Sweep a qubit to entangle states and to gauge its environment

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prologue: the "standard" Landau-Zener problem



Landau, Zener, Stückelberg, Majorana (1932)

time-dependent two-level system

$$H(t) = -\frac{vt}{2}\sigma_z + \frac{\Delta}{2}\sigma_x$$

- diabatic states: $|\uparrow\rangle$, $|\downarrow\rangle$
- adiabatic states



finite times: numerical & analytical

(parabolic cylinder functions)





 $t \rightarrow \infty$: analytical

transition probability

$$P_{\uparrow \to \downarrow}(\infty) = 1 - \exp\left(-\frac{\pi\Delta^2}{2\hbar\nu}\right)$$

 large splitting Δ: adiabatic following, P_{↑→↓}(∞) = 1

Landau, Zener, Stückelberg, Majorana (1932)

alternative: complete summation of a perturbation series in $\frac{\Lambda}{2}\sigma_x$ Kayanuma (1984)

quantum information:

- qubit: two-level system $|\uparrow\rangle$, $|\downarrow\rangle$
- quantum gates: unitary operations

Landau-Zener transitions relevant for ...

✓ gate operations

🗸 ...

- ✓ manipulation of qubits
- ✓ quantum state preparation
- ✓ adiabatic quantum computing

PHYSICAL REVIEW A 69, 062320 (2004)

Cavity quantum electrodynamics for superconducting electrical circuits: An architecture for quantum computation

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TABLE I. Key rates and CQED parameters for optical [2] and microwave [3] atomic systems using 3D cavities, compared against the proposed approach using superconducting circuits, showing the possibility for attaining the strong cavity QED limit $(n_{\text{Rabi}} \ge 1)$. For the 1D superconducting system, a full-wave $(L=\lambda)$ resonator, $\omega_r/2\pi=10$ GHz, a relatively low Q of 10⁴, and coupling $\beta=C_g/C_{\Sigma}=0.1$ are assumed. For the 3D microwave case, the number of Rabi flops is limited by the transit time. For the 1D circuit case, the intrinsic Cooper-pair box decay rate is unknown; a conservative value equal to the current experimental upper bound $\gamma \le 1/(2 \mu s)$ is assumed.

Parameter	Symbol	3D optical	3D microwave	1D circuit
Resonance or transition frequency	$\omega_{\rm r}/2\pi, \Omega/2\pi$	350 THz	51 GHz	10 GHz
Vacuum Rabi frequency	$g/\pi, g/\omega_{\rm r}$	220 MHz, 3×10^{-7}	47 kHz, 1×10^{-7}	100 MHz, 5×10^{-3}
Transition dipole	d/ea ₀	~1	1×10 ³	2×10 ⁴
Cavity lifetime	1/ĸ,Q	10 ns, 3×10^{7}	$1 \text{ ms}, 3 \times 10^8$	160 ns, 10 ⁴
Atom lifetime	$1/\gamma$	61 ns	30 ms	2 <i>µ</i> s
Atom transit time	t _{transit}	≥50 <i>μ</i> s	100 µs	8
Critical atom number	$N_0 = 2\gamma\kappa/g^2$	6×10 ⁻³	3×10 ⁻⁶	≤6×10 ⁻⁵
Critical photon number	$m_0 = \gamma^2/2g^2$	3×10 ⁻⁴	3×10 ⁻⁸	≤1 × 10 ⁻⁶
Number of vacuum Rabi flops	$n_{\text{Rabi}} = 2g/(\kappa + \gamma)$	~10	~5	~10 ²

circuit QED





solid-state analogue of a two-level atom in an optical resonator (cavity QED)



Cooper-pair box = charge qubit resonator = (*LC*) oscillator tuneable parameters: gate voltage V_g magnetic flux ϕ_{ext}

circuit QED





• resonator as *LC* oscillator: $H = \hbar \Omega b^{\dagger} b$

• Cooper-pair box (CPB): charging energy

$$H_{\rm el} = 4E_{\rm C}(\hat{N} - N_{\rm g})^2$$
, $N_{\rm g} = \frac{C_{\rm g}V_{\rm g}}{2e}$

