The ring of Brownian motion: the good, the bad, and the simply silly

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Brownian motion

Gypsum cristals in a closterium moniliferum

Movie

source: IWF Wissen und Medien gGmbH, 1949

Why you should not do Brownian motion

- You know nothing about the subject

- Many very good people worked on it

  (Einstein, Langevin, Smoluchowski, Ornstein, Uhlenbeck, Wiener, Onsager, Stratonovich, …)

- You don’t have your own pet theory yet

Why you should do Brownian motion

- You know nothing about the subject
- Many very good people worked on it
- You still can do your own pet theory

Robert Brown (1773-1858)

Source: www.anbg.gov.au

Source: permission kindly granted by Prof. Brian J. Ford
http://www.brianjford.com/wbbrowna.htm

1827 – irregular motion of granules of pollen in liquids

- Brown, Phil. Mag. **4**, 161 (1928)
- Deutsch: *Did Robert Brown observe Brownian Motion: probably not*, Sci. Am. **256**, 20 (1991)
- Ford: *"Brownian movement in clarkia pollen: a reprise of the first observations"*,
  The Microscope **39**, 161 (1991)

Jan Ingen-Housz (1730-1799)

Source: www.americanchemistry.com

Johann Ingen-Houß

K. K. Hofraths und Leibarztes, der Königl. Gesellschaft der Wissenschaften zu London, der Batavischen Gesellschaft der Experimentalphilosophie zu Rotterdam 2c. 2c. Mitglieds

Vermischte

Schriften

physisch-medicinischen Inhalts.

Uibersetzt und herausgegeben

von

Nicolaus Carl Molitor.

Zweyte, verbesserte und mit ganz neuen Abhandlungen vermehrte Auflage.

Mit Kupfertafeln.

Zweyter Band.

WIEN,

gedruckt und verlegt bey Christian Friderich Wappler.

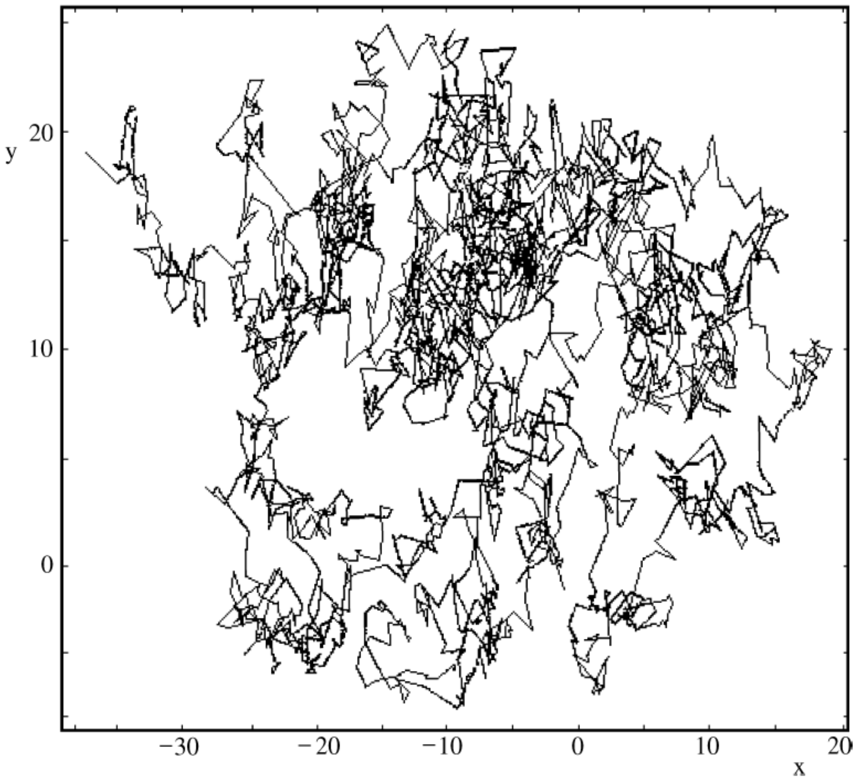
1784.

To see clearly how one can deceive one's mind on this point if one is not careful, one has only to place a drop of alcohol in the focal point of a microscope and introduce a little finely ground charcoal therein, and one will see these corpuscules in a confused, continuous and violent motion, as if they were animalcules which move rapidly around.

Mean squared displacement

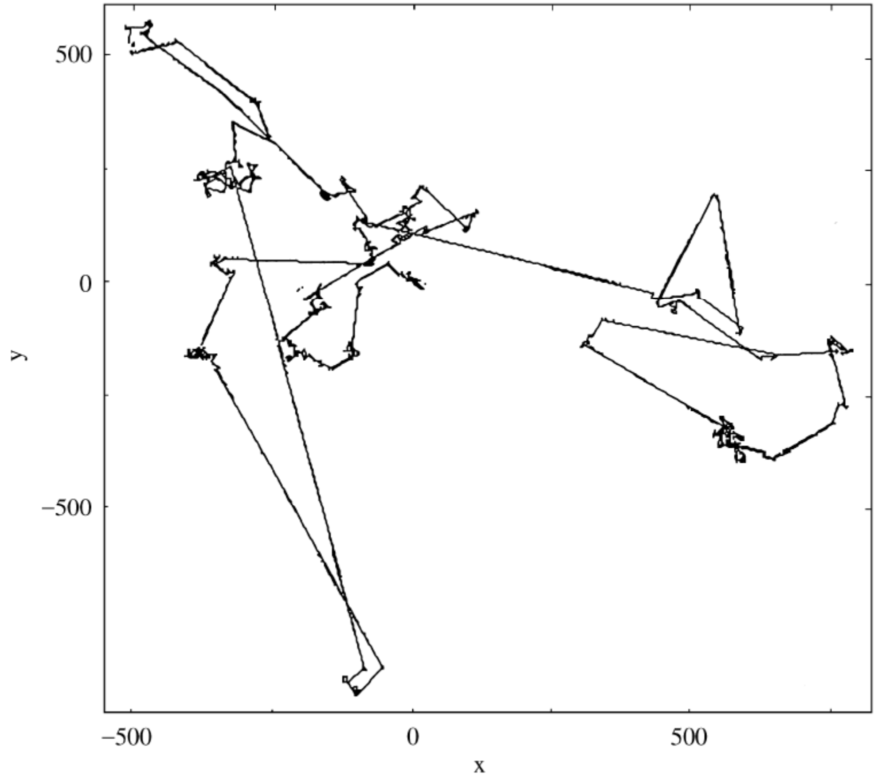
$$\langle x^2(t) \rangle \propto t^\alpha$$

Brownian movement $\alpha = 1$



Source: Physica A **282**, 13 (2000)

Lévy-Brownian movement $\alpha = \frac{4}{3}$



Source: Physica A **282**, 13 (2000)

Theory of Brownian motion

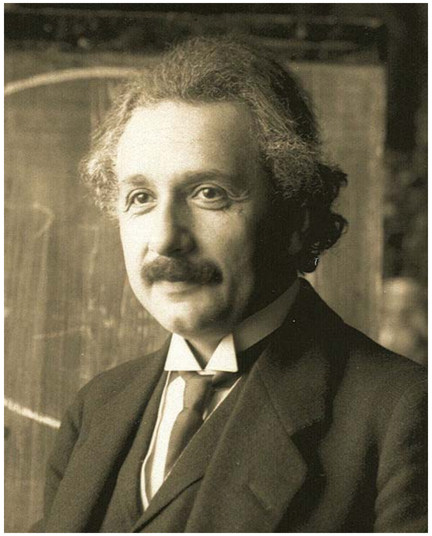
W. Sutherland (1858-1911)

Source: www.theage.com.au

$$D = \frac{RT}{6\pi\eta a C}$$

Phil. Mag. **9**, 781 (1905)

A. Einstein (1879-1955)



Source: wikipedia.org

$$\langle x^2(t) \rangle = 2Dt$$

$$D = \frac{RT}{N} \frac{1}{6\pi k P}$$

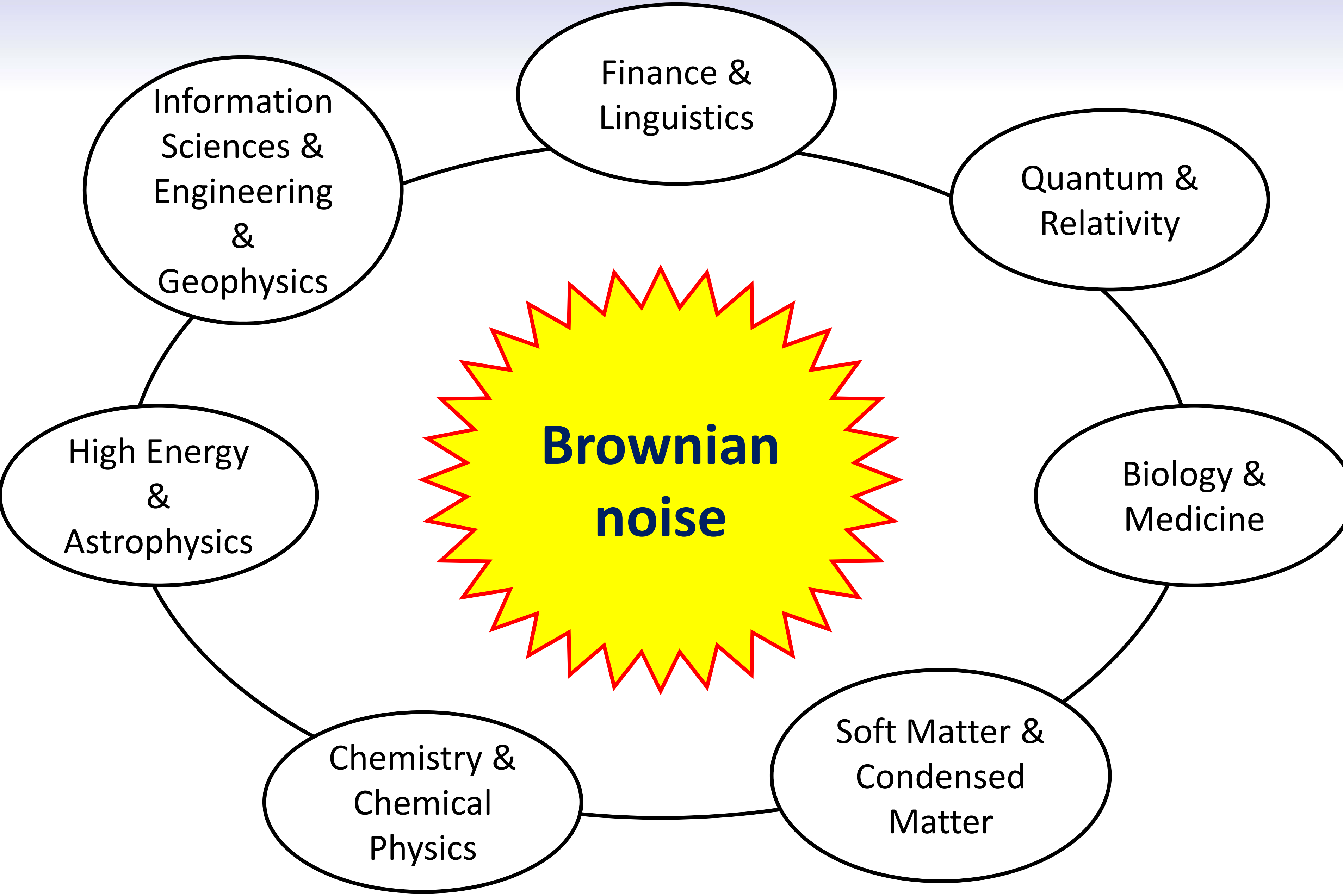
Ann. Phys. **17**, 549 (1905)

M. Smoluchowski (1872-1917)

Source: wikipedia.org

$$D = \frac{32}{243} \frac{mc^2}{\pi\mu R}$$

Ann. Phys. **21**, 756 (1906)


Brownian noise
Finance & Linguistics
Quantum & Relativity
Biology & Medicine
Soft Matter & Condensed Matter
Chemistry & Chemical Physics
High Energy & Astrophysics
Information Sciences & Engineering & Geophysics

Quantum-Mechanics
= Brownian Motion ?
= Stochastic Mechanics ?

(E. Nelson; 1966, 1986)

$$p(x,t) = |\Psi(x,t)|^2$$

Schrödinger-
gleichung

$$\Psi(x,t) = |\Psi(x,t)|\, e^{iS(x,t)}$$

$$\dot{p}(x,t) = -\frac{\hbar}{m}\nabla\left[(\nabla \ln|\Psi(x,t)| + \nabla S(x,t))\, p(x,t)\right] + \frac{\hbar}{2m}\nabla^2 p(x,t)$$

$$\mathbf{f}_1 \geq 0\,,\ \mathbf{f}_2 \geq 0\ :\quad \frac{1}{2}\langle \mathbf{f}_1(t_1)\mathbf{f}_2(t_2) + \mathbf{f}_2(t_2)\mathbf{f}_1(t_1)\rangle \geq 0$$

QM: **NO !**

H. Grabert, P.H., P. Talkner, Phys. Rev. A **19**, 2440 (1979)

Diffusion: space-time only

classical diffusion (Markovian)

$$p(t,x|t_0,x_0) = \left[\frac{1}{4\pi\,\mathcal{D}(t-t_0)}\right]^{1/2} \exp\left[-\frac{(x-x_0)^2}{4\mathcal{D}(t-t_0)}\right].$$

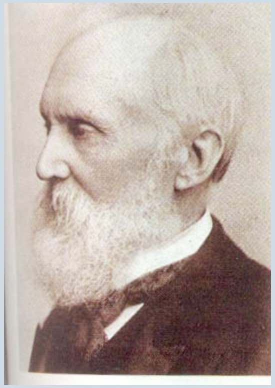
telegraph equation (non-Markovian)

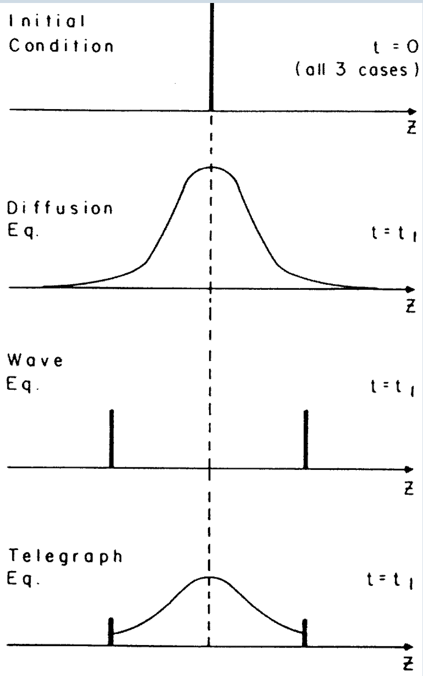
$$\tau_v \frac{\partial^2}{\partial t^2}\varrho + \frac{\partial}{\partial t}\varrho = \mathcal{D}\,\nabla^2\varrho,$$

alternative approach

$$\boxed{p(\bar{x}|\bar{x}_0) \propto \int_{a_-(\bar{x}|\bar{x}_0)}^{a_+(\bar{x}|\bar{x}_0)} \mathrm{d}a\;\exp\left(-\frac{a}{2\mathcal{D}}\right)}$$

