First-principles study of the electronic and magnetic structures of the tetragonal and orthorhombic phases of Ca$_3$Mn$_2$O$_7$

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On the basis of density functional theory electronic band structure calculations using the augmented spherical wave method, the electronic and magnetic properties of the orthorhombic and tetragonal phases of Ca$_3$Mn$_2$O$_7$ were investigated and the spin exchange interactions of the orthorhombic phase were analyzed. Our calculations show that the magnetic insulating states are more stable than the nonmagnetic metallic state for both polymorphs of Ca$_3$Mn$_2$O$_7$, the orthorhombic phase is more stable than the tetragonal phase, and the ground state of the orthorhombic phase is antiferromagnetic. The total energies calculated for the three spin states of the orthorhombic phase of Ca$_3$Mn$_2$O$_7$ led to estimates of the spin exchange interactions $J_{nn}=−3.36$ meV and $J_{nnn}=−0.06$ meV. The accuracy of these estimates was tested by calculating the Curie-Weiss temperature within the mean-field approximation.

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I. INTRODUCTION

Due to their potential technological application, magnetic ternary manganese oxides have received much attention in the past decade.$^{1-3}$ It is an important theoretical issue to understand their electronic and magnetic properties.$^{4,5}$ In accounting for such properties of magnetic oxides, density functional theory (DFT)$^{6,7}$ electronic structure calculations using the local spin density approximation (LSDA)$^8$ and the generalized gradient approximation (GGA)$^9$ have proven very reliable. Reduced dimensionality perovskite based systems are found in the Ruddlesden-Popper (RP) family: Ca$_{n+1}$Mn$_n$O$_{3n+1}$. The diamagnetic CaO rocksalt layers alternate with the magnetic (CaMnO$_3$)$_n$ layers along the crystallographic $c$ direction, where $n$ represents the number of MnO$_3$ perovskite sheets in each (CaMnO$_3$)$_n$ layer. To emphasize this structural feature, the formulas for these oxides can be rewritten as (CaO)(CaMnO$_3$)$_n$ with $n=1,2,3,...,\infty$. Known examples of this RP family are Ca$_2$MnO$_4$($n=1$), Ca$_3$Mn$_2$O$_7$($n=2$), and Ca$_4$Mn$_3$O$_{10}$($n=3$).$^{11}$ Note that the $n=\infty$ member is simply the perovskite CaMnO$_3$. In a recent electronic structure study,$^{14}$ it was shown that the $n=1$ RP member, Ca$_2$MnO$_4$, and its reduced phase Ca$_2$MnO$_{3.5}$ are antiferromagnetic (AF) insulators with band gap of $\sim 1$ eV. According to a neutron diffraction study at room temperature,$^{15}$ the $n=2$ RP member, Ca$_3$Mn$_2$O$_7$, exists as a mixture of the majority orthorhombic and the minority tetragonal phases. The main structural difference between the two systems is depicted in Fig. 1. Due to the tilting of the MnO$_6$ octahedra, the Mn-O-Mn angles between two adjacent corner-sharing octahedra are considerably smaller than $180^\circ$ (i.e., 159.1$^\circ$, 163.2$^\circ$, and 163.9$^\circ$). Ca$_3$Mn$_2$O$_7$ undergoes a three-dimensional magnetic ordering below $T_N \sim 110–115$ K with a $G$-type AF structure. In view of this coexistence of phases, which is an unusual feature in such reduced dimensionality manganese oxide systems, it is relevant to assess differences between the two crystal structures in terms of the electronic structure description. In the present work, the two polymorphs of Ca$_3$Mn$_2$O$_7$ are examined with first principles DFT calculations.

