,, \tag{7}$$

where the kernels $K^{(\pm)}(t,t')$ are defined by a series expression in $\Delta^2$. In particular, within the NIBA [5], which is formally obtained by neglecting both the interblip correlations ($\Lambda_{j,k} = 0$) and all blip-sojourn correlations ($X_{j,k} = 0$ for $j \neq k+1$), the kernels in (7) reduce to the expressions

$$\begin{aligned} K^{(+)}(t,t') &= \Delta^2 e^{-Q'(t-t')} \cos[Q''(t-t')]\, C_1^{(+)}(t,t') \,, \\ K^{(-)}(t,t') &= \Delta^2 e^{-Q'(t-t')} \sin[Q''(t-t')]\, C_1^{(-)}(t,t') \end{aligned} \tag{8}$$

It is interesting to observe that the polaron transformation approach discussed in[10,14] leads , if applied to the Hamiltonian (1), to a master equation analogous to (7) and with identical kernels (8).

Equation (7) is conveniently solved by Laplace transformation. Introducing the Laplace transform $\hat{P}(\lambda) = \int_0^\infty dt e^{-\lambda t} P(t)$ of $P(t)$, one finds

$$\lambda \hat{P}(\lambda) = 1 + \int_0^\infty dt e^{-\lambda t}[\hat{K}_\lambda^{(-)}(t) - \hat{K}_\lambda^{(+)}(t)P(t)] \,, \tag{9}$$

where $\hat{K}_\lambda^{(\pm)}(t) = \int_0^\infty dt' e^{-\lambda t'} K^{(\pm)}(t+t',t)$. For periodic driving the kernels $\hat{K}_\lambda^{(\pm)}(t)$ have the periodicity of the external field and can be expanded in Fourier series, i.e.,

$$\hat{K}_\lambda^{(\pm)}(t) = \sum_{m=-\infty}^{\infty} k_m^{\pm}(\lambda) e^{-im\Omega t} \,, \tag{10}$$

hence allowing a recursive solution[12,18]. In particular for $\alpha = 1/2$ *exact* analytical solutions are available[11]. For arbitrary Ohmic coupling and temperatures one has to resort to approximate solutions of the dissipative dynamics. For strong coupling $\alpha > 1$, or weak coupling $\alpha < 1$ *and* high enough temperatures, the bath-induced correlations between tunneling transitions may be treated within the NIBA [5,6]. On the other hand, for weak coupling $\alpha$ *and* low temperatures NIBA breaks down, and a systematic weak coupling calculation of the kernels $K^{(\pm)}$ appearing in (7) is needed[13].

In the next two sections applications of these results are discussed in relation to the phenomena of quantum stochastic resonance and dynamical localization, respectively.

## 3 Long time dynamics and quantum stochastic resonance

An analysis of the poles of the recursive solution of eq. (9) reveals that the asymptotic dynamics is *periodic* in time with the periodicity $2\pi/\Omega$ of the driving force, i.e.,

$$\lim_{t\to\infty} P(t) = P^{(\mathrm{as})}(t) = \sum_m p_m e^{-im\Omega t} \,, \tag{11}$$

where the coefficients $p_m$ have been evaluated explicitly in Refs. [11,12] for the case $\alpha = 1/2$ and within NIBA, respectively. In particular, *selection rules* related to spatial symmetry properties of the kernels $k_m^{\pm}(\lambda)$ imply that, for a *symmetric* TLS, all the Fourier components of $P^{(\mathrm{as})}(t)$ with *even* index vanish. These results will now be used to discuss control of tunneling via quantum stochastic resonance (QSR).

Stochastic resonance (SR) is a cooperative effect of noise and periodic driving in bistable systems, resulting in an increase of the response to the applied periodic signal for some optimal value of the noise. Since its discovery in 1981 this intriguing phenomenon has been the object of many investigations in classical systems [19]. Classically, the maximal enhancement in the output signal is assumed when the thermal hopping frequency is near the frequency of the modulation. Hence, the term *resonance*. In the deep quantum regime, where tunneling is the only channel for barrier crossing, qualitative new features occur as compared to the classical case. While classical SR is maximal for a symmetric bistable system [19], QSR may occur *only* in presence of a potential asymmetry between forward and backward transitions paths [15]. Moreover, the quantum noise may succeed in enhancing the periodic output (QSR) in the first harmonic response, and at the same time *suppressing* the (nonlinear) higher harmonic responses [16]. This anomalous suppression can indeed be utilized for a distortion-free amplification in quantum systems.