Josephson energy:

Blais *et al.* (2004) Wallraff *et al.* (2004)

$$H_{\rm J} = -\frac{E_{\rm J}}{2} \cos\left(\frac{2\pi\phi_{\rm ext}(t)}{\phi_0}\right) \sum_N \left(|N\rangle\langle N+1|+|N+1\rangle\langle N|\right)$$

• CPB-oscillator coupling: $H_{\text{int}} = \gamma (b^{\dagger} + b) \hat{N}$

 $E_{\rm C} \gg E_{\rm J}$ and $N_0 \le N_{\rm g} \le N_0 + 1$ \Rightarrow charge qubit: effective two-state system $|N_0\rangle$, $|N_0 + 1\rangle$ the qubit



"computational basis" in charge-degeneracy point $N_{\rm g} = N_0 + 1/2$:

$$|\uparrow\rangle = \frac{|N_0\rangle + |N_0 + 1\rangle}{\sqrt{2}} \qquad |\downarrow\rangle = \frac{|N_0\rangle - |N_0 + 1\rangle}{\sqrt{2}}$$

so that

$$H = \frac{vt}{2}\sigma_z + \gamma(b^{\dagger} + b)\sigma_x + \hbar\Omega b^{\dagger}b$$

typical parameter values:

- maximal Josephson energy $E_{J,max}/\hbar = v t_{max}/\hbar$: 10¹⁰ Hz, minimal switching time: 1 μ s
- oscillator frequency Ω : 10^8-10^9 Hz
- coupling strength $\gamma/2\pi\hbar$: 10⁶–10⁷ Hz; $\gamma/2\pi\hbar\Omega \simeq 10^{-3}$ –10⁻²
- [compare atom in cavity: $\gamma/2\pi\hbar\Omega \lesssim 10^{-6}$]



coupling to a single quantum oscillator



Saito, Wubs, Kohler, Hänggi, Kayanuma, Europhys. Lett. 76, 22 (2006)

qubit coupled to a single quantum oscillator



$$H(t) = \frac{vt}{2}\sigma_z + \gamma\sigma_x(b^{\dagger} + b) + \hbar\Omega b^{\dagger}b$$

- diabatic states: $|\uparrow, n\rangle$, $|\downarrow, n\rangle$
- initial state: |↑,0⟩
 (diabatic ground state)
- coupling $\sigma_x(b^{\dagger} + b)$ selection rule: $|\uparrow, 2\ell + 1\rangle$ and $|\downarrow, 2\ell\rangle$ never populated

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dynamics: numerical solution





- all $|\uparrow, n \neq 0\rangle$ finally unpopulated
- selection rule for odd *n* only



? hidden selection rule ?



series for $\langle \uparrow, n | U(\infty, -\infty) | \uparrow, 0 \rangle$ with perturbation $H_{\text{int}} = \gamma \sigma_x (b^{\dagger} + b)$



- Generalization of method by Kayanuma (1984)
- no contribution to $\langle \uparrow, n | U(\infty, -\infty) | \uparrow, 0 \rangle$
- only contribution: repeated jumps between $|\! \uparrow, 0 \rangle$ and $|\! \downarrow, 1 \rangle$

Volkov & Ostrovsky ('05); Dobrescu & Sinitsyn ('06); Saito et al. ('06)

no-go theorem





consequences:

- **1** no-go theorem for $t \to \infty$
- perturbation series for P_{↑→↑} consists of only the states |↑,0⟩ and |↓,1⟩
 - → no-go theorem for series
 - → same perturbation series as for standard LZ problem with $\Delta/2 \longrightarrow \gamma$

$$P_{\uparrow \rightarrow \downarrow}(\infty) = 1 - \exp\left(-\frac{2\pi\gamma^2}{\hbar v}\right)$$

