The Driven Spin-Boson System as a Model for Quantum Stochastic Resonance

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Abstract

Stochastic resonance (SR) is the process whereby noise operates on a bistable system enhancing the response to an external periodic signal. While this phenomenon has been intensely studied in classical systems, the role of quantum fluctuations has only started to be explored.
We investigate quantum stochastic resonance (QSR) in the periodically driven spin-boson system with Ohmic dissipation. Qualitative new features occur as compared to the classical case. While classical SR is maximal for a symmetric system, QSR vanishes as the equality between forward and backward transitions is approached. Further, QSR occurs only in parameter regimes where *incoherent* tunneling dominates over coherent transitions (arising either because of quantum interference effects at low temperatures, or of driving-induced correlations at nonadiabatic frequencies). Moreover, the quantum noise may induce an enhancement in the first harmonic of the periodic output, and *simultaneously* suppress higher order harmonics.

Stochastic resonance is a cooperative effect of noise and periodic driving in bistable systems, resulting in an increase of the response to the applied periodic signal for some optimal value of the noise. Since its discovery in 1981 this intriguing phenomenon has been the object of many investigations in classical systems [1]. Classically, the maximal enhancement in the output signal is assumed when the thermal hopping frequency is near the frequency of the modulation. Hence, the term *resonance.* Upon decreasing the temperature, quantum fluctuations become increasingly important and provide an additional escape mechanism (quantum tunneling) out of one of the metastable states. Their role on SR has only started to be explored. Here, we shall focus on the deep quantum regime, where tunneling is the only channel for barrier crossing. Qualitative new features occur as compared to the classical case. While classical SR is maximal for a symmetric bistable system [1], QSR may occur only in presence of a potential asymmetry between forward and backward transitions paths [2]. Moreover, the quantum noise may succeed in enhancing the periodic

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output (QSR) in the first harmonic response, and at the same time *suppressing* the (nonlinear) higher harmonic responses [3]. This anomalous suppression can indeed be utilized for a distortion-free amplification in quantum systems.

As a working model we consider the driven spin-boson Hamiltonian $H = H_{\rm TLS}(t) + H_B$, where ($\hbar = 1$)

$$H_S(t) = -\frac{1}{2}\Big(\Delta\sigma_x + \epsilon_0\sigma_z\Big) - \frac{\hat{\epsilon}}{2}\cos(\Omega t)\sigma_z \tag{1}$$

represents the driven bistable system in a two-state-system (TLS) approximation with $(\hat{\epsilon}/a)\cos\Omega t$ being the applied harmonic force. The σ's are Pauli matrices, and the eigenstates of σ_z are the basis states in a localized representation while a is the tunneling distance. The tunneling splitting energy of the symmetric TLS is given by Δ while the asymmetry energy is ϵ_0. Within the spin-boson model [4, 5] the environment is modeled by a term H_B describing an ensemble of harmonic oscillators with a bilinear coupling in the TLS-bath coordinates. The effects of the bath are captured by the spectral density $J(\omega)$ of the environment coupling, and we make the specific choice of Ohmic dissipation $J(\omega) = (2\pi/a^2)\alpha\ \omega e^{-\omega/\omega_c}$, where α is the dimensionless coupling strength and ω_c is a cutoff frequency [4, 5].

The relevant theoretical quantity describing the dissipative dynamics under the external perturbation is the expectation value $P(t) = \langle\sigma_z(t)\rangle$. On the other hand, the quantity of experimental interest for QSR is the averaged power spectrum $S(\omega)$, defined as the Fourier transform of the correlation function $\overline{C}(\tau) = \frac{\Omega}{\pi}\int_0^{2\pi/\Omega} dt\langle\sigma_z(t+\tau)\sigma_z(t) + \sigma_z(t)\sigma_z(t+\tau)\rangle$. The combined influence of dissipative and driving forces render extremely difficult an evaluation of the correlation function $\overline{C}(\tau)$ (and hence of the power spectrum) at short times. Matters simplify for times t, τ large compared to the time scale of the transient dynamics, where $P(t)$ and $\overline{C}(\tau)$ acquire the periodicity of the external perturbation. Expanding then $P^{(\rm as)}(t) = \lim_{t\to\infty} P(t)$ in Fourier series as

$$P^{(\rm as)}(t) = \sum_{m=-\infty}^{\infty} P_m(\Omega, \hat{\epsilon})e^{-im\Omega t}\ , \tag{2}$$

it is readily seen that the amplitudes $|P_m|$ determine the weights of the δ-spikes of the power spectrum in the asymptotic state $S^{(\rm as)}(\omega)$ via the relation $S^{(\rm as)}(\omega) = 2\pi\sum_{m=-\infty}^{\infty}|P_m(\Omega,\hat{\epsilon})|^2\delta(\omega - m\Omega)$. In particular, to investigate the nonlinear QSR, we shall examine the scaled power amplitude η_m in the m-th frequency component of $S^{(\rm as)}(\omega)$, which reads

$$\eta_m(\Omega, \hat{\epsilon}) = 4\pi|P_m(\Omega, \hat{\epsilon})/\hat{\epsilon}|^2\ . \tag{3}$$