The relevant theoretical quantity describing the dissipative dynamics under the external perturbation is the expectation value $P(t) = \langle\sigma_z(t)\rangle$. On the other hand, the quantity of experimental interest for QSR is the averaged power spectrum $S(\omega)$, defined as the Fourier transform of the correlation function $\overline{C}(\tau) = \frac{\Omega}{\pi}\int_0^{2\pi/\Omega} dt \langle\sigma_z(t+\tau)\sigma_z(t) + \sigma_z(t)\sigma_z(t+\tau)\rangle$. The combined influence of dissipative and driving forces render extremely difficult an evaluation of the correlation function $\overline{C}(\tau)$ (and hence of the power spectrum) at short times. Matters simplify for times $t, \tau$ large compared to the time scale of the transient dynamics, where $P(t)$ and $\overline{C}(\tau)$ acquire the periodicity of the external perturbation. In fact, it is readily seen that the amplitudes $|p_m|$ in (11) determine the weights of the $\delta$-spikes of the power spectrum in the asymptotic state $S^{(as)}(\omega)$ via the relation $S^{(as)}(\omega) = 2\pi \sum_{m=-\infty}^{\infty} |p_m(\Omega, \hat{\epsilon})|^2 \delta(\omega - m\Omega)$. In particular, to investigate the nonlinear QSR, we shall examine the scaled power amplitude $\eta_m$ in the $m$-th frequency component of $S^{(as)}(\omega)$, which reads

$$\eta_m(\Omega, \hat{\epsilon}) = 4\pi |p_m(\Omega, \hat{\epsilon})/\hat{\epsilon}|^2 \,. \tag{12}$$

Hence, the quantitative study of QSR requires to solve the asymptotic dynamics of the nonlinearly driven dissipative bistable system! In this contribution we shall discuss some characteristics of QSR as they emerge from the study of the *exact* solvable case $\alpha = 1/2$ of the Ohmic strength and, for general Ohmic coupling, within the NIBA. For the special value $\alpha = 1/2$ the resulting fundamental power amplitude $\eta_1$ is plotted in Fig. 2 as a function of the temperature for different driving strengths $\hat{\epsilon}$.

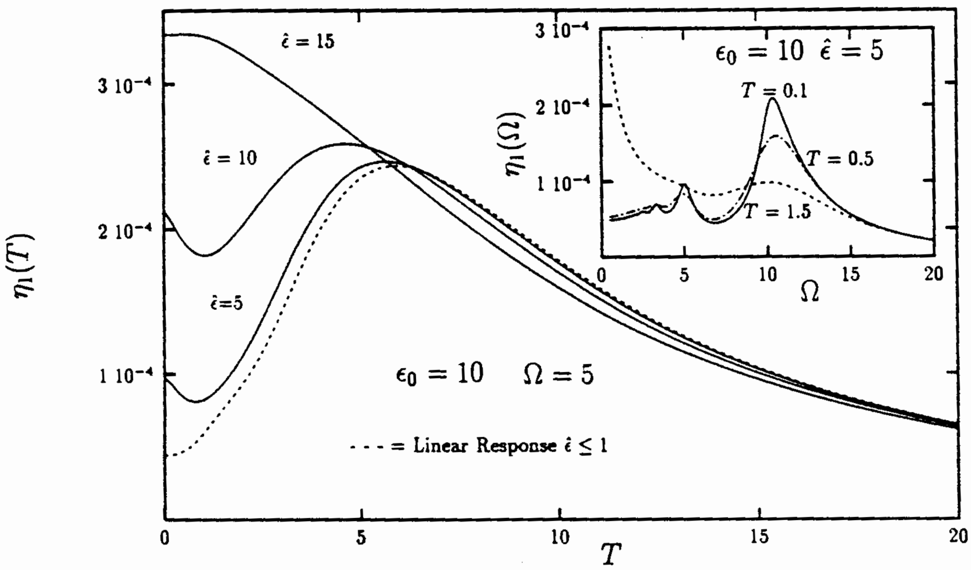


**Figure 2: Amplification vs. temperature of the fundamental amplitude $\eta_1$, via quantum SR, for different driving strengths $\hat{\epsilon}$ in the *exactly* solvable case $\alpha = 1/2$ of the Ohmic coupling strength. The inset depicts $\eta_1$ vs. driving frequency $\Omega$ for different temperatures. As the temperature is decreased, resonances are found at submultiples $\Omega = \epsilon_0/n$ of the static bias (dashed and full line). These denotes the occurence of driving-induced coherence.**

