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Suppression of tunneling in periodically driven bistable systems

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The phenomenon of tunneling is investigated for a symmetric double-well potential perturbed by a monochromatic driving force. The analysis is based on a numerical treatment of the quantum map that propagates the system over one period of the external force, and of the spectrum of its eigenphases (quasi-energies). At driving frequencies between the bare tunneling frequency and the harmonic well frequency, we find for specific parameter values of the driving force a localization of the wave packet in one of the wells (coherent destruction of tunneling).

The tunnel effect was recognized long ago during the heydays of quantum mechanics [1]. As we will show in this article, the influence of external periodic forces on the tunneling dynamics leads to unexpected phenomena and therefore deserves special attention. We report on analytical and numerical investigations of an archetype model, a particle moving in a symmetric double well, and driven by a monochromatic classical force. The Hamiltonian defining this model reads

$$H(x, p) = \frac{p^2}{2} - \tfrac{1}{4}x^2 + \frac{x^4}{64D} + xS \sin \omega t \,. \qquad (1)$$

Here, we use dimensionless units. In particular, $D = E_B/\hbar\omega_0$ denotes the barrier height E_B in units of $\hbar\omega_0$, with ω_0 the angular frequency of harmonic oscillations on the bottom of each well, and t is measured in units of the corresponding period $2\pi/\omega_0$. This model Hamiltonian characterizes the physics of a wide class of systems, such as the intramolecular rearragements in pyramidal molecules [1] and macroscopic quantum coherence phenomena in SQUIDs [2].

In the present work, we attempt to gain insight into the deep quantum regime of this system. That is, we focus on the parameter range of low barriers, such that D is of order unity and, in the corresponding unperturbed problem, there are only a few levels below the barrier. In addition, we do not restrict ourselves to small amplitudes S of the driving force. Consequently, we refrain from the use of semiclassical or perturbative methods. Our approach is based on the Floquet formalism and the concept of quasi-energies, as pioneered for the physics of atoms in intense laser fields [3–7]. To provide an adequate language, we adopt the concepts of the temporal autocorrelation function (probability to stay) and the local spectrum, well-known, e.g., in solid state physics [8] and quantum chaos [9, 10].

Consider the propagator for the operator in (1) over a single period $T = 2\pi/\omega$ of the external periodic force. This unitary operator U is the generator of a quantum map, i.e. applied iteratively to some initial state $|\psi_0\rangle$, it provides a stroboscopic, discrete-time evolution of the wave function. In view of the Floquet theorem, the eigenstates of the unitary operator U take the form $|\psi_k(nT)\rangle = \exp(-in\varepsilon_k T)|\Phi_k(0)\rangle$, where n denotes the number of time steps, and $|\Phi_k(t + T)\rangle = |\Phi_k(t)\rangle$. The quantities ε_k, defined modulo ω, are refered to as quasi-energies [6–10]. They are functions both of the driving amplitude S and the driving frequency ω. The generalized parity transformation $P: x \to -x$, $t \to t + T/2$, leaves the Hamiltonian $\mathcal{H} = H - i\partial_t$ invariant. Thus, the Floquet functions, being eigenfunctions of $\mathcal{H}$, can be classified into states of even and odd parity, respectively.

Given an initial wave packet $|\psi_0\rangle$ and its time evolution under U, the temporal autocorrelation function is defined by

$$P_n = |\langle \psi_n | \psi_0 \rangle|^2 . \tag{2}$$

Expanding both $|\psi_0\rangle$ and $|\psi_n\rangle$ in the Floquet basis, and using the role of the Floquet states as eigenfunctions of U, one finds

$$P_n = \xi^{-1} + \sum_{\alpha \neq \beta} \exp(in(\varepsilon_\alpha - \varepsilon_\beta)T) \times |\langle \Phi_\alpha | \psi_0 \rangle|^2 |\langle \Phi_\beta | \psi_0 \rangle|^2 , \tag{3}$$

where $\xi^{-1} = \lim_{N\to\infty} N^{-1} \Sigma_{n=0}^{N} P_n$ denotes the long time average of P_n. The spectral counterpart of the autocorrelation function P_n is the two-point correlation function $P_2^{\mathrm{loc}}(\eta)$ of the *local* Floquet spectrum [10]. It is related to P_n by Fourier transformation and thus contains all the frequencies involved in the time evolution of P_n, weighted according to their relative significance for this dynamics.

In the following, we will consider time evolutions starting from one particular type of initial state $|\psi_0\rangle$: a Gaussian centered, say, in the left well, equivalent to the ground state of the harmonic approximation of that well. This initial state is defined independently of the Floquet basis, and can readily be realized both in numerics and experiments. Moreover, with this initial state, the deviation of P_n from unity provides a first clue of the probability flow into the opposite well. A quantity serving the same purpose, but more specifically tailored to the symmetric double-well problem is the *occupation probability* in the left well,

$$\rho_n^{\mathrm{left}} = \int_{-\infty}^{0} \mathrm{d}x |\psi(x, nT)|^2 . \tag{4}$$

The concepts introduced in eqs. (3)–(7) can be used to discuss the varieties of the Floquet spectrum, as they show up in the (S, ω)-parameter space, and their consequences for the tunneling behavior.

The focus of this work is the range of intermediate driving frequencies between Δ, the lower characteristic frequency scale of the unperturbed system, and ω_0, its upper characteristic frequency scale. Other regimes of the external frequency such as the adiabatic or the extreme high frequency case are discussed in ref. [11]. For intermediate frequencies the Floquet spectrum may lose any similarity to the energy spectrum in the unperturbed case. Features of particular significance are close encounters of levels, as functions of the parameters, since they lead to exceptionally long time scales in the tunneling dynamics. If two quasi-energies, approaching each other, belong to different parity classes, they form an exact crossing, whereas in the opposite case, a crossing will be avoided.

