

MASTER EQUATION AND FLUCTUATION-THEOREMS

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Fluctuation theorems play an important role in the study of the dynamic behaviour of open many-particle systems /1/. They provide a relation between the response  $\phi_{ij}(\tau)$  to a small perturbation and the correlations  $\langle x_i(\tau)x_j(0) \rangle_0$  in the stationary state of the system. As a consequence, the renormalization perturbation scheme for nonlinear classical processes /2,3/ is considerable simplified.

The linear-response formalism is developed for systems with a time dependence described by a master equation. We give general conditions for the validity of fluctuation theorems of the form

$$\phi_{ij}(\tau) = \theta(\tau) \sum_{n=0}^{\infty} \sum_{\ell} \alpha_{j\ell}^{(n)} (\partial/\partial\tau)^n \langle x_i(\tau) x_{\ell}(0) \rangle_0,$$

and obtain classes of Liouville operators satisfying such conditions. This classification includes the case of a Fokker-Planck equation studied in Ref. /4/. From these fluctuation theorems, we derive various moments and sum rules.

We have also investigated the consequences of the principle of detailed balance. We find that it leads to a set of "potential conditions" for the stationary distribution and to symmetry conditions for the Onsager coefficients. For the special case of a Fokker-Planck equation, such potential conditions were already studied in Ref. /5/.

It is interesting to note, that also in the special case of a non-stationary Gauss-Markov-process, the response function  $\phi_{ij}(t, \Delta)$  is related to the fluctuations  $\langle x_i(t) x_k(\Delta) \rangle_0$  in the unperturbed non-stationary state.

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