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First-passage time for randomly flashing diffusion

Jerzy Łuczka
Department of Theoretical Physics, Silesian University, 40-007 Katowice, Poland

Mariusz Niemiec
Chair of Biophysics, Silesian School of Medicine, 40-952 Katowice 6, Poland

Peter Hänggi
Department of Physics, University of Augsburg, Memminger Strasse 6, D-86135 Augsburg, Germany
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The mean-first-passage time (MFPT) of a non-Markovian process that switches randomly between deterministic flow and a Fokker-Planck process (i.e., randomly flashing diffusion) is considered. The problem is formulated in an extended phase space in which the corresponding process is Markovian. It is shown that (boundary and natural) conditions for integration of differential equations determining the MFPT depend strongly on the class of domains from which the process is to escape. Exact solutions are obtained for the MFPT of a linear flow driven by randomly flashing white noise.

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I. INTRODUCTION

Processes driven by a single noise have attracted considerable attention in the past few decades. On the other hand, processes driven by *composite* noises, understood here as a product of at least two "elementary" noises, have been investigated rather rarely. Early references to such processes are related to studies of multistate random walks and multistate diffusion processes; see part III B of the review paper [1], and references therein. For example, a system is considered to be in one of the multistate N_i, $i=1,2,3,\ldots$, and subject to diffusion in each of them but with different diffusion coefficients D_i, $i=1,2,3,\ldots$. The duration of stay in each of the state is a random variable. It implies, for example, that even if the diffusion in each state is zero, the effective behavior is diffusional due to the random time spent in each of the states. In Ref. [2], the first-passage time (FPT) problem was considered for a two-state (dichotomic) diffusion process: a Brownian motion jumps between a diffusion process of strength D_1 and a diffusion process of strength D_2. A special class of processes, when a system jumps between a diffusional state and a deterministic state, was considered in detail in Refs. [3–7]. Such processes are referred to as *randomly flashing diffusion* because jumps between diffusional and deterministic states are random and steered by an exponentially correlated two-state $\{0,1\}$ Markovian process. In the papers, an evolution equation for a one-dimensional probability distribution was derived, solved, and analyzed in special cases.

In the present paper, we wish to study the FPT problem for flashing diffusion. The process considered is non-Markovian. The difficulties encountered in analyzing the FPT for non-Markovian processes are known. Let us quote here van Kampen [8]: "... it is not clear whether there exists a single quantity with a similar role, nor that it is approximately equal to the first-passage time ... ." Much attention has been devoted to the investigation of mean-first-passage times for non-Markovian processes, and several techniques have been worked out to calculate them [9–12]. Here, the problem is formulated in an extended phase space in which the process is Markovian [9]. The construction of correct boundary conditions for differential equations determining the MFPT is not a trivial task because in the general case the boundary conditions depend inevitably on deterministic (noiseless) dynamics, the kind of noise acting on the system and the interval $[A,B]$ from which the system is to exit. It should be emphasized that the *specification of boundary conditions should be based on physical assumptions,* and it should not be a purely mathematical choice.

The paper is organized as follows. In Sec. II we detail the dynamics of the system. In Sec. III we present equations for the MFPT. In Sec. IV we analyze the MFPT for a linear dynamically stable flow.

II. RANDOMLY FLASHING DIFFUSION

The system we consider is described by the stochastic equation in the Stratonovich sense [13],

$$\dot{x}_t=f(x_t)+g(x_t)\Gamma(t), \quad x\in(x_1,x_2)\ , \tag{1}$$

for a given relevant variable $x_t \equiv x(t)$. The interval (x_1,x_2) can be the real line, the half-line, or a finite interval. The functions $f(x)$ and $g(x)$ are deterministic, $\Gamma(t)$ is randomly flashing white noise defined by the relation [2,3,5–7]

$$\Gamma(t)=\tfrac{1}{2}[1+\xi(t)]\eta(t)\ , \tag{2}$$

with $\eta(t)$ being Gaussian white noise,

$$\langle\eta(t)\rangle=0, \quad \langle\eta(t)\eta(s)\rangle=2D\delta(t-s) \tag{3}$$

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and $\xi(t)$ being a symmetric two-state Markov process [14], with

$$\xi(t)=\{-1,1\}\ ,$$
$$\langle\xi(t)\rangle=0\ , \tag{4}$$
$$\langle\xi(t)\xi(s)\rangle=\exp(-2\nu|t-s|)\ .$$

