Quantum Dissipation: A Primer

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QUANTUM DISSIPATION

\[ L = \frac{1}{2} m_0 e^{\gamma t} x^2 - \frac{1}{2} m_0 e^{-\gamma t} \dot{x}^2 \]

\[
\frac{d}{dt} \frac{\partial L}{\partial \dot{x}} = \frac{d}{dt} m_0 e^{\gamma t} \dot{x} = m_0 e^{\gamma t} \ddot{x} + m_0 e^{\gamma t} \dot{x}
\]

\[- \frac{\partial L}{\partial x} = m_0 e^{\gamma t} \omega_0^2 x \]

\[
\Rightarrow e^{-\gamma t} [m_0 \ddot{x} + m_0 \gamma \dot{x} + m_0 \omega_0^2 x] = 0
\]

QM: \[ L \rightarrow H = \frac{p^2}{2m_0} e^{-\gamma t} + \frac{1}{2} m_0 e^{\gamma t} \omega_0^2 x^2 \]
QUANTUM DISSIPATION

\[ L = \frac{1}{2} m_0 e^{\gamma t} x^2 - \frac{1}{2} m_0 e^{\gamma t} \omega_0^2 x^2 \]

\[ \frac{d}{dt} \frac{\partial L}{\partial \dot{x}} = \frac{d}{dt} m_0 e^{\gamma t} \dot{x} = m_0 e^{\gamma t} \ddot{x} + m_0 e^{\gamma t} \dot{\gamma} \]

\[ - \frac{\partial L}{\partial x} = m_0 e^{\gamma t} \omega_0^2 x \]

\[ \Rightarrow e^{\gamma t} [ m_0 \ddot{x} + m_0 \dot{\gamma} \dot{x} + m_0 \omega_0^2 x ] = 0 \]

QM: \[ L \Rightarrow H = \frac{p^2}{2m_0} e^{-\gamma t} + \frac{1}{2} m_0 e^{\gamma t} \omega_0^2 x^2 \]

\[ [q_1, p] = +i \hbar e^{-\gamma t} \]
NOISE-INDUCED ESCAPE

rate = A(y) \frac{\omega_0}{2\pi} \exp(-\Delta V/D)

RMP 62: 251 (90)
\[ \Gamma = \pi \cdot \frac{\omega_0}{2n} \exp\left(-\frac{E_b}{kT}\right) \]

\[ \text{TST} \]

thermal equilibrium

P. H., P. TALKNER, M. BORKOVEC
REV. MOD. PHYS. 62: 251 (1990)
Reaction-rate theory: fifty years after Kramers

Peter Hänggi, Peter Talkner, Michal Borkovec

RMP 62: 251 (90)
THE PROBLEM

potential

thermal activation

coordinate $q$

$M\ddot{q} + \frac{dU}{dx} + \eta \dot{q} = 0$

ENVIRONMENT
**FACTS**

- **H on (110) tungsten [Gomer (82)]**

- **H₂ & HD sorbed in zeolites [Bouchard et al. (82)]**

- **CO-migration in hemoglobin [Frauenfelder]**

- **Tunneling in a Josephson junction subjected to memory friction [Esteve et al. (79)]**

---

**Graphs:**

1. **Graph (a):**
   - Log-log plot of reaction rate constants ($Т^{-1}$) as a function of temperature ($1000/Т$).
   - The graph shows two distinct regions: Arrhenius and tunneling.

2. **Graph (b):**
   - Log-log plot of diffusion coefficient ($D$) as a function of temperature ($1000/Т$).
   - Two datasets are shown: $D(H₂)$ and $D(HD)$, with corresponding fit lines.