II. CALCULATIONS

Our DFT calculations employed the all-electron augmented spherical wave (ASW) method in its scalar...
relativistic implementation. As preliminary calculations revealed that the GGA describes the effects of exchange and correlation better than the LSDA, hence leading to a more accurate description of the energy differences between different ordered magnetic states, we opted for a use of the GGA in the form proposed by Perdew et al. In the ASW method, the wave function is expanded in atom-centered augmented spherical waves, which are Hankel functions and numerical solutions of Schrödinger’s equation, respectively, outside and inside the so-called augmentation spheres. In order to optimize the basis set, additional augmented spherical waves were placed at carefully selected interstitial sites. The choice of these sites as well as the augmentation radii were automatically determined using the sphere-geometry optimization algorithm. The Ca 4s, Mn 4s, Mn 3d, and O 2p states were treated as valence states, and the low-lying O 2s states as core states. The basis set was complemented by wave functions of s, p, and possibly d symmetries centered at the interstitial sites. A sufficiently large number of k points was used to sample the irreducible wedge of the Brillouin zone, and the energy and charge differences between successive iterations were converged below $\Delta E = 10^{-8}$ Ry and $\Delta Q = 10^{-8}$, respectively, to obtain accurate values of the magnetic moments and accurate total energy differences between various ordered magnetic states. To extract more information about the nature of the interactions between the atomic constituents from electronic structure calculations, the crystal orbital overlap population (COOP) or the crystal orbital Hamiltonian population (COHP) may be employed. Both approaches provide a qualitative description of the bonding, nonbonding, and antibonding interactions between two atoms. A slight refinement of the COHP was recently proposed in the form of the “energy of covalent bond” (ECOV), which combines COHP and COOP to calculate quantities independent of the choice of the zero of potential. Both COOP and ECOV give similar general trends, but COOP, when defined within plane-wave basis sets, exaggerates the magnitude of antibonding states. In the present work, the ECOV was used for the chemical bonding analysis. It is noted that negative, positive, and zero values of ECOV are associated with bonding, antibonding, and nonbonding interactions, respectively.

III. RESULTS AND DISCUSSION

A. Nonmagnetic configuration

To gain insight into the chemical bonding in the two forms of Ca$_3$Mn$_2$O$_7$, their nonmagnetic (NM) states were calculated by performing non-spin-polarized electronic structure calculations by enforcing spin degeneracy for all species. Our calculations show charge transfer from the cation sites (Ca$^{2+}$, Mn$^{4+}$) to the anionic sites of oxygen as well as toward the voids, but the amount of charge transfer is not significant of an ionic character of the atomic constituents. The plots of the projected density of states (DOS) for the constituent atom sites are presented in Fig. 2. The projected DOS plots calculated for the five Mn 3d orbitals are presented in Fig. 3, and the ECOV plots calculated for the various Mn-O bonds in Fig. 4.

![Figure 2](http://example.com/fig2.png)

**FIG. 2.** (Color online) Projected DOS plots of the atomic sites calculated for the NM states of the (a) minority tetragonal and (b) majority orthorhombic phases of Ca$_3$Mn$_2$O$_7$.

The comparison of Figs. 2 and 4 shows that the valence bands (VB’s) are dominated by Mn-O bonding. The Mn-O interactions are bonding in the major part of the VB’s with strong contributions in the $-8$, $-6$ eV region (Fig. 2), i.e., where the Mn $e_g$ orbitals (Fig. 3) make $\sigma$ bonding with the O 2p orbitals. The $s$-like Ca states are smeared into the whole range of the VB’s (Fig. 2). For both polymorphs of Ca$_3$Mn$_2$O$_7$, the Fermi level $E_F$ occurs near the DOS peak dominated by the Mn 3d states so that the DOS value at the Fermi level, $n(E_F)$, is large. The DOS peak around $E_F$ arises from the $t_{2g}$-block bands in which the Mn $t_{2g}$ orbitals make weak $\pi$-antibonding interactions with the O 2p orbitals. The existence of Mn-O antibonding states in this DOS peak is clearly seen from the ECOV plot (Fig. 4).