*PRD **75**:043001 (2007)*



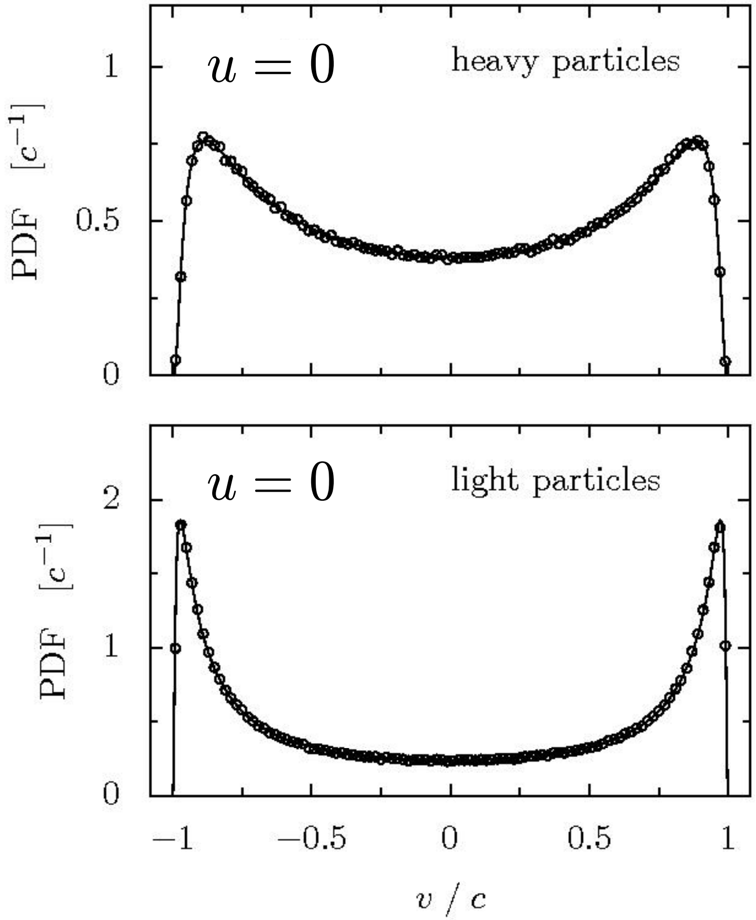


J Masoliver & G H Weiss
*Eur J Phys **17**:190 (1996)*

Jüttner Gas

$$f_{\mathrm{Maxwell}}(\vec{p}) = [\beta/(2\pi m)]^{d/2} \exp\left(-\beta p^2/2m\right)$$

$$f_{\mathrm{Jüttner}}(\vec{p}) = Z_d^{-1} \exp\left[-\beta_{\mathrm{J}}(m^2c^4 + p^2c^2)^{1/2}\right]$$



$$\langle \vec{p}\cdot\vec{v}\rangle = dk_{\mathrm{B}}\mathcal{T} = d/\beta_{\mathrm{J}}$$

statistical realtivistic temperature

$$T = \mathcal{T} = (k_{\mathrm{B}}\beta_{\mathrm{J}})^{-1}$$

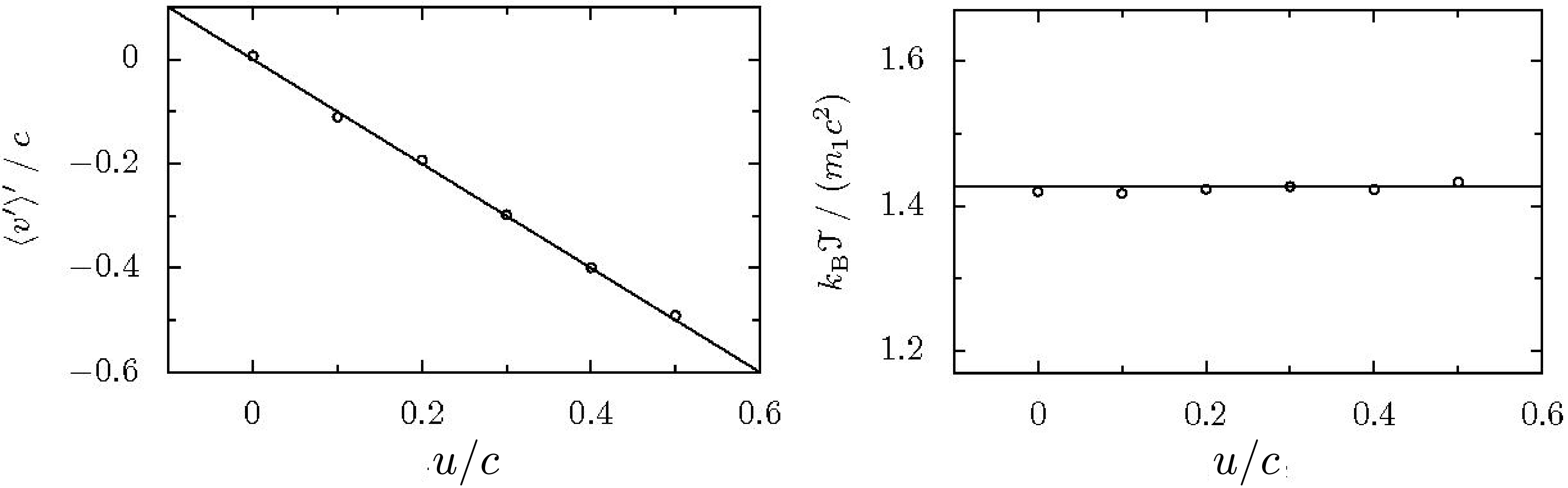
J. Dunkel & P.H., Phys. Rep. **471**, 1-73 (2009)

Measuring temperature in Lorentz invariant way

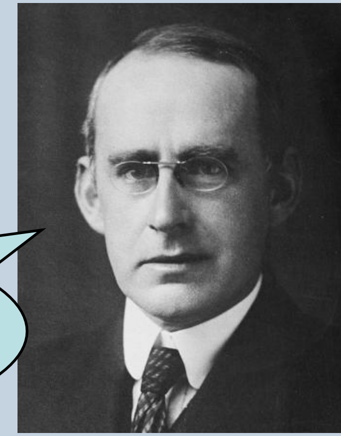
Lorentz invariant equipartition theorem

$$k_B \mathcal{T} = m\gamma(u)^3 \langle \gamma(v')(v'+u)^2 \rangle_{t'}$$

with $u = -\langle v' \rangle_{t'}$



“Temperature” problem in RTD?

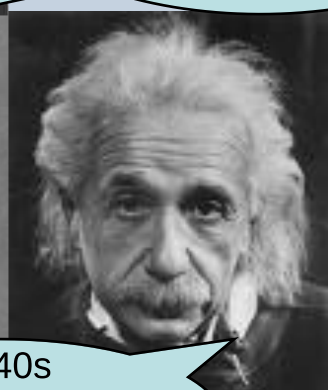


1923/1963

.. hotter!

1907/08

moving bodies appear cooler



1940s

maybe ... not

$$T'(w) = T\,(1 - w^2)^{\alpha/2} \qquad \alpha = \begin{cases} +1 & \text{Planck, Einstein} \\ 0 & \text{Landsberg, van Kampen} \\ -1 & \text{Ott} \end{cases}$$

T=T’ 1966-69

CK Yuen, Amer. J. Phys. 38:246 (1970)

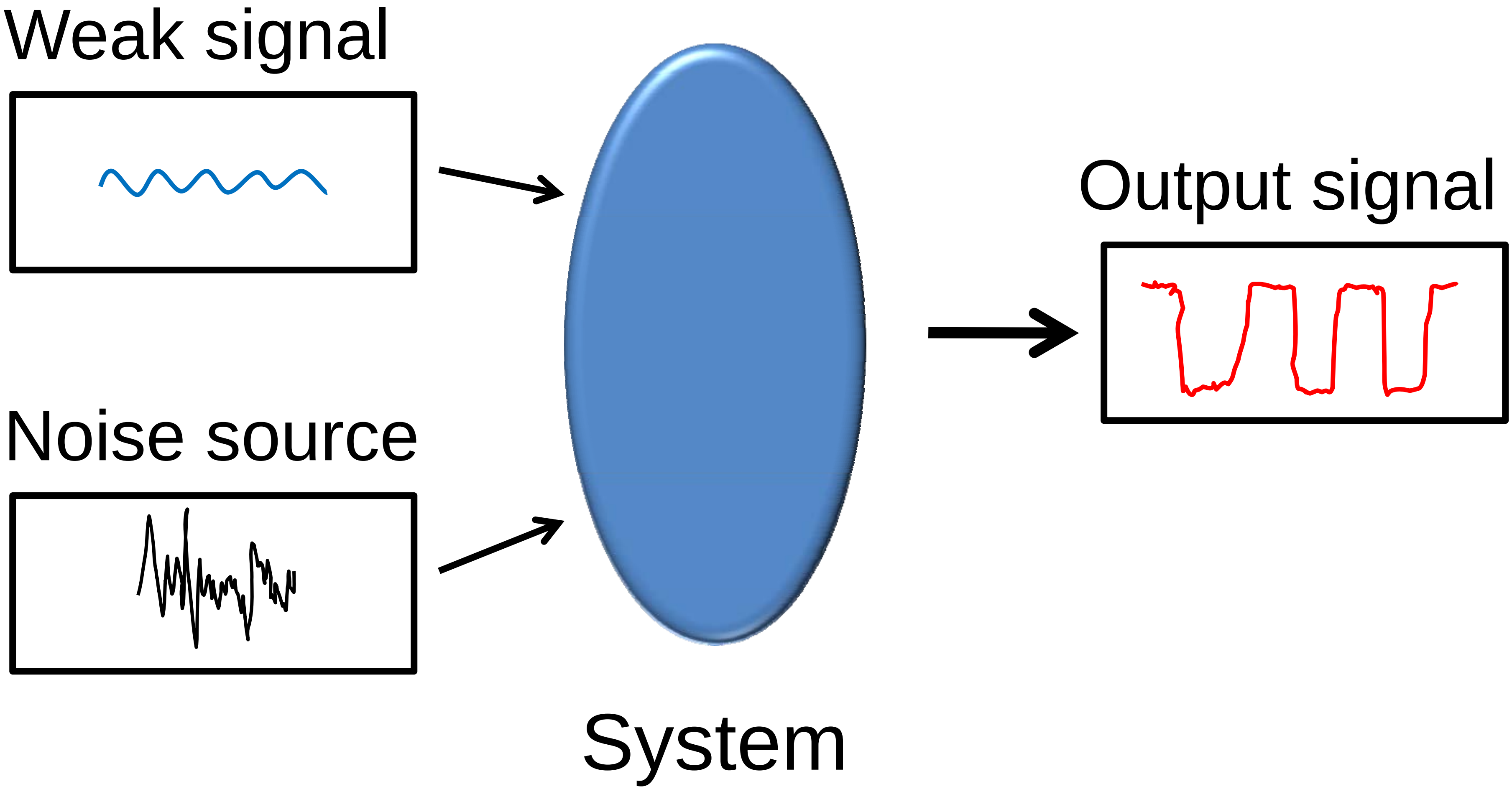
Two prominent examples

Stochastic Resonance

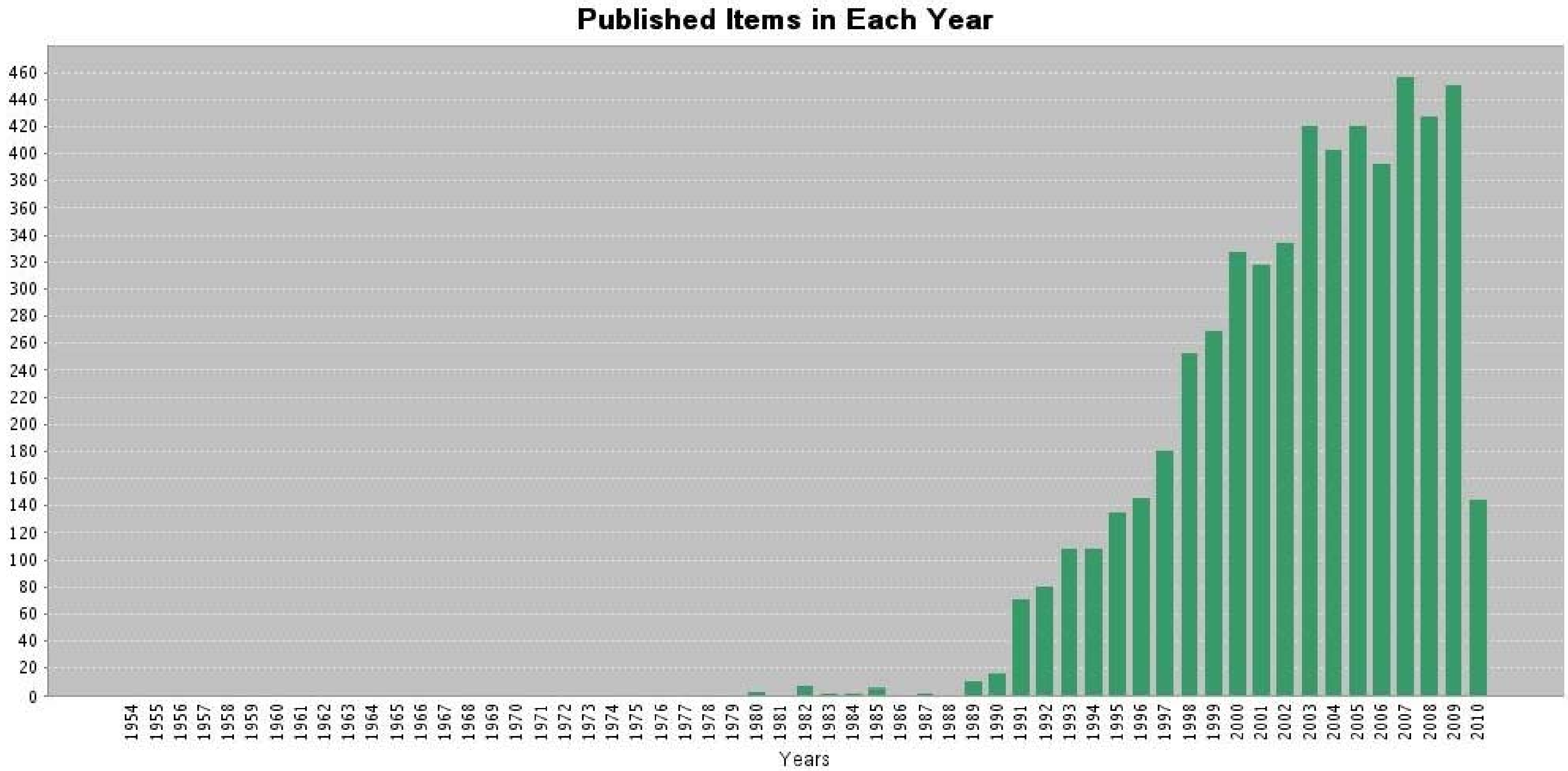
Brownian Motors

Stochastic Resonance

(in a nutshell)



SR - Citations



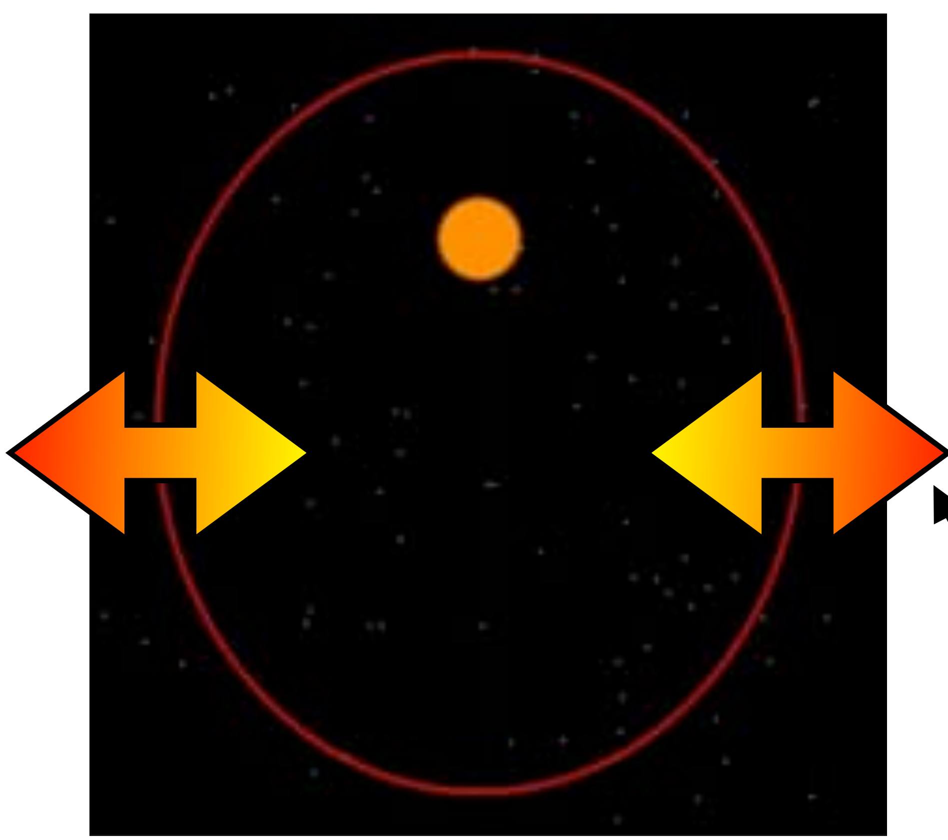
\# papers in 2009: ≈ 460

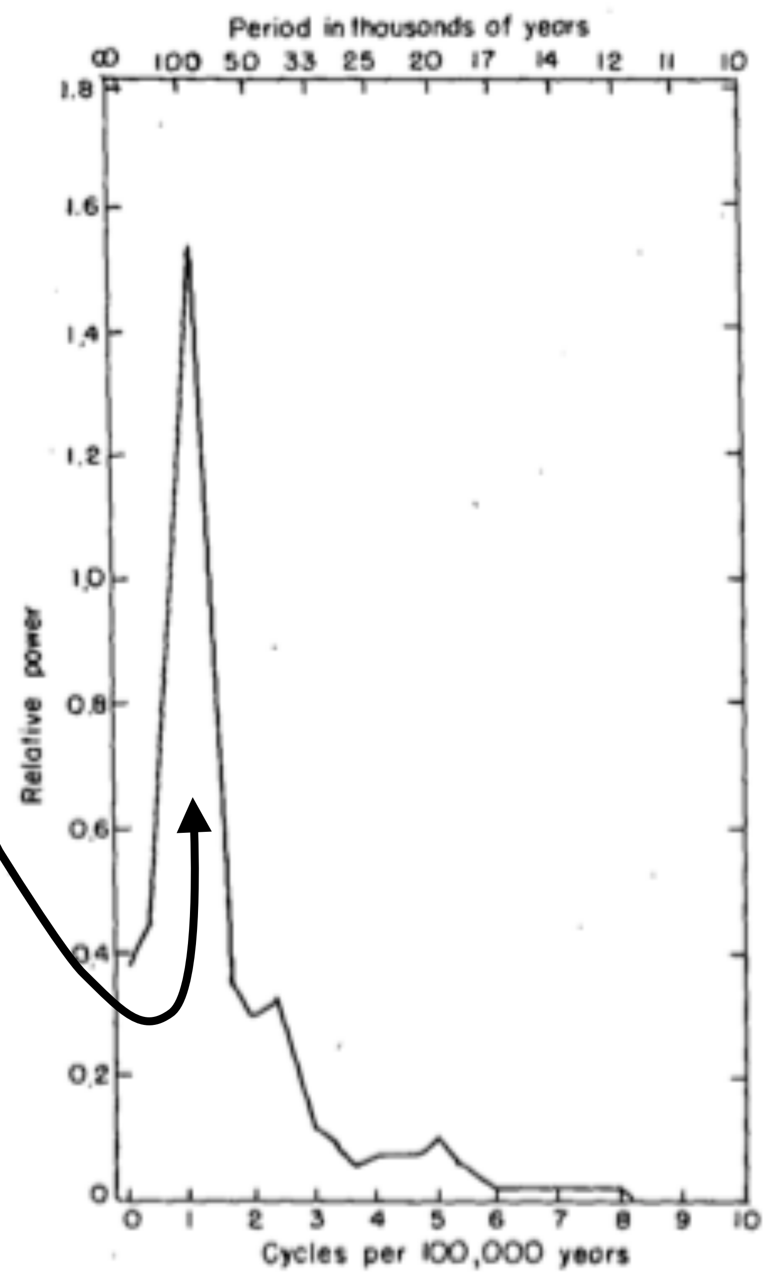
\> 85000 cites in total

Why are the ice-ages so periodic ?