3. **Graph (c):**
   - Log-log plot of tunneling times ($τ$) as a function of temperature ($t_0$).
   - The graph shows a clear exponential decay as temperature decreases, reaching $T = 18 mK$. The term "Theory!" is added as a note.
### Results

<table>
<thead>
<tr>
<th>Quantum Tunneling</th>
<th>Crossover</th>
<th>Quantum Corrections</th>
<th>Thermal Activation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k = A \exp(-B)$</td>
<td>2-0-modes</td>
<td>$k = T^2 \sigma$</td>
<td>$k = A(\eta) e^{-E_0/\delta T}$</td>
</tr>
<tr>
<td>$B = S_B(T, \gamma)$</td>
<td>$\frac{S_B}{\hbar} = \frac{E_B}{kT_0}$</td>
<td>quantum enhancement</td>
<td>$\gamma = \eta / \mu$</td>
</tr>
<tr>
<td>$B(T=0) = B(\eta=0)$</td>
<td>smooth!</td>
<td></td>
<td>Kramers</td>
</tr>
<tr>
<td>$A(\gamma) = A(\gamma)$</td>
<td>Erfc-behavior</td>
<td></td>
<td>$\left{ \frac{\nu^2}{4} + \omega_0^2 \right}^\frac{1}{2}$</td>
</tr>
<tr>
<td>$\propto A_0(1+2.30y)$, $y \to 0$</td>
<td>$E_0 \to E_0 - \frac{c}{T}$</td>
<td></td>
<td>$\nu = \frac{\omega_0}{2\pi}$</td>
</tr>
</tbody>
</table>
Quantum transmission coefficient

\[ \kappa = \frac{k}{k_{cl,TST}} \]

\[ k_{cl,TST} = \frac{1}{2\pi \hbar} \frac{k_b T}{Z} e^{-\beta E_b} \]
The calculation of rate coefficients is a discipline of nonlinear science of importance to much of physics, chemistry, engineering, and biology. Fifty years after Kramers' seminal paper on thermally activated barrier crossing, the authors report, extend, and interpret much of our current understanding relating to theories of noise-activated escape, for which many of the notable contributions are originating from the communities both of physics and of physical chemistry. Theoretical as well as numerical approaches are discussed for single- and many-dimensional metastable systems (including fields) in gases and condensed phases. The role of many-dimensional transition-state theory is contrasted with Kramers' reaction-rate theory for moderate-to-strong friction; the authors emphasize the physical situation and the close connection between unimolecular rate theory and Kramers' work for weakly damped systems. The rate theory accounting for memory friction is presented, together with a unifying theoretical approach which covers the whole regime of weak-to-moderate-to-strong friction on the same basis (turnover theory). The peculiarities of noise-activated escape in a variety of physically different metastable potential configurations is elucidated in terms of the mean-first-passage-time technique. Moreover, the role and the complexity of escape in driven systems exhibiting possibly multiple, metastable stationary nonequilibrium states is identified. At lower temperatures, quantum tunneling effects start to dominate the rate mechanism. The early quantum approaches as well as the latest quantum versions of Kramers' theory are discussed, thereby providing a description of dissipative escape events at all temperatures. In addition, an attempt is made to discuss prominent experimental work as it relates to Kramers' reaction-rate theory and to indicate the most important areas for future research in theory and experiment.

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Acknowledgments

LIST OF SYMBOLS

A (T) temperature-dependent quantum rate prefactor
C (t) correlation function
D diffusion coefficient
E energy function
E_b activation energy (= barrier energy with the energy at the metastable state set equal to zero)
E_A Hessian matrix of the energy function at the stable state
E_S Hessian matrix of the energy function around the saddle-point configuration
I action variable of the reaction coordinate
J Jacobian
K (x, x') transition probability kernel
M mass of reactive particle
P(E) period of oscillation in the classically allowed region
P(E, E') classical conditional probability of finding the energy E, given initially the energy E'
Q quantum correction to the classical prefactor
S_b dissipative bounce action
T temperature
T_c crossover temperature
T(E) period in the classically forbidden regime
U(x) metastable potential function for the reaction coordinate
V volume of a reacting system
Z partition function, inverse normalization
Z_0, Z_A partition function of the locally stable state (A)
Z^+ partition function of the transition rate
H Hamiltonian function of the metastable system
F complex-valued free energy of a metastable state
L Fokker-Planck operator
L^* backward operator of a Fokker-Planck process
j total probability flux of the reaction coordinate
h Planck's constant
h (2π)^{-1}
K_B Boltzmann constant
k reaction rate
k^+ forward rate
k^- backward rate
k_{TST} transition-state rate
k(E) microcanonical transition-state rate, semiclassical cumulative reaction probability
k_S mass of ith degree of freedom
p(x, t) probability density
p_0(x) stationary nonequilibrium probability density for the reaction coordinate
p_i configurational degree of freedom
q_i quantum reflection coefficient
r(E) density of sources and sinks
s(x) quantum transmission coefficient
t(E) mean first-passage time to leave the domain \Omega, with the starting point at x
t_{MFP} constant part of the mean first-passage time to leave a metastable domain of attraction
v = \dot{x} velocity of the reaction coordinate
x location of well minimum or potential minimum of state A, respectively
x_b barrier location
x_T location of the transition state
\beta inversion temperature (k_B T)^{-1}
**microscopic approach**