The occurrence of the Fermi level around a sharp DOS peak with a large $n(E_F)$ value indicates an instability toward spin polarization. This can be inferred from Stoner’s theory analysis (see Ref. 22 for a review on magnetic oxides). Within the Stoner theory, the large DOS at the Fermi level is related to the instability of the nonmagnetic state with respect to the onset of intraband spin polarization when $n(E_F)\Gamma > 1$. In this so-called Stoner criterion I is the Stoner integral calculated and tabulated by Janak for the elemental systems: $I(Mn) = 0.408$ eV. With $n_{Mn}(E_F) = 8.5$ eV$^{-1}$, the $n(E_F)$ value of 3.47 clearly points to an unstable non mag-
eral and orthorhombic phases of Ca$_3$Mn$_2$O$_7$ are summarized in Table I as well as in Fig. 6. For both phases, the FM state is substantially more stable than the NM states (i.e., $E_{FM} - E_{NM}$, and the total moment per f.u. is 6.0 $\mu_B$ (Table I), as expected for the high-spin Mn$^{3+}(d^7)$ ions in Ca$_3$Mn$_2$O$_7$. The total energy obtained for the orthorhombic phase is by 170 meV/f.u. lower than that of the tetragonal phase, which is consistent with the finding that the orthorhombic and tetragonal phases are the majority and minority phases, respectively. The projected DOS plots calculated for the FM state of the orthorhombic Ca$_3$Mn$_2$O$_7$ are presented in Fig. 6(a). The up-spin and down-spin bands are strongly split such that the up-spin $t_{2g}$-block bands are completely filled, while the down-spin $t_{2g}$-block bands are empty, as expected for the high-spin Mn$^{3+}(d^7)$ ions. The down- and up-spin bands have band gaps of about 1.0 and 0.3 eV, respectively. Given the fact that both the LSDA and the GGA underestimate band gaps, the FM state of Ca$_3$Mn$_2$O$_7$ should be insulating experimentally. However, the FM state is not the magnetic ground state of Ca$_3$Mn$_2$O$_7$, as will be discussed in the next section. Figure 6(b) shows the ECOV plots for the Mn-O bonds calculated for the up-spin and down-spin bands of the orthorhombic phase. The VB’s exhibit Mn-O bonding interactions up to $-2$ eV in both up-spin and down-spin bands. The energy region between $-2$ eV and the Fermi level shows Mn-O antibonding interactions due to the $\pi$-type antibonding interactions of the $t_{2g}$-block bands in the up-spin bands, but this feature is missing in the down-spin bands because the down-spin $t_{2g}$-block bands are raised above the Fermi level. Thus, the exchange splitting in the FM state enhances Mn-O bonding interactions in Ca$_3$Mn$_2$O$_7$. 

FIG. 3. (Color online) Projected DOS plots of the Mn 3$d$ orbitals calculated for the NM states of the (a) minority tetragonal and (b) majority orthorhombic phases of Ca$_3$Mn$_2$O$_7$.

FIG. 4. (Color online) ECOV plots of the Mn-O bonds calculated for the NM states of the (a) minority tetragonal and (b) majority orthorhombic phases of Ca$_3$Mn$_2$O$_7$. 

B. Magnetic states

Spin-polarized electronic structure calculations were carried out for the ferromagnetic (FM) state of the orthorhombic and tetragonal phases of Ca$_3$Mn$_2$O$_7$, and for the AF state of the orthorhombic phase of Ca$_3$Mn$_2$O$_7$. There are a number of possible AF spin arrangements. Although experimental findings$^{15}$ point to a $G$-type AF arrangement, in the present study, we considered two types of AF orderings for the sake of establishing comparisons and for testing the theory against the experiment, i.e., the $G$- and $A$-type AF states shown in Fig. 5. In the $G$-type AF state, the antiferromagnetically ordered planes of Mn sites are antiferromagnetically coupled. In the $A$-type AF arrangement, the ferromagnetically ordered planes of Mn sites are antiferromagnetically coupled within each perovskite layer.