For this process, the transition probabilities $+1\to-1$ and $-1\to+1$ during a small time dt equal $\nu\,dt$. Initial probabilities for $\xi(t)$ are chosen as

$$p_1(0)\equiv\Pr\{\xi(0)=1\}=\tfrac{1}{2}\ ,$$
$$p_{-1}(0)\equiv\Pr\{\xi(0)=-1\}=\tfrac{1}{2}\ . \tag{5}$$

It is assumed that an initial value x_0 of the process (1) is statistically independent of $\Gamma(t)$. It is also assumed that $\eta(t)$ and $\xi(t)$ are statistically independent of each other. Hence from (3) and (4) it follows that

$$\langle\Gamma(t)\rangle=0,\quad \langle\Gamma(t)\Gamma(s)\rangle=D\delta(t-s),\quad D>0\ . \tag{6}$$

The *non-Gaussian* noise $\Gamma(t)$ has zero correlation time and intensity D. $\Gamma(t)$ is white noise but the process x_t driven by it is *non-Markovian.* The higher-order correlation functions of $\Gamma(t)$ can be obtained by making use of the Gaussian character of $\eta(t)$ and properties of $\xi(t)$ [3]. Let us notice that the intensity of the noise $\Gamma(t)$ is a half of the intensity of the Gaussian white noise (3).

$\Gamma(t)$ can be interpreted as a Langevin force switched on and off at random time-instants. If t is a spatial variable, then $\Gamma(t)$ can model a stochastic two-layer medium: one layer is a medium with a diffusion coefficient $D_1=D$ [15,3], and the other is vacuum (surroundings characterized by a diffusion coefficient $D_2=0$). It is the simplest model of randomly stratified media and can be a starting point of generalizations for N layers with different diffusion coefficients D_i $(i=1,2,\ldots,N)$ for each layer [15].

As possible applications of Eq. (1) one can mention the problem of multiple scattering of particles through plates of matter separated by vacuum gaps [3]; transport phenomena in sponge-type structures with empty places (vacua) and matter randomly distributed in space; wave propagation in randomly stratified media [16], and so on. Equation (1) is a particular case of two-state models in which transitions from one state [deterministic: $\dot{x}_t=f(x_t)$] to the other state [diffusional: $\dot{x}_t=f(x_t)+g(x_t)\eta(t)$] and vice versa occur at random moments. Such models are mentioned by van Kampen [17].

Equation (1) generates a non-Markovian stochastic process x_t with probabilistic characteristics determined by all finite-dimensional probability distributions, multitime correlation functions, etc. It is impossible to find explicitly these quantities. Nevertheless, some characteristics can be obtained. One of the fundamental problems concerning the process x_t in (1) is to determine its one-dimensional probability density $P(x,t)$ assuming that an initial distribution density $P(x,0)=P_0(x)$ is known. In [5] it was shown that an evolution equation for $P(x,t)$ has the form

$$\frac{\partial}{\partial t}P(x,t)=-\frac{\partial}{\partial x}f(x)P(x,t)+\frac{D}{2}\frac{\partial}{\partial x}g(x)\frac{\partial}{\partial x}g(x)P(x,t)$$
$$+\frac{D^2}{4}\frac{\partial}{\partial x}g(x)\frac{\partial}{\partial x}g(x)\int_0^t ds\,\exp[-2\nu(t-s)]\int_{-\infty}^{\infty}dy\,\mathcal{G}(x,t|y,s)\frac{\partial}{\partial y}g(y)\frac{\partial}{\partial y}g(y)P(y,s)\ , \tag{7}$$

where $\mathcal{G}(x,t|y,s)$, $t\geq s$ is a solution of the problem

$$\frac{\partial}{\partial t}\mathcal{G}(x,t|y,s)=\hat{B}(x)\mathcal{G}(x,t|y,s)\ , \tag{8}$$

$$\mathcal{G}(x,t|y,t)=\delta(x-y) \tag{9}$$

and $\hat{B}(x)$ is a parabolic diffusion (Fokker-Planck-Kolmogorov) infinitesimal generator,

$$\hat{B}(x)=-\frac{\partial}{\partial x}f(x)+\frac{D}{2}\frac{\partial}{\partial x}g(x)\frac{\partial}{\partial x}g(x)\ . \tag{10}$$

In fact, $\mathcal{G}(x,t|y,s)$ is a conditional probability density of the corresponding parabolic diffusion. The evolution equation (7) is nonlocal with respect to space and nonlocal in time. Let us notice that the first two terms on the right-hand side of Eq. (7) form the diffusion generator (10). The remainder describes a deviation from parabolic diffusion and is connected with effects in which the correlation time

$$\tau_c=\frac{1}{2\nu} \tag{11}$$

of the telegraphic noise $\xi(t)$ plays a crucial role. Indeed, if the correlation time τ_c tends to zero ($\nu\to\infty$), then the last term on the right-hand side of Eq. (7) tends to zero so that (7) reduces to a Fokker-Planck equation.