\[
H_{\text{total}} = \frac{1}{2} M \dot{q}^2 + U(q)
\]

**system**

\[
+ \frac{1}{2} \sum_\alpha m_\alpha \dot{q}_\alpha^2 + \sum_\alpha m_\alpha \omega_\alpha^2 q_\alpha^2
\]

**(harmonic) bath**

\[
+ q_f \sum_\alpha c_\alpha q_\alpha
\]

**linear coupling**

\[
+ q^2 \sum_\alpha \frac{c_\alpha^2}{2 m_\alpha \omega_\alpha^2}
\]

**compensation of frequency shift**

- path integral approach to density matrix at temperature \( T \)
- trace out environment
dissipation

\[ H^T = H_{\text{system}} + H_{\text{bath}} + H_{\text{Int}} \]

\[ \ddot{q} = -\frac{1}{M} \frac{\delta U}{\delta q} - \int_{0}^{t} \tilde{g}(t-s) \dot{q}(s) \, ds \]

\[ S_E = S_{\text{rev. motion}} + S_{\text{(nonlocal)} \text{ dissipation}} \]
QUANTUM NOISE
QUANTUM LH-EQ.

\[ |0\rangle_{S+B} \neq |0\rangle_S |0\rangle_B \]

**DECOHERENCE AT** \( T = 0 \)

\[ H_{S+B} = H_S + H_{S-B} + H_B \]

\[ = \frac{\mathbf{p}^2}{2m} + V(\mathbf{x}) + \sum \left[ \frac{\mathbf{p}^2}{2m_x} + \frac{m_x c^2}{2} \left( \frac{\mathbf{q}_x - \mathbf{c}_x}{m_x c} \right)^2 \right] \]

\[ S_S \neq 2^{-1} \exp \left( -\frac{H_S}{\hbar T} \right) ! \]

\[ S_{Total} = S_{S+B} = 2^{-1} \exp \left( -\frac{H_{S+B}}{\hbar T} \right) \]
\[ i \dot{s} = [0, H_t] \]

\[ m \ddot{x} + m \int_0^t \int_0^s y(t-s) \dot{x}(s) + \frac{\partial V(x)}{\partial x} \]

\[ = \eta(t) - m y(t-0) \times 10 \]

INITIAL SLIP

\[ \gamma(t-s) = \frac{1}{m} \sum \frac{c_i^2}{m^2 a_i^2} \cos(a_i(t-s)) \]

\[ = \gamma(s-t) \]

\[ \eta(t) = \sum c_i [q_i(0) \cos(a_i t) + \frac{m_i}{m^2 a_i^2} \sin(a_i t)] \]
\[ [\eta(+) , \eta(s) ] = -i \hbar \sum \frac{e^2}{m} \sin (kx(t-s)) \neq 0 \]
\[ \sigma_B = 2^{-1} \exp \left\{ -\beta \left[ \sum \left( \frac{1 \sigma^2}{2m} + \frac{m \beta^2}{2} \right) \right] \right\} \]
\[ < \eta(+) > \sigma_B = 0 \]

\[ \frac{1}{2} < \eta(t) \eta(s) + \eta(s) \eta(+) > = C(t-s) \]
\[ = C(\tau) = \frac{1}{2} \sum \frac{e^2}{m} \coth \left( \frac{\pi \hbar k}{2kT} \right) \cos (\omega \tau) \]

\[ \hbar \tau \gg \frac{\hbar k}{2} \]
\[ \rightarrow kT \gamma (\tau) \]
\[ \hat{\delta}(z) = \int_0^L \exp(-zt) j(t) \, dt \]

\[ \delta(\omega) = \delta(z = -i\omega) \]

