R. Bartussek, P. Jung, and P. Hänggi, *Stochastic Resonance in Optical Bistable Systems*,
AIP Conf. Proc. Vol. 285: Noise in Physical Systems and 1/f Fluctuations, P.H. Handel, and A.L. Chung, eds.
(American Institute of Physics, New York, 1993)

STOCHASTIC RESONANCE IN OPTICAL BISTABLE SYSTEMS

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ABSTRACT

We investigate cooperative effects of noise and periodic forcing in an optical bistable system. It has been demonstrated in a recent experiment [1] that noise induced switching between low and high output intensity can be synchronized via the stochastic resonance effect by a small periodic modulation of the input intensity. Here we present theoretical results for stochastic resonance in optical bistable systems.

MODEL AND BASIC EQUATIONS

A model for optical bistability was introduced by Bonifacio and Lugiato [2]. For the amplitude y of the input light and the transmitted amplitude x, they have derived the equation of motion

$$\dot{x} = y - x - 2c\frac{x}{1+x^2} + \sqrt{D}\frac{x}{1+x^2}\Gamma(t), \tag{1}$$

where Γ represents δ-correlated, Gaussian distributed noise with zero mean. A weak periodic modulation of the input intensity is taken into account by adding a periodic term to y, i.e. $y \to y + A\sin(\Omega t + \psi)$. For the probability density of the transmitted amplitude, $P(x,t)$, we find the Fokker-Planck equation

$$\begin{aligned}\frac{\partial}{\partial t}P(x,t) &= -\frac{\partial}{\partial x}\left[y - x - \frac{2cx}{1+x^2} + D\frac{x(1-x^2)}{(1+x^2)^3} + A\sin(\Omega t + \psi)\right]P(x,t)\\ &\quad + \frac{\partial^2}{\partial x^2}D\frac{x^2}{(1+x^2)^2}P(x,t).\end{aligned} \tag{2}$$

The spectral density of the transmitted amplitude has δ-spikes at multiples $n\Omega$ of the driving frequency [3] with the corresponding weights w_n being a measure for the output power at the frequency $n\Omega$. They can be expressed in terms of the Fourier coefficients of the time periodic, asymptotic mean value [4]

$$\langle x(t)\rangle_{as} = \sum_{n=-\infty}^{\infty} |M_n| \exp\left[in(\Omega t + \psi + \varphi_n) - i\frac{\pi}{2}\right] \tag{3}$$

by

$$w_n = 2\pi|M_n|^2. \tag{4}$$

AMPLIFICATION OF THE OPTICAL SIGNAL

The amplification of the periodic signal is given by the ratio of the transmitted power at the driving frequency and the input power [4]

$$\eta_1(\Omega) = 4\frac{|M_1|^2}{A^2}. \tag{5}$$

The numerically calculated results for η_1 are shown in Figs.1 for various frequencies by the solid lines. Fig.1a corresponds to choosing the dc input intensity y such that $P(x,t)$ shows two peaks of nearly equal height in the limit $D \to 0$ what we call the "symmetric case". Fig.1b corresponds to a "asymmetric case", where the peaks of the stationary probability have different probabilistic weights.