III. FPT: GENERAL CASE

The pair $(x_t,\xi(t))$ forms a stationary Markov process in the extended phase space $(x_1,x_2)\times\{-1,1\}$, with the conditional probability density $f(x,\xi,t|x_0,\xi_0,0)$. Let $\mathrm{Prob}\{x_t\in[A,B]|x_0=x,t=0\}$ be a probability that the process having the value $x_0=x\in[A,B]$ at initial time $t=0$ is still in the interval $[A,B]$ at instant t. It can be expressed by

$$\mathrm{Prob}\{x_t \in [A,B] | x_0 = x, t=0\} = \tfrac{1}{2}\mathcal{P}^t_{AB}(x;+1) + \tfrac{1}{2}\mathcal{P}^t_{AB}(x;-1) , \quad (12)$$

where

$$\mathcal{P}^t_{AB}(x;\pm 1) = \int_A^B [f(y,1,t|x,\pm 1,0) + f(y,-1,t|x,\pm 1,0)]dy \quad (13)$$

denote conditional probabilities for the realization of the stochastic process x_t that started out initially at $x_0 = x \in [A,B]$, with $+1$ or -1 value of $\xi(t)$, respectively, to be found at time t still in the domain $[A,B]$, $A<B$. In Eq. (12), the initial preparation (5) for the process $\xi(t)$ has been taken into account. The sojourn probabilities $\mathcal{P}^t_{AB}(x;+1)$ and $\mathcal{P}^t_{AB}(x;-1)$ obey the backward equations (the Stratonovich prescription) [2],

$$\frac{\partial}{\partial t}\mathcal{P}^t_{AB}(x;+1) = \left[f(x) + Dg(x)g'(x) + Dg^2(x)\frac{\partial}{\partial x}\right] \times \frac{\partial}{\partial x}\mathcal{P}^t_{AB}(x;+1) - \nu[\mathcal{P}^t_{AB}(x;+1) - \mathcal{P}^t_{AB}(x;-1)] , \quad (14)$$

$$\frac{\partial}{\partial t}\mathcal{P}^t_{AB}(x;-1) = f(x)\frac{\partial}{\partial x}\mathcal{P}^t_{AB}(x;-1) - \nu[\mathcal{P}^t_{AB}(x;-1) - \mathcal{P}^t_{AB}(x;+1)] . \quad (15)$$

The initial condition at time $t=0$ of preparation for $\mathcal{P}^t_{AB}(x;\pm 1)$ reads

$$\mathcal{P}^{t=0}_{AB}(x;\pm 1) = \Theta(x-A)\Theta(B-x), \quad A<B , \quad (16)$$

where Θ is the Heaviside step function. The MFPT, $T_{AB}(x)$, is given by the relation

$$T_{AB}(x) = \int_0^\infty t\phi_t(x)dt , \quad (17)$$

where

$$\phi_t(x) = -\frac{\partial}{\partial t}\mathrm{Prob}\{x_t \in [A,B] | x_0 = x, t=0\} = \tfrac{1}{2}\phi_t(x;+1) + \tfrac{1}{2}\phi_t(x;-1) \quad (18)$$

is a first-passage-time density and

$$\phi_t(x;\pm 1) = -\frac{\partial}{\partial t}\mathcal{P}^t_{AB}(x;\pm 1) \quad (19)$$

are conditional first-passage-time densities.

If we introduce conditional first moments $T_{AB}(x,\pm 1)$ defined as

$$T_{AB}(x;\pm 1) = \int_0^\infty t\phi_t(x;\pm 1)dt , \quad (20)$$

then, in virtue of (18), one has

$$T_{AB}(x) = \tfrac{1}{2}[T_{AB}(x;+1) + T_{AB}(x;-1)] . \quad (21)$$

It is worth stressing that $T_{AB}(x)$ is a quantity that can be measured. Neither $T_{AB}(x;+1)$ nor $T_{AB}(x;-1)$ can be detected. These two quantities are "hidden variables" and constitute only auxiliary quantities for determining $T_{AB}(x)$. From (14) and (15), equations for the moments (20) can be derived, yielding

$$\left[f(x) + Dg(x)g'(x) + Dg^2(x)\frac{d}{dx}\right]\frac{d}{dx}T_{AB}(x;+1) - \nu[T_{AB}(x;+1) - T_{AB}(x;-1)] = -1 , \quad (22)$$

$$f(x)\frac{d}{dx}T_{AB}(x;-1) - \nu[T_{AB}(x;-1) - T_{AB}(x;+1)] = -1 . \quad (23)$$