**Ohmic Dissipation**

\[ J(\omega) = \gamma \omega \exp\left(-\omega / \omega_c\right) \]

**Kondo-Parameter**

\[ \gamma = \frac{2\pi \hbar}{a^2} \times \omega \exp\left(-\omega / \omega_c\right) \]

\[ a = 2q a : \text{tunneling length} \]
1. QLE OPERATES IN FULL HILBERT SPACE OF $e \otimes b$

\[ \delta(t) = \sum_{\omega} \epsilon^{i \omega t} f(t) \text{dt} = \frac{i}{2m} \sum \frac{\epsilon_{\omega}}{\omega} \left[ \frac{1}{\omega^2 - \omega_{\alpha}^2} + \frac{1}{\omega^2 - \omega_{\beta}^2} \right] \]

\[ \frac{1}{\omega + i \eta} = P\left( \frac{1}{\omega} \right) - i \eta \delta(x) \quad \text{Im} \eta > 0 \]

\[ \text{Re} \delta(t = \omega + i \eta) = \frac{\pi}{2m} \sum \frac{\epsilon_{\omega}}{\omega^2} \left[ d(\omega - \omega_{\alpha}) + d(\omega + \omega_{\alpha}) \right] \]

\[ C(t) = \frac{m}{\pi} \int_0^\infty dw \, \text{Re} \delta(w + i \eta) \cos(\omega t) \cdot \coth \left( \frac{\hbar \omega}{2kT} \right) \]

2. with $\delta(t) = \hbar f(t) - m g(t) \times (0)$

\[ \delta_B = 2^{-1} \exp \left\{ - \beta \left[ \sum \left( \frac{\epsilon_{\omega}^2}{2m_{\epsilon}} + \frac{\epsilon_{\omega}^2}{2} \left( \frac{\epsilon_{\omega} - \epsilon_{\omega_{\alpha}}}{m_{\epsilon} \omega_{\alpha}} \right)^2 \right] \right\} \]

\[ \Rightarrow \langle \delta(t) \rangle_B = 0 \]

\[ \frac{1}{2} \langle \delta(t) \delta(0) + \delta(0) \delta(t) \rangle_B = C(t) \]
4. DEPHASING AT $T = 0$ !

$$\langle x(0) \tilde{y}(t) \rangle_{\beta} \neq 0$$

$$\langle \mathbf{H}_{\text{INT}} \rangle_{\beta} \neq 0$$

5.

$\tilde{y}(t) \rightarrow \text{C-NOISE } \tilde{y}(t)$

WITH CORRELATION $C(\tau)$

IS INCONSISTENT
\[ \hat{H}(t) = \hat{H}_0 - F(t)\hat{A} \quad \text{and} \quad g_\beta = Z^{-1} \exp(-\beta \hat{H}_0) \]

\[
<\hat{B}(t)> - <\hat{B}(t)>_\beta = <\delta\hat{B}(t)> = \int_{t_0}^{t} \chi(t-s) F(s) ds
\]

**Kubo:**

\[
\chi_{BA}(\tau) = \Theta(\tau) \frac{i}{\hbar} \left< [\hat{B}(\tau), \hat{A}(0)] \right> = -\Theta(\tau) \int_0^\infty \left< \hat{A}(\tau + \lambda) \hat{B}(\tau) \right> d\lambda
\]

Classical limit: \(-\Theta(\tau) \beta \left< \hat{B}(t) \hat{A}(0) \right>\)
\[ \hat{H}(t) = \hat{H}_0 - F(t) \hat{A} \; ; \; g_\beta = Z^{-1} \exp(-\beta \hat{H}_0) \]

\[ \langle \hat{B}(t) \rangle - \langle \hat{B}(t) \rangle_0 = \langle \delta \hat{B}(t) \rangle = \int_0^t \chi(t-s) F(s) ds \]

**Kubo:** \[ \chi_{BA}^{(\tau)}(\tau) = \Theta(\tau) \frac{i}{\hbar} \langle [\hat{B}(\tau), \hat{A}(0)] \rangle_\beta \]

\[ = -\Theta(\tau) \oint \langle \hat{A}(-i\lambda) \hat{B}(\tau) \rangle_\beta d\lambda \]