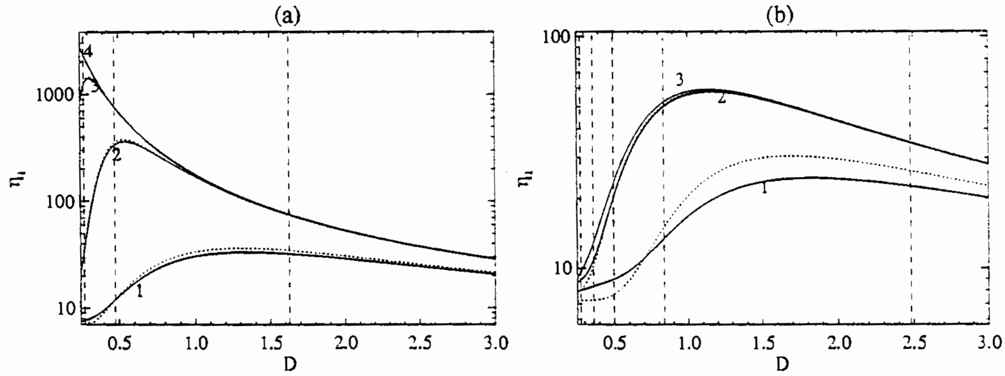

Fig. 1: Spectral amplification η_1 at $c = 6$, $A = 10^{-4}$ for $y = 6.72584$ (a) and $y = 6.8$ (b). Curve 1 corresponds to $\Omega = 10^{-1}$, curve 2 to 10^{-2}, curve 3 to 10^{-3} and curve 4 to 10^{-4}. The dotted lines correspond to results within linear response approximation (Eqs. (6) - (8)).

In the symmetric case we observe stochastic resonance [5] very much like in the quartic double well potential, i.e. a peak in the amplification of the signal (modulation) as a function of the noise intensity when the sum of the mean sojourn times in both stable states equals the period of the driving (these values of D are indicated as vertical dashed lines in Figs.1).

In the asymmetric case, the peak of the amplification is suppressed, because - in contrast to the symmetric case - the corresponding contribution (i.e. the weight g_T in Eq. (8)) of hopping motion to the response of the system disappears exponentially for small noise. The remaining maximum is only the tail of the amplification by synchronisation at large noise.

The numerical results are compared in Figs.1 with those obtained within linear response approximation [4,5] (dotted lines). In this approximation we find in terms of the response function $R(t)$

$$\langle x(t)\rangle_{as} - \langle x\rangle_{st} = \int_{-\infty}^{\infty} R(t-t')A\sin(\Omega t' + \psi)\mathrm{d}t' - \int_0^{\infty} xP_{st}(x)\mathrm{d}x, \tag{6}$$

with the stationary solution $P_{st}(x)$ of the undriven system. The response function $R(t)$ is expressed via a fluctuation theorem by a correlation function $K(t)$ of the undriven system

$$R(t) = \frac{\mathrm{d}}{\mathrm{dt}} \langle x(t)h(x(0))\rangle \equiv \frac{\mathrm{d}}{\mathrm{dt}} K(t) \tag{7}$$

with $h(x) = \frac{1}{D}\left(-\frac{1}{x} + 2x + \frac{1}{3}x^3\right)$. $K(t)$ is approximated by a sum of exponentials with the typical time scales of the system λ_T and $\lambda_{1,2}$ - stemming from hopping and local motion in the potential wells respectively, i.e.

$$K(t) \simeq \sum_{i=1,2,T} g_i e^{\lambda_i t}. \tag{8}$$

The weights g_i are determined by the correlation function $K(t)$ and its derivatives at $t = 0$.

GENERATION OF HIGHER HARMONICS

The generation of the n-th harmonic in the output due to the nonlinearities is characterized by the ratio

$$\eta_n(\Omega) = 4\frac{|M_n|^2}{A^2}. \tag{9}$$

The second harmonic depends on the noise strength as shown for the symmetric and asymmetric case in Figs.2. In the symmetric case (Fig.2a) a "dip" appears which becomes sharper with decreasing frequencies. In the asymmetric case (Fig.2b) we do not observe such a behaviour.

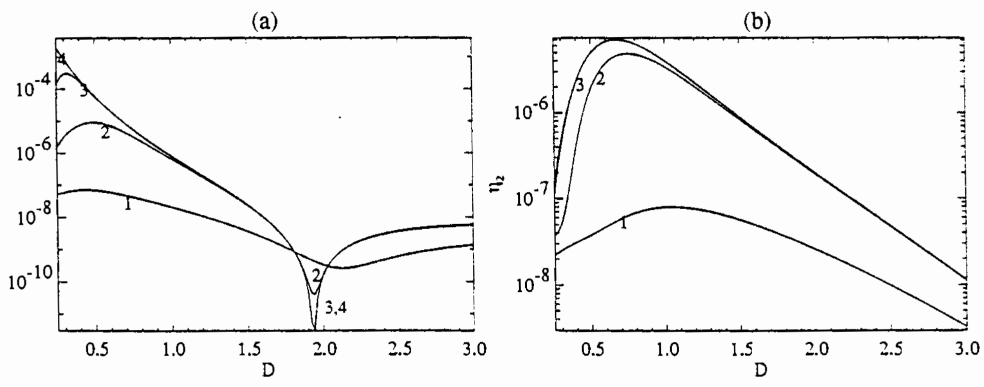

Fig. 2: Higher harmonic η_2, parameters as in Figs.1.