Here, we restrict ourselves to the case of exact crossings of quasi-energies, bearing surprising consequences, as we will discuss now. Consider the two lowest eigenstates of the unperturbed case: they form the well-known doublet of a symmetric and an antisymmetric state which is responsible for the familiar tunneling phenomenon. With the driving force S switched on, they evolve into two Floquet eigenstates Φ_1, Φ_2, respectively, with similar shape and, in particular, with the same parity as their unperturbed counterparts. These two "lowest" Floquet states, therefore, allow for *exact* crossings of their respective quasi-energies, as functions of S and ω. In fact, we find a one-dimensional manifold in the (S, ω)-plane where they cross. The consequences of such crossings are intriguing: the time scale for a wave packet prepared as a superposition $(\Phi_1 \pm \Phi_2)/\sqrt{2}$ to cross the barrier diverges, and so it will remain localized in the initially populated well! In contrast, for the case of an asymmetric unperturbed potential, generalized parity is lost, and consequently no exact crossings between quasi-energies occur.

The results of a numerical investigation of the localization phenomenon are presented in fig. 1. The parameter values chosen here are $S = 3.17 \times 10^{-3}$, $\omega = 0.01$, and $D = 2$. Figure 1(a) shows the time evolution of P_n over the first 10^3 time steps. The fact that P_n remains near unity, within 9%, is a first indication of a coherent suppression of tunneling. A more reliable measure of the transfer of probability to the opposite

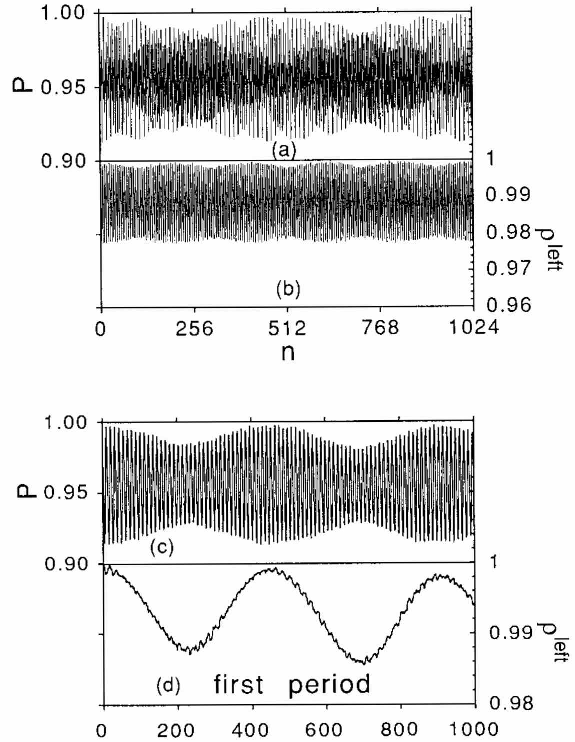

Fig. 1. Driven tunneling at an exact crossing of the two "lowest" quasi-energies (see text): (a) time evolution (quantum map), over the first 10^3 time steps, $T = 2\pi/\omega$, of the autocorrelation function P_n and (b) the occupation probability ρ_n^{left} in the initially populated well; resolution of the first period of P_n (c) and ρ_n^{left} (d) into 1000 time steps. The parameter values are $\omega = 0.01$ and $S = 3.17 \times 10^{-3}$.

well is ρ_n^{left}, plotted in fig. 1(b) over the same time window. Its deviation from unity does not even exceed 2.5%. Finally, we also generated a finely resolved time evolution, over one period of the driving force, of the two diagnostic quantities mentioned (figs. 1(c, d)). It clearly excludes the possibility of fast tunneling with the frequency of the driving force, which could have escaped the stroboscopic description used in all the other simulations. The beats in fig. 1(c) and the corresponding oscillation in fig. 1(d) occur at a frequency which corresponds to the difference between the quasi-energies ε_4 and ε_3. This reveals that the initial wave packet is not completely exhausted by the two "lowest" Floquet functions but has also some very minor contributions from such "higher" states.

At the end, some remarks are in order, concerning the experimental realization of the localization phenomenon. For ammonia, the archetype bistable system from molecular physics, one deals with $D \approx 2$. Using the dipole moment $\mu = 1.47$ Db, we can directly estimate the required wavelength of the external maser signal, $\lambda = 2$ mm, and intensity $P \approx 10^{13}$ W/m^2 to localize the nitrogen atom. In solid state physics, dealing with RF-SQUIDs, one can generate a symmetric bistable potential by applying an external flux $\phi_x = \phi_0/2$, with $\phi_0 = h/2e$. For this system, characterized by a capacitance of $C = 10^{-15}$ F, a self-inductance of $L = 10^{-9}$ H, and a critical current of $I_c = 10^{-6}$ A, the plasma frequency equals $\omega_p = 10^{12}$ Hz. To suppress flux tunneling (see fig. 1) in this device one has to inject an external AC-current of frequency $\omega = 10^{10}$ Hz with an amplitude of $I = 10^{-10}$ A. Likewise coherent flux tunneling is suppressed if one applies a time-varying external flux $\phi_x = \phi_0/2 + \delta\phi_x \sin \omega t$, where $\delta\phi_x = 10^{-19}$ Wb.

The results reported in this paper show that external driving of a bistable quantum system gives rise to unexpected modifications of the notion of tunneling. Periodic driving may slow down tunneling by *any* desired degree or even *suppress* it altogether, in a perfectly coherent way. This surprising effect enables to localize an otherwise bistable quantum system in one of its metastable states. Since it occurs along a one-dimensional manifold in the parameter space spanned by frequency and amplitude of the driving force, it should be readily observable experimentally.

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