**Classical limit:** \[ \rightarrow -\Theta(\tau) \beta \delta \langle \hat{B}(\tau) A(0) \rangle_\beta \]

\[ \hat{B} = \hat{A} = \hat{q} \; ; \; F(t) = A \cos \Omega t \]

\[ \langle \delta q(t) \rangle = P_1 e^{-i\Omega t} + P_{-1} e^{i\Omega t} \]

\[ P_{1,-1} = \frac{A}{2} e^{\pm i\Omega t} \chi(\pm \Omega) \]
QUANTUM - FDT

\[ S_{BA}(\tau) = \frac{1}{2} < (\hat{B}(t) - <\hat{B}>_\rho) (\hat{A}(0) - <\hat{A}>_\rho) + (\hat{A}(0) - <\hat{A}>_\rho) (\hat{B}(t) - <\hat{B}>_\rho)_\rho \]

\[ \chi_{BA}(\tau) = \chi'_{BA}(\tau) + i \chi''_{BA}(\tau) \]
\[ \frac{1}{2} [\chi_{BA}(+) + \chi_{AB}(-t)] \]
\[ -\frac{i}{2} [\chi_{BA}(+) - \chi_{AB}(-t)] \]

\[ \chi_{BA}(\omega) = \sum_{-\infty}^{\infty} \chi_{BA}(t) e^{i\omega t} dt \]

\[ \chi''_{BA}(\omega) = \frac{1}{\hbar} \tanh(\hbar \omega \beta/2) S_{BA}(\omega) \]
\[ S_{BA}(\omega) = \hbar \coth(\hbar \omega \beta/2) \chi''_{BA}(\omega) \]
\[ \hbar \omega \ll 1 \Rightarrow 2 \chi''_{BA}(\omega)/(\beta \hbar) \]

**NOTE:** \[ \chi''_{BA}(\omega) = \frac{1}{2} [\chi^*_{AB}(\omega) - \chi_{BA}(\omega)] \]
\[ \neq \text{Im} \chi_{BA}(\omega) \text{, except } \lambda = \beta \]

\[ \hat{A} = \hat{B} = \hat{a} : \ S_{99}(\omega) = \hbar \coth(\hbar \omega \beta/2) \text{Im} \chi_{99}(\omega) \]
**EQ. CURRENT NOISE**

\[ h = b \]

\[ I = \frac{dh}{dt} \]

\[ \langle \delta I(t) \rangle = \frac{1}{c} \int_{-\infty}^{\infty} Z(t-s) \delta(s) ds \]

\[ Z(t-s) = \frac{dX(t)}{dt} \]

\[ X''_{AA}(\omega) = \frac{1}{\omega} \text{Im} \left( \frac{2\omega}{\epsilon} \right) = -\frac{1}{\omega} \text{Re} \ Z(\omega) \]

\[ S_{II}(\omega) = -\omega^2 S_{BB}(\omega) \]

\[ S_{II}(\omega) = (k\omega) \text{coth} \left( \frac{k\omega}{2kT} \right) \text{Re} \ Z(\omega) \]

\[ kT \gg k\omega : S_{II}(\omega) \rightarrow 2kT \text{Re} \ Z(\omega) \rightarrow 2kT/R \]

**JOHNSON-NYQUIST (1928)**

\[ kT \ll k\omega \rightarrow k\omega \text{Re} \ Z(\omega) \]

quantum-zero point fluct.

\[ S_{II}(\omega = 0) = 0 \text{ at } \omega = 0 \]
1900-1951

J.B. Johnson

Thermal agitation of electricity in conductors.

Phys. Rev. (1928) 32 (July) 97-109

H. Nyquist

Thermal agitation of electric charge in conductors.

Phys. Rev. (1928) 32 (July) 110-113

L. Onsager

Reciprocal relation in irreversible process.

Phys. Rev. (1931) 32 (February) 405-426

H.B. Callen, T.A. Welton

Irreversibility and Generalized Noise.