Saito, Wubs, SK, Hänggi, Kayanuma, EPL 76, 22 (2006)





consequences:

- "no-go-up theorem" for $t \to \infty$
- Perturbation series for P_{↑→↑}(∞) involves only |↑,0⟩ and |↓,1⟩
 → same series as for standard LZ problem

$$P_{\uparrow \to \uparrow}(\infty) = \exp\left(-\frac{2\pi\gamma^2}{\hbar\nu}\right)$$

 beyond RWA, but ... independent of frequency Ω

quantum state preparation: single-photon generation



- $\gamma \ll \hbar \Omega$
- slow sweep $(v \ll \gamma^2/\hbar)$ $\rightarrow |\uparrow, 0\rangle \rightarrow |\downarrow, 1\rangle$ single-photon generation $P_{\uparrow \rightarrow \downarrow} = 1$

- slow sweep + cavity decay
 - → "single-photon-cycle"

K. Saito, M. Wubs, S. Kohler, P. Hänggi, Y. Kayanuma, Europhys. Lett. 76, 22 (2006)



• general sweep \rightarrow

 $\psi(\infty) = \alpha(v) |\! \uparrow, 0\rangle + \beta(v) |\! \downarrow, 1\rangle$

• controllable qubit-oscillator entanglement in circuit QED

•
$$P_{|\downarrow,1\rangle} = |\beta(\nu)|^2 = 1/2 \rightarrow$$

qubit-oscillator Bell state

M. Wubs, S. Kohler, P. Hänggi, Physica E 40, 187 (2007)



... coupled to two quantum oscillators



$$H(t) = \hbar\Omega b_1^{\dagger} b_1 + \gamma \sigma_x (b_1^{\dagger} + b_1) + \frac{\nu t}{2} \sigma_z + \gamma \sigma_x (b_2^{\dagger} + b_2) + \hbar(\Omega + \delta\omega) b_2^{\dagger} b_2$$

Wubs et al., Physica E 40, 187 (2007)

two transmission lines







R. Gross et al., WMI Garching

adiabatic energies:

- diabatic states: $|\uparrow, n_1, n_2\rangle, |\downarrow, n_1, n_2\rangle$
- initial state: $|\uparrow, 0, 0\rangle$

transition probability







no-go theorem:

- forbidden: many-photon states
- only contribution: jumps between oscillator ground state and single-photon states
- → spin-flip probability

$$P_{\uparrow \to \downarrow}(\infty) = 1 - \exp\left(-\frac{2\pi(\gamma_1^2 + \gamma_2^2)}{\hbar \nu}\right)$$

for $g_i \ll \Omega_1 = \Omega_2$: $|\psi_{\text{final}}\rangle = \alpha(v)|\uparrow, 0, 0\rangle + \beta(v)(|\downarrow, 1, 0\rangle + |\downarrow, 0, 1\rangle)$



- slow switching, $v \ll g^2/\hbar$: $\alpha(v) \approx 0$
 - → oscillator-oscillator entanglement
- intermediate $v: \alpha(v) = \beta(v) = \frac{1}{\sqrt{3}}$ \rightarrow W state

Wubs, Kohler, Hänggi, Physica E 40, 187 (2007)



coupling to a quantum heat bath



Caldeira-Leggett model: coupling to bath of harmonic oscillators

$$H = H_{\text{system}}(t) + X \sum_{\nu} \gamma_{\nu} (a_{\nu}^{\dagger} + a_{\nu}) + \sum_{\nu} \hbar \omega_{\nu} a_{\nu}^{\dagger} a_{\nu}$$

