which may be regarded as a geometrical purity in the entangled phase-space, promised by the system under consideration. In this model, the Hamiltonian is defined by
\[ H = \sum_{i} \frac{p_i^2}{2m} + U(x_i) \]

where \( U(x_i) \) is the potential energy function.

Termination of tunneling due to the driving in the deep quasiregime. In Section 3, we discuss the model and its symmetries.

2. The model and its symmetries

The harmonically driven quartic double well is described by the Hamiltonian
\[ H(x,p) = \frac{p^2}{2m} + \frac{1}{2} k(x^2 - \langle x \rangle^2)^2 + \frac{1}{4} \alpha (x^4 - 2\langle x \rangle^2 x^2 + \langle x \rangle^4) \]

where \( k \) and \( \alpha \) are the spring and quartic force constants, respectively, and \( \langle x \rangle \) is the mean position of the particle.

With the dimensionless variables used, the only parameter controlling the unperturbed tunnelling is the ratio \( \Gamma = \frac{E}{\Delta} \), where \( E \) is the energy and \( \Delta \) is the energy barrier.

Accordingly, the classical limit amounts to letting \( E \rightarrow -\infty \). The driving is then described by the system of the Hamiltonian
\[ H(x,p) = \frac{p^2}{2m} - \hbar \omega x + \hbar \omega (\sin \theta - \cos \theta) \]

where \( \theta \) is the phase of the driving force. The quantum tunnelling probability is then calculated, and the results are discussed in the context of the present model.

The final result is expressed in terms of the Bessel functions and is given by
\[ P_{\text{tunnel}} \propto J_0(\sqrt{2\pi \Gamma}) \]

where \( J_0 \) is the zeroth-order Bessel function of the first kind.

The energy spectrum is the probability of the states and is given by
\[ \varepsilon_n = \frac{n\pi \hbar}{2L} \]

where \( L \) is the length of the double well.

Energy level densities of the states are calculated, and the results are compared with the classical and quantum predictions. The agreement is found to be excellent, thereby validating the model.

The study is concluded with a brief discussion on the implications of the results for the understanding of the tunnelling phenomenon in more complex systems, such as those involving multiple degrees of freedom or interactions with external fields.
The experimental determination of the hyperbolic function in various conditions and procedures to understand its impact on the properties of materials and systems. The analysis of data suggests that the function's behavior is influenced by several factors, including temperature and pressure. Further investigations are needed to confirm these findings and explore potential applications.

![Graph showing the relationship between variable A and variable B](image_url)

### Table 1: Properties of Material X under Different Conditions

<table>
<thead>
<tr>
<th>Condition</th>
<th>Temperature</th>
<th>Pressure</th>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20°C</td>
<td>1 atm</td>
<td>Property A</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>30°C</td>
<td>2 atm</td>
<td>Property A</td>
<td>0.6</td>
</tr>
<tr>
<td>3</td>
<td>40°C</td>
<td>3 atm</td>
<td>Property A</td>
<td>0.7</td>
</tr>
</tbody>
</table>

The table above illustrates the variation of a specific property with changes in temperature and pressure. Further studies are recommended to validate these observations.
the classical phase-space transport. There is another conspicuous modification of the phase-space structure, the growth of the regular zone generated by the first resonance of the driving with the unperturbed oscillation [39], which does not, however, affect the coherent dynamics as substantially as the onset of chaos does.

Quantum mechanically, phase-space transport by chaotic diffusion competes with tunneling [22–27]. In the present section we investigate how these two processes influence each other. Applying ideas from Einstein-Brillouin-Keller (EBK) quantization for periodically driven systems [38], as well as from random-matrix theory for mixed (regular and chaotic) systems [39], to the present context, we arrive at the following simple expressions [33–35]. Even with the driving, the two isolated regular regions within the wells remain related by a discrete symmetry, the generalized parity $P$ (see Eq. 5). Accordingly, Floquet states residing within these regions should form a more or less regular ladder of tunnel-induced doubles. For states mainly residing within the chaotic layer, in contrast, random-matrix theory predicts level repulsion. We therefore expect that, as soon as one of the pairs of quantizing tori pertaining to the symmetry-related regular regions resides in the spreading chaotic layer, the exponentially small splitting of the corresponding doublet widens until it reaches a size of the order of the mean level separation. As a consequence, the coherent tunneling on an extremely long time scale will give way to a more irregular dynamics on shorter time scales, forming the quantum counterpart of deterministic diffusion along the separatrix.

We emphasize that the breakup of the tunnel doublets in the chaotic layer is not a direct consequence of a local property, the positive Lyapunov exponent. Rather, it depends on the fact that diffusive spreading connects all parts of the chaotic layer, even across the symmetry plane. Furthermore, one should keep in mind that the disintegration of a classical torus, looked at closely, is not an abrupt event but proceeds through an intermediate "leaky" stage with fractal dimension ("fractal torus") [41].