Results of our calculations for the FM state of the tetragonal and orthorhombic phases of Ca$_3$Mn$_2$O$_7$ are summarized in Table I as well as in Fig. 6. For both phases, the FM state is substantially more stable than the NM states (i.e., $E_{FM} - E_{NM}$, and the total moment per f.u. is 6.0 $\mu_B$ (Table I), as expected for the high-spin Mn$^{3+}(d^7)$ ions in Ca$_3$Mn$_2$O$_7$. The expected for the high-spin Mn$^{3+}(d^7)$ ions in Ca$_3$Mn$_2$O$_7$. The
FIG. 5. (Color online) Schematic representations of the Mn spin arrangements in the (a) FM, (b) the G-type AF, and (c) A-type AF states of Ca$_3$Mn$_2$O$_7$. Dark (blue) and white (open) spheres refer to up- and down-spin alignments, respectively.

Results of calculations for the $G$- and $A$-type AF states of the orthorhombic phase of Ca$_3$Mn$_2$O$_7$ are summarized in Table II and Fig. 7. Both AF states are more stable than the FM state, and the $G$-type AF state is slightly more stable than the $A$-type AF state (Table II). The latter is consistent with the neutron diffraction study[13] which found a $G$-type AF ordering for Ca$_3$Mn$_2$O$_7$. While the resulting magnetization is zero as expected in AF order, the atomic magnetic moments and the resulting up-spin and down-spin sublattice magnetizations are reduced with respect to their value in the ferromagnetic configuration.

The DOS and band structure are shown in Fig. 7 for one magnetic sublattice. The similarities with the features observed in Fig. 6 for the ferromagnetic case are present here too. So at least from the point of view of the electronic band structure, some relationship is persistent, although the AF too. So at least from the point of view of the electronic band structure, some relationship is persistent, although the AF too. Consequently, in terms of the $J_{nn}$ and $J_{nnn}$, the energies per Mn site of the FM, G-type AF, and A-type AF states are written as shown in Eq. (1),[23–25] where $N$ is the number of unpaired spins at each Mn$^{4+}$ site (i.e., $N=3$). Just like the analysis done for the similar Sr based layered system,[4] the $J$ values of

change $J_{nnn}$ interactions of the (CaMnO$_3$)$_2$ double-perovskite thin films in the orthorhombic phase of Ca$_3$Mn$_2$O$_7$. The spin exchange interactions between the adjacent double-perovskite layers are expected to be negligible, so we consider only those interactions within a double-perovskite layer. Then, each Mn site has five nn and eight nnn spin exchange interactions. For simplicity, we assume that all five nn interactions are identical, and so are all the eight nnn interactions.

IV. ANALYSIS OF THE SPIN EXCHANGE INTERACTIONS

It is of interest to estimate the nearest-neighbor ($nn$) spin exchange $J_{nn}$ and the next-nearest-neighbor ($nnn$) spin ex-

change $J_{nnn}$ interactions of the (CaMnO$_3$)$_2$ double-perovskite thin films in the orthorhombic phase of Ca$_3$Mn$_2$O$_7$. The spin exchange interactions between the adjacent double-perovskite layers are expected to be negligible, so we consider only those interactions within a double-perovskite layer. Then, each Mn site has five nn and eight nnn spin exchange interactions. For simplicity, we assume that all five nn interactions are identical, and so are all the eight nnn interactions. Consequently, in terms of $J_{nn}$ and $J_{nnn}$, the energies per Mn site of the FM, G-type AF, and A-type AF states are written as shown in Eq. (1),[23–25] where $N$ is the number of unpaired spins at each Mn$^{4+}$ site (i.e., $N=3$). Just like the analysis done for the similar Sr based layered system,[4] the $J$ values of

### Table I. Total energies and magnetic moments of the atomic sites calculated for the ferromagnetic states of the tetragonal and orthorhombic phases of Ca$_3$Mn$_2$O$_7$. The energies are given in units of eV per half the f.u. (i.e., per Mn) with respect to those of the non-spin-polarized state. The magnetic moments are given in units of Bohr magneton ($\mu_B$).