Hence, the quantitative study of QSR requires to solve the asymptotic dynamics of the nonlinearly driven dissipative bistable system. In doing so we shall take advantage of novel results for the driven dynamics obtained using a real-time path integral approach [6, 7]. In this short contribution we shall discuss some characteristics of QSR as they emerge from the study of the case $\alpha = 1/2$ of the Ohmic strength and, for general Ohmic coupling, within the non-interacting-blip approximation (NIBA) for the stochastic forces [4, 5].

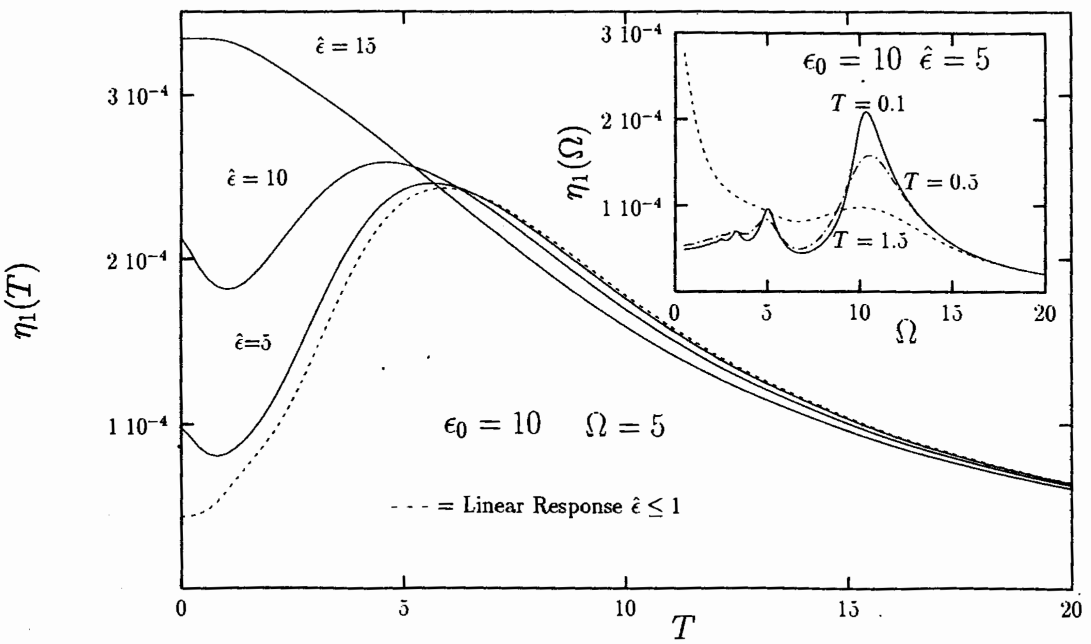

Fig. 1: *Amplification vs. temperature of the fundamental amplitude* η_1, *cf. (3), via quantum SR, for different driving strengths* $\hat{\epsilon}$ *in the* exactly *solvable case* $\alpha = 1/2$ *of the Ohmic strength. The inset depicts* η_1 *vs. driving frequency* Ω *for different temperatures. As the temperature is decreased, resonances are found at submultiples* $\Omega = \epsilon_0/n$ *of the static bias (dashed and full line). These denotes the occurence of driving-induced coherence.*

For the special value $\alpha = 1/2$ *exact* analytical solutions are available [6]. The resulting fundamental power amplitude η_1 is plotted in Fig. 1 as a function of the temperature for different driving strengths $\hat{\epsilon}$. Here and in Fig. 2 frequencies are in units of the bath-renormalized tunneling splitting Δ_e (see below Eq. 4), temperatures in unit of Δ_e/k_B. For highly nonlinear driving fields $\hat{\epsilon} > \epsilon_0$ the power amplitude decreases monotonically as the temperature increases (upper most curve). As the driving strength $\hat{\epsilon}$ of the periodic signal is decreased, a shallow minimum followed by a maximum appears when the static asymmetry ϵ_0 equals, or slightly overcomes, the strength $\hat{\epsilon}$ (intermediate curves). For even smaller external amplitudes, the nonlinear QSR can be studied within the linear response theory (dashed curve). In the linear region the shallow minimum is washed out and only the principal maximum survives. It is now interesting to observe that, because for the undriven case the TLS dynamics for $\alpha = 1/2$ is *always incoherent* down to $T = 0$, the principal maximum arises at the temperature T at which the relaxation process towards thermal equilibrium is maximal. On the other hand, the minimum in η_1 appears in the temperature region where *driving-induced* coherent processes are of importance. This means that the power amplitude η_1 plotted *versus frequency* shows resonances when $\Omega \approx \epsilon_0/n$ ($n = 1, 2, ..$) (see inset in Fig. 1). Correspondingly, the dynamics is intrinsically non-Markovian! As the temperature is increased, the coherence is increasingly lost (note the behavior of the dot-dashed and dashed lines in the inset).