In order to solve Eqs. (22) and (23) we have to augment these equations with appropriate boundary conditions. These do depend on the form of deterministic functions $f(x)$ and $g(x)$. For another FPT problem, this fact was explicitly demonstrated in Ref. [11]. We next elucidate this feature, and other subtleties, for a linear dynamically stable flow.

IV. MFPT: LINEAR DRIFT

In this section we specify the system. It is a linear system with additive noise, i.e., with $f(x) = -ax$ and $g(x) = 1$ the stochastic flow reads

$$\dot{x}_t = -ax_t + \Gamma(t), \quad a>0, \quad x \in (-\infty, \infty) . \quad (24)$$

Now, we have to distinguish two classes of intervals. For the first class, the interval $[A,B]$ belongs to the positive or negative half-axes,

$$0<A<B, \quad A<B<0 . \quad (25)$$

For the second class,

$$A<0<B . \quad (26)$$

A. First class of intervals

For the first class of intervals the set of Eqs. (22) and (23) reduces to the form

$$DT_1''(x) - axT_1'(x) - \nu[T_1(x) - T_{-1}(x)] = -1 , \quad (27)$$

$$-axT_{-1}'(x) - \nu[T_{-1}(x) - T_1(x)] = -1 , \quad (28)$$

where the prime denotes a derivative with respect to x and the abbreviations

$$T_1(x) = T_{AB}(x,+1), T_{-1}(x) = T_{AB}(x,-1) \quad (29)$$

have been introduced. For these equations three boundary conditions have to be supplied. Let us consider, for example, the case $0<A<B$. Take into account (25) and notice that for $\xi(t)=1$, Eqs. (1) with (24) describe the Ornstein-Uhlenbeck process,

$$\dot{x}(t) = -ax(t) + \eta(t) . \quad (30)$$

Therefore, for absorbing boundary conditions [2], one has

$$T_1(A) = T_{AB}(A;+1) = 0, \quad T_1(B) = T_{AB}(B;+1) = 0 . \quad (31)$$

On the other hand, for $\xi(t)=-1$, Eq. (1) with (24) reduces to the deterministic equation

$$\dot{x}(t)=-ax(t)\ , \tag{32}$$

with the solution

$$x(t)=x_0e^{-at}\ , \tag{33}$$

where $x(t=0)=x_0\in[A,B]$. Because the solution (33) is a decreasing function of time, the realization $x(t)$ starting at the point B cannot leave the domain $[A,B]$ through the right side. On the other hand, the realization $x(t)$ starting at the point A leaves the interval $[A,B]$ through the left side. Hence the third absorbing boundary conditions for our problem is

$$T_{-1}(A)=T_{AB}(A;-1)=0\ . \tag{34}$$

So, Eqs. (27) and (28) with boundary conditions (31) and (34) determine the mean-first-passage time $T_{AB}(x)$ of the process x_t alone. There are two boundary conditions at the left endpoint A [for $T_1(x)$ and $T_{-1}(x)$] and one at the right endpoint B [for $T_1(x)$]. The value of $T_{-1}(B)$ follows then uniquely from Eqs. (27) and (28); its value is not known *a priori.*

To solve the set (27) and (28), let us differentiate (27) with respect to x and use (28) in order to eliminate $T_{-1}(x)$. The result reads

$$DaxT_1'''(x)+(\nu D-a^2x^2)T_1''(x)-ax(a+2\nu)T_1'(x)+2\nu=0\ . \tag{35}$$

We seek a solution of Eq. (35) in the form of the infinite series,

$$T_1(x)=\sum_{n=0}^{\infty} b_nx^n\ . \tag{36}$$

The coefficients of Eq. (35) are polynomials with respect to x. Therefore one can insert (36) into Eq. (35) and compare the coefficients at the same x^n, $n=0,1,2,\ldots$. It leads to the relations

$$b_2=-\frac{1}{D} \tag{37}$$

and

$$b_{k+2}=\frac{ka(ka+2\nu)}{D(k+1)(k+2)(ka+\nu)}b_k,\quad k=1,2,3,\ldots\ . \tag{38}$$