Milankowitch cycles:
Small changes in earth orbit eccentricity with 100k year periodicity

Changes are small! (<0.1% of solar constant)



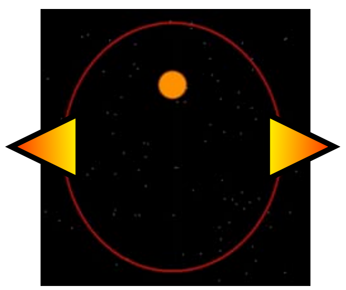


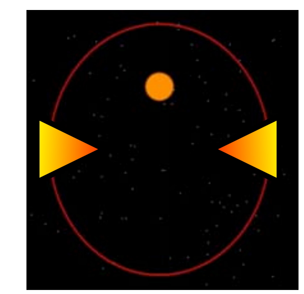
What can amplify those small changes ?

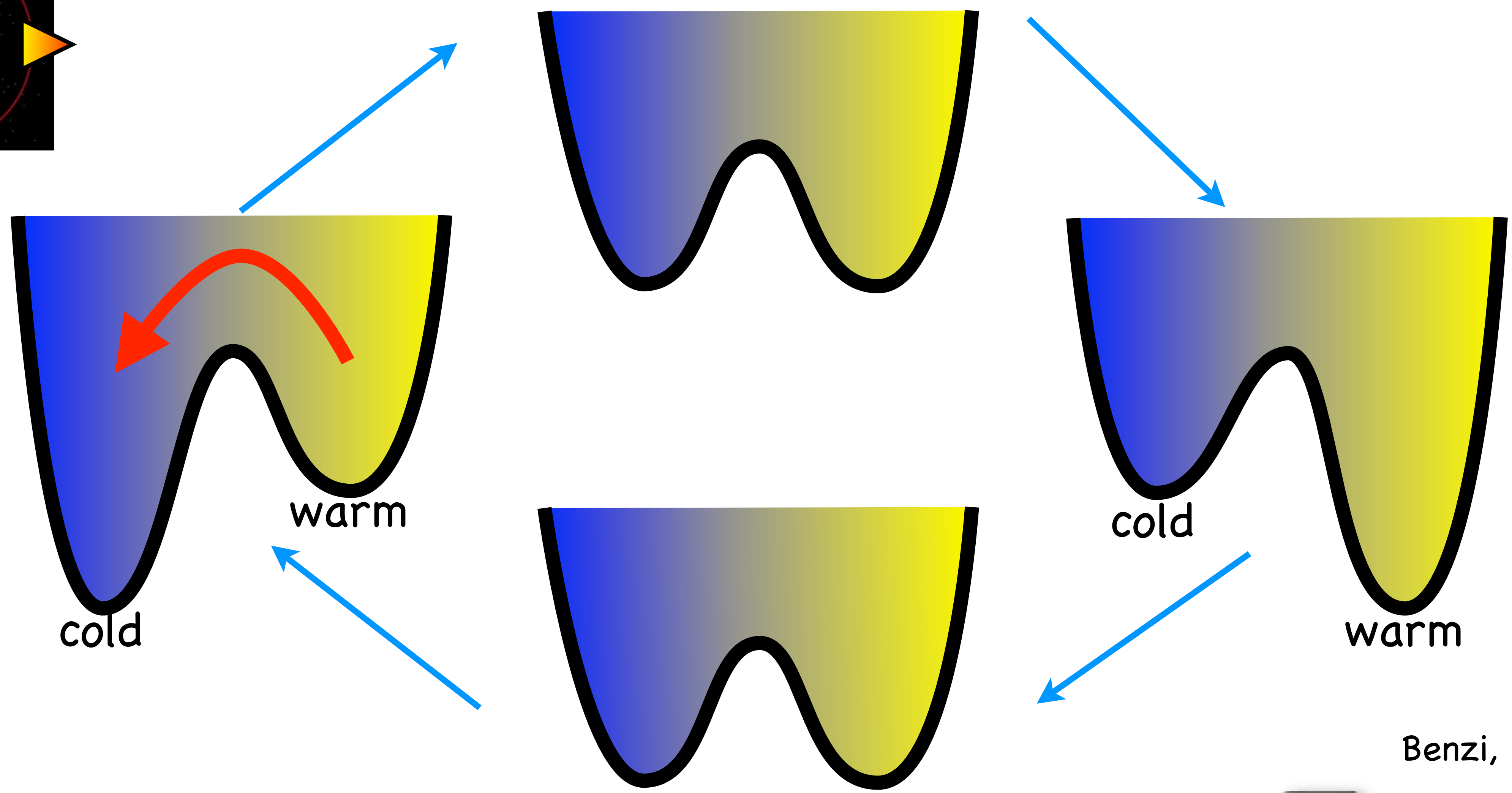
M. Milankowitch, Handbuch der Klimatologie I (1930)

Milankowitch Cycles and Bistability

Climate “landscape”



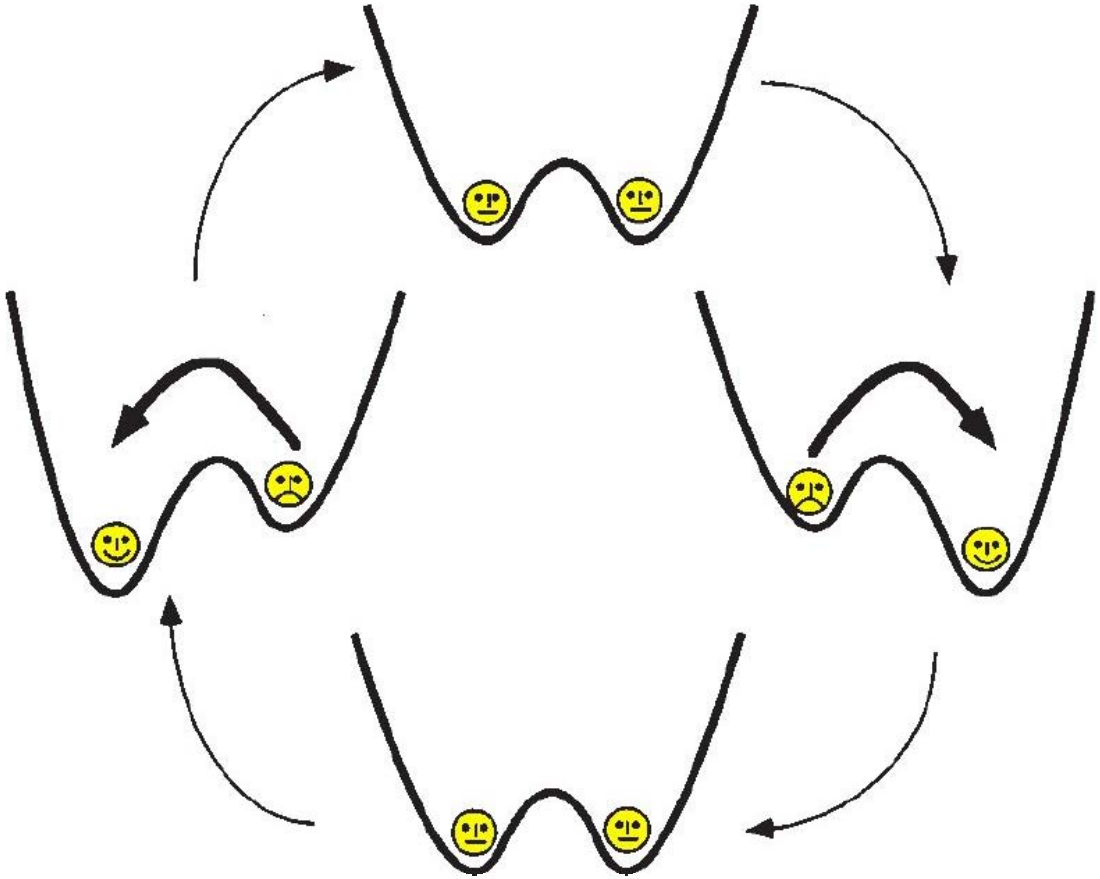




Benzi, Sutera and Vulpiani
(Tellus, 1981)
C. Nicolis and G. Nicoli
(Tellus, 1981)

- The 100ky cycles only bias the climate
- Fluctuations make climate switch
- small changes of conditions can have huge impact

Noise-assisted synchronized hopping

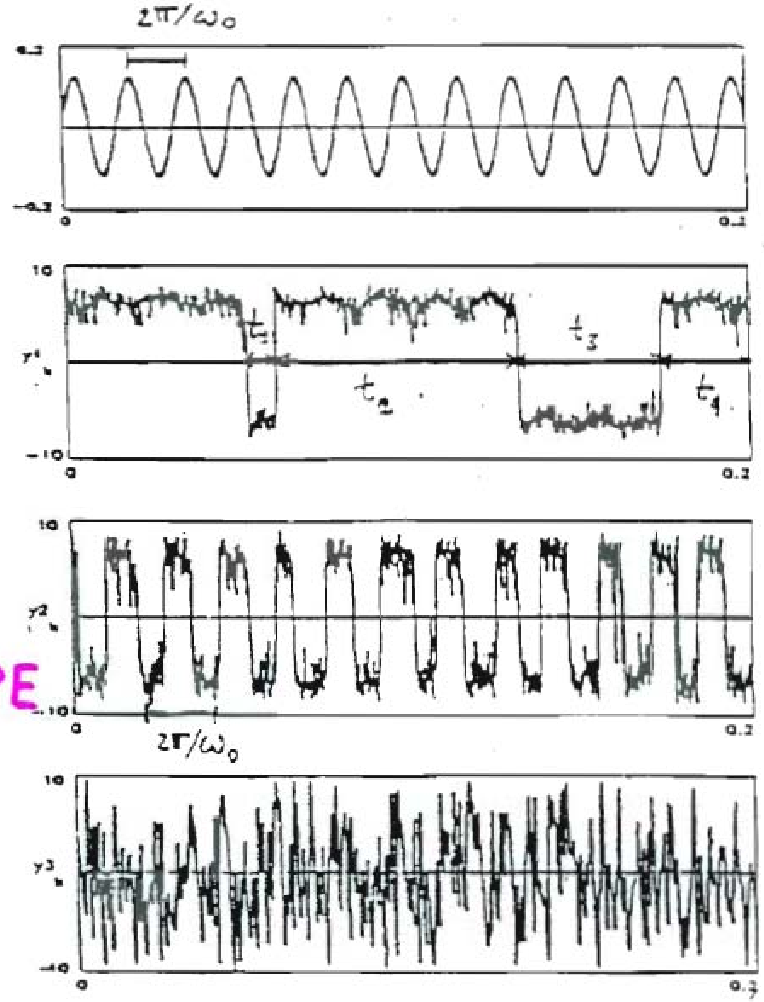


$$T_{\text{period}} \cong 2T_{\text{escape}}$$

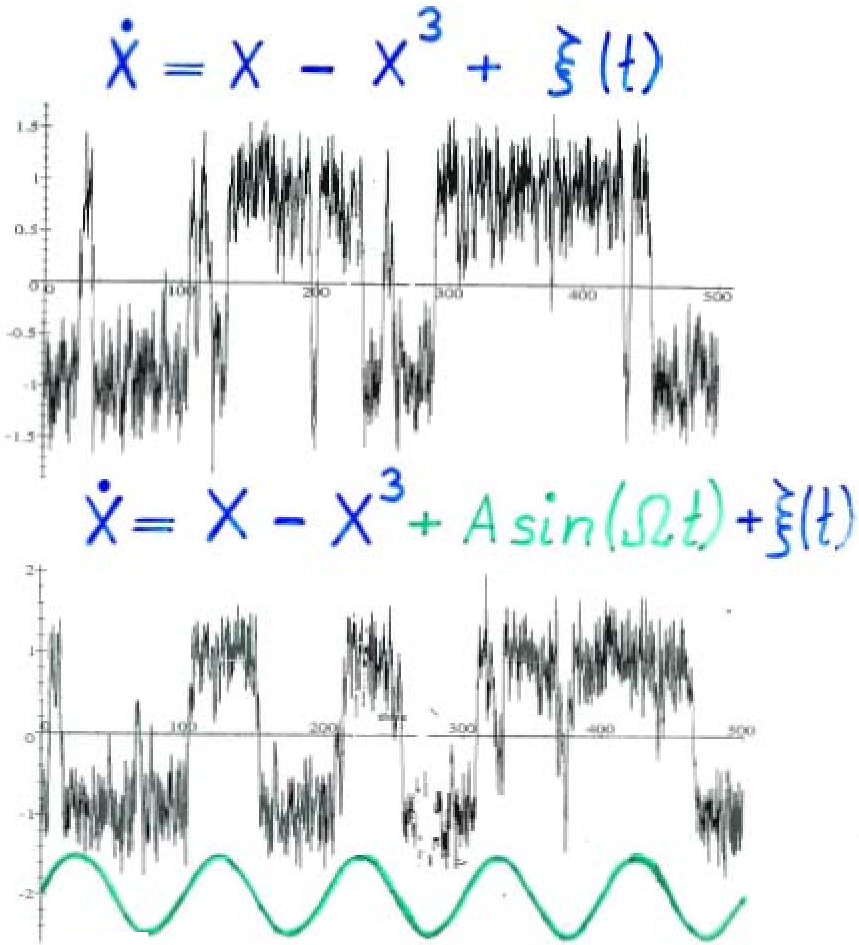
Synchronization

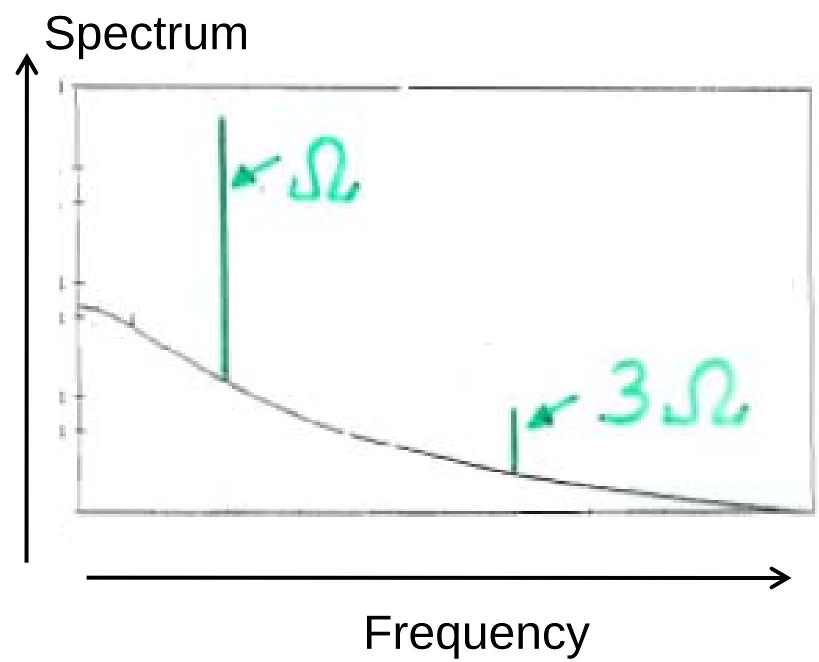
SIGNAL

$T_e \sim 2\Gamma_{ESCAPE}^{-1}$



Power spectral density





Measuring SR

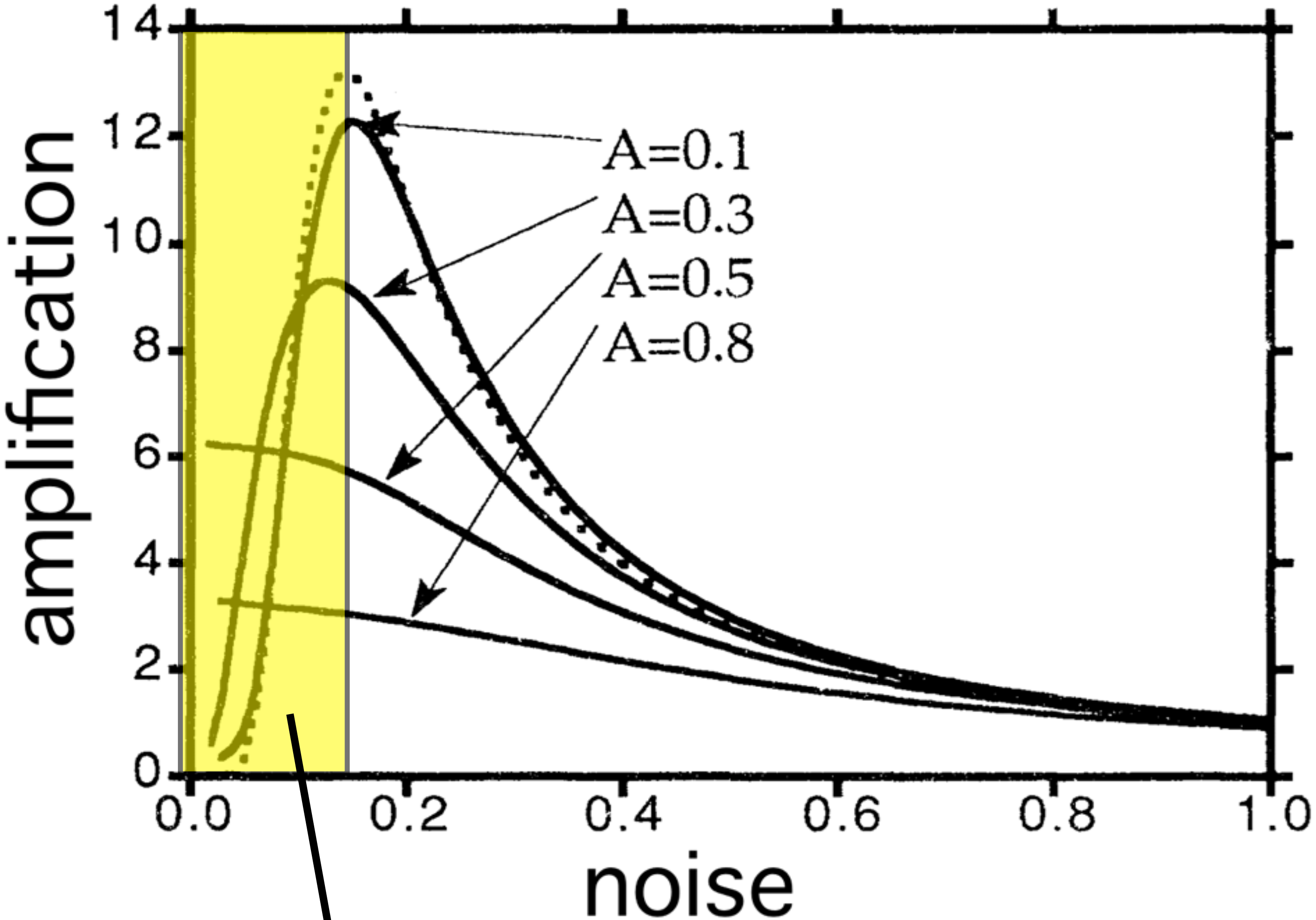
- Signal to noise ratio
- Spectral amplification
- mutual information
- cross-correlation: input ↔ output
- peak area, (phase-) synchronization, …