For arbitrary values of the Ohmic coupling strength, one has to resort to approximate solutions of the dissipative dynamics. For strong coupling $\alpha > 1$, or weak coupling $\alpha < 1$ *and* high enough temperatures, the bath-induced correlations between tunneling transitions may be treated within the NIBA [4, 5]. A set of coupled equations for the Fourier coefficients P_m can then be derived for *any* strength and frequency of the driving force [7]. The resulting dynamics is in general non-Markovian and not even time-translational invariant. In particular, driving-induced correlations may result in an highly coherent dynamics, leading to resonances in the power spectrum similar to those shown in the inset in Fig. 1. In this coherent regime QSR *never occurs*: The power amplitudes η_m always show a monotonic decay as the temperature is increased [3].

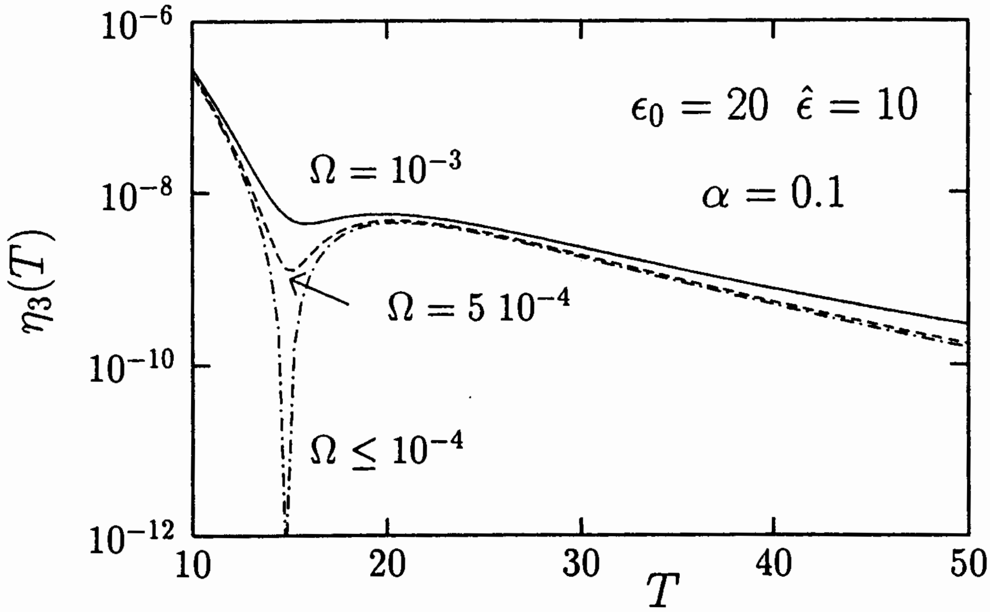

Fig. 2: *Noise-induced-suppression (NIS) of the third amplitude η_3 vs. temperature at low frequencies.*

Only in the low-frequency regime $\Omega \ll \alpha kT$, to leading order, driving-induced non-Markovian correlations do not contribute. The asymptotic dynamics, within the NIBA, is intrinsically incoherent and governed by the rate equation $\dot{P}^{(as)}(t) = -\gamma(t)[P^{(as)}(t) - P_{eq}(t)]$, with time-dependent rate $\gamma(t) = \mathrm{Re}\Sigma[\varepsilon(t)]$ and equilibrium value $P_{eq}(t) = \tanh[\varepsilon(t)/2kT]$. Here, $\varepsilon(t) = \epsilon_0 + \hat{\epsilon}\cos\Omega t$ plays the role of a time-dependent adiabatic asymmetry, and the rate is obtained from

$$\Sigma[\varepsilon(t)] = \frac{\Delta_e}{\pi}\left(\frac{\beta\Delta_e}{2\pi}\right)^{1-2\alpha}\frac{\Gamma(\alpha + i\beta\varepsilon(t)/2\pi)}{\Gamma(1-\alpha+i\beta\varepsilon(t)/2\pi)}, \tag{4}$$

where $\Gamma(z)$ denotes the Gamma function and $\Delta_e = \Delta(\Delta/\omega_c)^{\alpha/(1-\alpha)}[\cos(\pi\alpha)\Gamma(1-2\alpha)]^{1/(2-2\alpha)}$. The rate equation can then be solved in terms of quadratures [6, 7], and the nonlinear low-frequency power spectrum can be investigated. QSR indeed occurs in this incoherent tunneling regime [2, 3]. As for the case $\alpha = 1/2$, the