In order to check our hypothesis numerically, we have to quantify the distinction between "regular" and "chaotic" eigenstates, i.e., states located mainly in regions of a corresponding measure in classical phase space. We base this quantification on a quasi-mechanical probability density in phase space, the Husimi distribution [42, 43]. The overlap of the Husimi representation $Q_0(x,p,t)$ of a Floquet state $|\phi_0(t)\rangle$ with the chaotic layer [18, 19],

$$
F_0 = \frac{\omega}{2\pi} \int_{0}^{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dQ_0(x,p,t) \delta(x,F(x,p,t),
$$

can be used as a measure of "how chaotic that state is". Here, $\Gamma(x,p,t)$ denotes the characteristic function for the chaotic region. It can be determined numerically, e.g., by letting a trajectory started anywhere in this chaotic region "tick" loops in a coarse-grained phase space of the desired resolution. Since the Husimi distribution forms a normalised probability distribution over phase space, we have $0 \leq F_0 \leq 1$.

As an illustration of these concepts, we compare, in Fig. 4, the Husimi distribution for the quasistate $|\psi_0\rangle$ with the corresponding classical phase-space portrait, for (a) $S = 10^{-5}$ and (b) $S = 0.2$.
In this section, we are going to establish our working model, Eq. (1), in a way that it allows us to describe the influence of the double-coil structure and the role of the double-coil equivalence on the final result. The double-coil structure is characterized by its capacitance and inductance, which determine the resonance frequency and the quality factor, respectively. The double-coil equivalence is defined as the ratio of the double-coil capacitance to the inductance of the single coil. This ratio determines the efficiency of the double-coil structure, and its value is typically between 0 and 1. The model allows us to calculate the efficiency of the double-coil structure and to predict the performance of the double-coil equivalent circuit.
above, with the initial states prepared as coherent states at the location of either one of the maxima of the asymptotic distribution (see Fig. 10) within the left well, corresponding to nonresonant motion (a) and to the first resonance (b), respectively. In both cases, we observe a coherent oscillation decaying as the stationary state is approached. Fig. 12 reveals that these oscillations indeed form a remnant of tunneling within each the symmetry-related pair of regular regions. The stationary distribution among both pairs is reached only on the longer time scale of the classical relaxation. Clearly, we here observe tunneling between limit cycles.

6. Summary

The present paper is intended to highlight a number of facets of the nonlinear dynamics in a periodically driven double-well potential, at different stages of the transition from microscopic, coherent to macroscopic, incoherent behavior. Our main tool has been a numerical analysis on basis of the Floquet formalism, which allows to speak of quasienergies and quasienergy eigenstates of the driven system, in analogy to eigenenergies and eigenstates in the undriven case.

In the deep quantum regime, we find modifications due to the driving, of the familiar tunneling. They range from a mere acceleration of its rate, in the two extremes of slow and of fast driving, through complex quantum beats near resonances with the unperturbed system frequencies, to an almost complete suppression of tunneling by a coherent mechanism effective along one-dimensional manifolds in the parameter space spanned by amplitude and frequency of the driving.

Towards the semiclassical limit of the conservative system, the quantal behavior begins to exhibit clear traces of the classical dynamics. Specifically, we addressed the interplay between coherent transport by tunneling and diffusive transport along the chaotic layer developing in the vicinity of the separatrix of the undriven system. Eigenstate doublets residing within the paired, symmetry-related regular regions of the classical phase space exhibit exponentially small splittings and thus support tunneling. As the pair of quantizing tori pertaining to such a doublet resolves in the chaotic sea, the splitting widens and tunneling gives way to a more complex dynamics contributing to the quantal counterpart of chaotic diffusion. On a closer look, however, the scenario turns out to be less simple. For example, classical tori disintegrate only via intermediate steps, dubbed "canons" and "vague tori," with the consequence that the transition from a regular to a chaotic nature of a quasienergy eigenstate is not sharply defined, but rather proceeds in a smooth and retarded manner. Accordingly, a strict distinction between regular and chaotic regions is inadequate on the quantum-mechanical level.

The other principal ingredient of the crossover to macroscopic behavior, besides a small relative $\hbar$, is the coherence-disturbing effect of the ambient degrees of freedom, modeled microscopically as a coupling to a quasistationary reservoir. As an immediate consequence, the incoherent processes render the coherence effects observed in the deep quantum regime transients. Surprisingly, however, the coherent suppression of tunneling is stabilized if damping constant and reservoir temperature are in a specific regime, a result akin to the quantum zero effect and to the classical stabilization of instable equilibria by multiplicative noise. On the time scale of classical dynamics, the quantum system appears to be a driven classical system, with the quantum fluctuations contributing to the classical noise.