M. Wubs, K. Saito, S. Kohler, P. Hänggi, Y. Kayanuma, PRL **97**, 200404 (2006) K. Saito, M. Wubs, S. Kohler, Y. Kayanuma, P. Hänggi, PRB **75**, 214308 (2007)

coupling to a quantum heat bath



- $\cos \theta \neq 0$: displaced oscillator ground states
 - → diabatic states $|\uparrow, n_+\rangle$ and $|\downarrow, n_-\rangle$, note: generally $|n_+\rangle \neq |n_-\rangle$
 - → reorganization energy $E_0 = \sum_{\nu} \frac{\gamma_{\nu}^2}{4\hbar\omega_{\nu}}$
- effective coupling strength $S = \sum_{\nu} \gamma_{\nu}^2$
- zero temperature: initially in ground state of system-plus-bath





no-go-up theorem: (initial state: $|\uparrow, 0, 0, ...\rangle$)

- no contribution: many-photon states
- only contribution: jumps between oscillator ground state and single-photon states

$$P_{\uparrow \to \uparrow}(\infty) = \exp\left(-\frac{2\pi \sum_{\nu} \gamma_{\nu}^2}{\hbar \nu}\right)$$

M. Wubs, S. Kohler, P. Hänggi, Physica E 40, 187 (2007)



more general qubit-bath coupling:

$$H = \frac{vt}{2}\sigma_z + \frac{\Delta}{2}\sigma_x + (\sigma_z \cos\theta + \sigma_x \sin\theta) \sum_v \gamma_v (a_v^{\dagger} + a_v) + \sum_v \hbar \omega_v a_v^{\dagger} a_v$$

- σ_z "longitudinal coupling" $\Leftrightarrow \quad \theta = 0$
- σ_x "transverse coupling" $\Leftrightarrow \theta = \pi/2$

• reorganization energy
$$E_0 = \sum_{\nu} \frac{\gamma_{\nu}^2}{\hbar \omega_{\nu}} = \frac{\hbar}{4\pi} \int_0^\infty d\omega J(\omega) / \omega$$

• integrated spectral density
$$S = \sum_{\nu} \gamma_{\nu}^2 = \frac{\hbar^2}{4\pi} \int_0^\infty d\omega J(\omega)$$

• zero temperature: initially in ground state of system plus bath

- no-go-up theorem \rightarrow bath ends in ground state if qubit ends $|\uparrow\rangle$
- for $0 < P_{\uparrow \rightarrow \downarrow}(\infty) < 1 \rightarrow$ qubit-bath entanglement
- transition probability

$$P_{\uparrow \to \uparrow}(\infty) = \exp\left(-\frac{\pi W^2}{2\hbar v}\right), \quad W^2 = \left(\Delta - E_0 \sin\theta \cos\theta\right)^2 + S \sin^2\theta$$

exact solution for a dissipative quantum system

M. Wubs, K. Saito, S. Kohler, P. Hänggi, Y. Kayanuma, PRL 97, 200404 (2006)



- dependence on
 - reorganization energy *E*₀
 - total coupling strength S
- vary $\Delta \rightarrow$ determine E_0 , S... unless $\theta = 0$, sin $\theta = 0$

Sweep a qubit to entangle states and to gauge its environment

in the presence of a spin bath

... and beyond



Saito, Wubs, Kohler, Kayanuma, Hänggi, PRB 75, 214308 (2007)

spin bath



general coupling to a spin bath

$$H = -\frac{\nu t}{2}\sigma_z + \frac{\Delta}{2}\sigma_x + \sum_{i=x,y,z}\sigma_i\sum_{\nu}\gamma_{\nu}^i\tau_{\nu}^i + \sum_{\nu}\sum_{i=x,y,z}B_{\nu}^i\tau_{\nu}^i$$

• no-go theorem \rightarrow exact spin-flip probability

special case:

•
$$\gamma_v^x = \gamma_v^v = 0$$
 corresponds to $\theta = 0$
 $H_{\text{qubit-env}} = \sigma_z \sum_v \gamma_v^i \tau_z^i$

→ $W^2 = \Delta^2$, i.e. Landau-Zener probability bath-independent



? when is the LZ probability bath-independent ?