Here and in Fig. 3 frequencies are in units of the bath-renormalized tunneling splitting $\Delta_e$ (see below Eq. 14), temperatures in unit of $\Delta_e/k_B$. For highly nonlinear driving fields $\hat{\epsilon} > \epsilon_0$ the power amplitude decreases monotonically as the temperature increases (upper most curve). As the driving strength

$\hat{\epsilon}$ of the periodic signal is decreased, a shallow minimum followed by a maximum appears when the static asymmetry $\epsilon_0$ equals, or slightly overcomes, the strength $\hat{\epsilon}$ (intermediate curves). For even smaller external amplitudes, the nonlinear QSR can be studied within the linear response theory (dashed curve). In the linear region the shallow minimum is washed out and only the principal maximum survives. It is now interesting to observe that, because for the *undriven* case the TLS dynamics for $\alpha = 1/2$ is *always incoherent* down to $T = 0$, the principal maximum arises at the temperature $T$ at which the relaxation process towards thermal equilibrium is maximal. On the other hand, the minimum in $\eta_1$ appears in the temperature region where *driving-induced* coherent processes are of importance. This means that the power amplitude $\eta_1$ plotted *versus frequency* shows resonances when $\Omega \approx \epsilon_0/n$ $(n = 1, 2, ..)$ (see inset in Fig. 2). Correspondingly, the dynamics is intrinsically non-Markovian! As the temperature is increased, the coherence is increasingly lost (note the behavior of the dot-dashed and dashed lines in the inset).

For strong coupling $\alpha > 1$, or weak coupling $\alpha < 1$ *and* high enough temperatures, the bath-induced correlations between tunneling transitions may be treated within the NIBA.

The resulting dynamics is in general non-Markovian and not even time-translational invariant. In particular, driving-induced correlations may result in an highly coherent dynamics, leading to resonances in the power spectrum similar to those shown in the inset in Fig. 2. In this coherent regime QSR *never occurs*: The power amplitudes $\eta_m$ always show a monotonic decay as the temperature is increased [16].