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Tetragonal</th>
<th>Orthorhombic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta E_{FM-NM}$ (eV)</td>
<td>−1.442</td>
<td>−1.610</td>
</tr>
<tr>
<td>$\Delta E_{FM-NM}$ (eV)/Mn</td>
<td>−0.721</td>
<td>−0.831</td>
</tr>
<tr>
<td>$\mu$(Mn)</td>
<td>2.729</td>
<td>2.719</td>
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<tr>
<td>$\mu$(O1)</td>
<td>0.088</td>
<td>0.092</td>
</tr>
<tr>
<td>$\mu$(O2)</td>
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</tr>
<tr>
<td>$\mu$(O3)</td>
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<td>0.056</td>
</tr>
<tr>
<td>$\mu$(O4)</td>
<td>0.014</td>
<td>0.014</td>
</tr>
<tr>
<td>$\mu$(f.u.)</td>
<td>6.0</td>
<td>6.0</td>
</tr>
</tbody>
</table>

FIG. 6. (Color online) FM Ca$_3$Mn$_2$O$_7$. (a) Site and spin projected DOS and (b) chemical bonding (ECOV) for Mn-O interaction regrouping all four oxygen sublattices. Energy reference is with respect to the VB top $E_V$ due to the closely insulating character.
our system are not known experimentally to the best of our knowledge. Thus, the calculation of \( J_{nn} \) and \( J_{nnn} \) was carried out.

\[
E(\text{FM}) = \frac{1}{2} (5J_{nn} - 8J_{nnn}) \left( \frac{N^2}{4} \right) \left( -2.5J_{nn} - 4J_{nnn} \right),
\]

\[
E(G - \text{AF}) = \frac{1}{2} (5J_{nn} - 8J_{nnn}) \left( \frac{N^2}{4} \right) \left( 2.5J_{nn} - 4J_{nnn} \right),
\]

\[
E(A - \text{AF}) = \frac{1}{2} (3J_{nn}) \left( \frac{N^2}{4} \right) \left( -1.5J_{nn} \right).
\]

According to the results of our electronic band structure calculations (Table II), we have

\[
E(\text{FM}) - E(G - \text{AF}) = 37.8 \text{ meV/Mn},
\]

\[
E(A - \text{AF}) - E(G - \text{AF}) = 29.7 \text{ meV/Mn}. \tag{2}
\]

From Eqs. (1) and (2), it is estimated that \( J_{nn} = -3.36 \text{ meV} \) and \( J_{nnn} = -0.06 \text{ meV} \), i.e., \( J_{nn}/k_B = -39 \text{ K} \) and \( J_{nnn}/k_B = -0.7 \text{ K} \), where \( k_B \) is the Boltzmann constant. The nn interaction is strongly AF, and the nnn interaction is very weakly AF. Therefore, within the mean-field approximation, the

**V. CONCLUSION**

Our electronic structure study reveals that for both polymorphs of \( \text{Ca}_3\text{Mn}_2\text{O}_7 \), the magnetic states are more stable than the nonmagnetic states. The spin polarization makes \( \text{Ca}_3\text{Mn}_2\text{O}_7 \) insulating and lowers its energy mainly through exchange, leading to a magnetic moment on Mn expected for a high-spin Mn \(^{3+}\) \( (d^5) \) ion. In the spin-polarized configuration the down-spin states provide more Mn-O bonding than do the up-spin states because the down-spin Mn \( t_{2g} \)-block bands are raised above the Fermi level. In agreement with the experiment, our calculations show that the orthorhombic phase is more stable than the tetragonal phase, and the ground state of the orthorhombic phase is \( G \)-type AF. The total energies of the FM, \( A \)-type AF, and \( G \)-type AF states of \( \text{Ca}_3\text{Mn}_2\text{O}_7 \) obtained from our calculations lead to the estimates of the spin exchange interactions \( J_{nn}/k_B = -39 \text{ K} \) and \( J_{nnn}/k_B = -0.7 \text{ K} \). The Curie-Weiss temperature \( \Theta \) estimated from these spin exchange parameters is approximately half the experimental value, so that our calculations underestimated the values of the spin exchange parameters by a factor of approximately 2.
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