SR-reviews:

L. Gammaitoni, P. Hänggi, P. Jung, F. Marchesoni, Rev. Mod. Phys. **70**, 223 (1998)
P. Hänggi, ChemPhysChem **3**, 285 (2002)

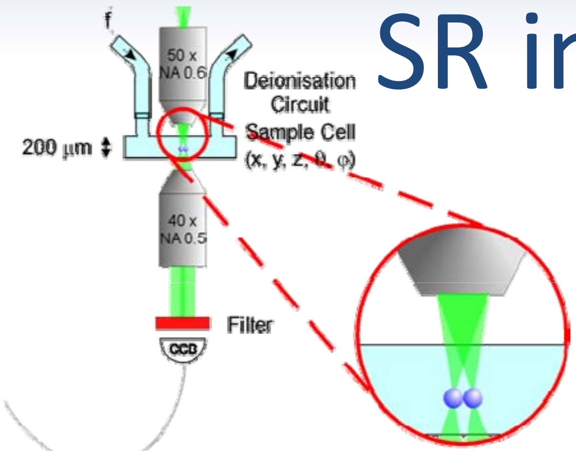
Amplification of small signals by noise

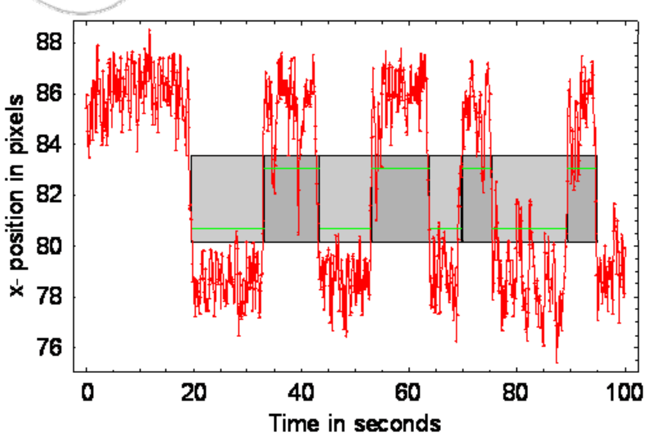
(P. Jung, P. Hänggi, Phys. Rev. A **44**, 8032 (1991))

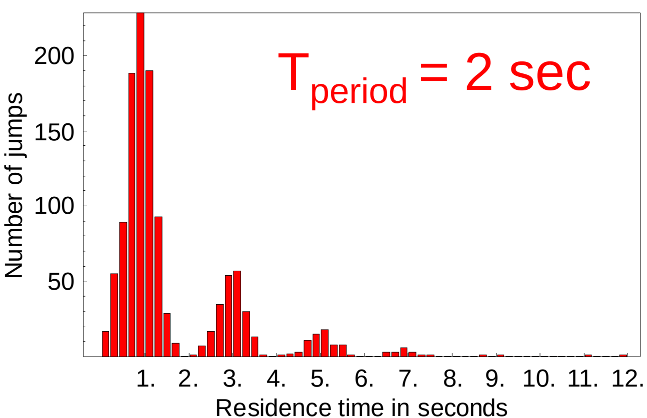


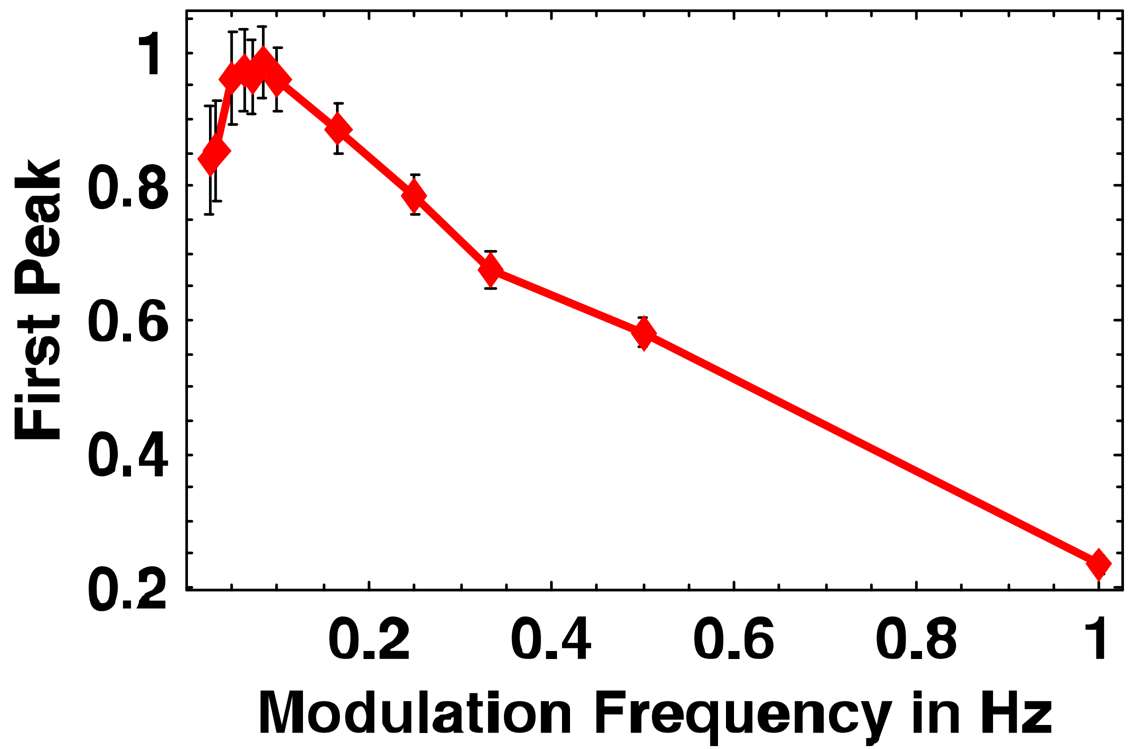
More noise , more signal !!

SR in colloidal systems









D. Babic, C. Schmitt, I. Poberaj, C. Bechinger, Europhys. Lett. **67**, 158 (2004)

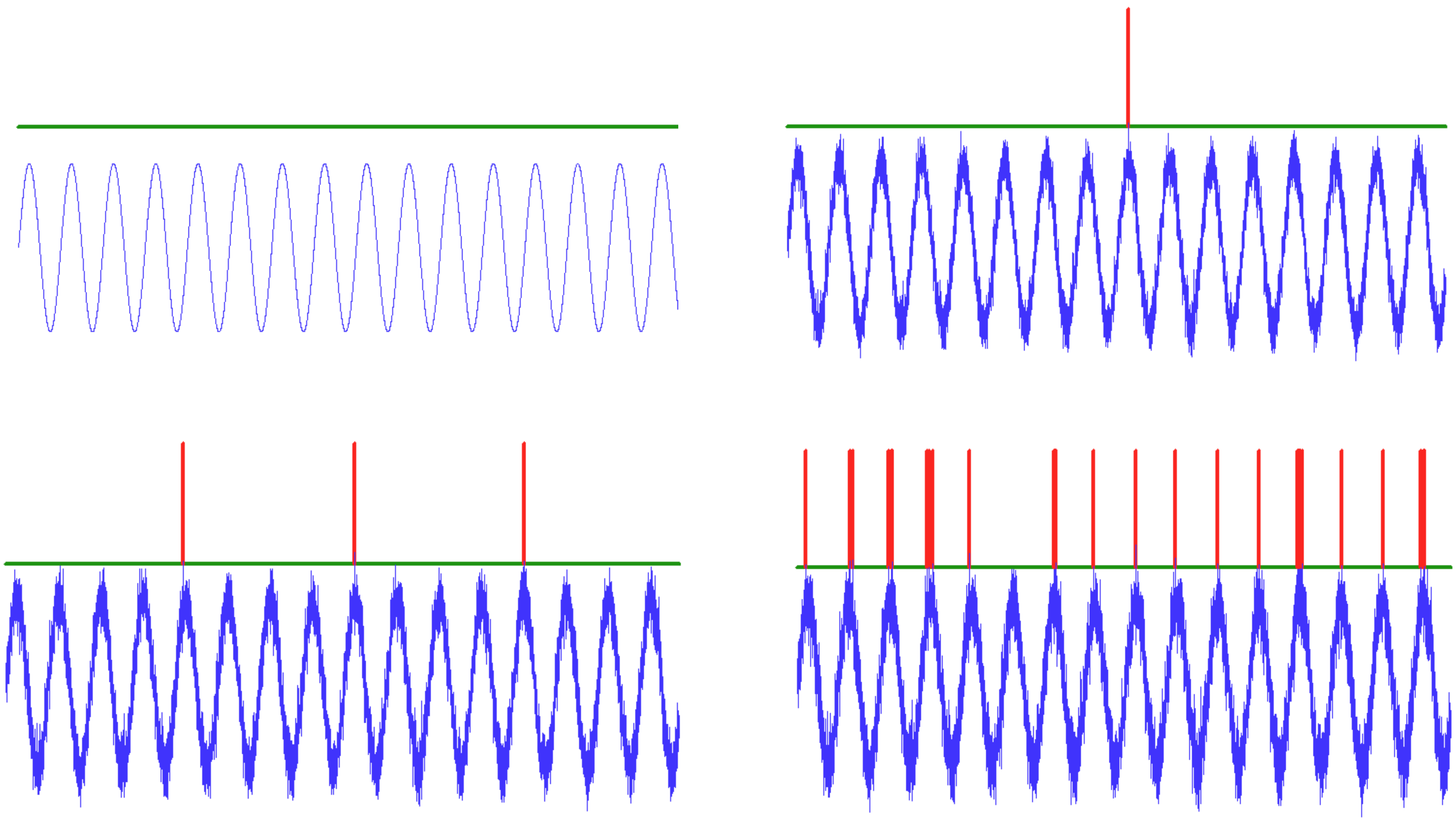
SR - Ingredients

- ✓ Threshold system
- ✓ Weak (subthreshold) signal
- ✓ Noise

⬇

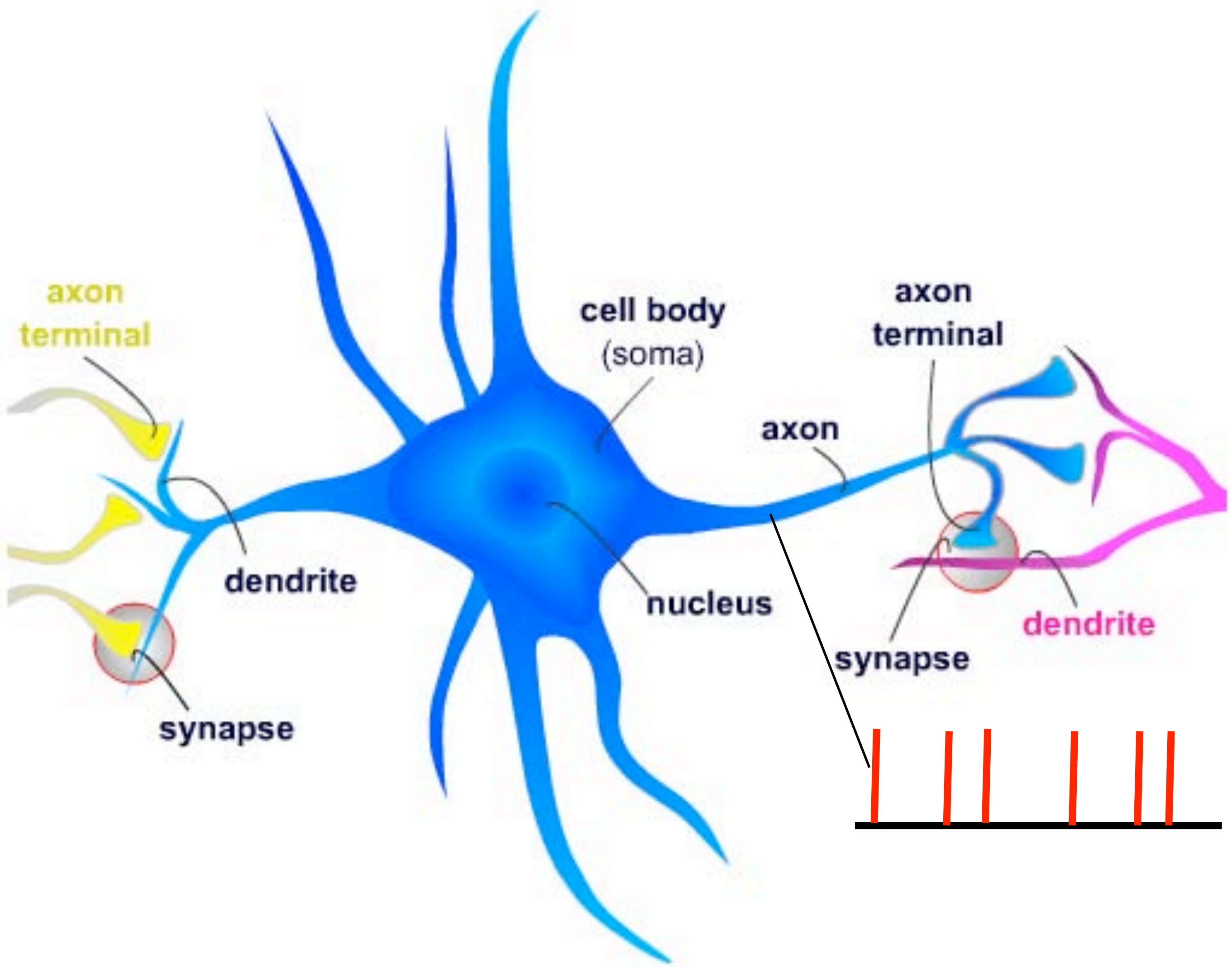
Anomalous amplification properties

Thresholds and Stochastic Resonance



P. Jung, Phys. Rev. E50, 2513 (1994), F. Moss and L. Kiss, EPL, 29 (1995)

Stochastic Resonance in Neurobiology



source: Consortium on Cognitive Science Instruction (CCSI)

Input: currents at synapses

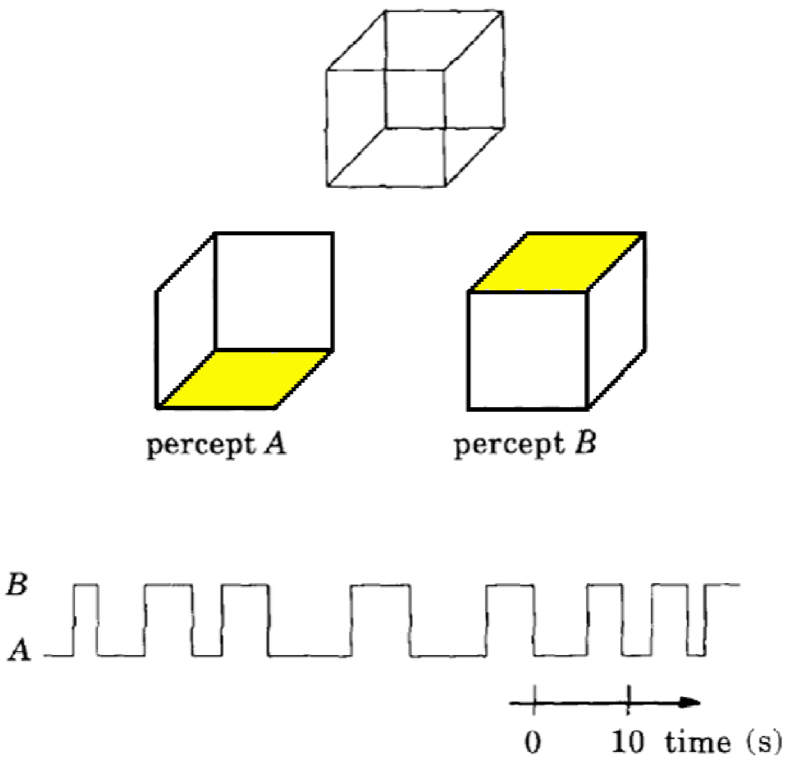
Processing: action potential if the sum of currents exceeds threshold