Because of the recurrence relation (38), the solution (36) can be presented in the form

$$T_1(x)=b_0+b_1F(x)-G(x)\ , \tag{39}$$

where

$$F(x)=x\sum_{n=0}^{\infty}\gamma_{2n}x^{2n},\quad G(x)=(x^2/D)\sum_{n=0}^{\infty}\beta_{2n}x^{2n}\ . \tag{40}$$

The coefficients γ_n and β_n read

$$\gamma_0=1,\quad \gamma_{2n}=(a/D)^nA_1A_3A_5\cdots A_{2n-1} \tag{41}$$

and

$$\beta_0=1,\quad \beta_{2n}=(a/D)^nA_2A_4A_6\cdots A_{2n}\ , \tag{42}$$

with

$$A_k=\frac{k(ka+2\nu)}{(k+1)(k+2)(ka+\nu)},\quad k=1,2,3,\ldots\ . \tag{43}$$

The coefficients b_0 and b_1 are determined from the boundary conditions (31) and are given by the following relations:

$$b_0=\frac{F(B)G(A)-F(A)G(B)}{F(B)-F(A)}\ , \tag{44}$$

$$b_1=\frac{G(B)-G(A)}{F(B)-F(A)}\ . \tag{45}$$

Thus, $T_1(x)$ is fully determined. To calculate $T_{-1}(x)$, let us notice that from Eq. (28) one can express $T_{-1}(x)$ by $T_1(x)$ as follows:

$$T_{-1}(x)=\frac{1}{a}x^{-\nu/a}\int_A^x y^{-1+\nu/a}[1+\nu T_1(y)]dy\ , \tag{46}$$

where the boundary condition (34) at $x=A$ has been taken into account.

Inserting $T_1(x)$ of the form (39) into (46) yields

$$T_{-1}(x)=W(x)-(x/A)^{-\nu/a}W(A)\ , \tag{47}$$

where

$$W(x)=\frac{1}{\nu}+b_0+b_1M(x)-R(x) \tag{48}$$

with the function $M(x)$ and $R(x)$ defined by the series

$$M(x)=x\sum_{n=0}^{\infty}\frac{\nu}{\nu+(2n+1)a}\gamma_{2n}x^{2n}\ , \tag{49}$$

$$R(x)=(x^2/D)\sum_{n=0}^{\infty}\frac{\nu}{\nu+(2n+2)a}\beta_{2n}x^{2n}\ . \tag{50}$$

Equations (39) and (47) with (21) yield the solution of the MFPT problem with absorbing boundaries for the first class (25) of intervals. In Figs. 1 and 2 we show the dependence of $T(x)=(\frac{1}{2})[T_1(x)+T_{-1}(x)]$ upon two

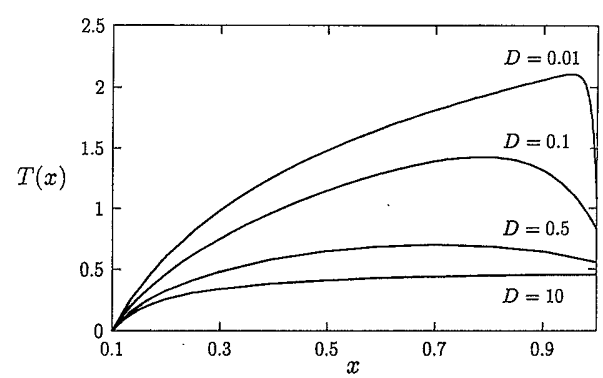

FIG. 1. MFPT $T(x)$ for the first class of intervals, $A=0.1$, $B=1$, and for four values of the noise intensity D and fixed $a=\nu=1$.

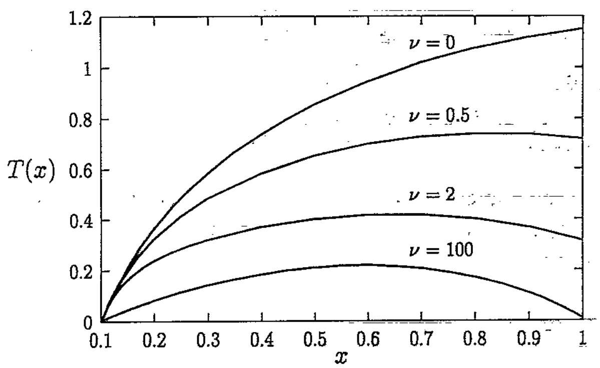

FIG. 2. Same as Fig. 1, but for various values of the inverse ν of correlation time of the dichotomic process $\xi(t)$ and fixed $D=a=1$.