generalized phase noise:

$$H(t) = -\frac{vt}{2}\sigma_z + \frac{\Delta}{2}\sigma_x + \sigma_z \mathscr{X} + H_{\text{env}}$$

for arbitrary environment H_{env} and bath operator $\mathscr X$

$$P_{\uparrow \to \uparrow} = \exp\left(-\frac{\pi\Delta^2}{2\hbar\nu}\right) \qquad \text{for } T = 0$$

→ spin-flip probability bath-independent provided that

- *H*_{env} has a unique ground state
- the bath couples to σ_z only

adiabatic quantum computing:

- result encoded in ground state of a Hamiltonian H_f
- reach ground state by adiabatic time-evolution with H(t) with $H(t_{\rm f}) = H_{\rm f}$ \rightarrow requires ideal adiabatic following
- problem: LZ transitions at avoided crossings



 $|\downarrow\rangle$, adiabatic following

influence of heat bath?



$$P_{\uparrow \rightarrow \uparrow} = \exp\left(-\frac{\pi W^2}{2\hbar v}\right)$$
 with $W^2 = \left(\Delta - E_0 \sin\theta \cos\theta\right)^2 + S \sin^2\theta$

- $W > \Delta$: bath supports adiabatic following
- $W < \Delta$: bath increases error probability

bath with ohmic spectral density: $J(\omega) \propto \omega e^{-\omega/\omega_c} \rightarrow S = \hbar \omega_c E_0$



at zero temperature:

- $\theta = 0, \pi$: LZ probability bath-independent
- low cutoff frequency
 → non-adiabatic transitions

"general belief": thermal excitations support $\left|\uparrow\right\rangle\longrightarrow\left|\uparrow\right\rangle$

Ohmic phase noise, weak coupling classical noise spin bath Ao & Rammer, PRL'89

Kayanuma, PRB'98

Wan & Amin, J.Quant.Inf.'09

→ AQC error expected to increase with temperature

weak coupling: $g = 0.04\Omega$:



- non-monotonic *T* dependence
- ✓ adiabatic following improves for
 - larger damping γ
 - $k_{\rm B}T \sim \hbar \Omega$
- \checkmark ... but not for $k_{\rm B}T > \hbar\Omega$
- → AQC error may decrease with kT
- cf. non-monotonic γ -dependence at finite kT

Nalbach & Thorwart, PRL (2009), Le Hur et al.

PRA (2010), R.S. Whitney et al. PRL (2011)



- exact LZ transition probability for ground state
- quantum state preparation in circuit QED
 - single-photon generation
 - Bell states, W states
- dissipative LZ transitions at T = 0
 - adiabatic quantum computing
 - spin bath
- qubit-oscillator-bath at finite *T*
 - non-monotonic temperature dependence



Landau-Zener tunneling ...





coupling to a dissipative oscillator



Zueco, Hänggi, SK, New J. Phys. 10, 115012 (2008)

effective spectral density





- diagonalize oscillator-bath Hamiltonian
- → qubit plus bath with peaked spectral density
- → exact solution for $P_{\uparrow \rightarrow \downarrow}$ at T = 0

- ? oscillator dynamics
- ? finite temperatures



→ master equation for qubit + oscillator

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho = -\frac{\mathrm{i}}{\hbar}[H_{\mathrm{qubit}}(t) + H_{\mathrm{osc}},\rho] + \mathcal{L}_{\mathrm{diss}}[\rho]$$

• note: \mathcal{L}_{diss} depends on qubit+oscillator dynamics



test case:

- zero temperature
- weak q-osc coupling: $g = 0.04\Omega$
- ✓ reliable for $\gamma ≤ 0.1\Omega$ (error < 1%)

thanks to ...