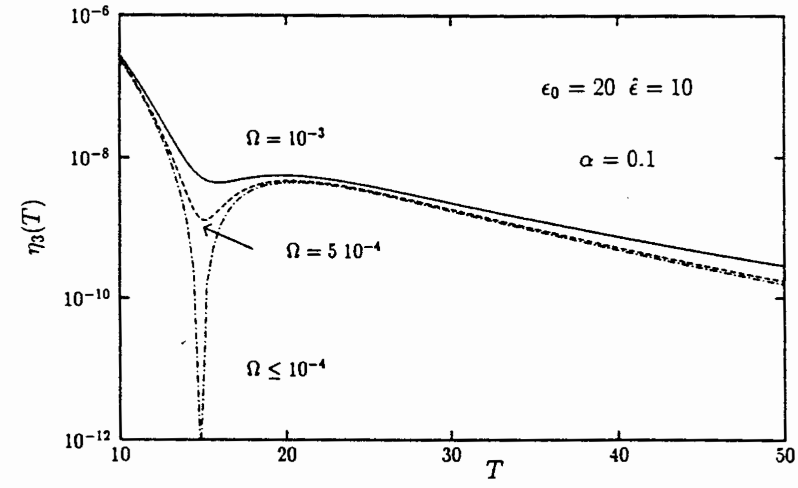


**Figure 3: Noise-induced-suppresion (NIS) of the third power amplitude $\eta_3$ vs. temperature at low frequencies.**

Only in the low-frequency regime $\hbar\Omega \ll \alpha kT$, to leading order, driving-induced non-Markovian correlations do not contribute. The asymptotic dynamics, within the NIBA, is intrinsically incoherent and governed by the rate equation $\dot{P}^{(as)}(t) = -\gamma_L(t)[P^{(as)}(t) - P_{eq}(t)]$, with time-dependent *low frequency* rate $\gamma_L(t) = \mathrm{Re}\Sigma[0;\varepsilon(t)]$ and equilibrium value $P_{eq}(t) = \tanh[\varepsilon(t)/2kT]$. Here, $\varepsilon(t) = \epsilon_0 + \hat{\epsilon}\cos\Omega t$ plays the role of a time-dependent adiabatic asymmetry, and the rate is obtained from

$$\Sigma[\lambda;x] = \frac{\Delta_e}{\pi}\left(\frac{\hbar\beta\Delta_e}{2\pi}\right)^{1-2\alpha}\frac{h(\lambda;x)}{\alpha+\hbar\beta(\lambda+ix)/2\pi}, \tag{13}$$

$$h(\lambda;x) = \Gamma[1+\alpha+\hbar\beta(\lambda+ix)/2\pi]/\Gamma[1-\alpha+\hbar\beta(\lambda+ix)/2\pi], \tag{14}$$

where, for later convenience we expressed the rate in a form useful to investigate the coupling regime $\alpha < 1$. Here $\Gamma(z)$ denotes the Gamma function and $\Delta_e = \Delta(\Delta/\omega_c)^{\alpha/(1-\alpha)}[\cos(\pi\alpha)\Gamma(1-2\alpha)]^{1/(2-2\alpha)}$ is the bath-renormalized tunneling splitting when $\alpha < 1$. The rate equation can then be solved in terms of quadratures [11,12], and the nonlinear low-frequency power spectrum can be investigated. QSR indeed occurs in this incoherent tunneling regime [15,16]. As for the case $\alpha = 1/2$, the QSR maximum appears only when the static asymmetry $\epsilon_0$ overcomes the external strength $\hat{\epsilon}$ [16].

Moreover Fig. 3, which shows the behavior of the third power amplitude $\eta_3$ versus temperature, reveals another striking effect: As the driving frequency is decreased, a noise-induced suppression (NIS) of higher harmonics occurs in correspondence of the SR maximum in the fundamental harmonic. A numerical evaluation shows that the NIS indeed appears when $\Omega \ll \min\{\gamma_L(t)\}$, so that the quasi-static expression holds

$$p_m = \frac{1}{2\pi}\int_0^{2\pi} dx \tanh[\hbar\beta(\epsilon_0 + \hat{\epsilon}\cos x)/2]\cos(mx) .$$

In contrast to classical SR, where the enhancement is maximal for symmetric bistable systems, we found the necessity of a non-zero bias for QSR. To understand this behavior, we qualitative investigate the predictions for QSR within a linear response approach (see also Fig. 2). Within linear response, only the harmonics $0, \pm 1$ of $P^{(as)}(t)$ in (11) are different from zero, $p_0$ being just the thermal equilibrium value in the absence of driving and $p_{\pm1} = \hat{\epsilon}\chi(\pm\Omega)$ being related to the linear susceptiblility $\chi(\Omega)$ by Kubo's formula. With increasing strength $\hat{\epsilon}$ higher harmonics become important. In the regime where incoherent transitions dominate the dynamics the susceptibility is explicitly obtained in the form

$$\chi(\Omega) = \frac{\beta}{4}\frac{1}{\cosh^2(\hbar\beta\epsilon_0/2)}\frac{1}{1-i\Omega\gamma_0^{-1}} . \tag{15}$$