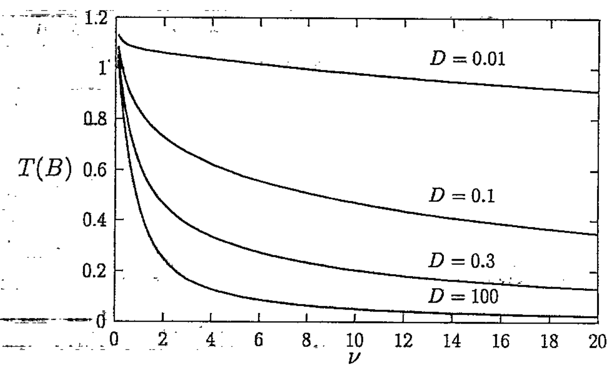

FIG. 4. Dependence of $T(B)$ on ν for several values of D and fixed $a=1$.

basic parameters of the model: the strength of the noise D and the relation ν/a between two characteristic times $\tau_c=1/(2\nu)$, the correlation time of the noise, and $\tau_d=1/a\equiv 1$, the deterministic relaxation time of the system. The main statement following from these figures is that

$$T(B)\neq 0 \tag{51}$$

for any finite values of D and ν. The dependence of $T(B)$ on the noise strength D and the constant ν is presented in Figs. 3 and 4. Why $T(B)\neq 0$? Let us notice that

$$\begin{aligned} T_{AB}(B)&=\tfrac{1}{2}[T_{AB}(B;+1)+T_{AB}(B;-1)]\\ &=\tfrac{1}{2}T_{AB}(B;-1) \end{aligned} \tag{52}$$

because of (31). If $\xi(t)=-1$ and $x_0=B$ at $t=0$, then for some time t,

$$x(t)=Be^{-at}<B\ . \tag{53}$$

So, the system cannot suddenly cross the left point $x=A$ according to the deterministic evolution. After some time, $\xi(t)=1$ and the system jumps to the stochastic dynamics (30). Then of course it can leave the domain $[A,B]$ due to fluctuations. The system switches between two (deterministic and stochastic) states with the mean frequency ν. The quantity $1/\nu$ is a mean waiting time for the switching to be made.

Now, the explanation of results visualized in Figs. 1–4 is simple. For $\nu\to\infty$ (the correlation time of the noise $\tau_c\to 0$), the system (1) with (24) behaves as the Ornstein-Uhlenbeck process and due to fluctuations $T(B)\to 0$ as $\nu\to\infty$. If $\nu\to 0$ ($\tau_c\to\infty$), then with probability $\frac{1}{2}$ the system is deterministic as in (32) and with the same probability it is stochastic as in (30). So,

$$T_{AB}(B)=\tfrac{1}{2}T_{AB}^{\rm det}(B)\quad\text{for } \nu=0\ , \tag{54}$$

where

$$T_{AB}^{\rm det}(B)=\frac{1}{a}\ln(B/A) \tag{55}$$

is a deterministic time of leaving the interval $[A,B]$ through the boundary $x=A$.

B. Second class of intervals

If $A<0<B$ and $\xi(t)=-1$, then any realization $x(t)$ starting at $x\in[A,B]$ cannot leave the interval $[A,B]$ neither through A nor through B. Therefore, analogically as in the previous case, $T_{-1}(B)\neq 0$ and similarly $T_{-1}(A)\neq 0$ because, on average, during the waiting time $1/\nu$ the system behaves in a deterministic way, as in (53). For this case, Eqs. (27) and (28) can be utilized with two absorbing boundary conditions (31) for $T_1(A)=T_1(B)=0$. To find the third condition, we follow the reasoning in Ref. [11], by noting that with $A<0<B$ the point $x=0$ provides a "natural boundary condition," i.e., the point $x=0$ is a singular point for (27) and (28). It then follows that at $x=0$,

$$T_{-1}(0)=\frac{1}{\nu}+T_1(0)\ . \tag{56}$$

This is the third condition for the set (27) and (28) to be

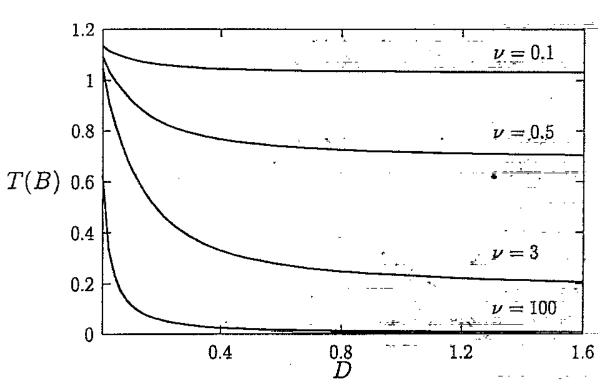

FIG. 3. MFPT at the right boundary, $T(B)$, as a function of the noise intensity D for various values of ν and fixed $a=1$. The boundaries are the same as in Fig. 1.