Phys. Rev. (1951) 83 (1) 34-40
QUANTUM NOISE

NO QUANTUM EQ.-PARTITION-TH.

\[ \mathcal{S} \xrightarrow{H_{\text{INT}}} \mathcal{B} \]

FEYNMAN-PATH-INT.

QUANTUM LANG.-EQ.

GME

STOCH.-L=\nu N.-EQ.

\[ \psi := |\psi_1(t)\rangle < \psi_2(t) | \quad \mu := \frac{2}{\hbar} \sum \omega \frac{\Delta \omega}{c} \]

\[ i \hbar \dot{\psi} = [H_0, \psi] + \frac{\hbar}{2} \left[ x, \psi \right] - \frac{1}{\hbar} \left[ x, \psi \right] \]

\[ \langle \psi(t) \psi(t') \rangle = Re \mathcal{L}(t-t'), \quad \langle \psi(t) \nu(t') \rangle = \frac{2}{\hbar} \delta(t-t')Im \mathcal{L}(t-t'), \quad \langle \nu(t) \nu(t') \rangle = 0 \]

COMPLEX VALUED NOISE
PITFALLS

MARKOV MASTER EQ

\[ \frac{\partial}{\partial t} \rho = -i \hbar \left[ H, \rho \right] + \Gamma(t) \]

BLOCH-REDFIELD

\text{i.e. NO DET. BALANCE}

ROTATING WAVE APPROX.

(LINDBLAD; DAVIES-APPROX.)

DET. BALANCE \& O.K.

BUT

\bullet \text{WRONG EHRENFEST EQ.}

\bullet \text{NO FDT}

\bullet \text{NO KMS-COND.} \quad \langle u(t) v \rangle = \langle v u(t + \tau) \rangle
Schematic of stochastic resonance. The cross-hatched oval represents a black-box system which receives two inputs: one weak and periodic, the other strong and random. The output is relatively regular with small fluctuations.
NOISE-ASSISTED SYNCHRONIZED HOPPING
Bistable Model

\[ \dot{x} = x - x^3 + A \cos(\Omega t + \varphi) + \xi(t) \]

\[ \langle \xi(t) \rangle = 0 \]

\[ \langle \xi(t) \xi(t') \rangle = 2D \delta(t-t') \]

\[ T_e = \frac{2\pi}{\Omega} \]

SIGNAL

\[ T_e = 2 \Gamma^{-1} \]

ESCAPE
MORE NOISE → MORE SIGNAL

AMPLIFICATION

\[ n \]

\[ D = \text{NOISE INTENSITY} \]

\[ \Omega = 0.1 \]

P. JUNG + P. H., PHYS. REV. A44: 8032 (1991)
AMPLIFICATION

LRT

\[ \Omega = 0.1 \]

\[ A = 0.1 \]
\[ A = 0.3 \]
\[ A = 0.5 \]
\[ A = 0.8 \]

\( D = \text{noise intensity} \)

MORE NOISE → MORE SIGNAL

\[ M_1 \sim \chi(t) = -\frac{1}{D} \frac{d}{dt} \langle j x(t) S S^\dagger \rangle \]

\[ |M_1|^2 \propto \frac{1}{D^2} \exp(-2\Omega U D) \]
SR

IN QUANTUM MECHANICS

QS R
\[ V_0 \gg \hbar \omega_0 \gg \hbar \varepsilon_0, kT \]

\[ \omega_0 \approx -\hbar \varepsilon_0 \]

\[ -\frac{\hbar}{2} \left( \varepsilon_c \sigma_z + \Delta \sigma_x \right) \]

\[ \frac{1}{2} \sum_{\alpha} \left( \frac{p_\alpha^2}{m_\alpha} + m_\alpha \omega_\alpha^2 x_\alpha^2 - c_\alpha x_\alpha \sigma_z \right) \]

\[ T \] 

\[ \text{Temperature} \]

\[ \Omega, \hat{\Omega} \]

\[ \frac{\hbar \hat{E}}{2} \cos(\Omega t) \sigma_z \]

\[ n \cdot \pi = \hat{\pi} \]

\[ t \rightarrow 0 \]
LINEAR RESPONSE & QSR

with \( P_1 = \frac{A}{2} \chi_{qq}(\Omega) = \frac{A}{2} \chi(\Omega) \)

\[
\eta_I = 4\pi |P_1|^2 = \pi A^2 |\chi(\Omega)|^2
\]

\[
\text{SNR} = \frac{\pi A^2 |\chi(\Omega)|^2}{S_{qq}(\Omega, A=0)} = \frac{\pi A^2 |\chi(\Omega)|^2}{\text{Im} \chi(\Omega) \coth(\hbar \Omega / 2)}
\]