This talk based on:

K. Saito, M. Wubs, S. Kohler, P. Hänggi, Y. Kayanuma, EPL **76**, 22 (2006)
M. Wubs, K. Saito, S. Kohler, P. Hänggi, Y. Kayanuma, PRL **97**, 200404 (2006)
K. Saito, M. Wubs, S. Kohler, Y. Kayanuma, P. Hänggi, PRB **75**, 214308 (2007)
M. Wubs, S. Kohler, P. Hänggi, Physica E (in press); cond-mat/0703425

thermally occupied initial state $|\uparrow, n\rangle$

- narrowly avoided crossings
- weak damping and weak coupling





thermally occupied initial state $|\uparrow, n\rangle$

- narrowly avoided crossings
- weak damping and weak coupling



- $|\uparrow,1\rangle$ has additional path to $|\downarrow\rangle$
- n = 1 most relevant for $k_{\rm B}T \sim \hbar \Omega$
- → $P_{\uparrow \rightarrow \downarrow}$ larger at intermediate temperatures





- upper level finally never populated → "no-go theorem"
 Brundobler & Elser (1993)
 Shytov (2004); Volkov & Ostrovsky (2005)
- here:
 - generalization to (infinitely) many states
 - corollary: "no-go theorem" for perturbation series

application to nanomagnets





• molecular Fe₈ cluster, spin S = 10

$$H = -DS_z^2 + E(S_x^2 - S_y^2) + g\mu_0 \vec{S} \cdot \vec{H}(t)$$

- determination of the very small splittings by LZ transitions Wernsdorfer & Sessoli, Science (1999)
- implicit assumption: individual LZ transition probabilities independent of dephasing Leuenberger & Loss, PRB (2000)

application to nanomagnets





• molecular Fe₈ cluster, spin S = 10

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- determination of the very small splittings by LZ transitions Wernsdorfer & Sessoli, Science (1999)
- implicit assumption: individual LZ transition probabilities independent of dephasing Leuenberger & Loss, PRB (2000)
- proof for T = 0 (K): this talk ($\theta = 0$)



• perturbation series in qubit-oscillator coupling: terms of the structure

$$\int_{-\infty}^{\infty} \mathrm{d}t_1 \int_{t_1}^{\infty} \mathrm{d}t_2 \int_{t_2}^{\infty} \mathrm{d}t_3 \dots \exp\left[\mathrm{i}\sum_{\ell} \left(\lambda_{\ell}\Omega t_{\ell} + \frac{\nu}{2\hbar}(t_{2\ell}^2 - t_{2\ell-1}^2)\right)\right]$$

where $\lambda_{\ell} = \pm 1$ (from b^{\dagger} and b)

• substitution to time differences

$$\int_{-\infty}^{\infty} \mathrm{d}t_1 \int_0^{\infty} \mathrm{d}\tau_2 \mathrm{d}\tau_3 \dots$$

• first integral provides $\delta\left(\nu \sum_{\ell} \tau_{\ell} + \Omega \sum_{\ell=1}^{2k} \lambda_{\ell}\right)$

• since all
$$\tau_{\ell} \ge 0 \Rightarrow \sum_{\ell}^{2\kappa} \lambda_{\ell=1} \le 0$$

• all terms with "<" have vanishing prefactors $\rightarrow \lambda_{\ell} = (-1)^{\ell}$



- What is a Landau-Zener transition?
- Experiment: circuit QED
- Qubit coupled to a quantum oscillator
- Dissipative Landau-Zener transitions

no-go theorem





upper level finally never populated
 → "no-go theorem"

Brundobler & Elser (1993) Shytov (2004); Volkov & Ostrovsky (2005); Dobrescu & Sinitsyn (2006)

• generalization to infinitely many levels

series for $\langle \uparrow, n | U(\infty, -\infty) | \uparrow, 0 \rangle$ with perturbation $H_{\text{int}} = \gamma \sigma_x (b^{\dagger} + b)$



- no contribution
- only contribution: repeated jumps between $|\!\uparrow,0\rangle$ and $|\!\downarrow,1\rangle$