QSR maximum appears only when the static asymmetry ϵ_0 overcomes the external strength $\hat{\epsilon}$ [3]. Moreover Fig. 2, which shows the behavior of the third power amplitude η_3 versus temperature, reveals another striking effect: As the driving frequency is decreased, a noise-induced suppression (NIS) of higher harmonics occurs in correspondence of the SR maximum in the fundamental harmonic. A numerical evaluation shows that the NIS indeed appears when $\Omega \ll \gamma(t)$, so that the quasi-static expression holds

$$P_m = \frac{1}{2\pi}\int_0^{2\pi} dx \tanh[\beta(\epsilon_0 + \hat{\epsilon}\cos x)/2]\cos(mx) \, .$$

In contrast to classical SR, where the enhancement is maximal for symmetric bistable systems, we found the necessity of a non-zero bias for QSR. To understand this behavior, we qualitative investigate the predictions for QSR within a linear response approach (see also Fig. 1). Within linear response, only the harmonics $0, \pm 1$ of $P^{(as)}(t)$ in (2) are different from zero, P_0 being just the thermal equilibrium value in the absence of driving and $P_{\pm 1} = \hat{\epsilon}\chi(\pm\Omega)$ being related to the linear susceptiblility $\chi(\Omega)$ by Kubo's formula. With increasing strength $\hat{\epsilon}$ higher harmonics become important. In the regime where incoherent transitions dominate the dynamics the susceptibility is explicitly obtained in the form

$$\chi(\Omega) = \frac{1}{4k_BT}\frac{1}{\cosh^2(\epsilon_0/2k_BT)}\frac{1}{1 - i\Omega\gamma_0^{-1}} \, . \tag{5}$$

Here $\gamma_0 = \lim_{\hat{\epsilon}\to 0}\gamma(t)$ is the sum of the forward and backward static relaxation rates, γ_+ and γ_- respectively, out of the metastable states. The factor $1/\cosh^2(\epsilon_0/2k_BT)$ expresses that the two rates are related by the detailed balance condition $\gamma_+ = e^{\beta\epsilon_0}\gamma_-$. It is now interesting to note that the *same* formal expression for the incoherent susceptibility (and hence for η_1) holds true for the classical case, with γ_+ and γ_- the forward and backward Arrenhius rates [1]. Hence, in the classical SR the maximum arises because of the competition between the thermal Arrenhius dependence of the rates and the algebraic factor $(k_BT)^{-1}$ that enters the linear susceptibility, and it is then obtained at the temperature such that the thermal hopping rate equals the driving frequency [1]. On the other hand, the quantum rate possesses a rather weak temperature dependence as compared to the Arrhenius rate [4, 5]. The crucial role is now taken by the Arrhenius-like exponential factor $1/\cosh^2(\epsilon_0/2k_BT)$, where in the incoherent two-state picture ϵ_0 is of the order of the energy difference between the energy levels. Hence, whenever $\epsilon_0 \ll k_BT$ the energy levels are essentially equally occupied and no response to the external signal occurs. The second consequence is that the maximum arises simply at the temperature such that $k_BT \simeq \epsilon_0$ over a wide frequency range. We observe that similar qualitative results, together with the occurence of NIS, are obtained also in the parameter region of low temperatures $kT \leq \Delta$ and weak coupling $\alpha \ll 1$ where overdamped quantum coherence occurs [3]. In this regime NIBA *fails* to predict the correct long-time behaviour because the neglected bath-induced correlations contribute to the dissipative effects to first order in the coupling strength. Nevertheless, a perturbative treatment allows an investigation on QSR even in this regime.

In summary, we found that quantum noise can substantially *enhance or suppress* the nonlinear response. In particular, the occurence of NIS allows a distortion-free amplification of signals in quantum systems. These novel phenomena could be detected by measuring the ac-conductance in mesoscopic metals [8], in ac-driven atomic force microscopy [9], investigating ac-driven hydrogen tunneling in metals [10], or in a SQUID system [11].

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