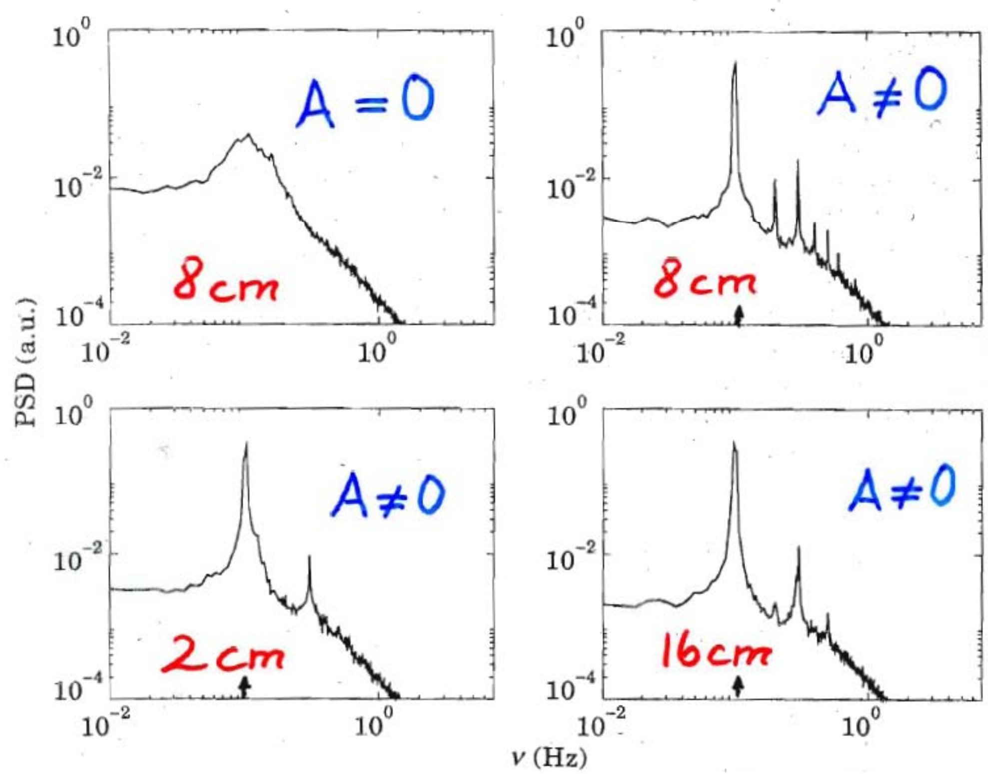
Output: electric pulses traveling down the axon

Basic idea: Signals below threshold can be detected in the presence of additional noise

SR in Visual Perception

Experiment:

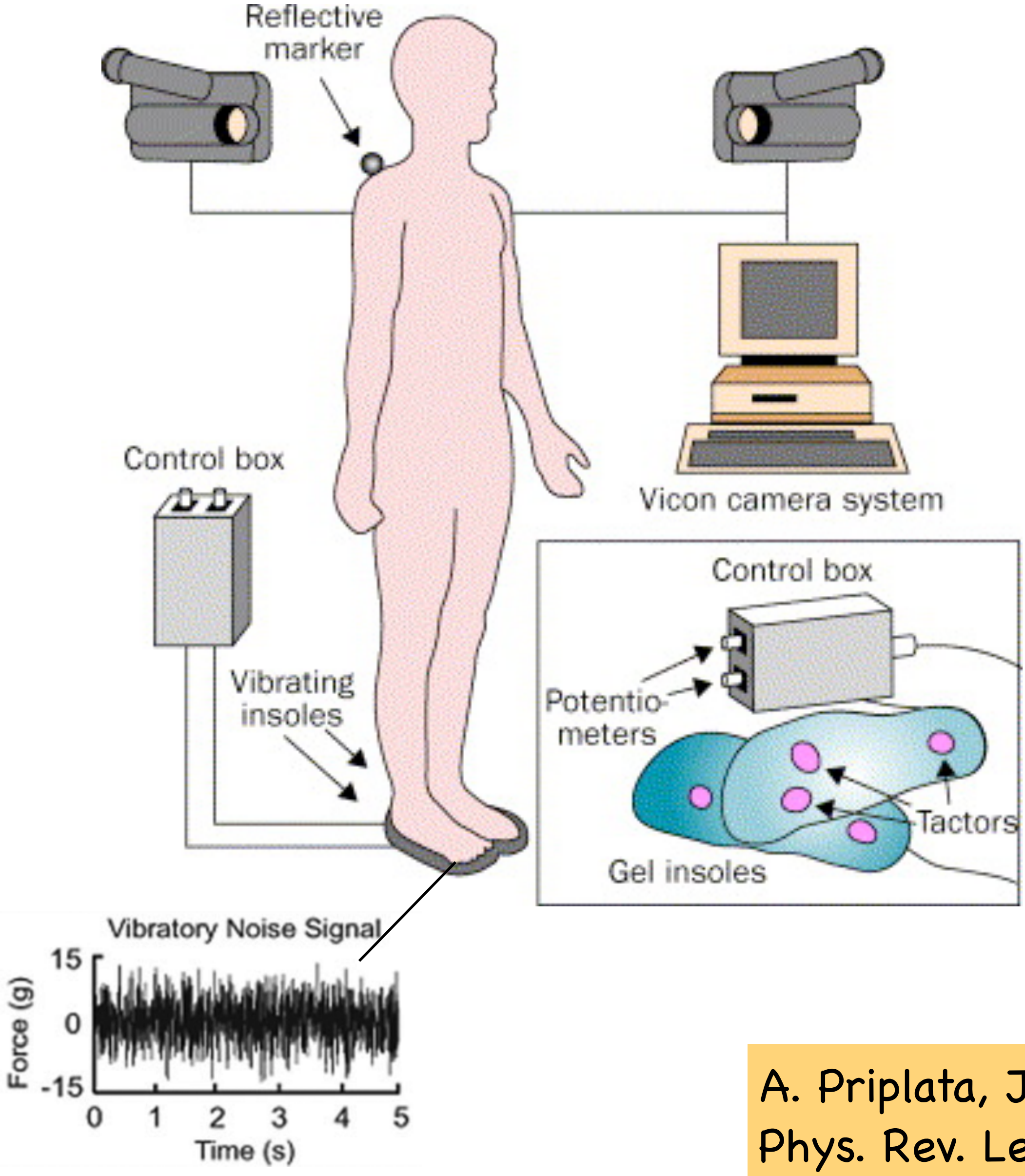




M. Riani, E. Simonotto, Nuovo Cimento D **17**, 903 (1995)

SR and human posture control

Somatosensory function declines with age and in diabetic patients. Can additional noise help restore function?



Reduction in sway of person

A. Priplata, J. Niemi, M. Salen, J. Harry, L.A. Lipsitz and J.J. Collins
Phys. Rev. Lett. 89 (2002)

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- Parkinson
- Multiple sclerosis (MS)
- Stroke / skull-brain-trauma
- Cross-section paralysis
- Depression
- Pain
- …

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- Depression
- Pain
- …

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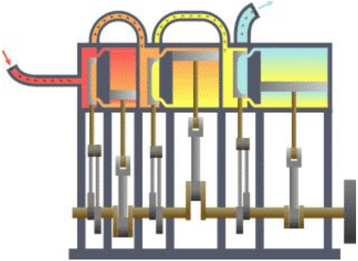
SR trends

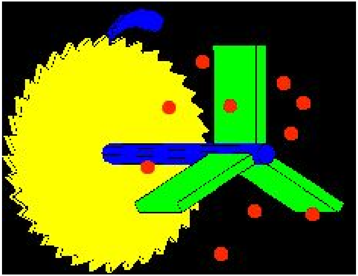
- Spatio – temporal SR
- Aperiodic SR
- Quantum SR

Motors $\Longrightarrow$ Brownian motors

Two heat reservoirs

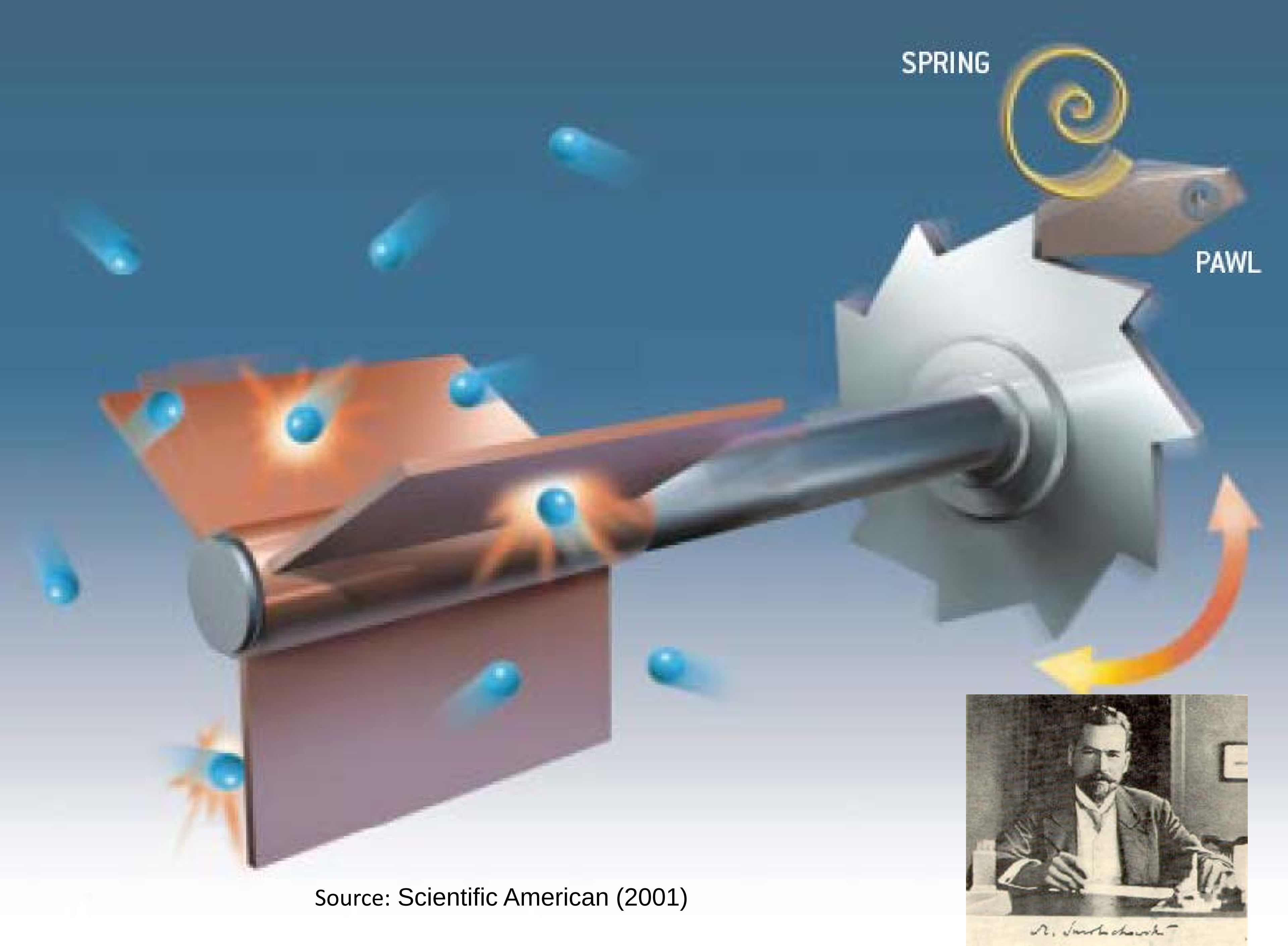


One heat reservoir



Perpetuum mobile of the second kind?

NO !

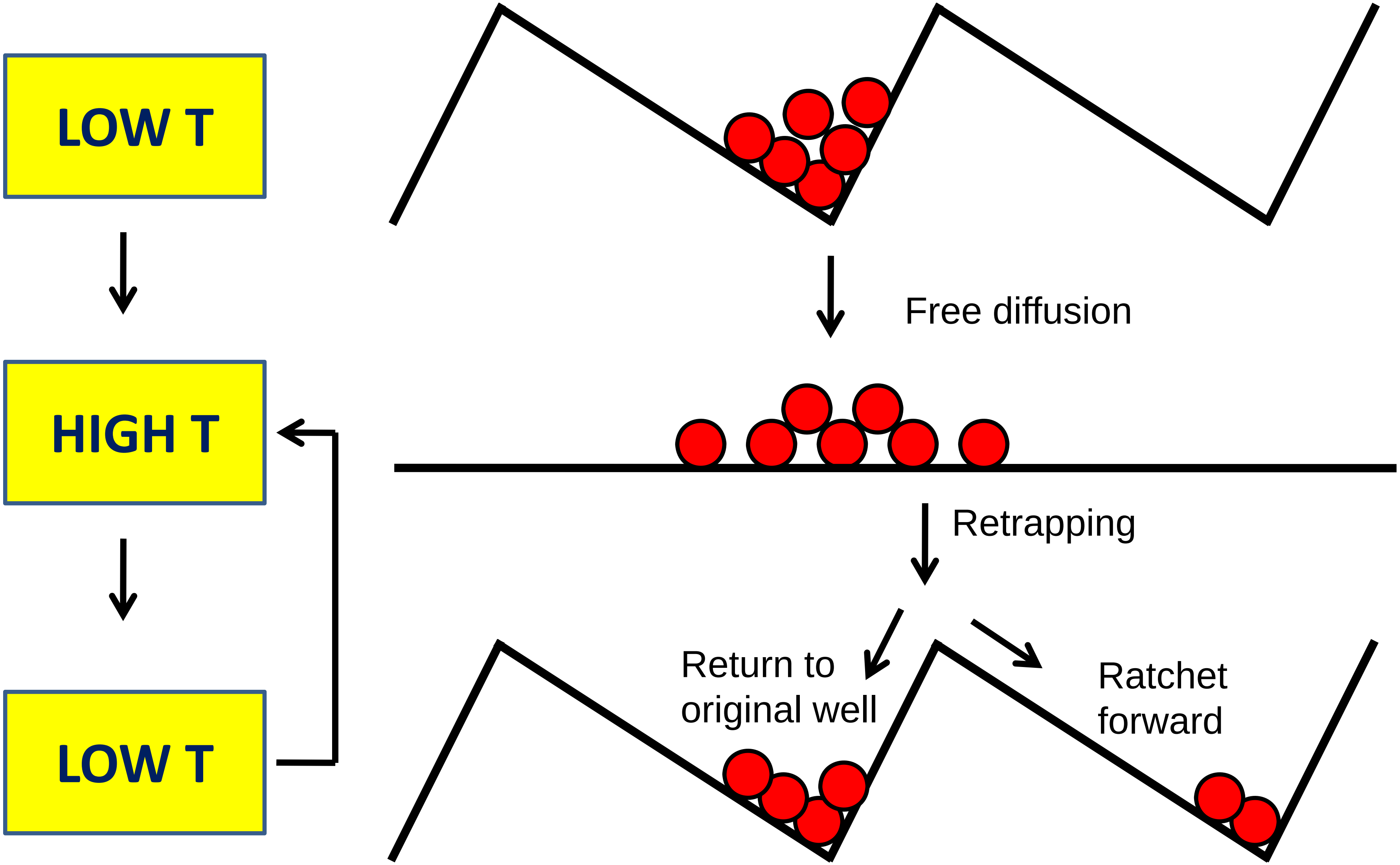


Source: Scientific American (2001)

Brownian motor

Movie

Temperature / Flashing Ratchet



Brownian motors - Characteristics

- Noise & AC-Input → **DC-Ouput**
- Non-equilibrium Noise → **Directed Transport**
- **Current reversals**
- **Applications:**
  - **Novel pumps and traps for charged or neutral particles**
  - **Brownian diodes & transistors**

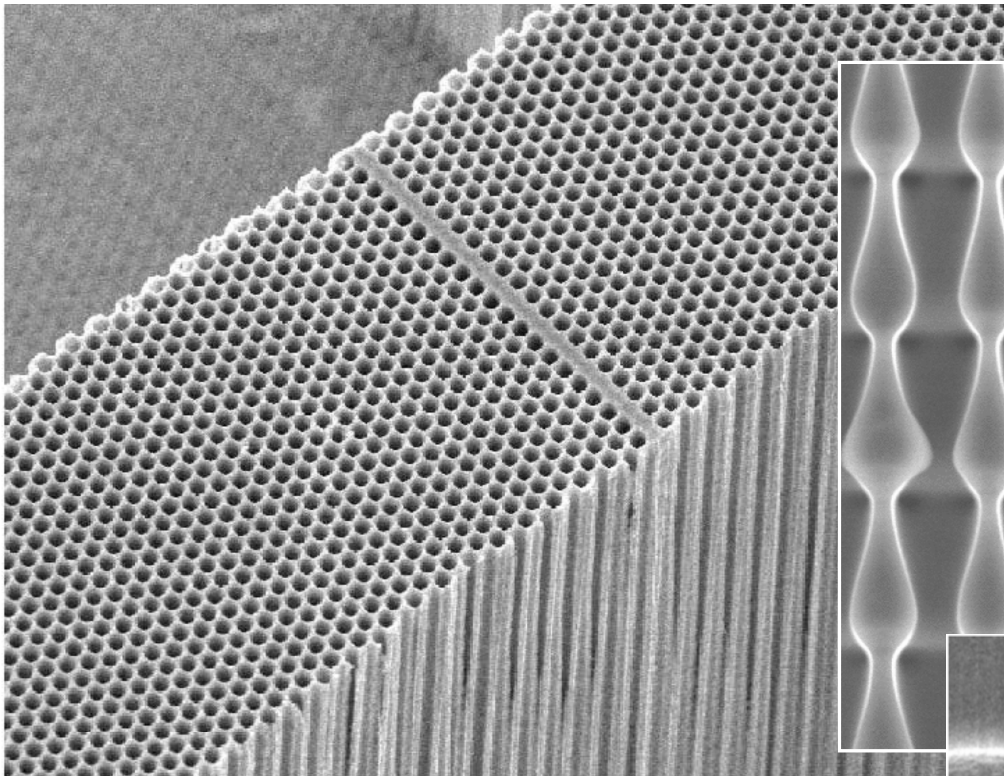
Ask not what physics can do for biology, ask what biology can do for physics