integrated. One should stress that (56) is not imposed on the boundary of the region $[A,B]$ but it includes the *internal point* $x=0\in[A,B]$.

Now, we can proceed along the same way as solving the problem for the first class of intervals. For $T_1(x)$ we get the same relation (39). For $T_{-1}(x)$ one obtains [cf. (46)]

$$T_{-1}(x)=\frac{1}{a}x^{-\nu/a}\int_0^x y^{-1+\nu/a}[1+\nu T_1(y)]dy\ , \tag{57}$$

where the condition (56) is involved. Insert $T_1(x)$ from (39) into (57). Then

$$T_{-1}(x)=W(x)\ , \tag{58}$$

with $W(x)$ given by Eq. (48). So, the MFPT problem for the second class of intervals is solved. The graphical form of the solution is presented in Figs. 5 and 6, where the dependence on the intensity D of white noise and the correlation time of the dichotomic process $\xi(t)$ is displayed.

The MFPT problem for the second class of intervals can also be treated in another way. Namely, consider Eqs. (27) and (28) and add a small diffusive perturbation of strength ε to the second equation so that we obtain

$$DT_1''(x)-axT_1'(x)-\nu[T_1(x)-T_{-1}(x)]=-1\ , \tag{59}$$

$$\varepsilon T_{-1}''(x)-axT_{-1}'(x)-\nu[T_{-1}(x)-T_1(x)]=-1\ . \tag{60}$$

This procedure corresponds to adding a small diffusion to the deterministic dynamics in (32) and reduces to a special case of a dichotomic Fokker-Planck process considered in Ref. [2], cf. Eqs. (3.1) and (3.2) therein and take $f_+(x)=f_-(x)=-ax$, $g_+(x)=1$ and $g_-(x)=\sqrt{\varepsilon/D}$. In the limit $\varepsilon\to 0$ we recover the starting equations. Now, since both for $\xi(t)=1$ and $\xi(t)=-1$ the dynamics is diffusionlike, i.e., with absorbing boundaries at A and B, one finds the relations [2]

$$T_1(A)=T_{AB}(A;+1)=0,\quad T_1(B)=T_{AB}(B;+1)=0\ , \tag{61}$$

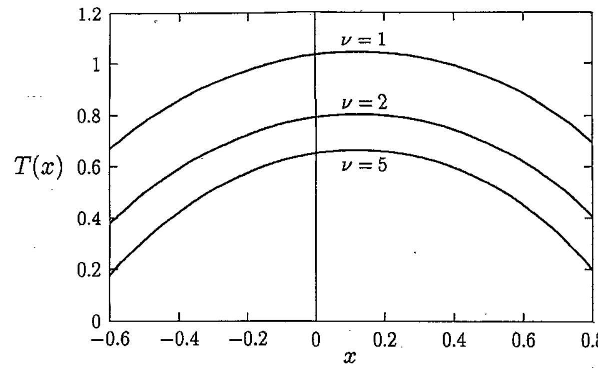

FIG. 6. Same as Fig. 5, but for various values of ν and fixed $D=a=1$.

$$T_{-1}(A)=T_{AB}(A;-1)=0,\quad T_{-1}(B)=T_{AB}(B;-1)=0\ . \tag{62}$$

Because coefficients of the differential equations (59) and (60) are at most linear functions of x, we can proceed along the same way as solving Eqs. (27) and (28) and expand $T_1(x)$ and $T_{-1}(x)$ into series. We get recurrence relations for coefficients of the expansions and, by use of the boundary conditions (61) and (62), the MFPT $T(x)$ can be determined. Unfortunately, these so obtained solutions read very complicated and are not of legible form. In Fig. 7, the influence of the diffusive perturbation on the MFPT $T(x)$ is visualized. For decreasing values of ε, a sharp increase of $T(x)$ from boundary values $T(A)=0$ and $T(B)=0$ is observed and in the limit $\varepsilon\to 0$, one recovers results obtained by use of the first method. However, for arbitrary small values of ε, the boundary conditions (61) and (62) hold and only in the limit $\varepsilon\to 0$, $T(A)\neq 0$ and $T(B)\neq 0$, and characteristic jumps at the boundaries are exhibited.