Here $\gamma_0 = \lim_{\hat{\epsilon}\to 0}\gamma_L(t)$ is the sum of the forward and backward static relaxation rates, $\gamma_+$ and $\gamma_-$ respectively, out of the metastable states. The factor $1/\cosh^2(\hbar\beta\epsilon_0/2)$ expresses that the two rates are related by the detailed balance condition $\gamma_+ = e^{\hbar\beta\epsilon_0}\gamma_-$. It is now interesting to note that the *same* formal expression for the incoherent susceptibility (and hence for $\eta_1$) holds true for the classical case, with $\gamma_+$ and $\gamma_-$ the forward and backward Arrenhius rates [19]. Hence, in the classical SR the maximum arises because of the competition between the thermal Arrenhius dependence of the rates and the algebraic factor $\beta = (kT)^{-1}$ that enters the linear susceptibility, and it is then obtained at the temperature such that the thermal hopping rate equals the driving frequency [19]. On the other hand, the quantum rate possesses a rather weak temperature dependence as compared to the Arrhenius rate [5,6]. The crucial role is now taken by the Arrhenius-like exponential factor $1/\cosh^2(\hbar\beta\epsilon_0/2)$, where in the incoherent two-state picture $\hbar\epsilon_0$ is of the order of the energy difference between the energy levels. Hence, whenever $\hbar\epsilon_0 \ll kT$ the energy levels are essentially equally occupied and no response to the external signal occurs. The second consequence is that the maximum arises, over a wide frequency range, simply at the temperature such that $kT \simeq \hbar\epsilon_0$. We observe that similar qualitative results, together with the occurence of NIS, are obtained also in the parameter region of low temperatures $kT \leq \hbar\Delta_e$ and weak coupling $\alpha \ll 1$ where damped quantum coherence occurs[16]. In this regime NIBA *fails* to predict the correct long-time behaviour because the neglected bath-induced correlations contribute to the dissipative effects to first order in the coupling strength. Nevertheless, a perturbative treatment allows an investigation on QSR even in this coherent regime[16].

## 4 Transient dynamics and dynamical localization

As shown by (7) or (9), the transient as well as the long time dynamics depends on a complicate interplay between the stochastic and driving forces. Some simplifications are allowed when a separation of time scales is applicable. Here we shall restrict to the interesting high frequency regime $\Omega \gg \tau_K^{-1}$, where $\tau_K$ is the characteristic time of the transient dynamics. In this approximation, the kernels $\hat{K}_\lambda^{(\pm)}(t)$ in (9) can be substituted with their average $k_0^\pm(\lambda)$ over a period. One obtains (for convenience we explicitly indicate the field dependence),

$$\hat{P}(\lambda) = \frac{1 + k_0^-(\lambda;\hat{\epsilon})/\lambda}{\lambda + k_0^+(\lambda;\hat{\epsilon})}\,, \tag{16}$$

where the condition $\Omega \gg \tau_K^{-1} \simeq |\lambda|$ has to be proofed self-consistently. Thus, a fast field suppresses the periodic long times oscillations and, as follows peaking up the $\lambda = 0$ pole in (16), the TLS approaches incoherently the steady value $p_0 = k_0^-(0;\hat{\epsilon})/k_0^+(0;\hat{\epsilon})$ with the ***high frequency*** relaxation rate $k_0^+(0;\hat{\epsilon}) \equiv \gamma_{\rm H}$ [see for comparison eq. (14) and the discussion above for the low frequency rate $\gamma_{\rm L}(t)$]. Whithin NIBA one has

$$\gamma_{\rm H} = \Delta^2 \int_0^\infty {\rm d}\tau \, \cos[Q''(\tau)]{\rm e}^{-Q'(\tau)} \cos(\epsilon_0\tau) J_0\left(\frac{2\hat{\epsilon}}{\Omega}\sin\frac{\Omega\tau}{2}\right) . \qquad (17)$$

where $J_0(z)$ is the zero order Bessel function of first kind. In the limit $\hat{\epsilon} \to 0$ the modified rate $\gamma_{\rm H}$ reduce to the static one $\gamma_0$ [5,6]. To make quantitative predictions, we restrict to the case of a TLS with zero intrinsic asymmetry ($\epsilon_0 = 0$) and Ohmic dissipation, where for high cut-off frequencies $\omega_c\tau \gg 1$, the functions $Q'$ and $Q''$ are defined by eq. (2).