For the third harmonic, η_3, we find a smooth curve in the symmetric case and a dip in the asymmetric case.

We have confirmed the results for the higher harmonics within an adiabatic approximation, valid for small driving frequencies.

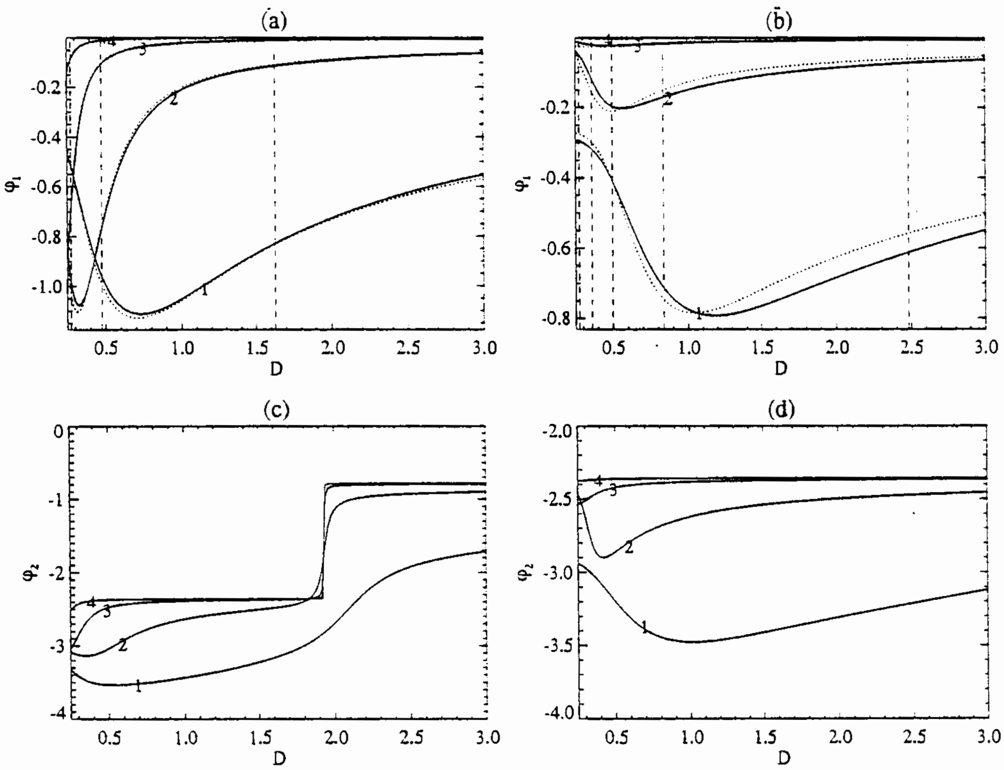

Fig. 3: Phase shifts, parameters as in Figs.1.

PHASE SHIFT OF THE OUTPUT SIGNAL

In Figs.3, the phase shifts of the first and second harmonic of the asymptotic mean value $\langle x(t)\rangle_{as}$ are shown for the symmetric (Figs.3a and 3c) and asymmetric case (Figs.3b and 3d). The results within linear response theory are shown by dotted lines. The phase shift in the symmetric case looks like in the quartic model: The maximum results from the competition between internal motion and hopping processes. In the asymmetric case the maximum is suppressed for small frequencies because the hopping disappears at small noise strength.

At values of D, for which a dip in a higher harmonic appears, the corresponding phase shift approaches a step function for small frequencies.

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