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WHAT MINIMAL NOISE IS NECESSARY FOR GENERATION OF TRANSPORT IN PERIODIC STRUCTURES ?

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Abstract

In spatially periodic structures, nonequilibrium fluctuations of specific statistics are able to generate non-zero current (Brownian ratchets). An unsolved problem is: What minimal statistics for the fluctuations is required for inducing finite transport in ratchet-type systems? In particular, is there a chance that a *symmetric* and *delta-correlated* additive noise does in fact yield directed motion?

In generic cases, random fluctuations (and other irregular forces) are reckoned to act destructively on processes, starting from physical through biological up to sociological ones. On the other hand, constructive influence of uncontrollable perturbations can be observed in nature. Examples are activation processes, stochastic resonance phenomena, Brownian ratchets, etc. In the latter, transport (non-zero current) in spatially periodic structures can be generated by random fluctuations of zero average values, without any field gradients and external bias forces.[1] The interest of such a transport mechanism is considerable: In biology (protein motors: transport of vesicles and ogranelles, locomotion, segregation of chromosomes),[2] material sciences (separation or pumping of particles), electronics (nano- and micro-technologies) and physics.

Periodic structures possess or do not possess a reflection symmetry. It means that systems can be described in terms of a spatially periodic potential $V(x) = V(x+L)$ with period L. For systems with a reflection symmetry, there is a constant C such that $V(C-x) = V(C+x)$. Fluctuations that act in systems can be symmetric or asymmetric. Symmetric noise $\xi(t)$ is characterized by the fact that all its odd numbered cumulant averages are identically vanishing; in contrast, asymmetric noise of zero mean can possess nonvanishing odd-numbered higher order cumulants.

From previous investigations, it has been known that in periodic structures:
(a) Transport can be induced by correlated symmetric noises in systems with a broken spatial symmetry (i.e., when the spatial potential is asymmetric).[3,4]
(b) Transport can be genereted by correlated asymmetric fluctuations in systems without broken spatial symmetry (i.e., when the spatial potential is symmetric).[5]
(c) Transport can be induced by uncorrelated (or δ-correlated) asymmetric shot noise in systems without broken spatial symmetry.[6]
(d) Transport cannot be generated by thermal fluctuations (symmetric Gaussian white noise).

An open question thus reads: What minimal statistics of noise should be sufficient for inducing a macroscopic current in periodic structures ? To be more precise, let us formulate the problem in the form of overdamped motion of Brownian particles (of unit masses) in spatially periodic potential $V(x)$, namely,

$$\dot{x} = f(x) + \Gamma(t) + \xi(t), \tag{1}$$

where $f(x) = -dV(x)/dx$. The process $\Gamma(t)$ represents thermal fluctuations: It is Gaussian delta-correlated noise of zero mean and of strength $D_T \equiv k_B T/\gamma$ with T and γ denoting the temperature of the system and the friction coefficient, respectively. The process $\xi(t)$ is a "driving force" and models another, nonequilibrum source of fluctuations.

Let us recast the question as follows: Is it possible to generate non-zero current by *symmetric and δ-correlated* fluctuations $\xi(t)$? According to Ref. 3: "... all that is needed to generate motion and forces in the Brownian domain is loss of symmetry and substantially long time correlations". In Ref. 4 it is stated: "... if (in our notation) $\xi(t)$ is another symmetric white noise process, the stationary state corresponds to a thermal equilibtium state satisfying the condition of detailed balance, in which case no net current is possible for any shape of the potential". So, taking the above statements virtually, the answer to the question stated above would be in the negative.

The first problem is to construct models of such δ-correlated fluctuations $\xi(t)$, which, by virtue of the statement in (d), should be non-equilibrium and non-Gaussian. There are several candidates for such stochastic processes as e. g. symmetric Poissonian white noises (white shot noises),[7] *composite* noises, in particular, multi-state diffusion processes[8] (in each state, the system is subject to diffusion with various diffusion coefficients and randomly jumps between states) or the so-called randomly interrupted (or flashing) Gaussian white noise: [9] Jumps between the Brownian diffusional state (a Feynman ratchet carrying zero current) and a deterministic flow (also carrying zero current) are steered by a two-state Markov process.

If $\xi(t)$ is Poissonian white noise, the output process $x(t)$ defined by (1) is a Markovian stochastic process. A master equation for the probability distribution $P(x,t)$ of it is a partial integro-differential equation with an integral kernel being a probability density of weights for the δ-kicks of shot noise. If statistical cumulants of odd order of the noise source are all equal to zero, and even numbered ones are non-zero, then the shot noise is symmetric. For $\xi(t)$ being composite noise, the resulting process $x(t)$ is non-Markovian, the treatment of which is more complicated. Then, the open problem is to obtain the stationary, *nonzero* probability current of the combined, output process $x(t)$; if so, the answer to the open problem is in the positive!

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