As a first feature, because $|J_0(z)| \leq 1$, it is apparent that for a *symmetric* TLS the effect of a fast asymmetry modulation is an overall reduction of the incoherent tunneling rate $\gamma_{\rm H}$ as compared to the static one $\gamma_0$, whenever $\alpha < 1/2$. Secondly, we study the modification of the quantum coherent motion by stochastic and driving forces, i.e., we explicitly determine the poles of (16) resulting from the equation $\lambda + k_0^+(\lambda;\hat{\epsilon}) = 0$. For our purposes it is convenient to express the kernel $k_0^+(\lambda;\hat{\epsilon})$ in terms of the static one $\mathcal{K}(\lambda) \equiv \lim_{\hat{\epsilon}\to 0} k_0^+(\lambda;\hat{\epsilon})$ as

$$k_0^+(\lambda;\hat{\epsilon}) = \sum_{n=-\infty}^{\infty} J_n^2(\hat{\epsilon}/\Omega)\mathcal{K}(\lambda + in\Omega) , \qquad (18)$$

and $\mathcal{K}$ is obtained from eq. (14) as $\mathcal{K}(\lambda) \equiv \Sigma[\lambda, 0]$. Using (13), the pole equation predicts for the static case with $\alpha \leq 1/2$ a destruction of quantum coherence by bath-induced incoherent transitions above a transition temperature $T_0(\alpha)$ [5,6]. For $\alpha \geq 1/2$ the dynamics is incoherent down to $T = 0$. For weak Ohmic damping $\alpha \ll 1$ one has from (14) that $h(\lambda; 0) \simeq 1$. Hence, the pole equation becomes just a quadratic equation in $\lambda$ and the transition temperature is determined by the condition of the solutions being real and degenerate [5,6]. Such a situation is expected to be strongly modified in the presence of ac-fields. Taking into account the high-frequency condition $\Omega \gg |\lambda|$, up to the order $O(|\lambda|/\Omega)^4$ and for $\alpha \ll 1$, we find

$$k_0^+(\lambda;\hat{\epsilon}) = \mathcal{K}(\lambda)\left[J_0^2(\hat{\epsilon}/\Omega) + \left(\frac{\alpha + \hbar\beta\lambda/2\pi}{\hbar\beta\Omega/2\pi}\right)^2 \sum_{n\neq 0} \frac{J_n^2(\hat{\epsilon}/\Omega)}{n^2}\right] .$$

This equation is of fundamental importance to understand the role of driving fields on the tunneling dynamics. Its two parts act in fact in preserving or suppressing quantum coherence, respectively. When the first contribution dominates, the effect of a fast field is roughly to renormalize the effective tunneling matrix element $\Delta_e$ as $\Delta_e \to \Delta_{\hat{\epsilon}}$, where $\Delta_{\hat{\epsilon}} = \Delta_e |J_0(\hat{\epsilon}/\Omega)|^{1/1-\alpha}$. Hence, from static considerations, we find the transition temperature $T_{\hat{\epsilon}}(\alpha) \simeq \hbar\Delta_{\hat{\epsilon}}/\pi\alpha k_B$ when $\alpha \ll 1$. Because $T_{\hat{\epsilon}}(\alpha) \leq T_0(\alpha)$, the effect of asymmetry driving is an overall reduction of quantum coherence [10]. Near the zeroes of $J_0(\hat{\epsilon}/\Omega)$ quantum coherence is completely suppressed and the particle tunnels incoherently with rate $\gamma_H = \gamma_0 \; (2\pi\alpha/\hbar\beta\Omega)^2 \sum_{n\neq 0} J_n^2(\hat{\epsilon}/\Omega)/n^2$ down to $T = 0$. Because $\gamma_H < \gamma_0 \ll \Omega$, suppression of tunneling may be stabilized for weak dissipation over several periods of the driving force in accordance with [9].

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18. The recursive solution represents the formal solution of (7) in the presence of arbitrarily fast and/or strong time periodic driving. It reads:

$$\hat{P}(\lambda) = \frac{1}{\lambda + k_0^+(\lambda)} \sum_{j=0}^{\infty} H_j(\lambda),$$

where the functions $H_j(\lambda)$ are recursively determined by the formula

$$H_{j+1}(\lambda) = \sum_{m \neq 0} \frac{-k_m^+(\lambda)}{\lambda + im\Omega + k_0^+(\lambda + im\Omega)} H_j(\lambda + im\Omega)$$

with the starting expression

$$H_0(\lambda) = 1 + \sum_{m'} \frac{k_{m'}^-(\lambda)}{\lambda + im'\Omega} .$$

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