\( \textbf{Valid at all temperatures!} \)

PROBLEM: QUANTUM \( \chi_{qq}(\Omega)^{FDT} S_{qq}(\Omega) \)

\[
S_{qq}(+) = \frac{1}{2} < \delta q(+) \delta q(0) + \delta q(0) \delta q(+) >_\beta
\]

\( \textbf{DIFFICULT!} \)
\textbf{LINEAR RESPONSE & QSR}

with \( P_1 = \frac{A}{2} \chi_{gg} (\Omega) \equiv \frac{A}{2} \chi (\Omega) \)

\( \eta_1 = 4\pi |P_1|^2 = \pi A^2 |\chi (\Omega)|^2 \)

\[
\text{SNR} = \frac{\pi A^2 |\chi (\Omega)|^2}{S_{gg} (\Omega, A=0)} = \frac{\pi A^2 |\chi (\Omega)|^2}{\text{Im} \chi (\Omega) \text{coth} (\hbar \beta / 2)}
\]

\( \text{Valid at all temperatures!} \)

\textbf{PROBLEM: QUANTUM } \chi_{gg} (\Omega)^{\text{FDT}} \equiv S_{gg} (\Omega)

\( S_{gg} (t) = \frac{1}{2} \left< d\xi (t)d\xi (0) + d\xi (0)d\xi (t) \right> \)

\( \textbf{DIFFICULT!} \)

\( \Rightarrow \text{2 LIMITS} \)

above \( \approx \) near \text{crossover to thermal hopping \text{at low T}}
\[ \alpha = 50 \]
\[ \frac{V_b}{\hbar \omega_b} = 0.2 \]
\( T = 0 \)

\( \mathcal{R}^c = (x_1^c, x_2^c) \)

\( x_1^c \Rightarrow q \)

Classically allowed

Classically forbidden

\[ U(q) \]

\( q_c (\tau = 0) \quad \gamma > 0 \)

Tunnelling

Thermal activation

\[ q_c (\tau) \]

Bounce

\[ \dot{q}_c (\tau) \]

Zero-mode

\[ \gamma = 0: \]

\[ M \frac{d^2 q_B (\tau)}{d\tau^2} = \frac{\delta U}{\delta q_B} \]

\[ - M \frac{d^2 q_B (\tau)}{d\tau^2} + \left( \frac{\delta^2 U}{\delta q_B^2} \right) \dot{q}_B (\tau) = 0 \]
\[-M \ddot{q}_B + \frac{\partial U(q_B)}{\partial q_B} + \int_{-\frac{\Theta}{2}}^{\frac{\Theta}{2}} k(\tau-\tau') q_B(\tau') d\tau' = 0\]

\[\Theta = \frac{\hbar}{kT}\]

\[q_B(\tau + \Theta) = q_B(\tau)\]
QUANTUM SR

(a) INCOHERENT TUNNELING

\[ \varepsilon = 0 \]

\[ \alpha \ll 1, \quad \varepsilon \neq 0 \]

HIGH FREQUENCIES
HIGH TEMPERATURES

driving induced coherence

(b) LOW FREQUENCIES
LOW TEMPERATURES

adiabatic quantum coherence

LOW FREQUENCIES
HIGH TEMPERATURES

incoherent regime
DRIVEN QUANTUM TUNNELING

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DRIVEN - TUNNELING - ZOO
SUPPR. vs. ENH.

CDT

EHG

CHAOS-ASSISTED

QSR

COHERENT TUNNELING CONTROL

- DRIVING (Ω, Λ, I)
- BATH SPECTRUM
- NOISE INPUT
HOMEPAGE
„HANGGI“

GO TO : FEATURE ARTICLES

• Quantum Dissipation and Quantum Transport

http://www.physik.uni-augsburg.de/theo1/hanggi/Quantum.html