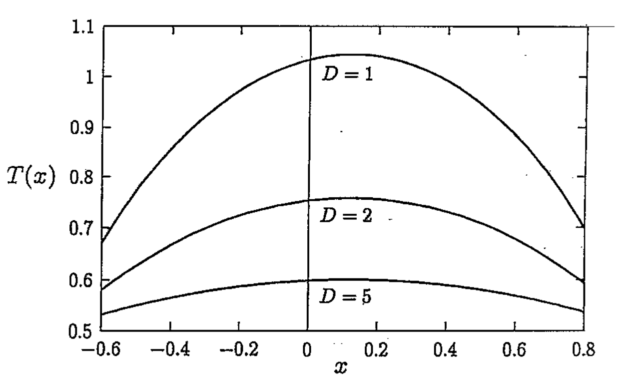

FIG. 5. MFPT $T(x)$ for the second class of intervals, $A=-0.6$, $B=0.8$, and for three values of the noise intensity D and fixed $a=\nu=1$.

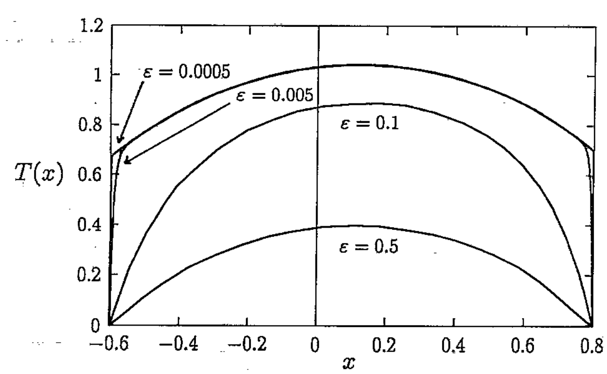

FIG. 7. MFPT $T(x)$ for the diffusively perturbed system (59) and (60) and for the second class of intervals, $A=-0.6$, $B=0.8$, for several values of the perturbation parameter ε and fixed $D=a=\nu=1$.

V. CONCLUSIONS

We elucidated the problem of evaluation of the mean-first-passage time for processes that are subjected to diffusion that is switched "on" and "off" with a Poissonian switching time distribution. We obtained explicit results for a linear relaxation dynamics with stochastically interrupted Gaussian white noise diffusion. As a primer, it has been demonstrated that the MFPT depends on (boundary) conditions that depend not only on the statistics of the driving noise process but as well on the character of the deterministic dynamics within the support $[A,B]$ of the stochastic dynamics. This leads to the consideration of two classes of intervals. For the first class, the boundary conditions for the MFPT $T(x)$ cannot be formulated directly, but are obtained only via the auxiliary pair $[T_1(x), T_{-1}(x)]$ of MFPT's, cf. Eqs. (31) and (34). The value of the composed MFPT $T(x)$ explicitly depends on the boundary conditions for these auxiliary MFPT's $T_1(x)$ and $T_{-1}(x)$, as well as on the choice of initial preparation of the jump dynamics, cf. Eqs. (5). In this case, three absorbing boundary conditions can be specified. For the second class of intervals, we formulated a complete set of boundary conditions, Eqs. (31) and (56), for the auxiliary pair $[T_1(x), T_{-1}(x)]$ of the conditional MFPT's. Two of them are absorbing boundary conditions, and we have no third condition involving endpoints but a "fixed point" or an attraction point $x=0$ of the deterministic dynamics. The boundary values of $T_{-1}(B)$ and/or $T_{-1}(A)$ cannot be specified *a priori* but arise from the stochastic dynamics. These values can be calculated either from Eq. (47) or from Eq. (58) for the first and the second class of intervals, respectively. In conclusion, one generally cannot use a single quantity like $T(x)$ but rather several auxiliary, intermediate quantities, such as $T_1(x)$ and $T_{-1}(x)$, as in the case considered herein. This is the rationale behind van Kampen's statement in Ref. [8], cited in the Introduction.

Finally we note that the stochastic relaxation with randomly flashing diffusion considered herein might find direct application in areas where information is transcoded along tottery transmission lines, e.g., in radiophysics, or in faulty neuron networks. Moreover, a flashing diffusion model might also find applications for control of chaos [18] or phase control [19].

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