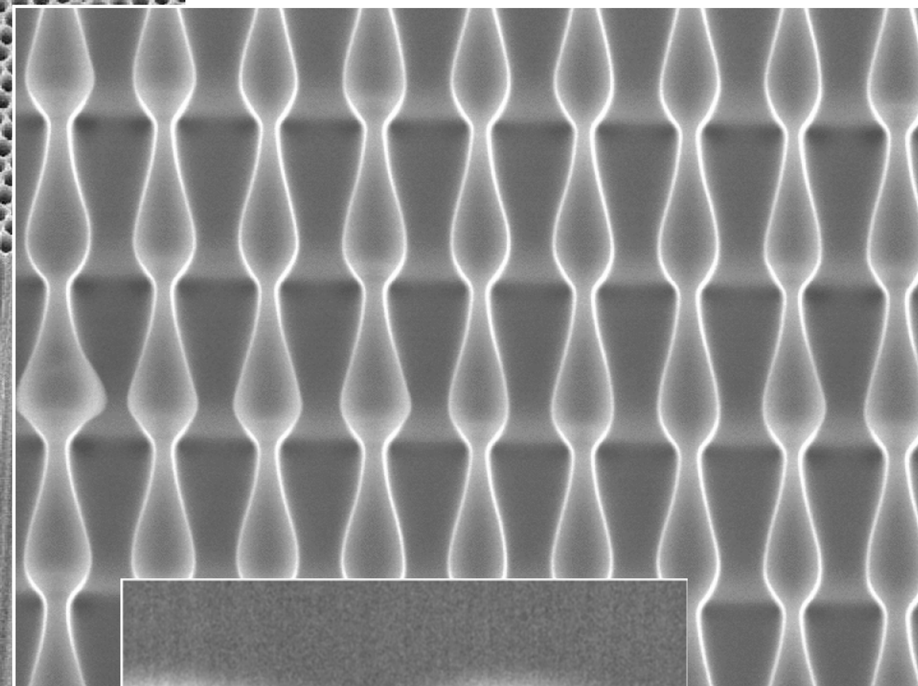
REVIEWS OF MODERN PHYSICS, VOLUME 81, JANUARY–MARCH 2009

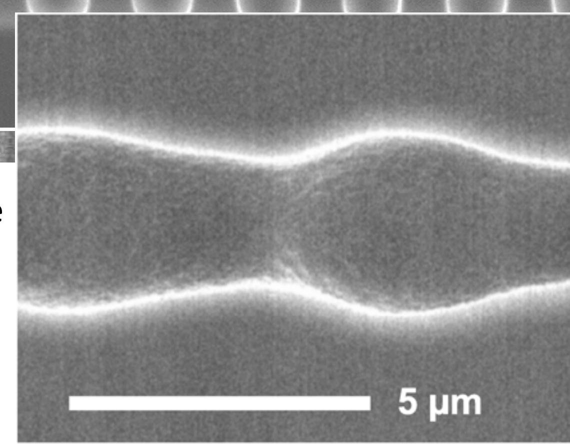
Artificial Brownian motors: Controlling transport on the nanoscale

P.H. and F. Marchesoni

Drift Ratchet - Device





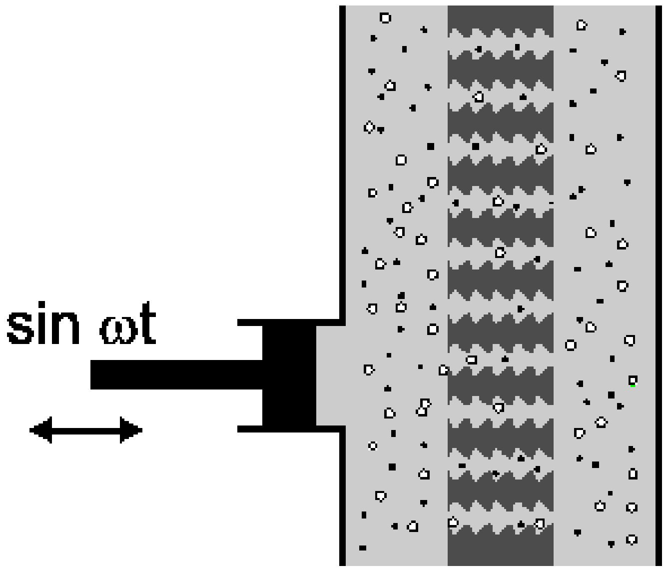


Source: F. Müller, MPI for microstructure physics, Halle

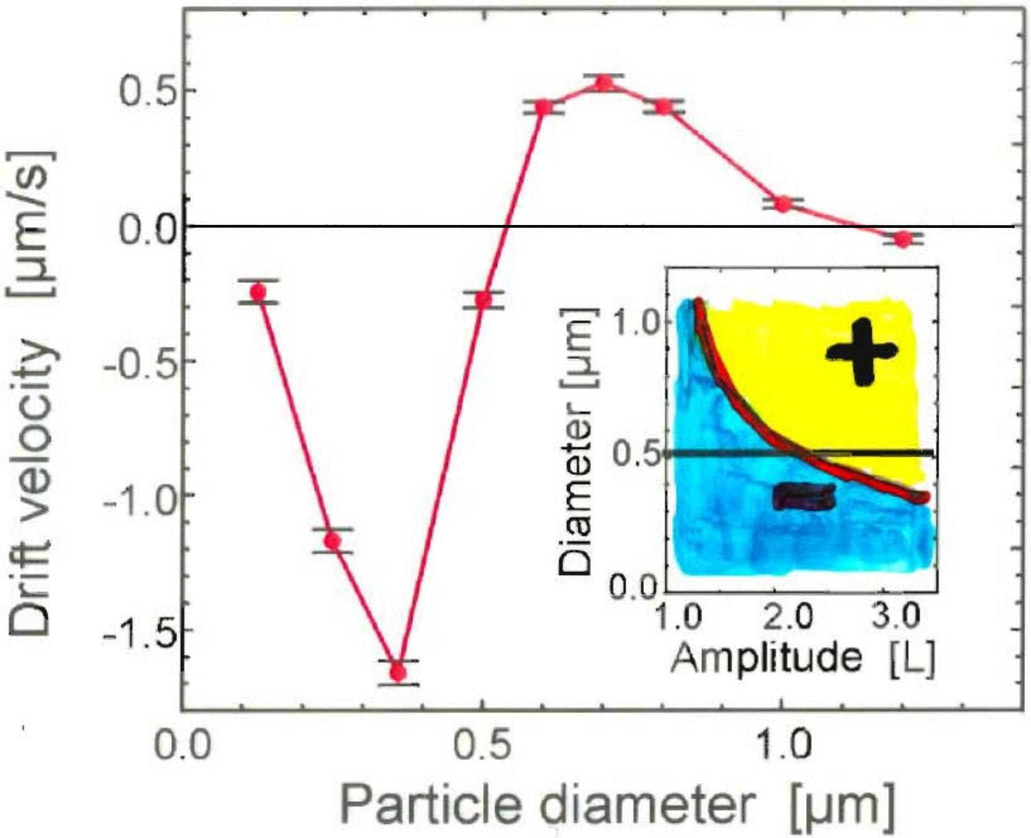
Drift Ratchet - Theory

C. Kettner, P. Reimann, P. H., F. Müller, Phys. Rev. E **61**, 312 (2000)

Setup

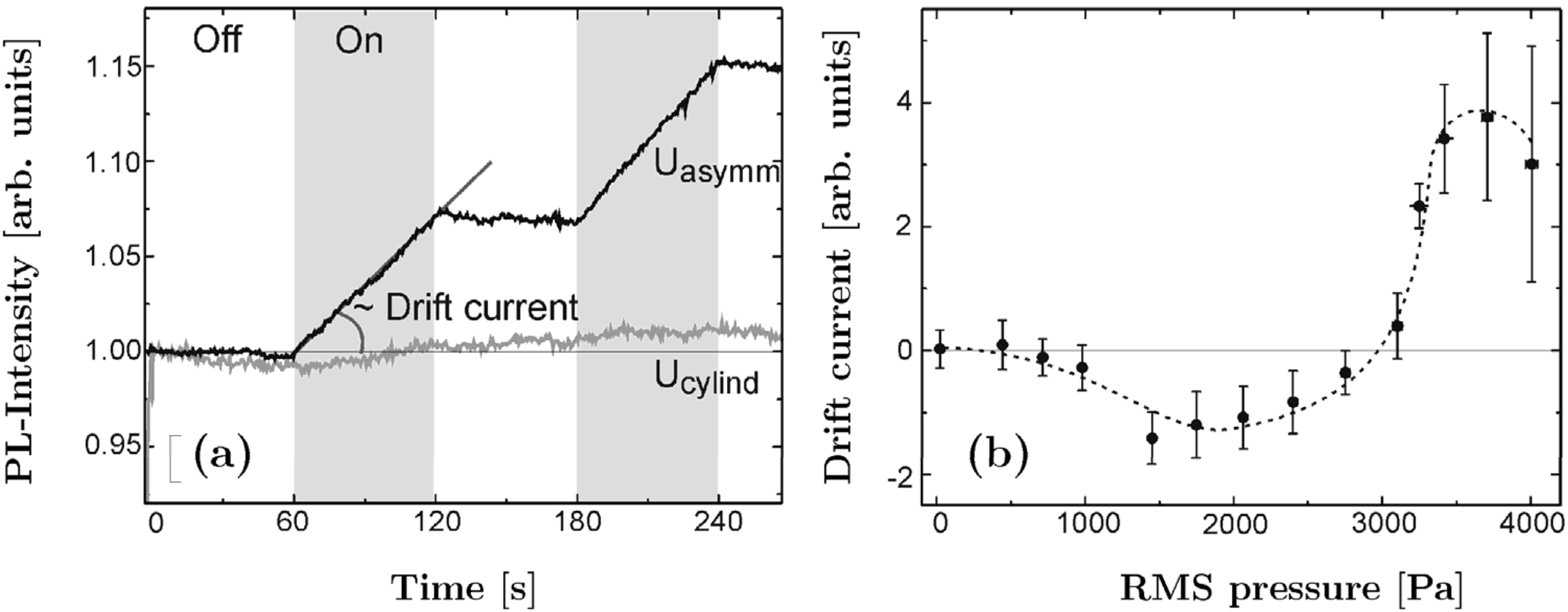


Particle Separation



Drift Ratchet – Experiment

S. Matthias, F. Müller, Nature **424**, 53 (2003)

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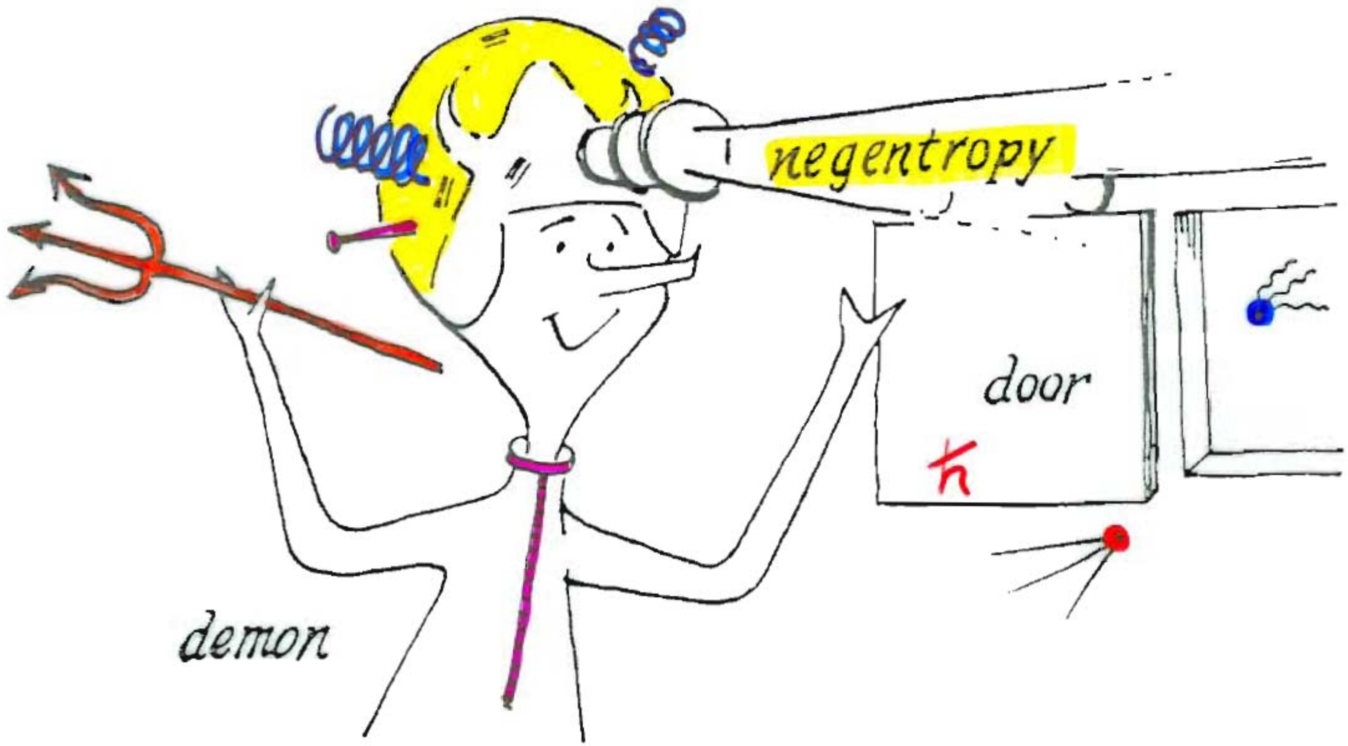
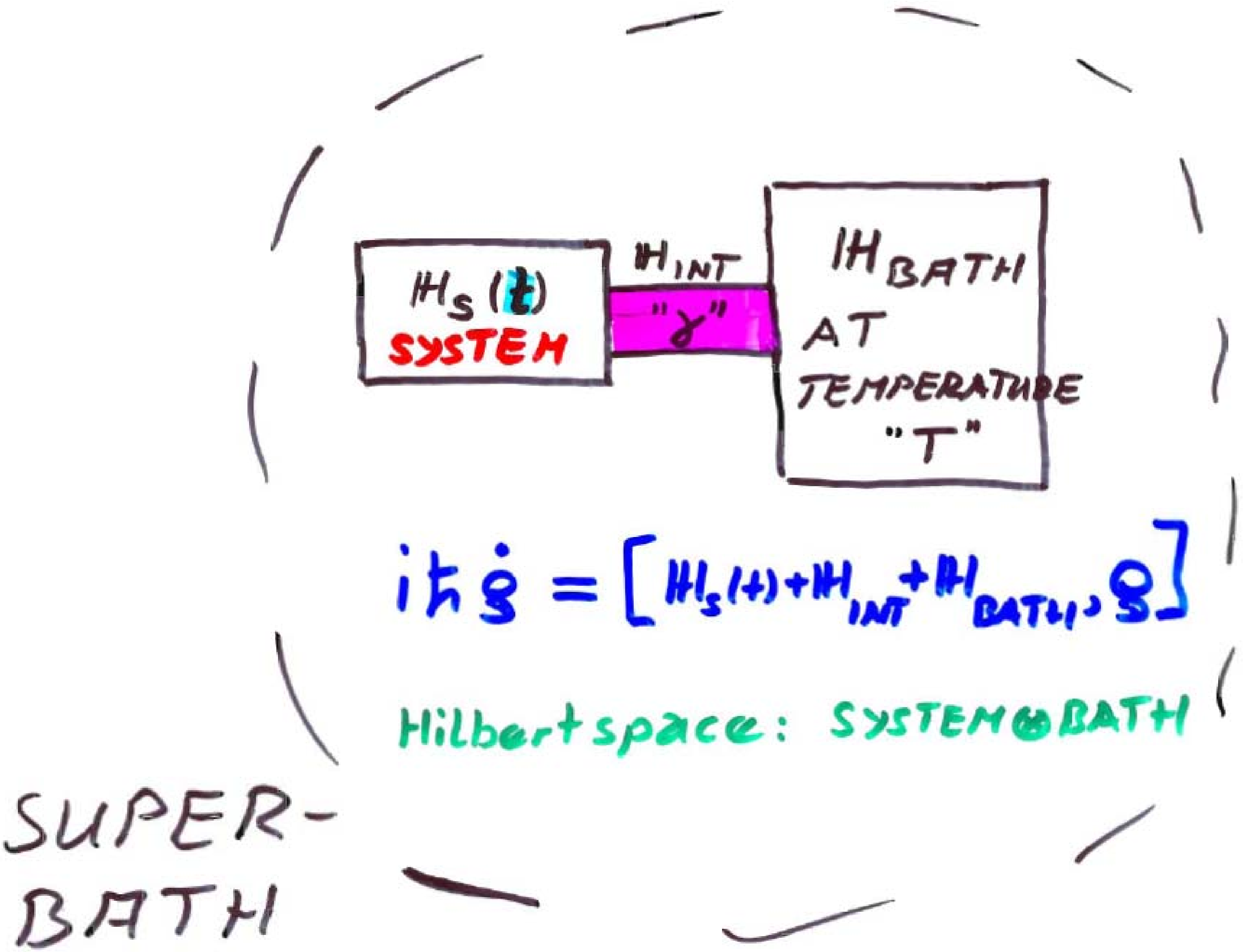
Quantum Demon ?

A measurement → Increase information → Reduction of entropy



Source: H.S. Leff, *Maxwell's Demon* (Adam Hilger, Bristol, 1990)

Quantum Brownian Motors



Quantum-Langevin-equation

$$m\ddot{\mathbf{x}}(t) + \int_{-\infty}^{t} \gamma(t-t')\,\dot{\mathbf{x}}(t')\,\mathrm{d}t' + V'(\mathbf{x};t) = \boldsymbol{\xi}(t)$$

$$\frac{1}{2}\langle \boldsymbol{\xi}(t)\boldsymbol{\xi}(s) + \boldsymbol{\xi}(s)\boldsymbol{\xi}(t)\rangle_{\text{bath}} = \frac{m}{\pi}\int_0^\infty \mathrm{Re}\hat{\gamma}(-i\omega+0^+)\,\hbar\omega\coth\left(\frac{\hbar}{2k_{\mathrm{B}}T}\right)\cos\left[\omega\left(t-s\right)\right]\,\mathrm{d}\omega$$

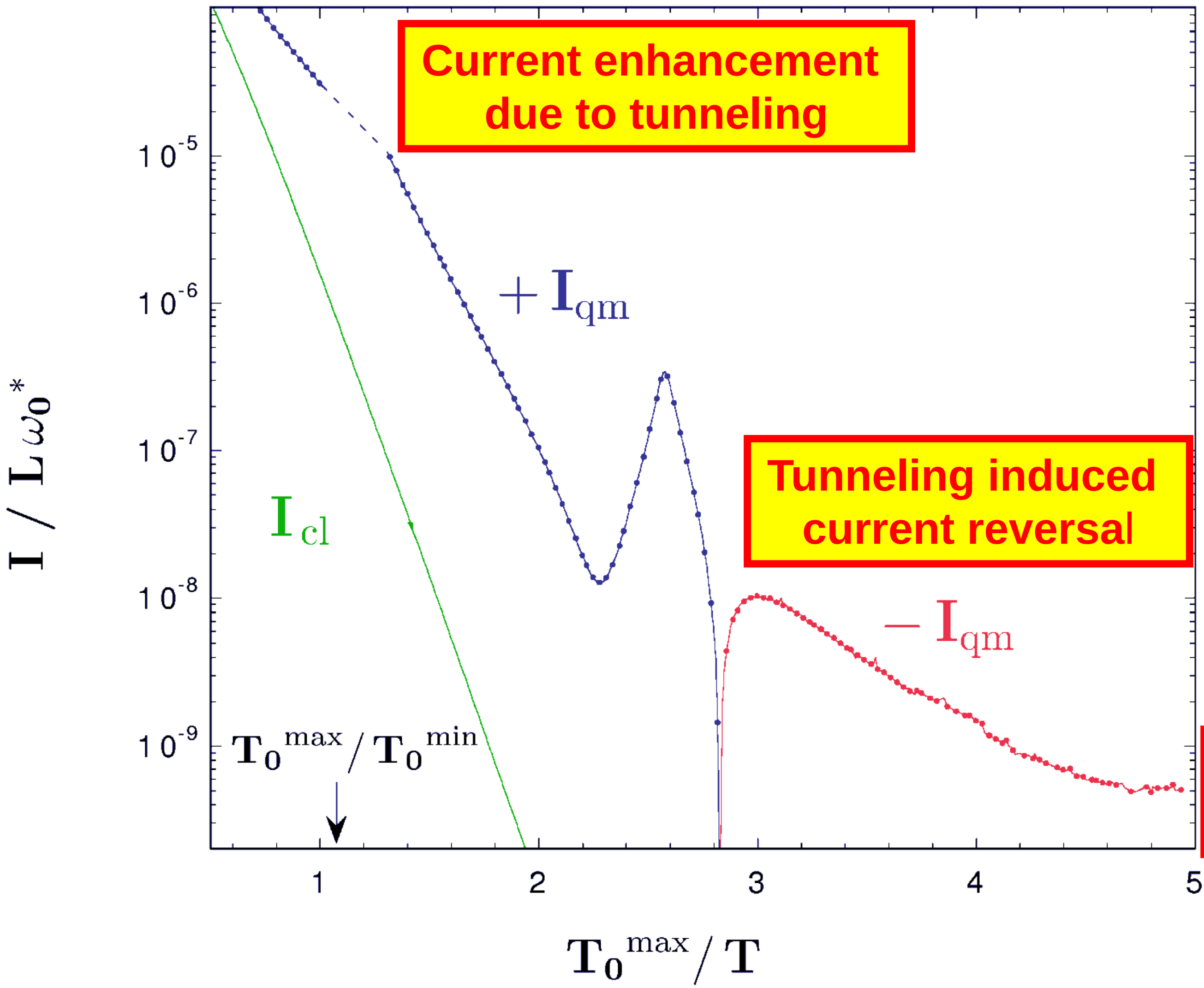
And:

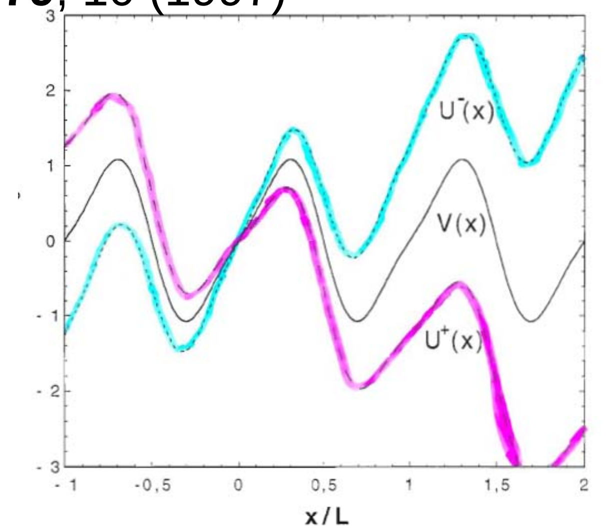
$$[\boldsymbol{\xi}(t),\boldsymbol{\xi}(s)] = -i\hbar\cdots \neq 0$$

!

Rocking Ratchet - Theory

P. Reimann, M. Grifoni, P. H., Phys. Rev. Lett. **79**, 10 (1997)





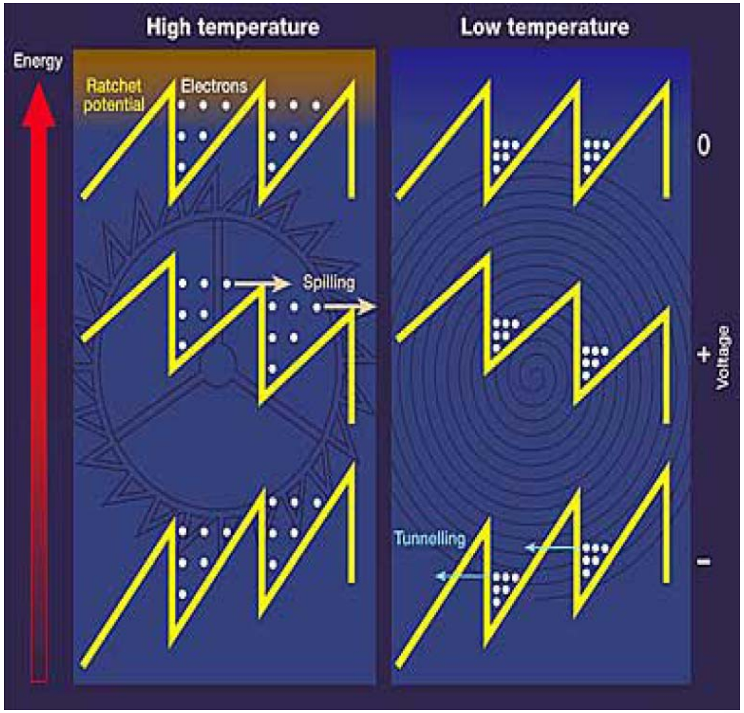
Current enhancement due to tunneling

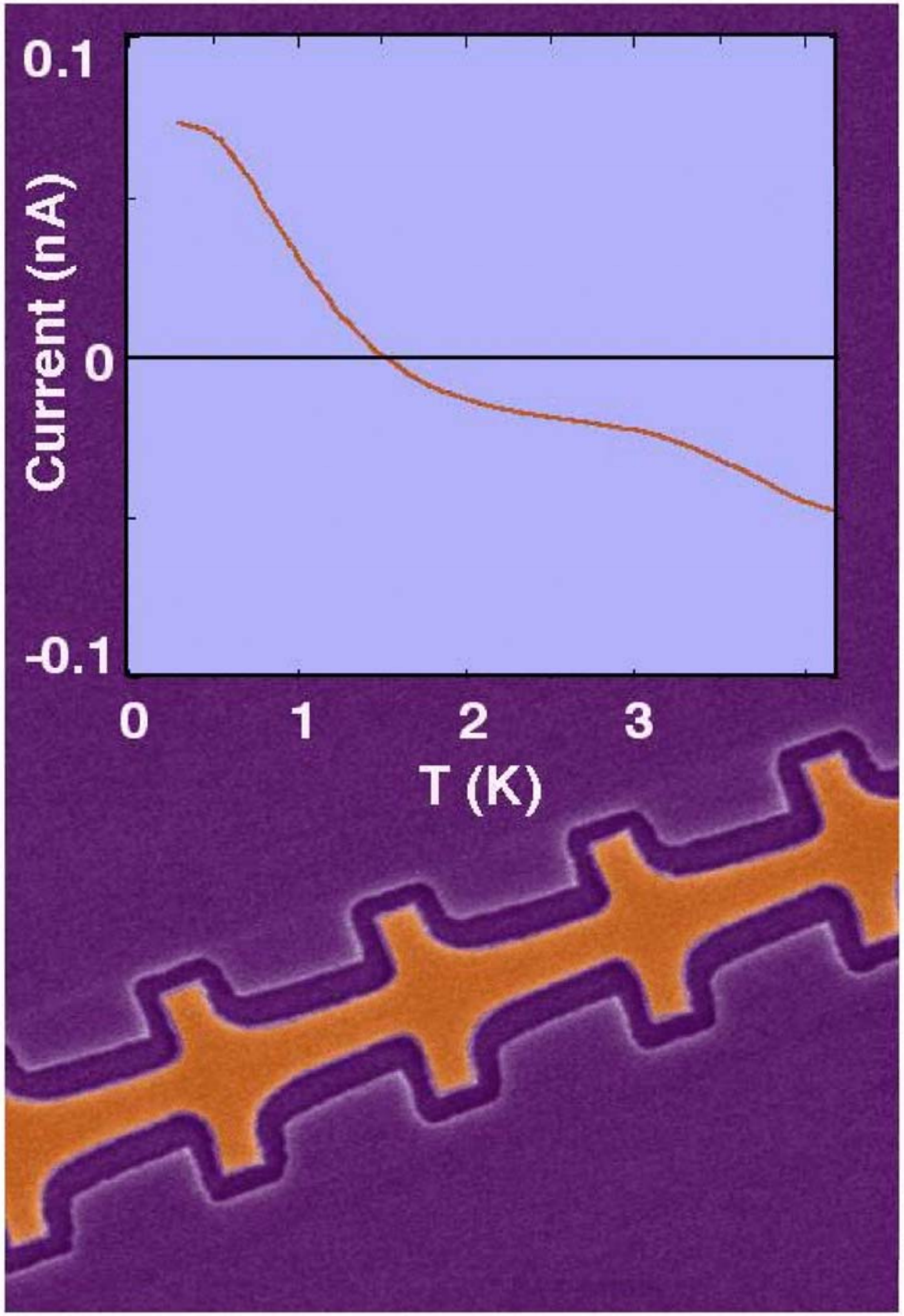
Tunneling induced current reversal

Finite current for T→0

Rocking QM Ratchet – Experiment

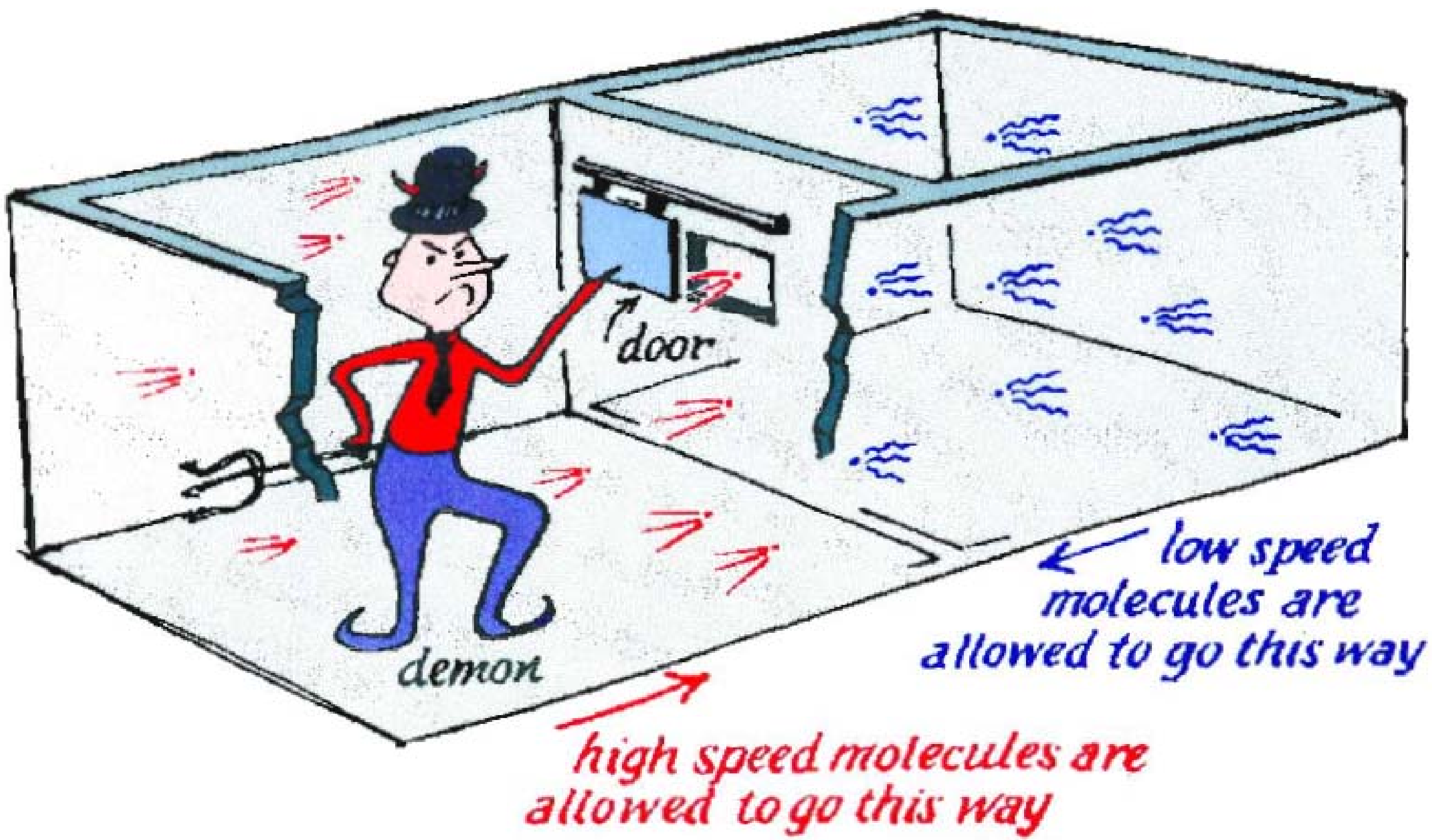
H. Linke, *et al.*,
SCIENCE **286**, 2314 (1999)





Brownian motors:

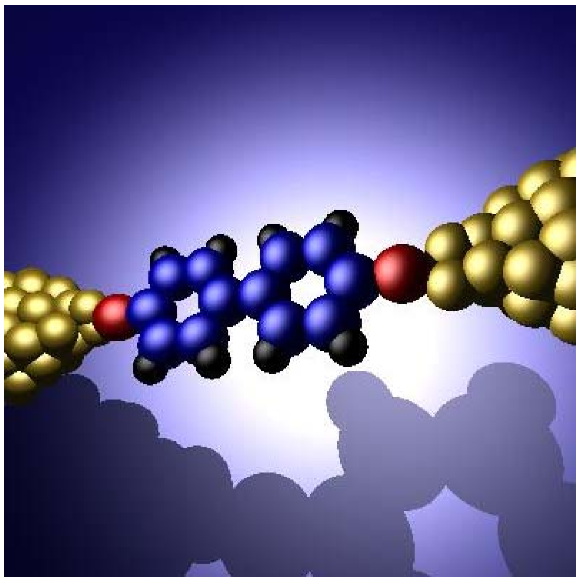
EX(E/O)RCISING DEMONS

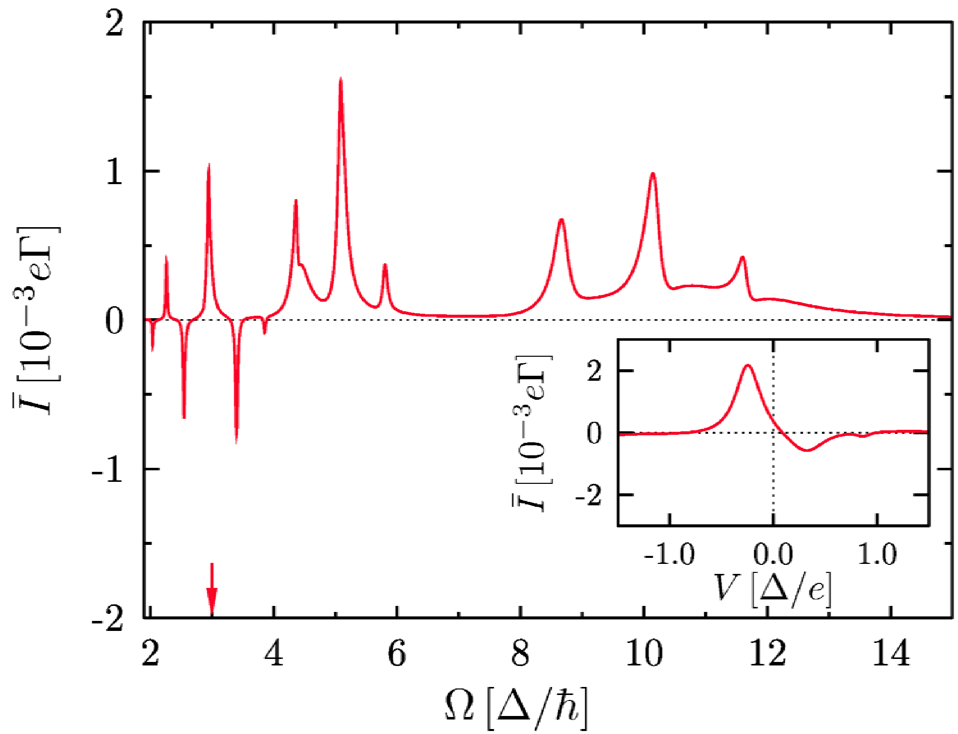


Source: H.S. Leff, *Maxwell's Demon* (Adam Hilger, Bristol, 1990)

Molecular wires

J. Lehmann, S. Kohler, P. H., A. Nitzan, Phys. Rev. Lett. **88**, 228305 (2002)

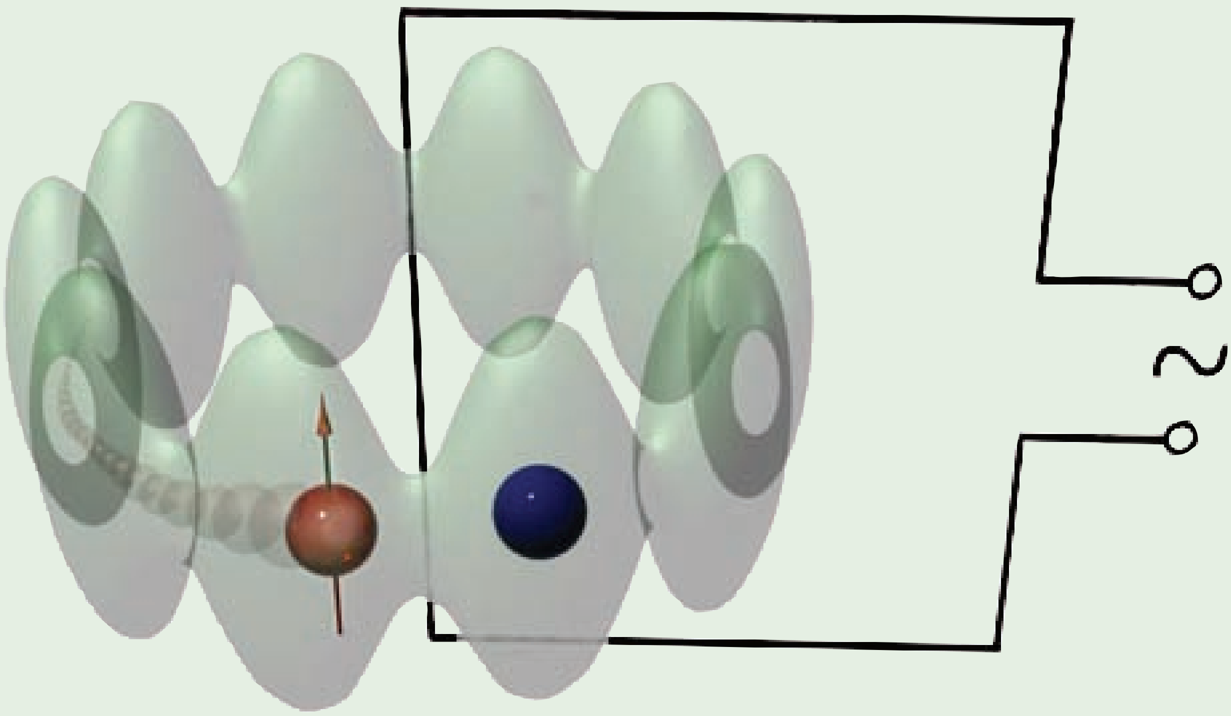




- Field strength E=106 V/cm
- Ω=3Δ corresponds to 4μm wavelength
- typical current: some nA

PHYSICAL REVIEW LETTERS

Articles published week ending 12 JUNE 2009



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Volume 102, Number 23

Quantum Gears: Driving Through Interactions

cold atoms (bosons, fermions)

$$\tilde{H} = H_R + H_B(t) + H_{RB}, \quad H_B(t+T) = H_B(t)$$

- targeting excitations
- experimentally feasible setup
- full many-body treatment

Generalizations of Brownian Motion

Brownian motion: Generalized Langevin-equation

Hamiltonian: $H_{\text{tot}} = H_{\text{sys}} + H_{\text{env}} + H_{\text{WW}}$

$$\Rightarrow \quad m\ddot{\mathbf{x}}(t) + \int_{-\infty}^{t} \gamma(t-t')\,\dot{\mathbf{x}}(t')\,\mathrm{d}t' + V'(\mathbf{x};t) = \boldsymbol{\xi}(t)$$

Asymptotically normal, anomalously fast, or anomalously slow
– via fractional Brownian motion –

$$\int_0^\infty \gamma(t)\,\mathrm{d}t = \begin{cases} \text{const} & \Rightarrow \text{normal} \\ 0 & \Rightarrow \text{superfast} \\ \infty & \Rightarrow \text{superslow} \end{cases}$$

Connection to the fractional Fokker-Planck-equation

Fractional Fokker-Planck equation

Subdiffusion (α<1):

$$\frac{\partial P(x,t)}{\partial t} = \left[\frac{\partial}{\partial x}\frac{V'(x,t)}{\gamma_\alpha} + K_\alpha \frac{\partial^2}{\partial x^2}\right] {}_0D_t^{1-\alpha} P(x,t)$$

Fat tails in the distribution of the residence times

Riemann-Liouville Operator

Superdiffusion (α>1):

$$\frac{\partial P(x,t)}{\partial t} = \left[\frac{\partial}{\partial x}\frac{V'(x,t)}{\gamma_\alpha} + K_\alpha \frac{\partial^{2/\alpha}}{\partial |x|^{2/\alpha}}\right] P(x,t)$$

Fat tails in the distribution of the jump lengths

Riesz-derivative

Fractional Fokker-Planck equation

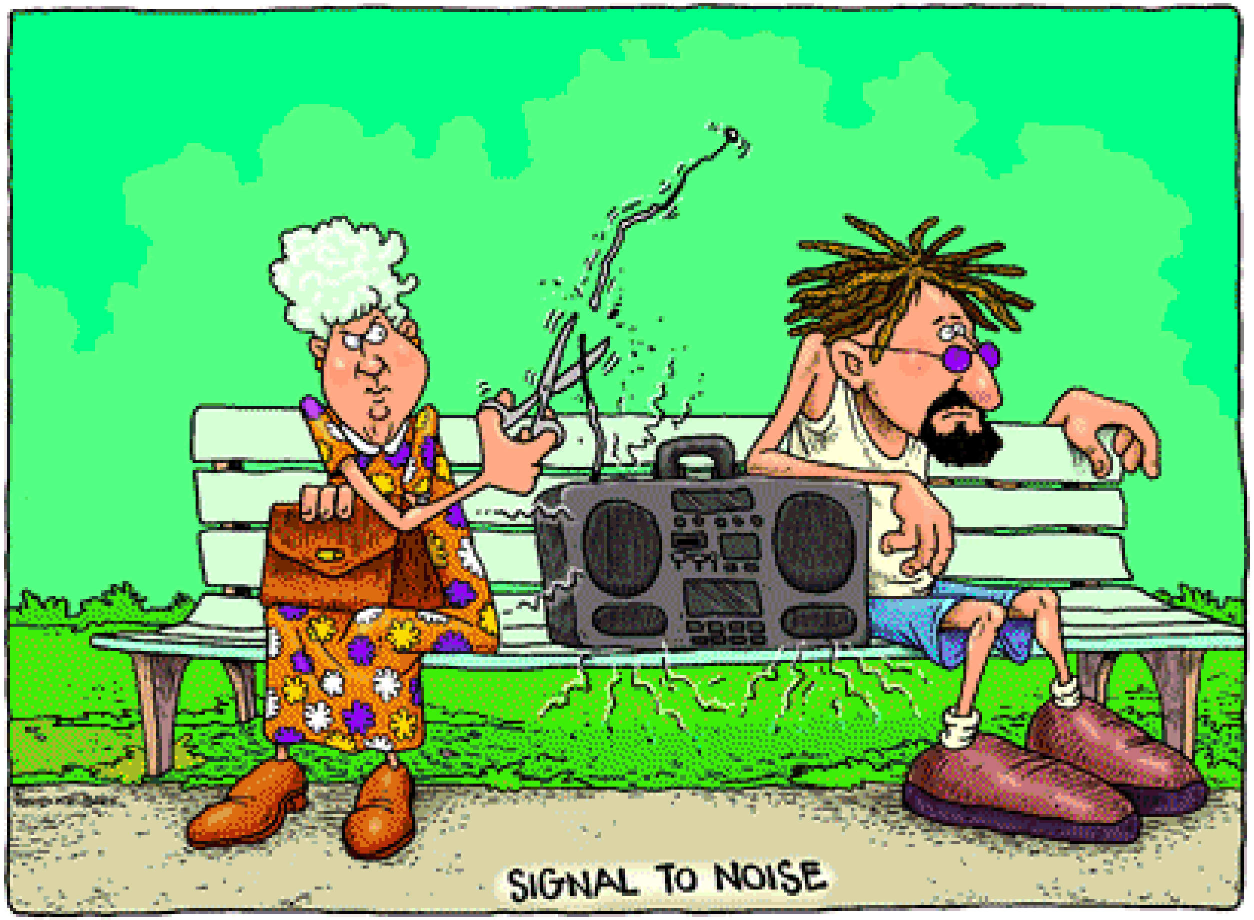
subdiffusive (α<1)

$$\frac{\partial P(x,t)}{\partial t} = \left[\frac{\partial}{\partial x}\frac{V'(x,t)}{\eta_\alpha} + K_\alpha \frac{\partial^2}{\partial x^2}\right] {}_0D_t^{1-\alpha} P(x,t)$$

Riemann-Liouville Operator

$${}_0D_t^{1-\alpha} f(t) = \frac{1}{\Gamma(\alpha)}\frac{d}{dt}\int_0^t \frac{f(t')}{(t-t')^{1-\alpha}} dt'$$

Noise – always bad ?



Source: Agilent Technologies

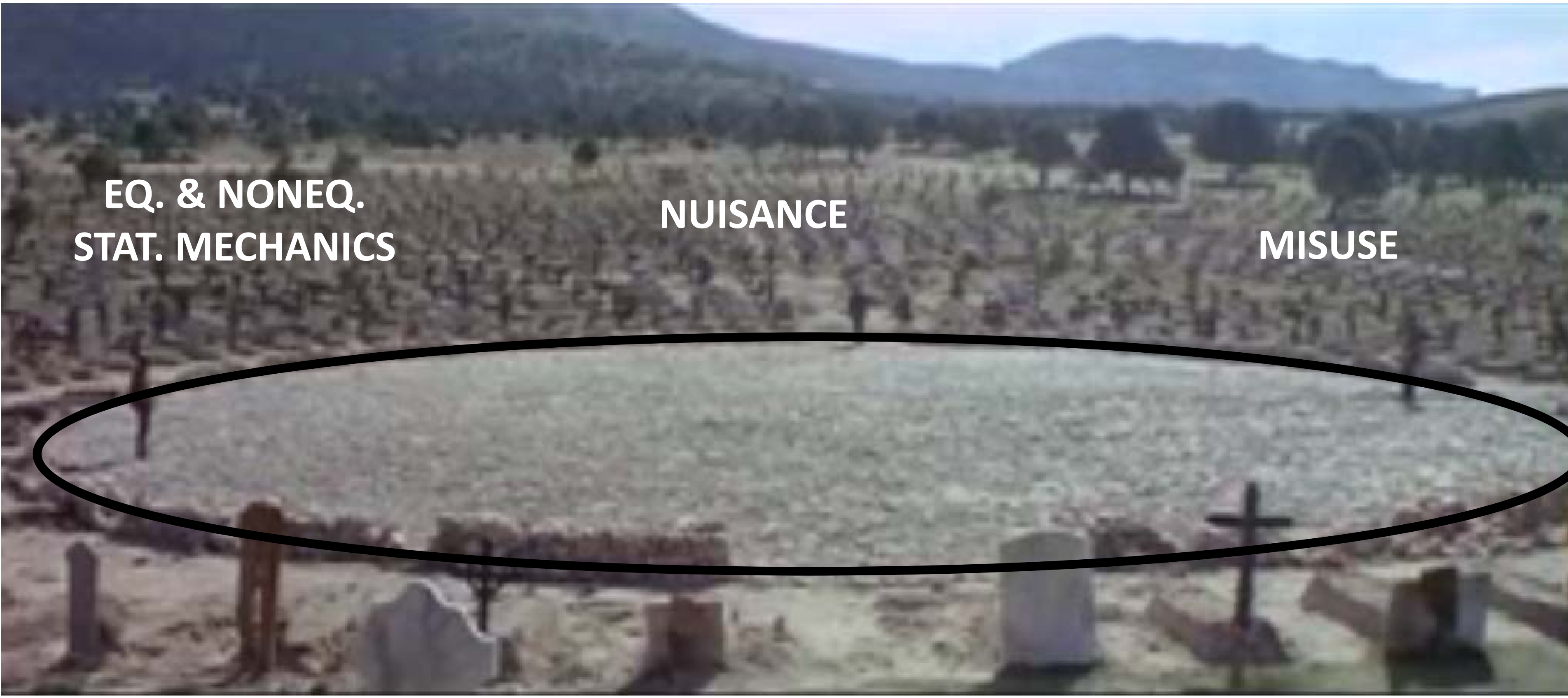
Brownian motion

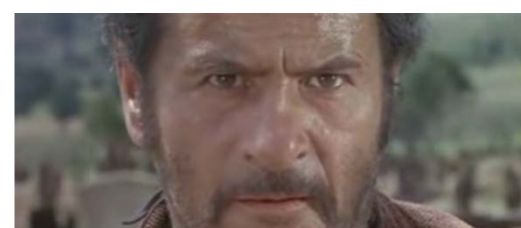
EQ. & NONEQ.
STAT. MECHANICS

NUISANCE

MISUSE

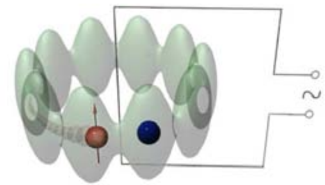
The good, the bad and the simply silly

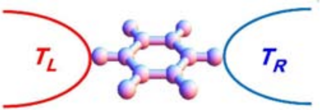




Source: www.youtube.com

Driven Many-Body Quantum Systems





- quantum machinery
- role of heat transport in nanodevices

What “is” thermodynamics ?

- ✓ non-local description in terms of symmetry (breaking) parameters

“Good” starting point in relativistic thermodynamics ?

- ✓ (non-)conserved tensor densities, Noether currents

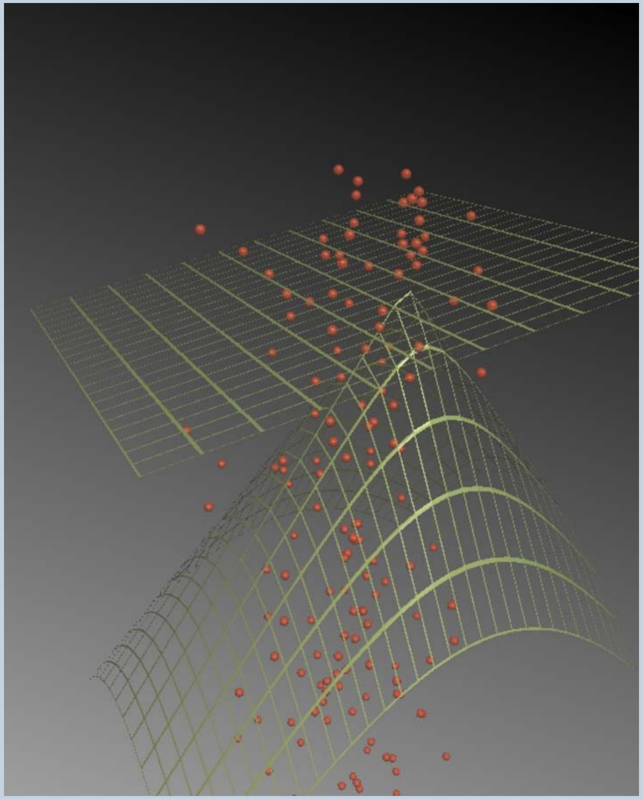
Origin of different temperature transformation laws ?

- ✓ choice of space-time hyperplanes
- ✓ definition of heat, formulation of 1st/2nd law

How should one define thermodynamic observables in special and general relativity?

- ✓ invariant manifolds, **lightcone integrals**

Observable consequence:
temperature-induced apparent drift



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